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Modelling the focal stage of the strain field distribution in highly stressed rock

The article deals with the non-Euclidian continuum model applied to describe the stress-strain state of highly stressed rock at the pre-failure stage. The oscillating character of the distribution of stresses both along the perimetre of the rock and along its height has been demonstrated in it. A satisfactory convergence of the results of the theoretical researches with the experimental data has been obtained.

Key words: sample of rock; uniaxial compression simulation; non-classical field of stress.

Introducion

Determination medium-term and short-term precursors of geodynamic phenomena requires research the regularities of deformation of rocks in the pre-failure state.

Therefore, it is important to simulate the deformation in strongly compressed samples of rocks around the perimeter of the sample and its height.

Mathematical modeling of deformations in rock samples in the pre-failure state on the basis of models of mechanics of defective mediums is performed [1, 2], where the rock is in the pre-fracture stage is regarded as a defective medium in conditions far from thermodynamic equilibrium, when the conditions of compatibility of elastic deformations in general are not performed

Experimental studies the deformation of rock samples

Experimental study of the nature of the deformation of rock samples in pre-fracture stage loading were realized on the samples of dacite and rhyolite cylindrical shape with fixing of strain gages on the basis of 10 mm and the use of recording equipment tensometric station UIU-2002 (Fig. 1). The source of destruction is pointed by red color on the figure 1. The symbol δ marked the acoustic sensors. Below all the calculations and graphs were made for a sample of rhyolite N 23.

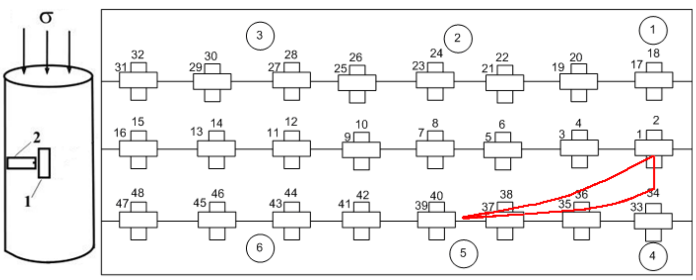

Fig. 1. Scheme loading of the sample. Location of pairs strain gauges and sensors AE

Researching patterns of the deformation in rock samples in the pre-failure state when $P > P^*$ (a threshold of dilatancy can be accepted as Critical load P^*) showed that the deformation of the sample have often reversible character, consisting in changing the sign of the increment of longitudinal and lateral strains, which were measured in local areas, the relevant scheme stickers sensors.

Research [3] revealed that the deformations of the sample along its height in the pre-failure stage have different signs consists in changing the sign of the increment of longitudinal and lateral strains in the area of formation cracks of tearing off and simultaneous strain of the same sign significantly (abnormally) exceeding the typical level for this breed.

Apart from of the increments of strains of different signs along height of the sample, the same character of strain observed along the perimeter.

Furthermore, it was found the geometrical position of source of destruction, and reversible (perilesional) areas I and II type.

Theoretical study of the deformation of rock samples

Theoretical studies of the patterns of deformation of samples at strong compression are performed using continuum model with the self-balanced stresses ([1]). In [1] describes the non-classical stationary distribution of the stress field in the cylindrical sample of rock using a non-Euclidean continuum model.

The non-classical normal uniaxial and tangential stresses T_{zz} and $T_{\varphi\varphi}$ in the cylindrical system coordinate are determined by equalities:

$$T_{zz}^{s,m,n} = -k_s^2 J_n\left(k_s r\right)\cos(n\varphi + \varphi_0)\cos q_m z\,,$$

$$T_{\varphi\varphi}^{s,m,n} = -\left(\frac{1}{r}\frac{dJ_n(k_s r)}{dr} - \frac{n^2}{r^2}J_n(k_s r) + k_s^2 J_n(k_s r)\right)\cos(n\varphi + \varphi_0)\cos q_m z,$$

where $k_s = \dfrac{z_s}{R}$, $s = 1, 2, \ldots$; $z_s -$ are roots of equation $J1(z) = 0$, and $q_m = \dfrac{\pi \cdot m}{h}$, $m = 1, 2, \ldots.$;

$R -$ is radius of the sample, $2h -$ *is height of the sample*, m is its height, m.

Stress components represented as the sum of three terms:

$$T_{zz}\left(r,\varphi,z\right) = B^{1,1,0}T_{zz}^{\ 1,1,0} + B^{1,1,1}T_{zz}^{\ 1,1,1} + B^{1,1,2}T_{zz}^{\ 1,1,2}\,, \qquad (1)$$

$$T_{\varphi\varphi}\left(r,\varphi,z\right) = A^{1,1,0}T_{\varphi\varphi}^{1,1,0} + A^{1,1,1}T_{\varphi\varphi}^{1,1,1} + A^{1,1,2}T_{\varphi\varphi}^{1,1,2}\,, \qquad (2)$$

where $T_{zz}^{1,1,0} = -k_1^2 J_0\left(k_1 r\right)\cos(\varphi_0)\cos q_1 z$, $T_{zz}^{1,1,1} = -k_1^2 J_1\left(k_1 r\right)\cos(\varphi + \varphi_0)\cos q_1 z$,

$$T_{zz}^{1,1,2} = -k_1^2 J_2\left(k_1 r\right)\cos(2\varphi + \varphi_0)\cos q_1 z\,;\quad T_{\varphi\varphi}^{1,1,0}\left(r,\varphi,z\right) = \left[\frac{k_1}{r}J_1\left(k_1 r\right) - k_1^2 J_0\left(k_1 r\right)\right]\cos\left(\varphi_0\right)\cos\left(q_1 z\right),$$

$$T_{\varphi\varphi}^{1,1,1}\left(r,\varphi,z\right) = \left[\left(\frac{2}{r^2} - k_1^2\right)J_1\left(k_1 r\right) - \frac{k_1}{r}J_0\left(k_1 r\right)\right]\cos\left(\varphi + \varphi_0\right)\cos\left(q_1 z\right),$$

$$T_{\varphi\varphi}^{1,1,2}\left(r,\varphi,z\right) = \left[\left(\frac{6}{r^2} - k_1^2\right)J_2\left(k_1 r\right) - \frac{k_1}{r}J_1\left(k_1 r\right)\right]\cos\left(2\varphi + \varphi_0\right)\cos\left(q_1 z\right).$$

Laboratory studies have shown that the source area of the sample is an ellipsoid.

The coordinates center of source area and the coordinates center area of the reversible deformations are $S\left(\frac{R}{2},0,-0,4\cdot h\right)$ and $B\left(\frac{R}{2},0,0\right)$ respectively.

In order to determine the six unknown coefficients of the expansion in the components of the stress need six equations. For the first of them use as fracture criterion:

$$K_I = M(-\beta T_{\varphi\varphi} + T_{zz}). \tag{3}$$

Here K_I – is Odintsev's stress intensity factor (MPa·m$^{1/2}$), at the top of the mezo-crack in the source of the sample; $\beta = \frac{\gamma_3}{\gamma_1}$, $\gamma_1 = 0,25$; $\gamma_3 = 0,2$; $M = \sqrt{\pi l}\cdot\gamma_1$.

The fracture toughness (MPa·m$^{1/2}$) of the rock sample are finding by formula

$K_{Ic}^* = M\cdot\sigma_c^*$, MPa$\cdot\sqrt{м}$, with the proviso that the load exceeds a certain critical P^* ($P > P^*$),

l – half length mezo-crack, m.

The normal uniaxial and tangential stresses $T_{\varphi\varphi}$ и T_{zz} are given by (1) и (2). Where $\sigma_c^* = \sigma_c - P^*$, P^* – is a critical load, above which in the sample begins to form mezo-cracking structure.

We assume that in the center of the source area the stress intensity factor reaches a value equal the fracture toughness of the rock sample:

$$K_I\big|_S = K_{Ic}^*. \tag{4}$$

The second equation is obtained on the assumption that the first invariant of the stress tensor at the point $(R,0,0)$ reaches σ_c^*, when the load exceeds a critical value P^*.

The remaining four equations derived from the conditions to achieve the extreme uniaxial and lateral stresses in the center of the source region and the center region of the reversing type I adjacent to the source region from the top along the axis of the sample.

Solving the system of equations, we find the coefficients of the equations (1) and (2):

$$T_{\varphi\varphi}(r,\varphi,z) = 0,0011\cdot T_{\varphi\varphi}^{1,1,0}(r,z) + 0,00012\cdot T_{\varphi\varphi}^{1,1,1}(r,\varphi,z) - 0,00015\cdot T_{\varphi\varphi}^{1,1,2}(r,\varphi,z),$$

$$T_{zz}(r,\varphi,z) = 0,0006\cdot T_{zz}^{1,1,0}(r,z) - 0,0024\cdot T_{zz}^{1,1,1}(r,\varphi,z) + 0,0012\cdot T_{zz}^{1,1,2}(r,\varphi,z).$$

On the Fig. 2.3 shows the distribution of linear stresses on the side surface of the sample. It is clearly seen that the stress distribution is oscillatory nature along the perimeter of the sample and its height.

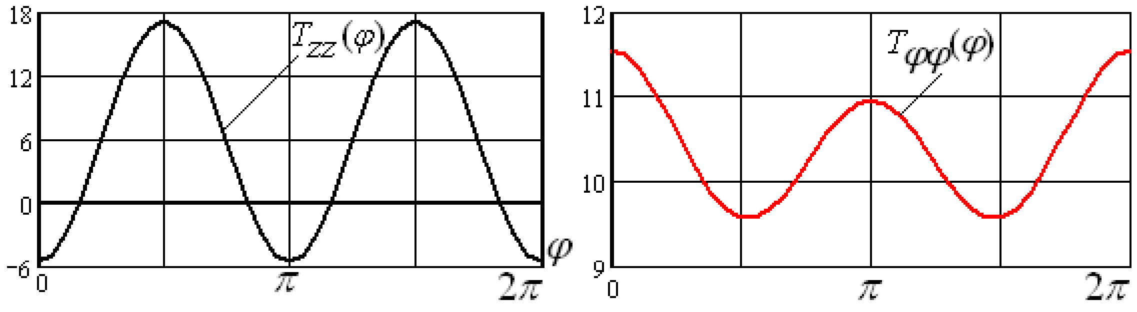

Fig. 2. The oscillatory nature of the stresses on the lateral surface of the highly compressed sample on its perimeter (the middle, $z = 0$)

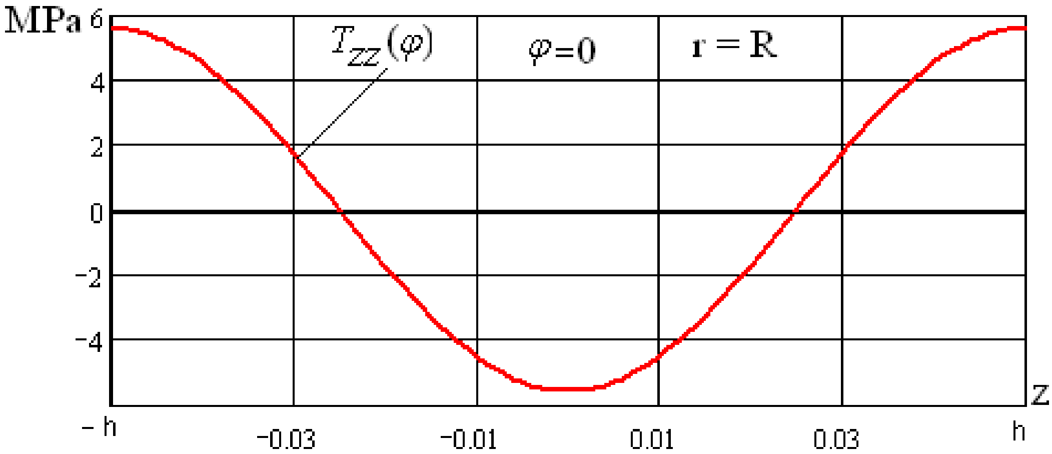

Fig. 3. The oscillatory nature of the uniaxial stresses on the lateral surface of the highly compressed sample at its height

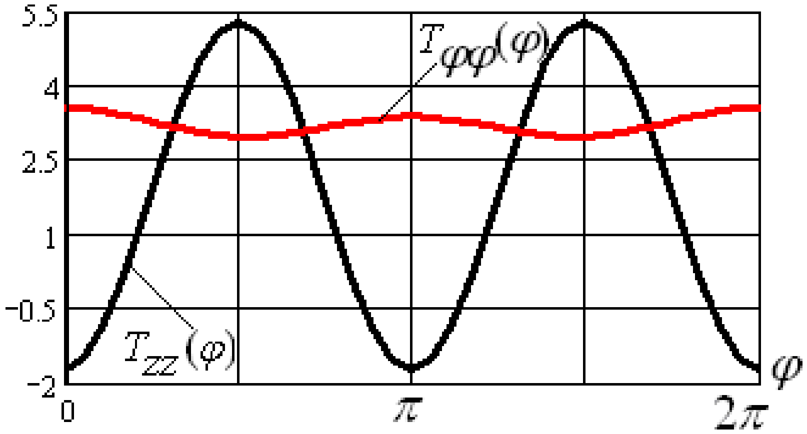

Fig. 4. The isolation of the source of failure region at the side surface of the sample

You can see that in the center of the fault zone longitudinal negative voltage that corresponds to the maximum compression of the material and the hoop stresses are positive, indicating that the tension in the circumferential direction of the rock in the fault zone.

From experiments follows:

$$E_z(\varphi)=\frac{1}{E}\left[T_{zz}(\varphi)-2\cdot\nu\cdot T_{\varphi\varphi}(\varphi)\right], \quad (5)$$

where $E=-66474\ MPa$ is the average modulus of total deformation; $\nu=0,4-$ the coefficient of transverse expansion. We assume that $T_{rr}(\varphi)=T_{\varphi\varphi}(\varphi)$.

Comparison the non-classical experimental and theoretical uniaxial deformations on the side surface of the sample in the neighbourhood of the source region shows that discrepancy between theoretical and experimental strains reaches 37%.

Conclusion

The values of experimental and theoretical deformations correlate satisfactorily. The maximum discrepancy between non-classical theoretical and experimental strains reaches 37%.

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Механизмы разрушения горных / Rock Failure Mechanism

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Моделирование очаговой стадии распределения поля деформаций в сильно сжатом образце горной породы

В работе применяется неевклидова модель сплошной среды для описания напряженно-деформированного состояния сильно сжатого образца в стадии предразрушения. Показан периодический характер распределения напряжений как по периметру образца, так и по его высоте. Получена удовлетворительная сходимость теоретических и экспериментальных деформаций.

Ключевые слова: образец горной породы, одноосное сжатие, неклассическое поле напряжений.

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