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MULTIVARIANCE OF THE VELOCITY MODEL FOR STRUCTURAL PLOTTING BASED ON SEISMIC AND BOREHOLE DATA

Anatoliy P. SYSOEV
Saint-Petersburg Mining University, Saint-Petersburg, Russia

The paper discusses the peculiarities of structural modelling (forecast of the depths of the reflecting horizons) based on the seismic and drilling data system. Seismic data are represented by vertical time values and the stacking velocity of borehole data that are the depth marks of the reflecting horizons. Vertical time and the depth of the reflecting horizons are bound by the equation of average velocity but the average velocity is not determined in a seismic experiment, therefore an issue of choosing a velocity model of a complex natural object arises. The task of structural modelling is solved by the selection of formal expressions containing correlations between the parameters of the underlying model and kinematic parameters of the wave field. The optimal decision on model selection is determined by the minimum discrepancy between the predicted and actual values of the depth of the sample boreholes. A practical example shows possible variants of the interpretation model.

An inverse kinematic problem on converting the vertical time of the reflected waves at the depth of horizons is solved in each production report on the results of seismic work and is probably the most common objective of seismic exploration. Considering the variety of research objects and the apparent obviousness of the solution, this topic is underrepresented in scientific literature.

Key words: method of the reflected waves; kinematic interpretation; velocity model; stacking velocity

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Introduction. Since the introduction method of reflected waves (MRW) and to the present day, structural maps of geological boundaries are the necessary and most important result of the seismic surveys. The success of solving tasks of structural geology defines unambiguous line of geological objects with the events of the wave field, which are connected by the equations of geometrical seismic surveys. Two types of environment settings are unknown in structural expressions: a velocity model and reflection point depths.

Let several reflecting horizons be described by a set of hodographs of reflected waves. For this system we define the thick-bedded model of the environment and the method of solution of direct kinematic problem. The task of the structural interpretation is to estimate parameters of the structural velocity model of the environment based on the minimisation of the functional describing the discrepancy between model and real hodographs of reflected waves. In such a setting, the KING system of kinematic interpretation of hodographs was implemented in 1980-s [5]. The task of selecting parameters in the depth-velocity model was solved on the basis of the mainframe in the absence of graphical controls solutions and interactivity.

In principle, in the same setting, but at a new technical level, the problem of selection of parameters of the structural velocity model for depth migration is solved on pre-stack of 3D-data. We note that the issue of finding minimum always has a solution, but in order that the solution satisfies a priori data (marks of the depths of the reflecting horizons – RH – is specified in borehole depths points), a formation model of the environment is amended by formal parameters, taking into account the velocity «anisotropy». But even with a significant complication of the model, the results of depth migration are presented not in a depth, but in a time scale, as in processing of seismic data, issues of full harmonisation of seismic and borehole data do not yet have satisfactory solutions [6, 8]. This fact determines the relevance of the problem analysis for structural expressions at the stage of seismic data interpretation.

The applied problems of structural interpretation address two kinematic parameter of the hodograph common midpoint: vertical time t_0 and stacking velocity v_s. The kinematic parameters (time sections of amplitudes of the reflected waves and the breakdown of stacking velocity) are obtained in the process of digital processing of seismic data, and the parameter t_0 appears as a result of phase

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correlation of reflection amplitudes. We assume that the phase correlation result is determined uniquely on temporary cuts of amplitudes, therefore, two kinematic parameters are the result of seismic data processing.

It is considered that the vertical time t_0 describes the time of passing the beam vertically from the line of harmonisation of seismic observations to the reflecting boundary and back without taking into account the refraction at the interfaces of the velocity medium model. Reference line can be defined above the surface of the observation – in this case the medium model introduces a dummy layer with predetermined velocity. In addition, compensation of lateral heterogeneity of the upper part of the section are procedures for replacing velocity – changing structure and parameters of the velocity medium model and, accordingly, changes the vertical time of the reflected waves.

In general, the vertical and effective velocity of the reflected waves are not the result of measurement of the parameters of the wave field during execution of field experiment and present the result of the reconstruction field based on equations describing the propagation of waves for a particular model of a medium. So, if variable surface topography compensates static corrections, then stacking velocity is distorted due to the limitations of this conversion method. Distortion effect of the reflected wave velocity occurs in the compensation of the static corrections for immersed permafrost inhomogeneities. We should also note the widely known fact that vertical distortion of time and effective velocity due to error in the measurement of low-frequency static corrections [7].

Even without these factors, values of the effective rate correspond to the average velocity only for homogeneous velocity model of a medium. For real environments, a vertical velocity gradient is always present, so the difference between the average and effective velocity increases with increasing RH depths. Let us consider the example on the materials of Tomsk Oblast. Here, in borehole points, the vertical time and the stacking velocity v_s of the reflected wave were found according to the GIS-defined depth values of the reference B horizon (roof of the Bazhenov formation) and results of seismic works. The data obtained allow perform of average and effective velocity comparative analysis. The average values for the sample was at the average velocity of 2511 m/s and 2721 m/s for stacking velocity (Fig.1, *a*). An analytical formula of recalculation of effective rate to the middle one do not exist. The use of Urupov-Dix formulas to calculate interval velocities and subsequent estimation of the average velocity reduces the difference between the velocity values, but does not solve the problem completely. We can assume that the effective velocity (or stacking velocity) is the problem of interpretation of the attribute of the wave field, that is only correlation-associated to the target parameter of the average velocity to the reflecting horizon.

The analysis of velocity graphs, shows that the difference between the two sequences does not describe a constant value, i.e., we are unable to bring the stacking velocity to the average to

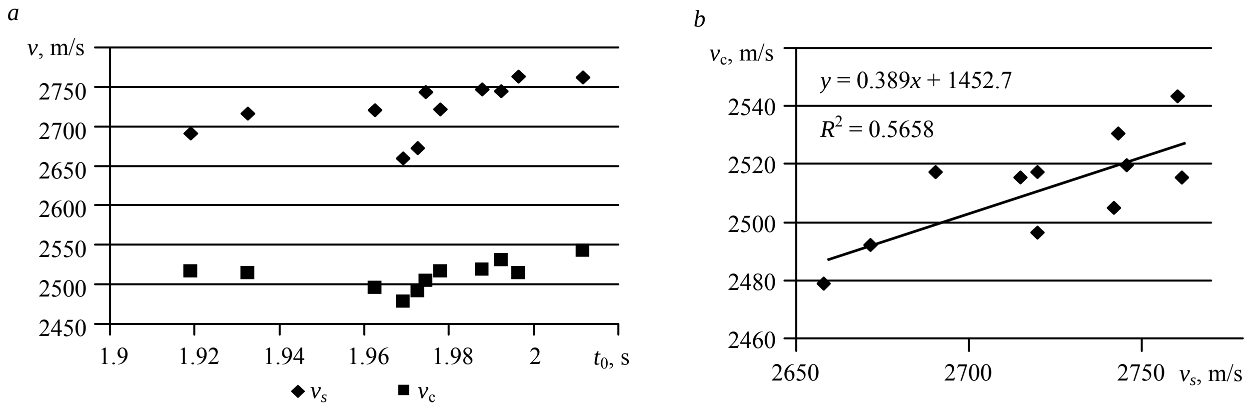

Fig.1. The average ratio of v_s and v_c are effective velocities at the B horizon

use the $v_c = v_s - a$ equation. The velocity dependence is also inadequately described by an equation $v_c = av_s$. A consistent description of the v_c distribution (v_s) is obtained by the linear regression equation (Fig. 1, *b*).

The definition of the structural modelling task. Following the S.V. Goldin's ideas, the problem of determining the depth of a reflecting horizon can be represented as follows [2]. *D* is for a study area in the plane (x, y). We assume that at any point $M(x, y)$ parameters $t_0(M)$ and $v_s(M)$ are defined as vertical and effective velocity of the reflected wave, which is single-valued functions of the variable *M*. The depth of the horizon is determined by the equation

$$h(M) = t_0(M)v_c(M)/2,$$

where $v_c(M)$ is the average velocity before the reflecting boundary.

To determine the unknown $v_c(M)$ parameter, we can consider two solutions. We relate the average velocity in the *M* point to the value: a) vertical time $v_c(M) = f(t_0(M))$; b) the effective velocity $v_c(M) = f(v_s(M))$.

The result provides two tasks of structural modelling. In the first case, the prediction is performed only at the vertical time of the reflected waves, and for the second, the source data are the vertical time and effective velocity (stacking velocity) of the reflected waves.

The specific formulation is characterised by a certain structure of the input data and the model, linking the unknown parameters with the original data. The source data for the considered problem include borehole and seismic information: 1) borehole information, the depth and position of target horizons intersection; 2) seismic data, i.e. time tracking of the reflecting horizons and stacking velocity of the RH.

We are to perform: 1) a forecast of the surface describing the depth of the target horizon within the study area; 2) an estimation of the forecast error.

Special conditions: the predicted depth values at the points of the layer intersection should exactly coincide with the borehole depth unless it has been proven the need for the borehole data error accounting of the.

Curves of deep wells logging are also directly related to the problem of structural constructions. Taking into consideration the vertical travel time curves of seismic logging cause a certain version of the three-dimensional velocity model, considered in [4].

Forecasting problem. To estimate the depth of the seismic horizon, it is necessary to determine the relationship between the temporal field variables (vertical time t_0, stacking velocity v_s) and the predicted parameters (depth *h* or average velocity v_c). Let's divide the solution of the forecast problem into four stages.

1. The formation of the training sample. Since the depth values are specified only at the boreholes points, it is necessary to determine the values of seismic parameters at these points to obtain a training sample. Additionally, the mean velocity values can be calculated from the depth and vertical time values at the boreholes points: $v_c = 2h/t_0$.

2. Model fitting. To solve the problem, it is necessary to determine at least one velocity model, which relates the kinematic parameters of the reflected waves with the average velocity or directly with the horizons depth. Estimation of model parameters is performed by minimizing the value of the function describing the deviation of the predicted and actual depths at the boreholes points. It is also acceptable to minimize the deviation between the predicted and observed time when selecting the parameters of a model.

3. Calculation of the depth-velocity model parameters in the context of seismic data determination.

4. Accordance of the forecast and actual depth values at the boreholes points (scatter of discrepancies).

The rationale for the linear dependence of the depth on the RH vertical time. The linear regression equation is most commonly used in the analysis of depth-time dependence of the RH.

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We show that this formal construction is physically meaningful to describe a certain type of structural model.

Let us assume that the depth h and the vertical time value of t_0 are known in K wells. Then determine the linear regression between depth and vertical time

$$h = at_0 + b. \tag{1}$$

Note that for the vertical hodograph of the seismic logging (SL), the $h(t)$ dependence is determined by a continuous increasing function that allows a piecewise linear approximation corresponding to the layered model. Consequently, the representation of the time-to-depth correlation by the linear regression equation within a uniform layer is reasonable in case of seismic logging data.

Equation (1) represents the unique dependence of the depth on the reflection time. Following [1], let's differentiate the function according to t_0:

$$\frac{dh}{dt_0} = a = \text{const.} \tag{2}$$

The a parameter characterises the rate of boundary depth change with the vertical time. For brevity, we call this parameter a gradient of the linear function (1). For the layered model with constant velocity layers $(h(x) = \sum h_i(x);\ t_0(x) = 2\sum h_i(x)/v_i)$, the gradient is given by

$$\frac{dh}{dt_0} = \frac{dh}{dx}\frac{dx}{dt_0} = \frac{dh}{dx}\bigg/\frac{dt_0}{dx} = \sum_i \frac{dh_i}{dx}\bigg/\sum_i \frac{2dh_i}{v_i dx}. \tag{3}$$

It is easy to show that the expression (3) has a constant value if the layer thicknesses are determined by the linearly dependent functions

$$h_i(x) = d_i f(x) + c_i$$

with arbitrary factors d_i, c_i. Then

$$\frac{dh}{dt_0} = \sum_i d_i \bigg/ \sum_i \frac{2d_i}{v_i} = a = \text{const}. \tag{4}$$

Let's consider a special case of the two-layered model, where thicknesses of two layers with constant velocity are described by arbitrary functions of the profile coordinates:

$$h_1(x) = d_1 f(x) + c_1;\ \ h_2(x) = d_2 f(x) + c_2.$$

Accordingly, the depth and vertical time for the lower boundary are given by:

$$h(x) = (d_1 + d_2) f(x) + (c_1 + c_2);$$

$$t_0(x) = 2\big((d_1/v_1 + d_2/v_2) f(x) + (c_1/v_1 + c_2/v_2)\big). \tag{5}$$

Equation (4) is modified to

$$\frac{dh}{dt_0} = 0{,}5\big(d_1 + d_2\big)/\big(d_1/v_1 + d_2/v_2\big) = a = \text{const.}$$

The obtained equation (5) shows that the dependence of $h(t_0)$ can be an increasing ($a > 0$), a decreasing ($a < 0$) and a constant ($a = 0$) function of vertical time.

Let's express the a gradient value as a function of the k factor characterising the ratio between thicknesses of layers of the depth model:

$$k = d_2/d_1,\ \Rightarrow\ a = \frac{1+k}{2(1/v_1 + k/v_2)}. \tag{6}$$

A positive k value corresponds to a direct linear dependence between the layer thicknesses, and negative shows the inverse effect. Direct dependence reflects an inherited development of the strata, the reverse relationship does not have a simple geological interpretation, but is generally interesting for formal analysis. The graph of $a(k)$ dependence for two-layered medium with layer velocity of $v_1 = 2000$ m/s and $v_2 = 3000$ m/s is shown in Fig.2.

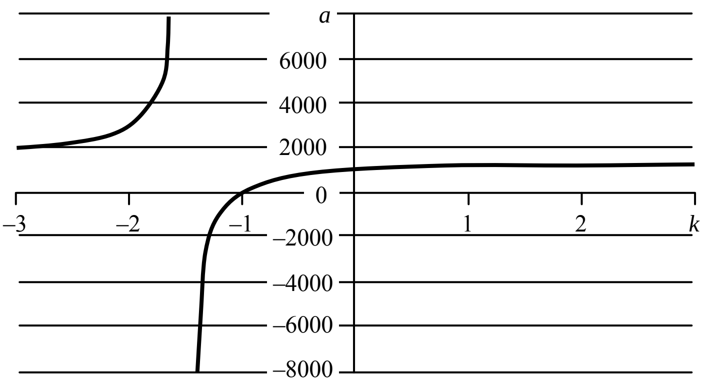

Fig.2. Dependence of the a gradient on the $k = d_2/d_1$ ratio of the depth model characteristics

We note the following features:

1. At the point $k = -1{,}5$ the function graph has a gap $\pm\infty$, here the denominator of the expression (6) is equal to zero:

$$1/v_1 + k/v_2 = 0, \;\Rightarrow\; k = -v_2/v_1, \;\Rightarrow\; k = d_2/d_1 = -v_2/v_1, \;\Rightarrow\; d_2 = -d_1 v_2/v_1.$$

Substituting the expression for d_2 in equation (5), we determine that the break point corresponds to the variable RH depth at a constant value of the RH vertical time (Fig.3):

$$h(x) = d_1(1 + d_2 v_2/v_1) f(x) + (c_1 + c_2);$$

$$t_0(x) = 2\big(c_1/v_1 + c_2/v_2\big) = \text{const}.$$

A sharp graph change at the $k = -v_2/v_1$ rupture point is defined by almost infinite depth change at a small vertical time change.

2. The a gradient a is equal to zero at the $k = -1$ point that corresponds to the $d_2 = -d_1$ relationship. From equations (5) we conclude that the value a = 0 corresponds to constant depth at a variable value of the vertical time (Fig.4).

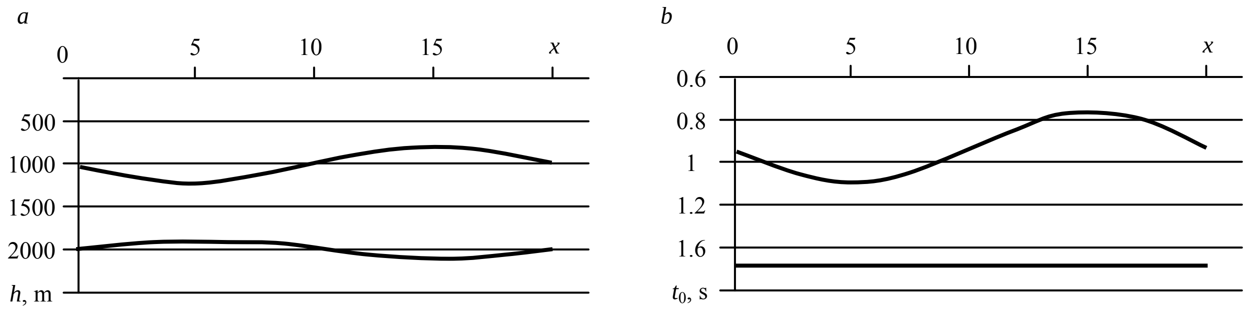

Fig.3. Depth model of $k = -v_2/v_1$ (a) and the RH vertical time graphs (b)

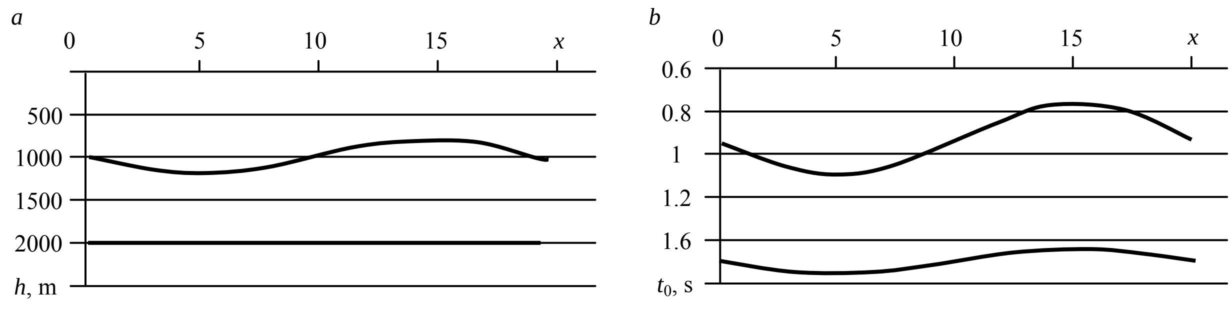

Fig.4. Depth model of $k = -1$ (a) and the RH vertical time graphs (b)

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3. In the range $-1.5 < k < -1$ of the gradient a is negative, which corresponds to the decrease of depth at the increasing RH vertical time (Fig.5).

4. The point of intersection with ordinate axis ($k = 0$) indicates the constant thickness of the second layer. The value of the $a = v_1/2 = 1000$ m/s gradient is determined by velocity of the first layer (Fig.6).

5. Asymptotics of $a(k \to \pm\infty) \to v_2/2 = 1500$ m/s corresponds to a model in which the thickness variation of the first layer is low, compared with thickness variation of the second one (Fig.7). Within limits, this model version corresponds to the constant thickness of the first layer.

To conclude this part, let us consider a model with low dependence between layer thicknesses. The linear $h(t_0)$ dependence is defined, but depth value deviations as compared to the regression line (Fig.8), are interpreted as forecast errors.

The results of model experiments allow conclusion that the equation of a linear relationship between depth and vertical reflection time corresponds generally to a multi-layered model with a

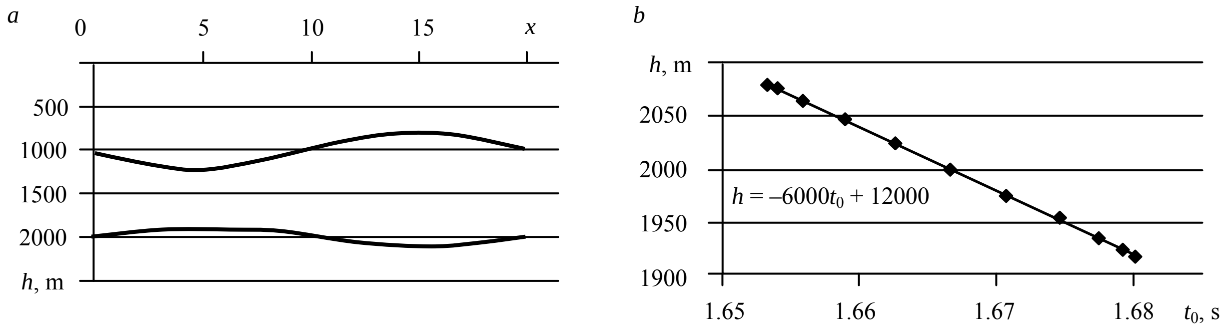

Fig.5. Depth model of k = –1,4 (a) and a graph of $h(t_0)$ (b)

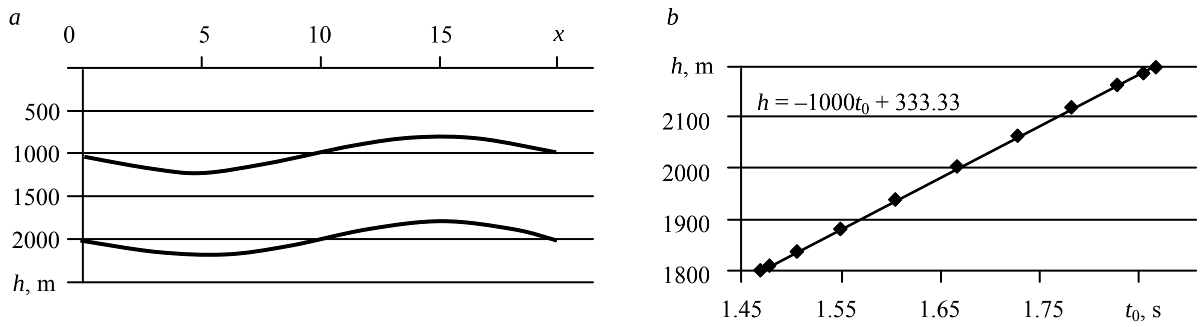

Fig.6. The depth model of k = 0 (a) and a graph of $h(t_0)$ (b)

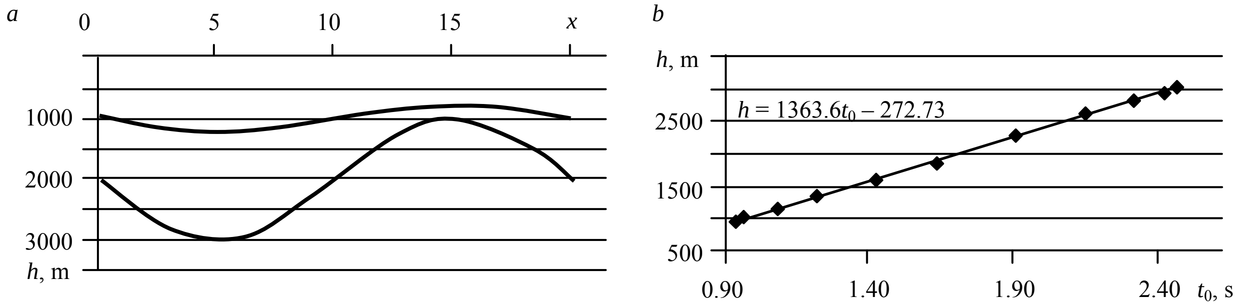

Fig.7. Depth model of k = 4 (a) and a graph of $h(t_0)$ (b)

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linear dependence between layer thicknesses. A linear dependence between the interval thicknesses or between the time values of reflecting horizons can serve as this model's indicators.

Selection of the interpretation model. Selection of the velocity model is demonstrated on the example of a training sample in Tomsk Oblast. Here, in 11 deep boreholes, the values of vertical time and horizon depths with conditional indexes G and B are determined. The RH time values with ascending second variables are shown in Fig.9. For the target B horizon, the values of stacking velocity are also determined and shown in Fig.2.

At the first analysis stage, we assume that the predicted h_k parameter, t_{0k} variable value and the calculated v_k value of the average velocity for the B horizon are known in *K* points of the boreholes. Even for this simple data systems several options for depth-velocity model are possible.

1. The distribution of $v(\mathbf{x})$ average velocity in study area can be obtained by interpolating the average velocity calculated in points of the boreholes.

2. The velocity value is determined as a function of vertical time of reflection: $v(\mathbf{x}) = f(t_0(\mathbf{x}))$.

3. Horizon depth is determined as a function of vertical time of reflection: $h(\mathbf{x}) = f(t_0(\mathbf{x}))$.

4. Vertical time of reflection is determined as a function of the horizon depth: $t_0(\mathbf{x}) = f(h(\mathbf{x}))$.

The average velocity model $v(\mathbf{x})$. The simplest decision is to interpolate the average velocity according to the v_k number of values. The average velocity model corresponding to this equation is described by the v_0 expression including constant and variable components:

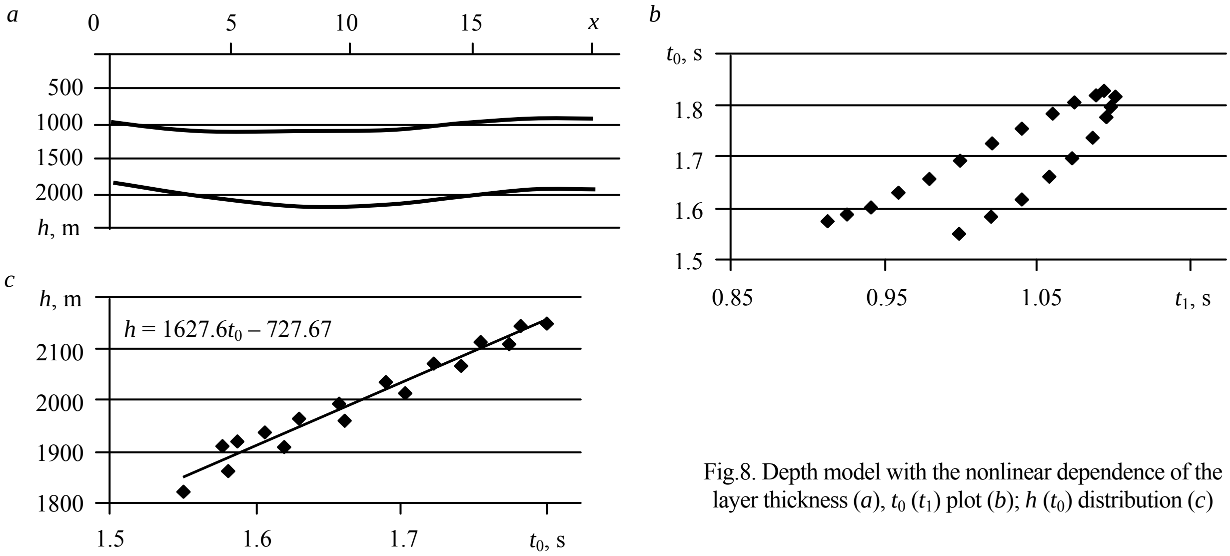

Fig.8. Depth model with the nonlinear dependence of the layer thickness (*a*), t_0 (t_1) plot (*b*); h (t_0) distribution (*c*)

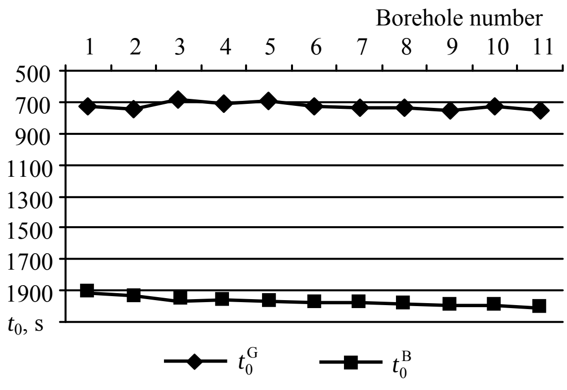

Fig.9. Vertical time values at borehole points

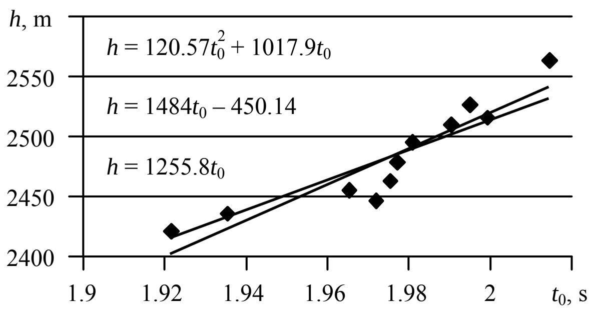

Fig.10. Distribution of $h(t_0)$ in the sample of boreholes

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$$v_k = v_0 + e_{vk}.$$

Estimation of the v_0 parameter is determined by minimising functional describing the discrepancy between actual and estimated depths of borehole points:

$$\Theta(v_0) = \sum (h_k - v_0 t_{0k}/2)^2 \to \min, \Rightarrow \hat{v}_0 = \sum_{k=1}^{K} h_k t_{0k} / \sum_{k=1}^{K} (t_{0k}^2/2).$$

Using $\hat{v}_0$ velocity estimation, we define the e_h discrepancy between the predicted and the actual depths and the σ_h value of the standard forecast error for each well:

$$e_{hk} = h_k - \hat{v}_0 t_{0k}/2; \ \ \sigma_h^2 = \sum_{k=1}^{K} e_{hk}^2 /(K-1).$$

Fig. 10 shows the average velocity model for the B horizon described by the $h = 1255.8t_0$ equation. The standard forecast error of the sample $\sigma_h = 17.9$ m.

A linear regression model $v(t_0)$. The equation of linear dependence of the average velocity on the reflection time is converted to dependency of depth on reflection time:

$$v = at_0 + b, \Rightarrow vt_0/2 = (at_0 + b)t_0/2, \Rightarrow h(t_0) = dt_0^2 + ct_0. \quad (7)$$

The depth is expressed by a parabolic equation with zero fixed term (Fig. 10). The standard depth forecast error of the sample $\sigma_h = 16.7$ m.

A linear regression model $h(t_0)$. The approximation result based on $h(t_0)$ the linear regression equation

$$h = at_0 - b + e_h \quad (8)$$

shown in Fig. 10. In the interval change of the t_0 variable, the regression line is almost superimposed on the parabolic equation (7). The standard forecast error of the sample, σ_h= 16.8 m.

The regression is $t_0(h)$. In the above linear regression equation (8) the random component e_h is defined as the measurement error of the h depth of horizons at borehole levels, and the explanatory variable t_0 is considered to be specified precisely. The assumption that the borehole depth is known exactly, and the time of reflections contains errors, is more consistent with the real situation. In [3] this situation is described as "errors of explanatory variables" that lead to biased estimates of parameters of the linear regression equation. If σ_t is a standard error of thet_0 variable, the offset of the a factor is estimated by the expression $a\sigma_t^2/(\sigma_t^2 + \sigma_h^2)$.

This model contradiction can be neutralised if using a linear regression equation to describe vertical time as a function of horizon depth:

$$t_0 = ah + b + e_t.$$

After estimating the factors, the $t_0(h)$ regression equation is modified to

$$h(t_0) = t_0/\hat{a} - \hat{b}/\hat{a}. \quad (9)$$

The regression factors are selected by minimizing the discrepancies between the forecast and actual values reflection time, so the depth forecast error increases to 18.2 m according to the equation (9).

Thus, two linear dependences of the horizon depth on the vertical time of the reflected wave were found with one sample. We assume that the vertical reflection time within the study area varies in the range of 1.9 and 2.1 s and calculate the depth horizon using the equation of the straight

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$h(t_0)$ and reverse $t_0(h)$ dependence (Fig.11). The discrepancy between the solutions for a specified reflection time change interval (0.2 s) is from –18 to 33 m. The $t_0(h)$ dependence determines a solution with more rapid changes of reflecting boundary depth.

The two-layer model. We introduce the problem statement of the vertical time value and the depth of the reflecting G horizon. We expect that the parameterisation of the depth model with two layers allows reduction of the forecast error for the target B horizon.

As it was previously shown, the linear regression equation describes correctly the dependence of the depth on the vertical time, if a linear correlation exists between the depth values (or vertical time boundaries). Fig.12 shows a graph of the $t_0^{\mathrm{B}}(t_0^{\mathrm{G}})$ joint distribution of the vertical times of two horizons. The linear dependence between time values of the two horizons is missing, which is the cause of significant depth forecast error with a linear regression equation.

If the temporary thickness of the two cross-section intervals varies independently, then it is logical to consider a model with an independent thickness estimation for each interval. Figure 13 presents two graphs characterizing the $h(t_0)$ distribution for horizon G and the correlation $d_h(d_t)$ between the B-G interval thickness and interval time.

For horizon G (Fig.13, *a*), the dependence $h(t_0)$ is described by the linear regression equation. The standard error value is σ_h = 4.2 m. For the B-G interval, the linear dependence is determined by the low coefficient of determination R^2 = 0.49 and the forecast error of the interval thickness is σ_h = 24 m. Thus, the complication of the model, performed by dividing the medium into two intervals, leads to an increase in the forecast error of the horizon B depth, compared with a linear dependence on one variable.

In the considered task, only two horizons are set, therefore, by the sweeping of the explanatory variables and model parameters, it is easy to establish that for estimating the B-G interval thickness,

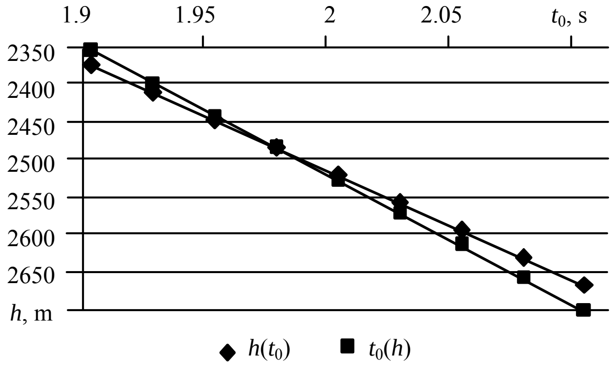

Fig.11. The discrepancy between the RH depth estimation for regression models $h(t_0)$ and $t_0(h)$

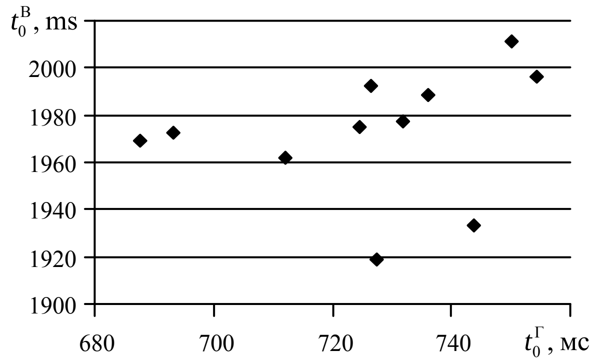

Fig.12. Distribution in $t_0^{\mathrm{B}}(t_0^{\mathrm{G}})$ boreholes points

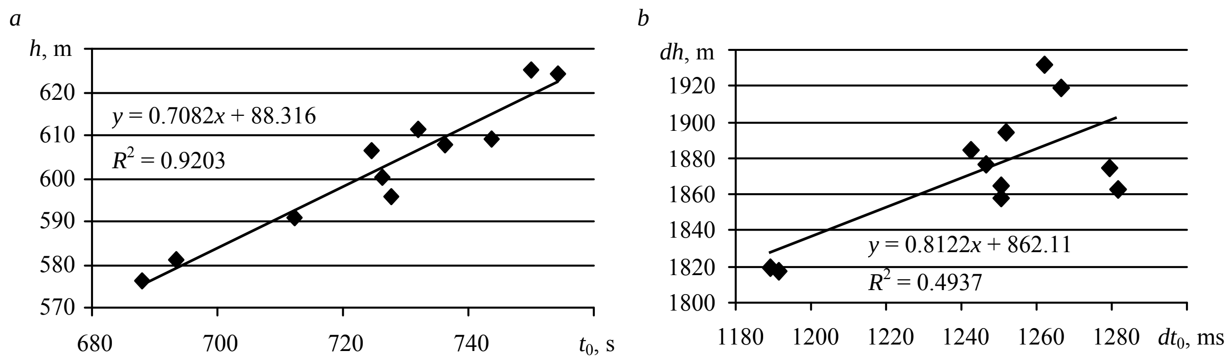

Fig.13. Analysis of dependencies for the reflecting horizon G (*a*) and B-G interval (*b*)

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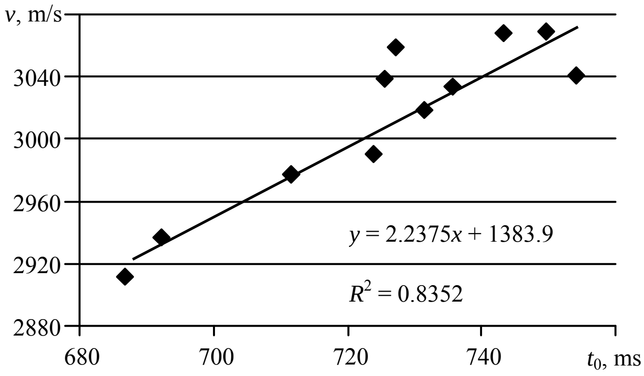

Fig.14. The dependence of the average velocity within the B-G interval from t_0 of G horizon

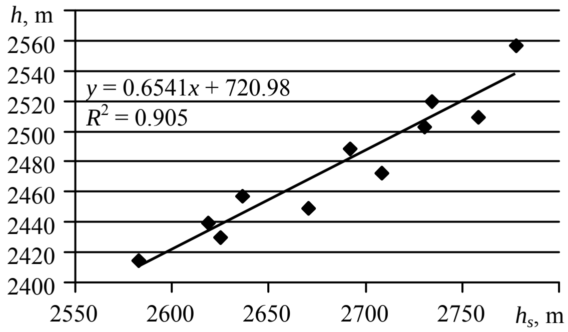

Fig.15. Dependence of the actual depth of the horizon on the «effective» one

a practically important variant of the model is determined as a linear dependence of the average velocity of B-G interval on the vertical time. G – top of the analysed interval (Fig.14),

$$v^{\mathrm{B-G}} = a_0 + a_1 t_0^{\mathrm{G}} \Rightarrow \hat{d}_h^{\mathrm{B-G}} = \hat{v}^{\mathrm{B-G}}(t_0^{\mathrm{B}} - t_0^{\mathrm{G}})/2 . \quad (10)$$

This solution suggests to estimate the first layer thickness by the linear regression equation and to describe the lower interval velocity model by the average velocity equation $v^{B-G}(t_0^{\mathrm{G}})$. The standard error of the of the B horizon depth forecast in case of a two-layered model $e_h = h - (\hat{h}^{\mathrm{G}} + \hat{d}_h^{\mathrm{B-G}})$ is 9.8 m.

Effective velocity model. Let's consider a problem when the seismic data is represented by the functions of the vertical time and stacking velocity of the reflected wave: $t_0(\mathbf{x})$, $v_s(\mathbf{x})$. In addition, the values of the average velocity v_c are calculated at the boreholes points.

To recalculate the stacking velocity to the average one, we use a formal linear regression equation $v_c = av_s + b + e_v$, where e_v – is the random component. The graph of the $v_c(v_s)$ distribution is shown in Figure 1, *b*. The depth of the horizon is determined by the formula

$$\hat{h}(\mathbf{x}) = \hat{v}_c(\mathbf{x})t_0(\mathbf{x})/2 = (\hat{a}v_s(\mathbf{x}) + \hat{b})t_0(\mathbf{x})/2 . \quad (11)$$

The value of the depth forecast standard error for this model is σ_h = 11.69 m.

Effective depth model. In the previous task, the stacking velocity was adjusted to the average one. Meanwhile, the ultimate goal is to select a velocity model that minimizes the divergence of the depths at the boreholes points.

The vertical time and stacking (effective) velocities values are known at the seismic prospecting points. This allows us to calculate the function, which we'll call «effective depth» of the horizon: $h_s = v_s t_0 / 2$. The h_s values are shifted relative to the true depths since the stacking velocities are shifted relative to the average ones. But the h_s values can be used as explanatory variables of the regression equation for the horizon depth forecasting (Fig.15):

$$h = d_1 h_s + d_2 = d_1(v_s t_0 / 2) + d_2 + e_h .$$

The $h(h_s)$ dependence is characterized by a higher coefficient of determination than in the average velocity estimation described in the previous example. While the value of the depth forecast error σ_h = 13.42 m is greater than for the effective velocity model.

How to take into account the randomness of the result. We take into account that the regression equation coefficients and forecast errors are random variables, i.e. the minimum error determined by the limited sample does not guarantee a better solution. It may be recommended to perform the calculation of all variants and estimate the discrepancy between the depth maps. It is possible that the discrepancy will be insignificant, then the problem of choosing a model is no longer required. With a significant difference, it is permissible to obtain weighted mean values of several solutions.

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If we accept the condition of independence of the forecast error for certain model types, then we can propose maps averaging with due regard to the forecast error of each implementation. Let's obtain, for example, two worthwhile results: the effective velocity model (10) and the two-layered model with an interval velocity estimation (11). The estimated solution errors, determined by a sample from 11 boreholes, are 11.7 and 9.8 m, respectively.

On the set of points **x**, two solutions $h_1(\mathbf{x}), h_2(\mathbf{x})$, with the estimated forecast error variance, are determined: D_1, D_2. The value of the required function at an arbitrary point **x** is defined as the weighted sum of the basic data:

$$h(\mathbf{x}) = w_1 h_1(\mathbf{x}) + w_2 h_2(\mathbf{x}),$$

where $w_1 + w_2 = 1$.

The variance of the sum of two independent variables is determined by the formula

$$D_e = w_1^2 D_1 + w_2^2 D_2 .$$

The determining of weights that ensure a minimum of variance w_1, w_2, considering an additional condition, is determined by the functional minimizing:

$$D(w_1, w_2, \lambda) = w_1^2 D_1 + w_2^2 D_2 + \lambda(w_1 + w_2 - 1),$$

where λ – undetermined Lagrange multiplier [1].

The minimum functional point is determined by the condition of equality to zero of the partial derivatives:

$$\frac{\partial}{\partial w_1} D(w_1, w_2, \lambda) = 0; \quad \frac{\partial}{\partial w_2} D(w_1, w_2, \lambda) = 0; \quad \frac{\partial}{\partial \lambda} D(w_1, w_2, \lambda) = 0.$$

It is easy to determine that the weight multipliers are determined by the

$$w_1 = D_2 /(D_1 + D_2); \quad w_2 = D_1 /(D_1 + D_2).$$

An estimation of the standard error of the forecast depth of the target horizon is calculated according to the formula (11) σ_h = 7.5 m.

Conclusion. Estimation of the seismic horizons depth is a standard forecasting problem, where the kinematic parameters of the wave field are used as explanatory variables: vertical time and stacking velocity of the reflected waves. The absence of a functional dependence between the predicted parameters of the medium and the parameters of the wave field determines the multivariance of the correlation between the horizon depth and the kinematic parameters of the wave field. Model variants are determined by the model's structure and basic data. The criterion of the adequate model is the minimum discrepancy between the predicted and actual depths at the boreholes points. As a rule, structural imaging is carried out according to a series of reflections, which determines the possibility of solving the problem within the framework of a single-layer or multilayer model.

The standard single-layer model is described by the linear dependence of the depth on the vertical time. It is shown that this equation corresponds to a multilayer model under the condition of a linear dependence of the layer thickness. It is noted that the values of the linear regression model parameters are consistent with the parameters of the thick-layer model, determined from seismic logging data, only in the ultimate versions of the velocity model.

Restrictions on the structure of the model are eliminated when using the effective velocity of the reflected waves. However, this parameter is rarely used in solving practical problems, since stacking velocity estimates are unstable to errors of the model parameters and methods of the heterogeneity compensation of the upper part of the section.

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The multilayer model assumes the sequential determination of the thickness or velocity of certain intervals of the section. The selection of explanatory variables for this model is limited only by the vertical time of reflections. The model is ideal for layers with a constant velocity, but the forecast result is unpredictable in the case of lateral velocity heterogeneities of the layers.

In each particular case, the velocity model selection is a non-trivial problem, also including an analysis of the wave field kinematic transformation in the seismic data processing.

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Author **Anatoliy P. Sysoev,** *Doctor of Engineering Sciences*, *Professor*, *sysoev@spmi.ru* (*Saint-Petersburg Mining University*, *Saint-Petersburg*, *Russia*).
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