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POWER LAW DISTRIBUTION OF VOLCANIC ERUPTIONS IN SIZE, A MECHANISM OF MAGMA ACCUMULATION IN THE EARTH'S LITHOSPHERE AND HAZARDS PREDICTION

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Abstract: The cumulative graphs of the probability distribution for duration of the activations (eruptions) at Klyuchevskoy and Karymsky volcanoes obey a power law. The graphs are approximated by two straight-line segments. At small and medium durations (from 1 day to 6 and 2 months) the tangent of slope angle of the repeatability graphs $\gamma = 0.53–0.55$ ($\gamma < 1$). 3) At more long activation duration, γ sharply increases by 1.6 -3 times, which probably indicates the presence of some ultimate eruption size for a given volcano or a gradual approach to such a size. An avalanche-like mechanism of magma accumulation when large floating magma-filed cracks absorb smaller overlying cracks in a permeable zone of the lithosphere is proposed. This may drastically change the law of their distribution in size from the initial exponential or normal to the power law one. Interestingly, the distribution of Volcanic Explosivity Index (VEI) for the Kamchatka volcanoes, on the one hand, and the seismic moment (M_0) of strong earthquakes in Kamchatka, on the other hand, obey an exponential law with similar indexes of $-\gamma = -0.7$ and -0.6, respectively. The frequency of occurrence of volcanic eruptions in Kamchatka in the range VEI = 2 – 5 is about 10% of the global one, which is quite a lot, since the length of the volcanic arc of Kamchatka is only about 2% of the sum of the lengths of all the volcanic arcs on Earth. The distribution of the ejected tephra (VT) for the eruptions of the volcanoes of the world and Kamchatka obeys the power law with close indexes of $-\gamma = -(0.7–0.75)$. In consumption of steady state volcanism the average intervals of occurrence of eruptions in Kamchatka is estimated as follows: every 15 years (*VEI* = 4), every 90 years (VEI = 5), every 350 years (VEI = 6, extrapolation) and every 1,400 years (*VEI* = 7, extrapolation).

Keywords: Magma accumulation, floating magma-filled crack, probability distributions of eruptions in size, long-term forecasting.

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СТЕПЕННОЕ РАСПРЕДЕЛЕНИЕ РАЗМЕРОВ ВУЛКАНИЧЕСКИХ ИЗВЕРЖЕНИЙ: МЕХАНИЗМ АККУМУЛЯЦИИ МАГМЫ В ЛИТОСФЕРЕ ЗЕМЛИ И ПРОГНОЗ ВУЛКАНИЧЕСКОЙ ОПАСНОСТИ

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Резюме: Кумулятивные графики распределения вероятностей для длительности активизаций (извержений) на Ключевском и Карымском вулканах на 15-летнем интервале подчиняется степенному закону. Графики аппроксимируются двумя отрезками прямых линий. Для малой и средней длительностей (от 1 дня до 6 и 2 месяцев) тангенс угла наклона графиков повторяемости $\gamma = 0.53 – 0.55$ ($\gamma<1$). Для более длительных активаций резко возрастает в 1,6–3 раза что, вероятно, указывает на наличие некоторого предельного размера извержения для данного вулкана или постепенного приближения к такому размеру. Предложена концептуальная модель лавинообразного укрупнения заполненных расплавом трещин, когда более быстро всплывающие в проницаемой зоне литосферы трещины большего размера поглощают вышележащие более мелкие трещины. Это приводит не только к аккумуляции магмы, но может радикально изменить закон распределения трещин по размерам с начального экспоненциального или нормального до степенного закона. Интересно, что распределение вулканического эксплозивного индекса (*VEI*) для извержений камчатских вулканов, с одной стороны, и сейсмический момент (M_0) сильных землетрясений на Камчатке, с другой стороны, подчиняются степенному закону с близкими показателями степени $-\gamma = -0.7$ и -0.6, соответственно. Частота вулканических извержений на Камчатке в диапазоне *VEI* = 2–5 составляет около 10% от общемировой, что довольно много, так как длина вулканической дуги Камчатки составляет лишь около 2% от суммы длин всех вулканических дуг на Земле. Распределение выброшенной тефры (VT) для извержений вулканов мира и Камчатки подчиняется степеннјму закону с близкими показателями степени, равными – 0.7…=0.75. В предположении «устойчивого вулканизма» средние интервалы повторяемости извержений на Камчатке оцениваются следующим образом: каждые 15 лет (*VEI* = 4), каждые 90 лет (*VEI* = 5), каждые 350 лет (*VEI* = 6, экстраполяция) и каждый 1400 лет (*VEI* = 7, экстраполяция).

Ключевые слова: аккумуляция магмы, заполненные магмой всплывающие трещины, распределение вероятностей размеров извержений, долгосрочный прогноз.

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Introduction

The articles published in issues 1 and 2, 2015 and 2, 2016 of "Istoriya i pedagogika estestvoznaniya" summed up the work on the prediction of volcanic eruptions in Kamchatka for 57 years (from 1955 to 2012) [1, 2, 3]. Over the years, interesting forecasting techniques were developed and a number of spectacular forecasts were issued (Figure 1). At the same time, there were many "goal passes" and "false alarms". One of the reasons for the relatively low efficiency of forecasting is the lack of understanding of the nature of magmatic and volcanic processes. When studying eruptions and their precursors, a number of questions arise. Why volcanic eruptions do not occur continuously, and are isolated in time events? Why is significant randomness typical for eruption size and time of its onset? Why does the probability distribution in size of the volcanic eruptions obeys a power law? These are widespread distributions with so-called "heavy tails", characteristic of complex systems. They have a paradoxical nature, since their first moment (mathematical expectation) diverges, and the remaining moments are not determined [6]. This involves a system with infinite energy, which is physically impossible.

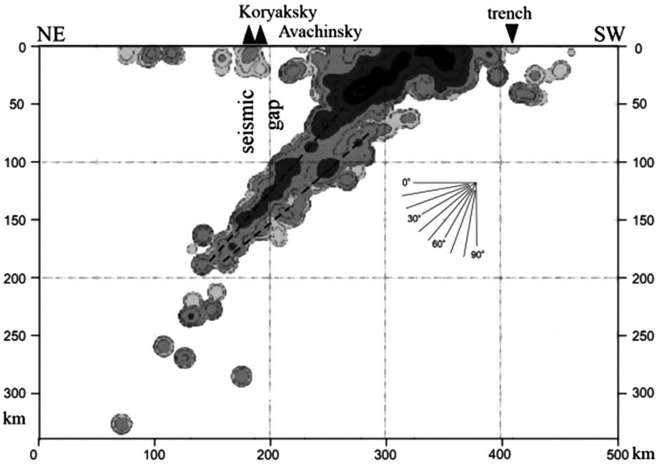
In this paper we will try to explain the possible origin of the power law distribution in size of volcanic eruptions. In our opinion, this type of distribution may be generated by a specific mechanism [7]. Such kind of distributions may contain important information about the processes of accumulation of magma. The study by the distribution method is important, since earthquakes can trace the movement of magma only in the highest echelons of the earth's crust. For the subduction zone of Kamchatka, there is a seismic gap between its seismic focal zone and the area of crust seismicity at depths from 100–150 km to 20–25 km (Figure 2). It is assumed that the region of magma generation is located in the seismic focal zone under the volcanic belt. A similar situation occurs in other subduction zones [10]. Knowing these distributions is also very helpful for long-term forecasting purposes.

In a subduction zone at great depths, at its origin, magma is generated in the intergranular spaces of rocks in the form of minute batches of melt ([11], pp. 41–42). The volumes in the astenosphere, containing a less dense melt, in the field of gravity, under the influence of Archimedes forces begin to rise extremely slowly in the asthenosphere, forming as a result of Rayleigh instability isolated «jets» (plumes) [12]. They form diapirs which are the roots of individual volcanoes or volcanic centers. With a decrease in lithostatic pressure due to a decrease in depth, the proportion of the melt increases. As a result of compaction processes, segregation melt into channels, interconnected networks of melt inclusions are formed. Anisotropic permeability structures can develop in deforming partially molten rocks ([13], p. 60). At the same time, the effective viscosity of the enclosing medium increases in five orders of magnitude, and the mechanism of magma propagation changes radically. In the elastic-brittle lithosphere, magma portions form floating cracks filled with it, which move to the surface of the earth. There multiple coalesces of floating cracks in larger ones take place. As a result, large dikes can be formed, feeding magmatic chambers or fissure eruptions [14].

The study of the distribution of eruptions by their Volcanic Explosivity Index (VEI) for the whole world was carried out by Simkin, Siebert [15], Mason et al [16], Sparks et al [17] and in Kamchatka by Tokarev [18], Gusev et al. [19] and Gusev [20].

Fig. 1. *Huge ash cloud of the direct blast during Bezymianny volcano eruption on March 30, 1956. Its height was up to 35–37 km [VEI =5]. The direct blast occurred against the background of a decrease in the number and energy of volcanic earthquakes. Nevertheless, G.S. Gorshkov informed the local authorities that a strong explosion is possible on the volcano [4]. VEI according to [5]. Photo by I.V. Erov (cortesy of V.A. Shamshin)*

Fig. 2. *Isolines of the distribution of energy density of weak (8 <KS <10) earthquakes of seismic focal zone and in the crust of Kamchatka along the transverse sector passing through the volcanoes Avachinsky and Koryaksky (according to Seliverstov ([8], p. 107)]. KS – energy class of earthquakes according to [9].*



In this work we will do the following:

1) Study the cumulative distributions of the activations (eruptions) in size for the two most active volcanoes of Kamchatka, Klyuchevskoy and Karymsky, over the past 15 years.

2) On this basis, propose a conceptual model of magma accumulation in the Earth's lithosphere in subduction zone which could explain power law distribution in size of volcanic eruptions.

3) Compare parameters of distributions volcanic eruptions in size and the of seismic moment of strong earthquakes M0 in Kamchatka.

4) Investigate the distributions for the volume of tephra (VT) ejected by all volcanoes of Kamchatka and the world over the past two centuries and to estimate what proportion is accounted for by the volcanoes of Kamchatka. According to this data, estimate the expected intervals between eruptions

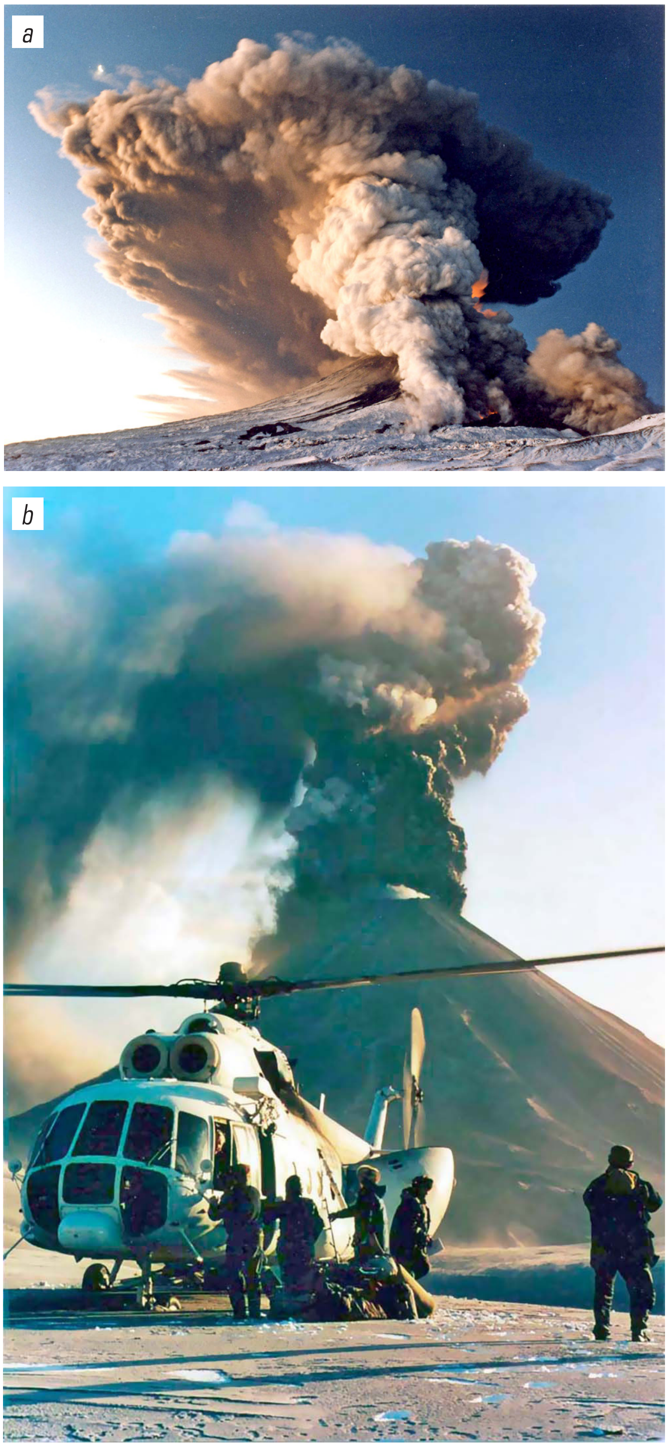
and the volumes of tephra which will be ejected by Kamchatka volcanoes for the future first hundreds of years (under the assumption of «steady state volcanism» [21]).

Some characteristic features of the power law distributions

For random variable $\{X\}$, which obeys the power law of distribution, the probability $P(X > x)$ that random variable greater than x, is described by the complementary distribution function [22]

$$P(x) = c\,x^{-\gamma}, \quad 0 \le x < \infty\,, \tag{1}$$

Fig. 3. *The paroxysmal phase of the eruption of Klyuchevskoy volcano 1.10.1994 (a) and Karymsky volcano 3.1.1996 (b). Photo by N.P. Smelov*



where c and γ – experimental positive constants. The complementary distribution function is also known as complementary cumulative distribution function.

Probability density $p(x)$ also follows a power law. As $P(x)$ denotes the probability $P(X > x) = 1 - P(X \le x)$, and taking into account that $p(x)$ is defined as the derivate of $P(X \le x)$, it follows that:

$$p(x) = \frac{d}{dx}(1 - P(X > x)) = c\gamma x^{-(\gamma+1)}. \tag{2}$$

Therefore $p(x)$ follows a power law with index greater than one (note that $m > 0$ as, for definition of $P(x)$, it needs to be obviously a decreasing function of x). The histogram of a power law yields a straight line on a log-log diagram. The probability density $p(x)$ decreases very slowly with x compared to normal or exponential laws of distribution. Therefore, power law ones are called distributions with «heavy tails» [6]. For these distributions, the 1-st moment (mathematical expectation) m diverges

$$m = M\{X\} = c\gamma\int_0^\infty x x^{-(\gamma+1)}dx = c\gamma\frac{x^{1-\gamma}}{1-\gamma}\Big|_0^\infty = +\infty. \text{ for } \gamma < 1 \tag{3}$$

This leads to the paradox that the sample average value of the random variable increases indefinitely with time

$$\underset{t\to+\infty}{\overline{X}} = \sum_{1}^{n} x_i \to +\infty. \tag{4}$$

For an active volcano, this means that the average mass or volume of the products ejected during its eruptions increases indefinitely. This is also the case for a volcanic zone consisting of many volcanoes. The 2nd moment (variance) of the power distribution $D\{X\} = M\{(X - m)2\}$ for $\gamma < 1$ is not defined, since m – is not defined. All moments of a higher order for power distribution with the index of $\gamma < 1$ are not defined either. This implies the existence of an infinite energy of process, which is physically impossible. This paradox is explained by the fact that random variables with such distributions are not infinite; for them there should be some limit values. It is of interest to consider the power distributions in scale of volcanic eruptions and possible mechanisms generating such distributions.

Fig. 4. *Klyuchevskoy volcano. Distribution of volcanic tremor parameters in 1994–1995: 1 – the daily average values of the ratio of amplitudes to the periods ((A/T)av., dotted curve); 2 – daily duration of tremor (DTtr, solid curve). The threshold for the parameter DTtr was taken 2 hours per day*

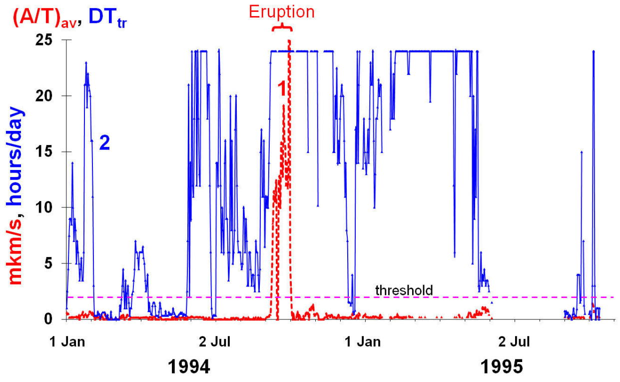
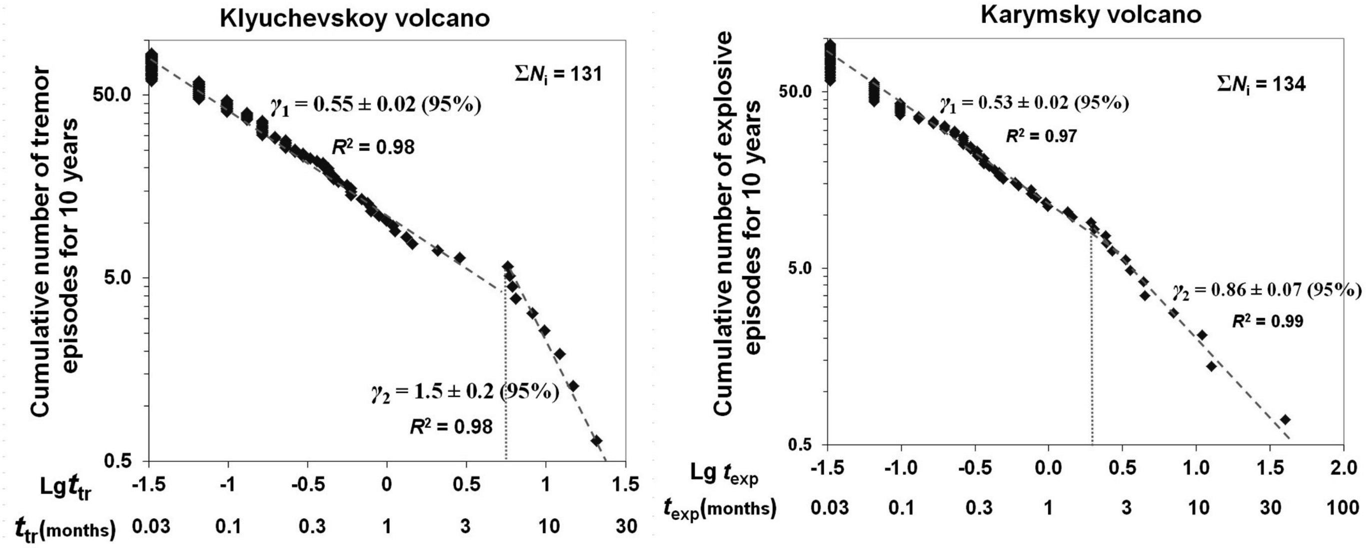


Fig. 5. *Cumulative log-log graphs of the durations of tremor (explosive) episodes t_{tr} and texp for Klyuchevskoy and Karymsky volcanoes normalized to a 10-year interval. Data is approximated straight dashed lines. γ_1 and γ_2 are tangents of tilt angles of repeatability graphs. Vertical dotted lines mark abrupt changes in the tilt angles of the approximating lines.*



The distributions of activations in size for Klyuchevskoy and Karymsky volcanoes

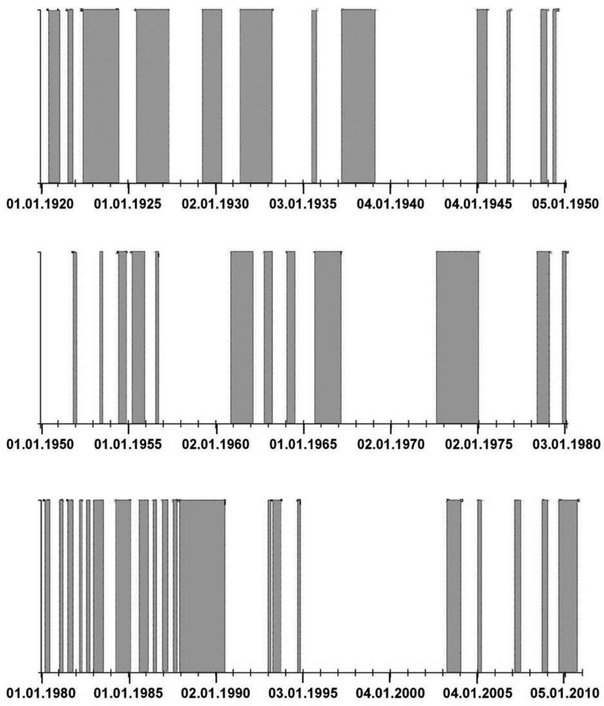
Volcanoes Klyuchevskoy and Karymsky are one of the most active volcanoes in Kamchatka, characterized by activations (eruptions) with durations from several days to tens of months (Figure 3). Giant Klyuchevskoy volcano is the highest volcano in Europe and Asia. It erupts relatively low viscosity basaltic and andesite-basaltic magmas in the form of extended lava flows and Strombolian and Strombolian-Vulcan pyroclastic explosions. On the contrary, Karymsky is characterized by eruptions of more viscous andesites, short, thick lava flows, as well as explosions of the Vulcanian type with the formation of a large amount of ash.

We analyzed two daily parameters: a) the duration of volcanic tremor (DT_{tr}) for Klyuchevskoy and b) the number of explosive volcanic earthquakes (DN_{exp}) for Karymsky. In Figure 4, the example of volcanic tremor parameters on Klyuchevskoy volcano is shown in 1994–1995. It was believed that activation would take place if the seismological parameters on a volcano exceed certain threshold values: (DT_{tr} > 2 hours/day) for Klyuchevskoy and (DN_{exp} >10) for Karymsky. The analysis was about 130 activations. The most extended activations contain eruptions. As a measure of the activation size is taken its duration in months: ttr for Klyuchevskoy and texp for Karymsky.

It is believed that the duration of activations for those two volcanoes is random variables $\{T_{tr}\}$ and $\{T_{exp}\}$. The complementary cumulative distribution functions of their activation durations ΣN_i ($T_{tr} \geq t_{tr}$) and ΣN_i ($T_{exp} \geq t_{exp}$) for Klyuchevskoy and Karymsky in 15-years intervals on a log-log diagrams are depicted in (Figure 5).

The following features of cumulative graphs in Figure 5 can be distinguished: 1) They are approximated by two straight-line segments. This indicates that distributions of the activations duration obey a power law. 2) At small and medium durations (from 1 day to 6 or 2 months) the tangent of slope angle of the repeatability graphs γ = 0.53–0.55, that significantly less than one. 3) At more long activations durations, γ sharply increases by 1.6–3 times, which probably indicates the presence of some ultimate eruption size for a given volcano or a gradual approach to such a size. 4) For Klyuchevskoy volcano, the ultimate duration of activations (eruptions) is estimated at 5.5–6 months, after which there is a sharp decline of the graph with γ = 1.5, that significantly more than one. This is preceded by an explicit rise in the cumulative graph in the range of 2 to 6.5 months (Figure 5, left). 5) For Karymsky volcano, starting from activation durations from 2 months, γ is increased 1.6 times, reaching 0.86, but still remains less than one. That is, it can be assume that the ultimate duration of activations (eruptions) for this volcano is more than 30 months (Figure 5, right).

Fig. 6. *The Klyuchevskoy volcano eruptions from January 1920 to September 2010 (44 eruptions). The width of the bars represent the duration of eruptions*



The viscosity of Karymsky andesitic magmas is significantly higher than the viscosity of Klyuchevskoy basalt and andesitic-basalt magmas. Consequently, the magma ascent rate at Karymsky volcano is much lower than at Klyuchevskoy volcano. Therefore, at Karymsky we should expect longer eruptions than at Klyuchevskoy.

Since the value of the parameter R^2 = 0.97-0.99 is close to 1, it indicates that for both distributions the power law is the best approximation. This kind of distribution is a paradoxical one; therefore it is important try to explain the possible physical mechanism that generates it.

An avalanche-like mechanism of magma accumulation in the lithosphere of a subduction zone

Eruptions of a certain volcano form a series of events of different sizes, separated by pauses (Figure 6). This indicates that magma rises through the lithosphere and the crust in the form of portions of different volumes, isolated from each other in space. Reaching the surface of the earth, such a portion either directly erupts onto the surface of the earth or, if a volcano has a magmatic reservoir, feeds it, which initiates volcanic eruption.

This is in good agreement with the Takada's model [14], which experimentally investigated the mechanism for coalescence of magma-filled cracks floating in a permeable zone of the Lithosphere under the influence of Archimedes forces. The density of melt (ρ_m) is less than the density of the host elastic-brittle medium (ρ_{rock}). It is supposed that magma-filed cracks are generated in the partially melted mantle. A quote: «One simple model for crack coalescence is adopted here as an example. Cracks of nearly the same size coalesce n times one after another (Two cracks of a volume M coalesce with each other so that the volume of a new crack becomes M x 2 (first coalescence)). Two cracks of a volume M x 2 coalesce so that the volume of a new crack becomes M x 2^2 (second coalescence). The volume of a magma-filled crack after the n-th coalescence amounts to M x 2^n» ([14], Figure 10, pp. 252–253].

Such a stepwise method of coalescence applied to floating cracks of different sizes does not change the law of their distribution. If the initial distribution of the size of floating cracks was a power law one, then after the coalescence, the distribution will remain the same. According to the Takada's model, if the initial distribution of floating cracks is exponential or normal, then the same distribution will be for cracks after n-th coalescence. We will try to propose a modified type of coalescence. The Takada's model assumes the existence of permeable zones in the Lithosphere, where rock strength in tension is negligible.

Consider the coalescence of magma-filed cracks of different sizes floating under the action of Archimedes forces in a certain permeable zone of the lithosphere.

A quote from ([14], pp. 251–252): «The maximum excess pressure caused by buoyancy in a vertical crack, P_{max}, is proportional to the crack height, h

$$P_{max} = \Delta\rho g h, \tag{5}$$

where $\Delta\rho = \rho_{rock} - \rho_m$, g – acceleration of gravity.

The ascent velocity of an isolated crack, V, in the case of laminar flow

$$V = \Delta\rho g w 2/3\eta, \tag{6}$$

where w and η are the crack width and the viscosity of injected liquid, respectively. The crack width is given by the function $w = H(h)$. Substituting into Eq. (6), the ascent velocity is

$$V \propto H(h^2). \tag{7}$$

For example, the ascent velocity of an isolated crack in gelatin

$$H(h) = h^2, \quad V \propto h^4. \tag{8}$$

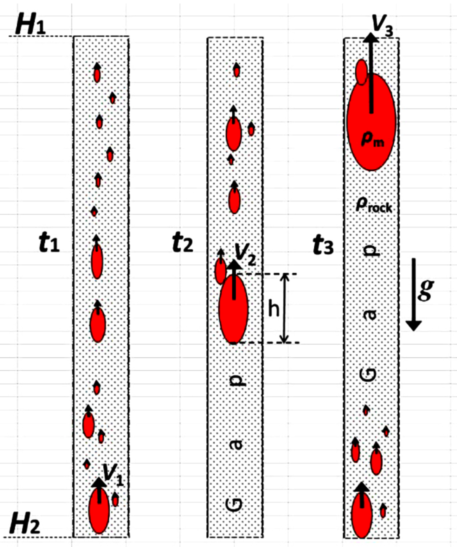
It is assumed that the ascent velocity becomes larger with increases in the height of an isolated crack. Thus, when the height of the upper crack is smaller than that of the lower main crack, the main crack can catch up with the upper crack» (end of quotation).

If the ratio of the width, height and thickness of floating cracks is kept constant, then from expression (8) it follows that its velocity V is proportional to the volume M of the melt in the crack to the 4/3 degree

$$V \propto M^{4/3}. \tag{9}$$

As larger cracks rise faster, they have a greater ability to absorb smaller overlapping cracks. As a result, larger cracks are accelerated significantly and their ability to absorb overlying cracks increases very much (Figure 7). That is why, unlike A. Takada, we believe that the floating cracks not only enlarge, but there is an avalanche-like mechanism of their coalescence. This may drastically change the law of their distribution in size from the initial exponential or normal to the power law one. A cardinal change in the distribution law of floating cracks can occur both due to the enlargement of large cracks, and by reducing the proportion of smaller cracks that have been absorbed by larger cracks. In our opinion, a similar mechanism of the avalanche-like enlargement of cracks can occur in the permeable zones of the lithosphere of any configuration.

Fig. 7. *Simplified schematic image of the absorption process by large buoyancy-driven cracks filled with melt, smaller cracks in the vertical pipe-like region of the lithosphere (magmatic channel). In this area, the strength of the rocks in tension is negligible. It is pictured in the plane of the cracks. H – depth, t – conditional time, ρ_m and ρ_{rock} – density of melt and rocks, respectively, g – acceleration of gravity. $H_1 < H_2$, $\rho_m < \rho_{rock}$. V_1, V_2, V_3 – velocity of a selected floating crack at t_1, t_2, t_3, respectively ($t_1 < t_2 < t_3$)*

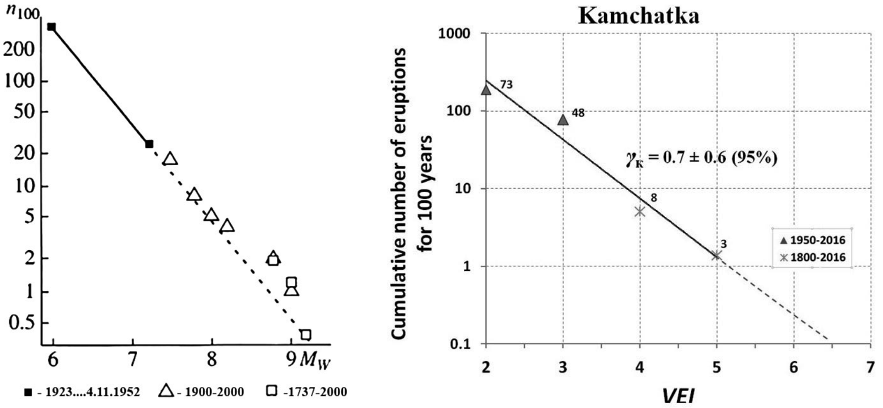


According to (Eq. 6) for $\Delta\rho = 10^2$ kg·m^{-3}, $g = 9.8$ m·s^{-2}, $w = 1$ m, $\eta = 10^2$ Pa·s, V is estimated equal about 3 m·s^{-1}.

Note that in the model in Figure 6, cracks do not coalesce with overlying cracks as a result of their attraction, but as a result of a collision of a larger crack that rises at a greater speed with a smaller one. Therefore, such a coalescence model also works in the case of the presence of significant differential stresses in the lithosphere, which prevent the attraction of neighboring cracks to each other [14]. The results of geomechanical modeling of the subduction zone show that under the associated volcanic belt should exist the conditions of extension of the Lithosphere [23].

Pop-up melt cracks move aseismically through such permeable zones. And only in the upper crust, due to a decrease in lithostatic pressure, the shear strength of rocks is significantly reduced, the rise of melts can be traced by volcanic earthquakes (Figure 2).

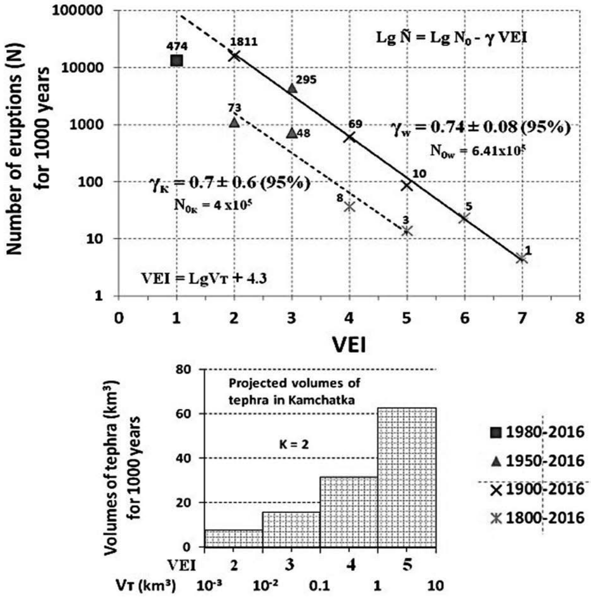
Fig. 8. *Left: summary of the cumulative repeatability graphs of earthquakes of Kamchatka in the scale of moment magnitudes. Black squares and the line connecting them are the estimate according to 1923–4.11.1952. The dashed line is the extrapolation of this data to large magnitudes. According to ([25], Figure 2, p.41). Right: cumulative repeatability graph of Volcanic Explosivity Index (VEI) of Kamchatka eruptions for 1800–2016. Numbers at the points indicate the number of eruptions in the sample for each VEI. All data are recalculated to a single 100 year interval*



This conceptual model allows us to explain the existence of the pause that occurs after the termination of an eruption (Figure 6). This is due to the emptying of the magmatic channel from magma over its considerable length as a result of the absorption of a whole echelon of smaller and higher floating cracks by of some larger floating magma-filled crack (the gap in Figure 7). Since the initial floating cracks are formed randomly in time and have random sizes, the result of their avalanche enlargement will also be random. It also follows from this model that the magnitude and time of the beginning of the future eruption have a random character, which makes it difficult to predict.

More detailed quantitative analysis of above mentioned process of magma accumulation is needed in the future.

Fig. 9. *Top graphs: The distributions of VEI and the corresponding volumes of tephra (VT, km^3) ejected during volcanic eruptions of the world (solid line) and Kamchatka (dashed line) in 1800–2016. The numbers above the dots indicate the number of eruptions in the sample for each VEI. The lower histogram: the most probable volumes of tephra, which will be ejected by all volcanoes of Kamchatka in the next 1000 years*



Comparison of probability distributions of the Volcanic Explosive Index for eruptions and the seismic moment of strong earthquakes in Kamchatka

As the second applicant for explaining the power law distribution of tephra volume, ejected by Kamchatka volcanoes, we consider the model of magma generation as a result of the movement of the oceanic lithosphere under the Eurasian plate which accompanied by an interplate earthquake in the subduction zone.

For volcanic eruptions in Kamchatka and the world we studied the cumulative graphs of Volcanic Explosivity Index (*VEI*) distributions from 1800 to 2016, which were taken from the database of the Smithsonian Institution [24]. Average value of ejected tephra ($\bar{V}_t$) may be calculated from *VEI* according to the formula

$$\bar{V}_t\ (\text{km}^3) \approx 10^{\,(VEI - 4.3)} \quad (\text{for } VEI \geq 2), \qquad (10)$$

which was deduced by us from the data of Table 6 of that article. It follows that if *VEI* is distributed exponentially, then V_T will be distributed according to a power law.

For strong earthquakes in the Kamchatka subduction zone we studied the cumulative graphs of their seismic moments (M_0) which considered as a logarithmic measure of the dropped seismic energy. The seismic moment was recalculated from the moment magnitude (M_w). Data about M_w was taken from the work of Gusev, Shumilina [25]. Comparison of cumulative graphs of the distributions of VEI and M_w for volcanic eruptions and strong earthquakes of Kamchatka is shown in Figure 8.

Graph of the moment magnitude (M_w) for Kamchatka earthquakes for 1923–4.11.1952 and M_w = 6–7 has a tangent

Table

The most probable intervals between eruptions and the total volumes of ejected tephra for future in Kamchatka

VEI	The average interval between eruptions	Volume of igneous tephra for 1000 years
4	15 years	30 km^3
5	90 years	60 km^3
6	350 years (extrapolation)	120 km^3 (extrapolation)
7	1400 years (extrapolation)	240 km^3 (extrapolation)

of the slope angle $\gamma_1 \approx 0.9$ (Figure 8, left). Taking into account the recalculation (M_w) in the decimal logarithm of the seismic moment (M_0) according to the formula from [26]

$$\lg M_0 \approx (3/2)\, M_w + 9.1, \qquad (11)$$

it turns out that M0 is distributed exponentially with index $(-\gamma) = (2/3)\gamma_1 \approx -0.6$. This is close to the values of the parameter γ for distribution of VEI for volcanic eruptions in Kamchatka, equal to 0.7 (Figure 8, right).

Gusev and Shumilina note [25] that before reaching the limiting magnitude of $M_w = 9$, there is a characteristic increase in the frequency of earthquakes (Figure 8, left). A similar increase takes place for the cumulative distribution function of the Klyuchevskoy volcano activations duration of before reaching its critical value of about 6 months (Figure 5, left).

It is believed that strong earthquakes and volcanism are generated by a single process – the subduction of the Pacific lithosphere under the continental Kamchatka. Does this mean that the volume of magma generated during subduction depends on the seismic movement of a strong inter-plate earthquake or is it a coincidence?

Analysis of the distribution of ejected tephra for volcanoes of the world and Kamchatka and estimation of repeatability of Kamchatka eruptions

The distributions of *VEI* and the corresponding volume of igneous tephra (V_T) for the eruptions of the world and Kamchatka from the beginning of the 19th century to 2016 are given in Figure 8, top graphs. Unlike the graph in Figure 7, right, they are differential graphs and approximated by a linear dependence of the form

$$\mathrm{Lg}\,\tilde{N} = \mathrm{Lg}N_0 - \gamma VEI = \mathrm{Lg}N_0 - \gamma(\mathrm{Lg}\,\bar{V}_t + 4.3) \qquad (12)$$

Hence the dependence of the number of eruptions $\tilde{N}$ can be written in the form of the following power function of $\bar{V}_t$ with the index of power ($-\gamma$)

$$\tilde{N} = N_0 \cdot 10^{-4.3\gamma}(\bar{V}_t)^{-\gamma}, \qquad (13)$$

where $\tilde{N}$ is the predicted number of eruptions normalized for 1000 years, γ is the tangent of the slope angle of the repeatability curve ($0 < \gamma < 1$), N_0 is the initial value depending on the activity level of the volcanic zone, Lg is the decima l logarithm. The comparatively large values of the parameter $R^2 = 0.99$ and 0.92, which are close to one, characterizing the deviation of the data from the power law, allow us to state that both distributions are well described by a power law. Obviously, the distribution of eruptions in size for a certain volcanic zone is the result of a superposition of such distributions for volcanoes composing this zone.

Forecast estimates for Kamchatka in consumption of steady state volcanism [21] are given in the Table

Conclusion

1. The cumulative graphs of duration distribution of the activations (eruptions) at Klyuchevskoy and Karymsky volcanoes obey a power law. The graphs are approximated by two straight-line segments. At small and medium durations (from 1 day to 2–6 months) the tangent of slope angle of the repeatability graphs $\gamma = 0.53$–0.55, that significantly less than one. 3) At more long activations durations, γ sharply increases by 1.6–3 times, which probably indicates the presence of some ultimate eruption size for a given volcano or a gradual approach to such a size.

2. We believe that the power-law distribution in size of activations (eruptions) may generated by an avalanche-like mechanism of magma accumulation when large floating magma-filled cracks absorb smaller overlying cracks in a permeable zone of the Lithosphere. This may drastically change the law of their distribution in size from the initial exponential or normal to the power law one. In future papers, this model should be quantified. Since the initial floating cracks are formed randomly in time and have random sizes, the result of their avalanche enlargement will also be random. It also follows from this model that the magnitude and time of the beginning of the future eruption have a random character, which makes it difficult to predict them.

3. Interestingly, the distribution of Volcanic Explosivity Index (*VEI*) for the Kamchatka volcanoes, on the one hand, and the seismic moment of strong earthquakes in Kamchatka, on the other hand, obey an exponential law with similar indexes equal –0.7 and –0.6, respectively. Does this mean that the volume of magma generated during subduction depends on the seismic movement of a strong inter-plate earthquake or is it a coincidence?

4. The frequency of occurrence of volcanic eruptions in Kamchatka in the range *VEI* = 2–5 is about 10% of the global one, which is quite a lot, since the length of the volcanic arc of Kamchatka is only about 2% of the sum of the lengths of all the volcanic arcs on Earth. This is due to the extremely high activity of the volcanoes of the Northern group of Kamchatka: Klyuchevskoy, Shiveluch, and the Tolbachinsky regional zone of slag cones. Presumably, this ratio will be valid for *VEI* = 6–7.

5. The distribution of the ejected tephra (V_T) for the eruptions of the volcanoes of the world and Kamchatka obeys the power law with close indexes $-\gamma = -(0.7$–$0.75)$. The data for Kamchatka have a much wider dispersion than the global data.

6. In consumption of steady state volcanism the average intervals of occurrence of eruptions in Kamchatka has estimated as follows: every 15 years (*VEI* = 4), every 90 years (*VEI* = 5), every 350 years (*VEI* = 6, extrapolation) and every 1,400 years (*VEI* = 7, extrapolation).

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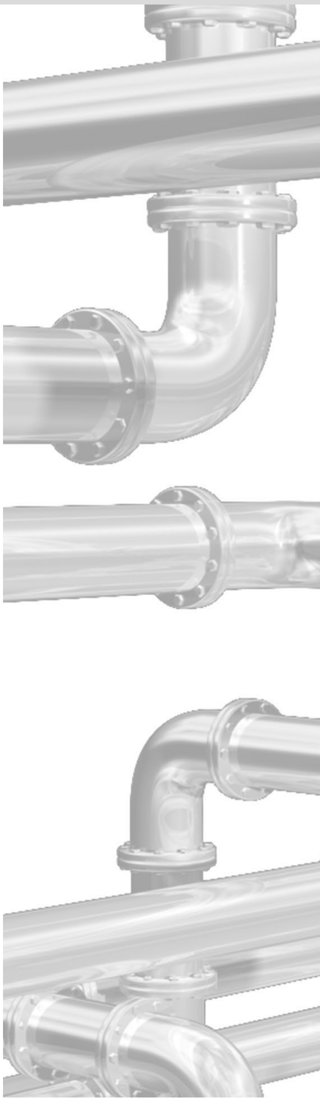

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