

YU GUANGMING, WANG JILIANG, SONG CHUANWANG, LU SHIBAO,

LI BINGBING

YU GUANGMING\* (contact, Author Introduction), PhD, Professor, Doctoral Supervisor, e-mail: [yu-guangming@263.net](mailto:yu-guangming@263.net); WANG JILIANG, SONG CHUANWANG, LU SHIBAO, LI BINGBING, *School of Civil Engineering, Qingdao University of Technology\*\**.  
*Qingdao, 266033, Shandong, China*

## **Research on Tunnel Rock Pressure Arch Deformation and Its Contribution for Surface Subsidence**

**Abstract:** Underground tunneling will cause surrounding rock stress redistributing and forming rock pressure arch. Therefore, in order to strike quantitative subsidence of tunnel vault. In this paper, the influence factors of tunnel surrounding rock pressure arch is analyzed, the scientific hypothesis is carried out, the pressure arch model is established, its stress is analyzed, and obtaining vault subsidence; meanwhile, taking Qingdao metro tunnel engineering cases, establishing empirical relationships between surface subsidence and dome subsidence. This achievement has important significance for tunnel excavation construction as well as control and design of surface environment.

*Key words:* tunnel, rock, pressure arch deformation, surface subsidence.

### **The introduction**

The tunnel will form a new free surface in excavation constraints tunnel face and wall after excavation. The upper part will be the sub-hole horseshoe arch. The initial stress field of surrounding rock or current stress field will be broken. The stress redistribution of surrounding rock will resist deformation. Both the stress field near the excavation contour surface offset and a stress increase belt will form in tunnel excavation sidewall. This phenomenon is called the arching effect of surrounding rock [1–3]. The maximum principal stress is compressive stress after readjusting, therefore the arching effect of surrounding rock is also known as the pressure arch effect.

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\* YU Guangming, China University of Mining Engineering Mechanics PhD graduate in 1997. Mainly engaged in the underground engineering, buildings protection and scientific research work such as mining subsidence.

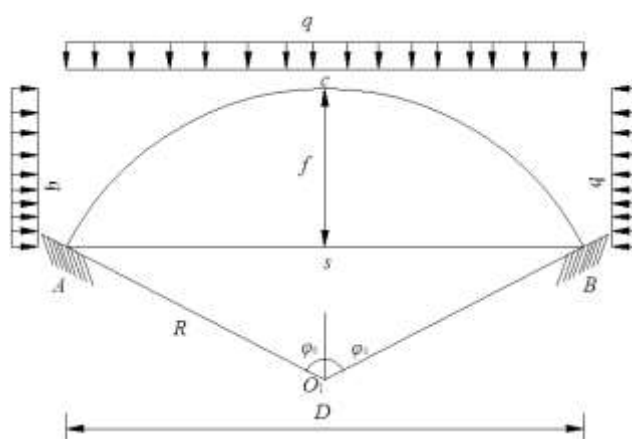
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However, the vault subsidence displacement values are mostly concerned in the process of tunnel excavation in the city [4, 5]. Therefore, we need to quantitatively study the vault subsidence and arch for surrounding rock stress of the relationship between the maximum subsidence and surface subsidence value. So this paper focuses on the city the stress of the surrounding rock pressure arch. The arch mechanics model is set up. Making use of the force method principle, force analysis is taken. So the pressure arch maximum subsidence values are calculated. Then surface subsidence and the experience of the vault sink relationship will be discussed.

### The pressure arch mechanical model

**Basic assumptions.** To analyze the stress of surrounding rock pressure of arch, the following hypothesis will be put forward according to the characteristics of urban tunnel surrounding rock and the stress environment [3, 6, 7].

1. Urban tunnel surrounding rock is similar to isotropic and homogeneous continuous medium.
  2. The center line of pressure arch of the subway tunnel surrounding rock is axial symmetry.
  3. Only considering the weight of the formation of the initial rock stress field.
  4. The mechanical calculation of the transverse plane of the horseshoe-shaped main section is regarded as a plane problem. Assuming that the size and distribution of the load along the longitudinal constant underground space, surrounding rock model is made up of 2m thickness and width of 1m rectangle to the axis instead.
  5. Axial force, shear and moment can be passed in the surrounding rock arch in the practical application with support reduced to a fixed end.
  6. Urban Tunnel pressure arch is simplified to withstand constant pressure of surrounding rock,  $q=\gamma h$  ( $h$  is depth subway for surrounding rock severe;  $\gamma$  is severe surrounding rock).
- Mechanical Model.** Based on the above assumptions, the tunnel in overlying rock pressure on arch structural mechanics model is shown in Fig. 1.



**Fig. 1. Pressure arch structural mechanics model:  $q$  – surrounding rock load;  $R$  – arch radius;  $\varphi_0$  – half the central angle;  $D$  – tunnel Dong Jing;  $f$  – high arch;  $C$  – vaulted point;  $O_1$  – arch center;  $S$  – dome projection points;  $A, B$  – Fixed endpoint.**

### Stress Analysis of the mechanical model of the pressure arch

**Internal Force Analysis.** There is a symmetrical non-hinged arch in the Fig. 1. The model is three times statically indeterminate structure. To simplify the calculations, two simplification are taken as follow [8]:

The first measure is to simplify the use of structural symmetry. Select the symmetrical basic system and cut open the vault. Thus the power law equation is simplified into following formula:

$$\delta_{11}X_1 + \delta_{12}X_2 + \Delta_{1p} = 0, \quad (1)$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \Delta_{2p} = 0, \quad (2)$$

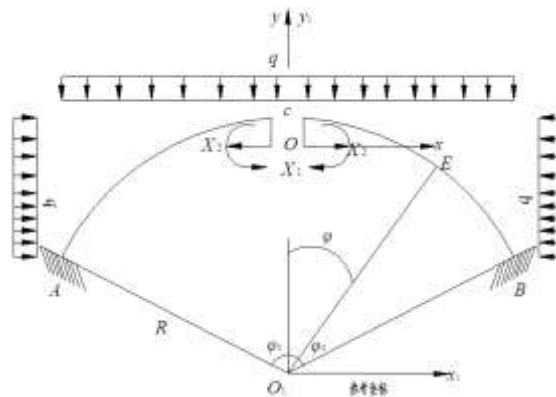
$$\delta_{33}X_3 + \Delta_{3p} = 0. \quad (3)$$

The second measure is to simplify the use of rigid arm in Fig. 2. The remaining pair of secondary factors  $\delta_{12}$  and  $\delta_{21}$  also are equal to zero, so that the force of law to simplify the equation into three separate one dollar equation:

$$\delta_{11}X_1 + \Delta_{1p} = 0, \quad (4)$$

$$\delta_{22}X_2 + \Delta_{2p} = 0, \quad (5)$$

$$\delta_{33}X_3 + \Delta_{3p} = 0. \quad (6)$$



**Fig. 2. Rigid arm calculation drawing: O – elastic center; x, y – coordinates;  $x_1, y_1$  – reference coordinate system;  $\varphi$  – central angle;  $X_1, X_2$  – basic unknown quantity.**

According to Fig. 2, x and y axes pass through the elastic center O. The other to take the reference axes  $x_1$  and  $y_1$  through the center  $O_1$ . Any point E on the arch coordinate axis x, y and  $x_1, y_1$  are expressed with its central angle as follows:

$$x_1 = x = R \sin \varphi, \quad (7)$$

$$y_1 = y + d = R \cos \varphi. \quad (8)$$

The distance between the elastic center O and the center O<sub>1</sub> can be expressed:

$$d = \frac{\int \frac{y_1}{EI} ds}{\int \frac{ds}{EI}} = \frac{2 \int_0^{\varphi_0} R \cos \varphi \cdot R d\varphi}{2 \int_0^{\varphi_0} R d\varphi} = \frac{R \sin \varphi_0}{\varphi_0}, \quad (9)$$

where EI is a cross-sectional stiffness model of surrounding rock arch.

The structural model is symmetrical, so X<sub>3</sub> = 0. When calculating the displacement, the impact moment is considered merely. When X<sub>1</sub> = 1 and X<sub>2</sub> = 1 respectively, moment equations of the basic structure are expressed:

$$\overline{M}_1 = 1, \quad (10)$$

$$\overline{M}_2 = -y = d - y_1 = R \left( \frac{\sin \varphi_0}{\varphi_0} - \cos \varphi \right), \quad (11)$$

$$EI \delta_{11} = \int \overline{M}_1^2 ds = 2 \int_0^{\varphi_0} R d\varphi = 2R\varphi_0, \quad (12)$$

$$EI \delta_{22} = \int \overline{M}_2^2 ds = 2 \int_0^{\varphi_0} R^2 \left( \frac{\sin \varphi_0}{\varphi_0} - \cos \varphi \right)^2 \cdot R d\varphi = 2R^3 \left( \frac{\varphi_0}{2} - \frac{\sin^2 \varphi_0}{\varphi_0} + \frac{1}{4} \sin 2\varphi_0 \right). \quad (13)$$

The basic structure moment equation is expressed under load q:

$$M_p = -\frac{q}{2} x^2 - \frac{q}{2} y^2 = -\frac{q}{2} R^2 \sin^2 \varphi - \frac{q}{2} R^2 \left( \cos \varphi - \frac{\sin \varphi_0}{\varphi_0} \right)^2, \quad (14)$$

so

$$\begin{aligned} EI \Delta_{1P} &= \int \overline{M}_1 M_p ds \\ &= 2 \int_0^{\varphi_0} \left[ -\frac{q}{2} R^2 \sin^2 \varphi - \frac{q}{2} R^2 \left( \cos \varphi - \frac{\sin \varphi_0}{\varphi_0} \right)^2 \right] R d\varphi \\ &= qR^3 \left( \frac{\sin^2 \varphi_0}{\varphi_0} - \varphi_0 \right), \end{aligned} \quad (15)$$

$$\begin{aligned} EI \Delta_{2P} &= \int \overline{M}_2 M_p ds \\ &= 2 \int_0^{\varphi_0} \left[ -\frac{q}{2} R^2 \sin^2 \varphi - \frac{q}{2} R^2 \left( \cos \varphi - \frac{\sin \varphi_0}{\varphi_0} \right)^2 \right] R \left( \frac{\sin \varphi_0}{\varphi_0} - \cos \varphi \right) R d\varphi \\ &= -qR^4 \left[ \sin \varphi_0 + \frac{\sin^2 \varphi_0 (\cos \varphi_0 - 2)}{\varphi_0} \right]. \end{aligned} \quad (16)$$

According to the above formula, the following equation can be expressed:

$$X_1 = -\frac{\Delta_{1p}}{\delta_{11}} = \frac{qR^2}{2} \left(1 - \frac{\sin^2 \varphi_0}{\varphi_0^2}\right), \quad (17)$$

$$X_2 = -\frac{\Delta_{2p}}{\delta_{22}} = -\frac{qR \left[ \sin \varphi_0 + \frac{\sin^2 \varphi_0 (\cos \varphi_0 - 2)}{\varphi_0} \right]}{2 \left( \frac{\varphi_0}{2} - \frac{\sin^2 \varphi_0}{\varphi_0} + \frac{\sin 2\varphi_0}{4} \right)}. \quad (18)$$

Equation (10), (11), (14), (17) and (18) can be drawn from the actual moment curve equation model arch under stress:

$$M_c = M_p + \overline{M}_1 X_1 + \overline{M}_2 X_2 = A \cos \varphi + B, \quad (19)$$

where A and B are formula for the coefficient of expression. They are expressed as follows:

$$A = \frac{qR^2 \sin \varphi_0}{\varphi_0} - RX_2 ;$$

$$B = X_1 + \frac{RX_2 \sin \varphi_0}{\varphi_0} - \frac{qR^2}{2} \left(1 + \frac{\sin^2 \varphi_0}{\varphi_0^2}\right).$$

**Calculation of dome subsidence.** According to the formula in structural mechanics, deflection dome subsidence formula can be drawn:

$$\Delta = \int \frac{\overline{M}_c}{EI} M_c ds = \frac{2R}{EI} \left[ \frac{AC \sin^2 \varphi_0}{2} + AD \left( \frac{\varphi_0}{2} + \frac{\sin 2\varphi_0}{4} \right) + AF \sin \varphi_0 + BC(1 - \cos \varphi_0) + BD \sin \varphi_0 + BF \varphi_0 \right], \quad (20)$$

where  $\overline{M}_c$  represents the unit load  $P = 1$ . The model of the moment arch curve equation is expressed:

$$\overline{M}_c = M_{p1} + \overline{M}_1 X_{p1} + \overline{M}_2 X_{p2} = C \sin \varphi + D \cos \varphi + F, \quad (21)$$

where C, D and F are coefficient formula, which are expressed as follows:

$$C = -\frac{R}{2} ;$$

$$D = -RX_{p2} ;$$

$$F = X_{p1} - RX_{p2} \frac{\sin \varphi_0}{\varphi_0},$$

where the process of solving process and solving are the same to section 3.1. The calculation is not taken again. The final formula is expressed:

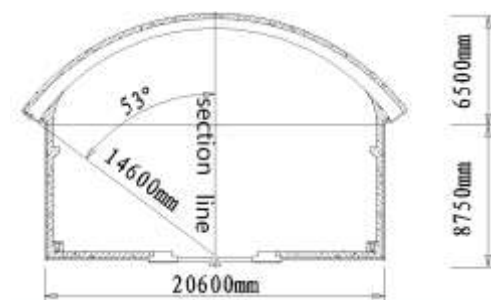
$$X_{p1} = -\frac{R(\cos \varphi_0 - 1)}{2\varphi_0}, \quad (22)$$

$$X_{p2} = -\frac{\frac{2 \sin \varphi_0 (\cos \varphi_0 - 1)}{\varphi_0} + \sin^2 \varphi_0}{4\left(\frac{\varphi_0}{2} - \frac{\sin^2 \varphi_0}{\varphi_0} + \frac{\sin 2\varphi_0}{4}\right)}. \quad (23)$$

In summary, the subsidence of dome can be drawn from the formula (20). So central angle subsidence of dome is related with and dome radius R, the model of surrounding rock arch sectional stiffness EI and close dome  $\varphi_0$ .

**The example of Qingdao subway tunnel calculation.** Through the above analysis, specific calculations on the vault subsidence under two different geological formations (hard rock, soft rock layer) conditions are analyzed.

1. Qingdao subway tunnel section station of Ling Qing is the first model. The degree of formation rock hard is hard rock. The station cross-sectional structure size is shown in Fig.3. Modulus of elasticity of hard rock E is 30000 MPa. The weight of hard rock is 25.4 kN/m<sup>3</sup>. The subway station depth h is 10 m. The model's cross-section inertia I is 2/3 m<sup>4</sup>. Under this geological condition, the vault subsidence calculated by the formula (20) is 33 mm. The station site monitoring values are consistent with the dome subsidence.



**Fig. 3. The station cross-sectional structure size.**

2. Qingdao subway tunnel Jiangxi Road station is weak rock layer, cross sectional dimensions of the station is similar to the section of station, and the elastic modulus soft rock mass E is 13000 MPa. The weight of soft rock is 23 kN/m<sup>3</sup>. The subway station depth h is 9.8 m. The model's cross-section inertia I is 2/3 m<sup>4</sup>. Under this geological condition, the vault subsidence calculated by the equation (20) is 68 mm. The station site monitoring values are consistent with the dome subsidence.

In summary, by comparison of the two cases, dome subsidence formula calculated by the tunnel pressure arch model is reasonable in an ideal hypothetical situation.

### **Empirical relationships of surface subsidence and dome subsidence**

Urban ground surface subsidence caused by tunnel excavation is an extremely complex process. However, surface subsidence begins to sink from the vault. The ground deformation will pass up through. Therefore, in order to explore the empirical relationship between the amount of

surface subsidence and dome settlement. We need to analyze surface subsidence in different geological formation conditions and vault settlement of relations. However, we can explore the empirical relationship of surface subsidence and dome subsidence according to section 3.3 Qingdao subway tunnel project examples:

1. Hard rock layer

The horizontal settling tank range is 50 m. The width of settling tank is 10m. The maximum surface subsidence value is about 5mm. The surface subsidence is extremely slight. The surface subsidence value accounts for one in sixth the dome settlement (33 mm). Mainly hard rock consolidation settlement is extremely small, causing dome settlement to pass upward through the formation, so the dome settlement is greater than the surface subsidence and the influence range of the surface subsidence is larger.

2. Weak rock layer

The horizontal settling tank range is 28 m. The width of settling tank is 5 m. The maximum surface subsidence value is 95 mm. The maximum surface subsidence is 1.4 times than the dome settlement amount (68 mm). The loss consolidation settlement weak rock layer is large. The vault sink value and consolidation settlement equals surface subsidence, which makes it larger than the dome surface subsidence sinking. However, the impact of surface subsidence range becomes smaller.

## Conclusion

Urban tunnel excavation in rock pressure arch force model was established and analyzed in this paper. The analysis of Qingdao subway tunnel project was verified. Conclusions are drawn as follows:

1. Pressure tunnel arch model is based on scientific assumptions and stress analysis. The formula derivation of dome subsidence was drawn. The vault subsidence was closely related with dome radius  $R$ , the model of surrounding rock arch sectional stiffness  $EI$  and dome central angle  $\varphi_0$ .
2. Qingdao subway tunnel project is taken as an example. By comparing the amount of dome settlement under two different geological conditions, the dome subsidence formula derived by the pressure arch tunnel stress analysis model is reasonable. The dome subsidence is consistent with the station site monitoring values.
3. When the layer is hard rock, the surface subsidence sinking is less than vault subsidence value; when the layer is soft, the dome settlement is greater than the surface subsidence in the Qingdao subway tunnel engineering.

Above all, the dome subsidence formula derived by the pressure arch tunnel stress analysis model has great significant efforts for the design, construction and protection of the tunnel.

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ЮЙ ГУАНМИН, ВАН ЦЗИЛЯН, СУН ЧУАНЬВАН, ЛЮ ШИБАО, ЛИ БИНБИН

ЮЙ ГУАНМИН, кандидат технических наук, профессор, научный руководитель,  
e-mail: yu-guangming@263.net; ВАН ЦЗИЛЯН, СУН ЧУАНЬВАН, ЛЮ ШИБАО, ЛИ  
БИНБИН – *Школа гражданского строительства, Университет технологий Цинда,  
Циндао, Шаньдун, 266033, Китай*

**Исследование напряженного состояния массива в тоннеле,  
обусловленного арочным деформационным эффектом,  
и его влияния на просадку земной поверхности**

**Аннотация:** Показана важность учета арочного эффекта деформирования массива горных пород при строительстве тоннелей для расчета величины оседания земной поверхности.

**Ключевые слова:** тоннель, массив горных пород, арочный эффект деформирования, оседание земной поверхности.