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РАЗРУШЕНИЕ ГОРНОЙ ПОРОДЫ. НЕЛОКАЛЬНАЯ ТЕОРИЯ УПРУГОСТИ

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Рассматривается разрушение слоя горной породы в рамках нелокальной теории упругости при пластическом течении. «Нелокальность» обусловлена зависимостью энергии деформируемой среды от производных метрического тензора деформаций. Показано: при продольном нагружении выделенного слоя пространственное распределение «поперечного» напряжения испытывает периодическое изменение. Максимальные значения напряжения достигаются в нестационарном режиме. Последующие разрушения горной породы ассоциируются с зонами максимального поперечного нагружения.

Ключевые слова: пластическое течение, разрушение горной породы, нелокальная теория упругости.

ROCK FAILURE AND NONLOCAL ELASTICITY THEORY

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The paper considers failure of a stratum within the framework of a nonlocal elasticity theory for a plastic flow. Here, the nonlocality is determined by dependence between deformation energy and metric deformation tensor derivatives. Our study has demonstrated that in case of a stratum under lateral stress, the spatial distribution of shear stress changes periodically. The maximum stress values are obtained in nonsteady regime. The following rock failures are associated with the zones of maximum shear loading.

Key words: plastic flow, rock fracture, nonlocal elastic theory.

In the classical elasticity theory, deformations are characterized by a metric tensor of elastic deformations g^{ik}. Plastic deformations also require considering a curvature tensor. IN their turn, nonelastic deformations take the deformation field out from Euclidean space. In [1] curvature is considered as a parameter of state having effect on deformation energy that makes the theory nonlocal. Unfortunately, the authors have provided no solid arguments for the introduction of additional degrees of freedom into the first law of thermodynamics, but still there is a whole class of problems in the failure theory, where the considered approach can be regarded as a reasonable one.

Let's consider a thin rod bended in a way its curvature radius at the local bend point is equal to $1/R$. If one draws a line along the rod, the rod's upper part will be extended by δl_1, while the lower part will be compressed by δl_2 relative to the rod. The first law of thermodynamics that unambiguously determines the process's dynamics, for a one –dimensional rod can be written as $dE_0 = TdS + f_1 dl_1 + f_2 dl_2$, where E_0 denotes the inner energy, T, S – temperature and entropy, $f_1 dl_1$, $f_2 dl_2$ – the forces affecting the rod while extension and compression. The parameters $\delta l_1, \delta l_2$ can be altered by the length of 'average' line and the curvature according to $2\delta l = \delta l_1 + \delta l_2,\ 2\pi \cdot \left(-\delta R / R^2\right) = \delta l_1 - \delta l_2$, so the first law will be expressed as: $dE_0 = TdS + \left(f_1 + f_2\right) dl + \pi \left(f_1 - f_2\right) dR / R^2$.

Hence, the assumption that the rod is bended requires a degree of freedom related to the curvature to be introduced. At the same time, there are indications that the classical elasticity theory is not complete in case rock failures in the vicinity of mines are interpreted [2]. Solving a problem of stress distribution around a radial plane under a given radial centripetal stress gives us the monotonic reduction of the stress tensor's angular component. The periodic zone of rock failure observed in mines may be the evidence of periodic stress distribution before the moment when a failure occurs. The classic solution does not describe such failures.

In [1] the authors suggest the equations describing nonelastic deformations with account for inner- energy dependence on curvature. In the paper, the curvature has been considered as a destruction parameter in a steady-state problem of medium deformation around a plane. It has been shown that the stress state is periodical in the radial direction, in other words, the spatial pattern observed around cylindrical mines has been described [2]. However, the authors have not studied the properties of the nonsteady solutions. In the current paper demonstrates that it is the nonsteady process that determines failure zone evolution in time.

Equations of motion for small infinitesimal strains with a destruction parameter are expressed as [1]:

$$\rho_0 \dot{v}_i = \partial_k \sigma_{ik},\ \dot{\varepsilon}_{ik} = \left(\partial_k v_i + \partial_i v_k\right)/2 - \bar{\varphi}_{ik} - \varphi_{\nu\nu}\delta_{ik}/3, -\frac{R}{2} = \Delta\varepsilon_{\mathbf{vv}} - \partial_i \partial_k \varepsilon_{ik},$$

$$\sigma_{ik} = \lambda \varepsilon_{\nu\nu}\delta_{ik} + 2\mu\varepsilon_{ik} + \beta\, R \cdot \delta_{ik}, \tag{1}$$

$$\varphi_{\nu\nu} = \zeta \cdot \left(\sigma_{\nu\nu} - 4\Delta\gamma\right), \bar{\varphi}_{ik} = \xi \cdot \left[\left(\sigma_{ik} - \sigma_{\nu\nu}\delta_{ik}/3\right) + 2\left(\partial_i \partial_k \gamma - \Delta\gamma\delta_{ik}/3\right)\right],\ \gamma = \beta\, \varepsilon + \alpha\, R.$$

Here $\mu, \lambda, \varsigma, \xi$ denote the elastic and plastic moduli, $R = g^{ik} R_{ik}$ – the curvature determined by the Ricci tensor, and α, β – the parameters of the state equation

$$E_0 / \rho = \text{const} + T_0(s - s_0) + \lambda \varepsilon_{\nu\nu}{}^2 / 2\rho_0 + \mu \varepsilon_{ik} \varepsilon_{ik} / \rho_0 + \beta R \cdot \varepsilon_{\nu\nu} / \rho_0 + \alpha R^2 / 2\rho_0 .$$

Alternative equations of destruction with plasticity are written as:

$$\frac{1}{2\zeta + \xi} \cdot \frac{\dot{R}}{2} = -\frac{4}{3}\left(\alpha - \frac{\beta^2}{\lambda + 2\mu}\right) \cdot \Delta^2 R + \frac{8}{3}\beta \frac{\mu}{\lambda + 2\mu} \cdot \Delta R -$$

$$- \mu \frac{\lambda + 2\mu/3}{\lambda + 2\mu} \cdot R + \left(-\frac{\xi}{2\zeta + \xi} + \frac{\lambda + 2\mu/3}{\lambda + 2\mu}\right)\dot{\Omega} - \frac{4}{3}\frac{\beta}{\lambda + 2\mu}\Delta\dot{\Omega}, \qquad (2)$$

$$\ddot{\Omega} - \frac{\lambda + 2\mu}{\rho_0} \cdot \Delta\Omega + \frac{3 \cdot \xi\zeta}{2\zeta + \xi}(\lambda + 2\mu) \cdot \dot{\Omega} = \left[\mu - (\lambda + 2\mu) \cdot \frac{3\zeta/2}{2\zeta + \xi}\right] \cdot \dot{R}$$

Changing of the density $\Omega = -\dot{\rho}$ under plastic conditions causes the failure *R* that satisfies the parabolic equation. As for the failure rate, it becomes a source in a wave equation determining the density change. The equations model density wave propagation and failure parameter diffusion as well as their mutual effect. The correctness of the parabolic equation is determined by the sign of the coefficient $\alpha - \beta^2 / (\lambda + 2\mu) > 0$.

The character of the nonsteady stress-strain state under external strength loading has been analyzed using a 1-D problem. A plate with the thickness $z \in [Z_1, Z_2]$ was considered. Its left boundary was under the compressive stress σ_{zz}, while its right boundary remained steady ($v_z = 0$). It was assumed that at the both boundaries $\gamma = \partial\gamma / \partial z = 0$. The system (1) had additional boundary conditions $\sigma_{zz} = -P_0, \gamma = \partial\gamma / \partial z = 0$ at $z = Z_1$ and $v_z = 0, \gamma = \partial\gamma / \partial z = 0$ at $z = Z_2$. The considered medium had the following material constants: $\rho =$ 2.2 kg/m^3, λ=12.63 GPa, μ=4.95 GPa, α = 3.11·10^{-4} kg·m^3/s, β=-5.7·10^{-4} kg·m^3/s, ξ=10^{-4} m·s/kg, ζ=10^{-4} m·s/kg. The thickness of the plate was 5 cm. The behavior of *R* can be seen in Fig. 1. In the process of reaching a steady state, the heterogeneities of maximum-amplitude destruction have been concentrated next to the boundary. In the figure its spatial distribution is given for three moments of time.

The maximum rate of amplitude growth has been achieved at the first maximum value. Given the considered boundary conditions, an external force application point formally corresponds to the first destruction boundary. However, it is the value of the second of destruction extremum that was of our immediate interest (Fig. 2). This domain corresponded to the maximum shear stress value associated with the deformation part of strain $\sigma^* = \lambda\varepsilon_{\nu\nu} + 2\mu\varepsilon_{11} = \sigma_{11} + |\beta| R$ (Fig. 2).

When an external force affects a rock at *z*=0, the stress σ^* determined by increasing destruction rate *R*, starts growing until the failure criterion is met (the Ishlinsky criterion -max|2σ^*|) in the zone where *R* reaches its maximum.

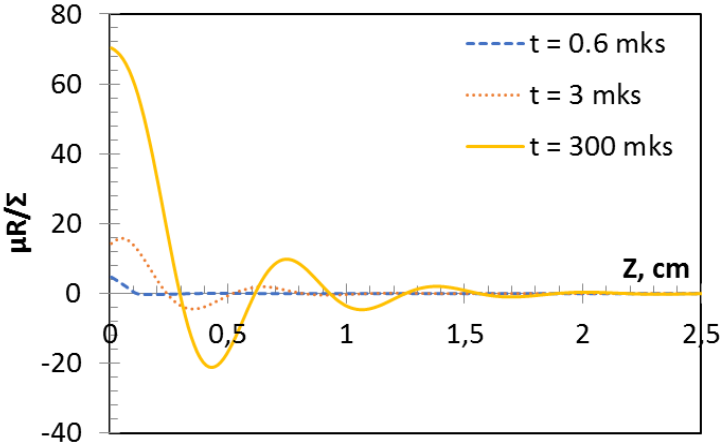

Fig. 1. Destruction process at the time moments of 0.6, 3 and 300 μs

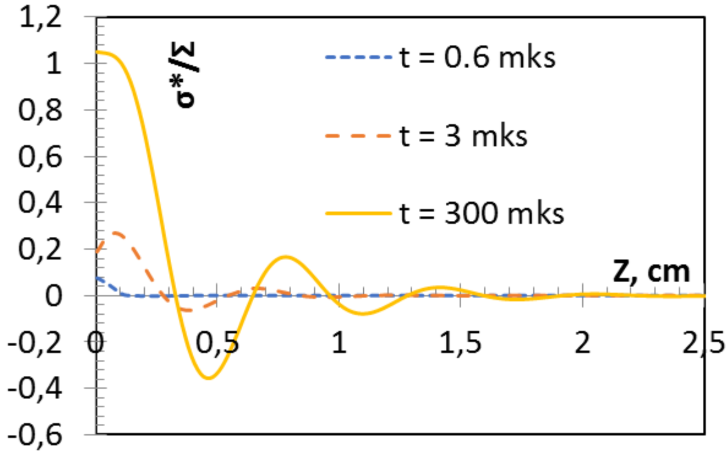

Fig. 2. The stress σ^* the time moments of 0.6, 3 and 300 μs

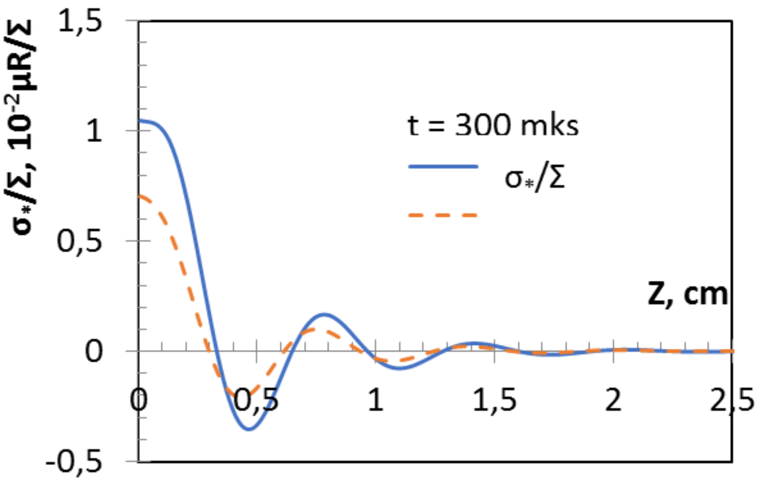

Fig. 3. The destruction R and the stress σ^* the time moments of 0.6, 3 and 300 μs

In the presented theory, rock failure is preceded by formation of periodic destruction zones in a medium. The maximum destruction value corresponds to the maximum destructing stress related to the deforming part of elastic stresses. The theory enables one to determine the characteristic scale of failure as a domain between two extreme values of destruction. The said failure scale can be achieved at the non-steady state of external strength loading put on a system.

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