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Estimation Method for Vector Field Divergence of Earth Crust Deformations in the Process of Mineral Deposits Development

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An essential requirement for effective and safe deposit development is good geomechanical software. Nowadays software packages based on finite element method are used extensively to estimate stress-strain state of the rock mass. Their quality use can only be assured if boundary conditions and integral mechanical properties of the rock mass are known. In mining engineering this objective has always been achieved by means of experimental observations. The main source of information on initial and man-induced stress-strain state of the rock mass is natural measurement of displacement characteristics. Measurement of geodetic data (coordinates, heights, directions) in the period between alteration cycles allows to plot a field of displacement vectors for the points in question. Taken together, displacement vectors provide information on the objective stress-strain state of the Earth crust. Basing on it, strain tensors, displacement components, directions and rates of compression and tension can be calculated in the examined area. However, differential characteristics of any physical vector field – namely, curl and divergence – need to be taken into account. Divergence is a single value (scalar) associated with a single point. Vector field as a whole can be described with divergence scalar field. Divergence indicates the sign (positive or negative) of volume changes in the infinitesimal region of space and characterizes vector flux in the nearest proximity and in all directions from a given point. In the paper authors propose a method to estimate divergence using discrete geodetic observations of displacement occurring on the surface of examined territory. It requires construction of formulas that model vector field for any point of the area. It is proposed to use power polynomials that describe displacement in three directions (x, y, z). These formulas allow to estimate field vectors in any given point, i.e. to form vector tubes. Then areas of input and output cross-section, as well as divergence values are calculated. This increases the quality of geodetic observation and provides opportunities for more precise modeling of the rock mass disrupted by mining operations, using modern software packages.

Key words: geodetic data; man-made rock mass; stress-strain state; vector field; divergence; vector tubes; polynomial models

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Introduction. Geomechanical provision of mining operations plays an important role in increasing the efficiency and safety of mineral resources extraction and to a great extent defines the selection of optimal parameters of deposit development and the strategy of raw material extraction at the mining plants [2]. Generalization of stress-strain state (SSS) of the rock mass is based on physical and mechanical properties of rock samples and their application in software packages on SSS modeling [8, 15]. At the same time, different conditions of rock formation, including tectonics, block composition, heterogeneity, three-phase interaction between the elements of the environment, require actual observations to adjust theoretical estimations. The main – and often the only – source of information on initial and man-induced stress-strain state of the rock mass is experimental measurement of displacement parameters [4, 5, 9, 17]. In this case rock mass displacement – a phenomenon associated with deposit development – is understood as a whole complex of deformation processes taking place in the rock mass during formation of initial stress-strain state outside the boundaries of mining operations influence, as well as its transformation within these boundaries [5, 9, 10, 12].

Problem statement. Main factors, defining the formation of stress-strain state of the rock mass, are: hierarchical block composition; kinematic activity; secondary structuring; concentra-

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tion of modern geodynamic movements on the boundaries of secondary structure blocks [2, 4, 5, 9, 13]. Under their influence a mosaic stress-strain state is formed, which is relatively homogenous across averaged integral parameters.

In order to identify formation parameters and dependencies of the initial stress-strain sate of the rock the following steps must be taken:

– experimentally assess the rate of modern geodynamic movements and parameters of formed stress-strain state, changing over time;

– study heterogeneity level of the stress-strain state, caused by secondary structuring of the rock mass under the influence of modern geodynamic movements and formation of the secondary stress field in the area of mining operations.

Hence two main types of information need to be instrumentally obtained: parameters of the integral rock mass movement, caused by natural and man-made factors, as well as data on hierarchical block structure of the rock mass and its dynamics over time.

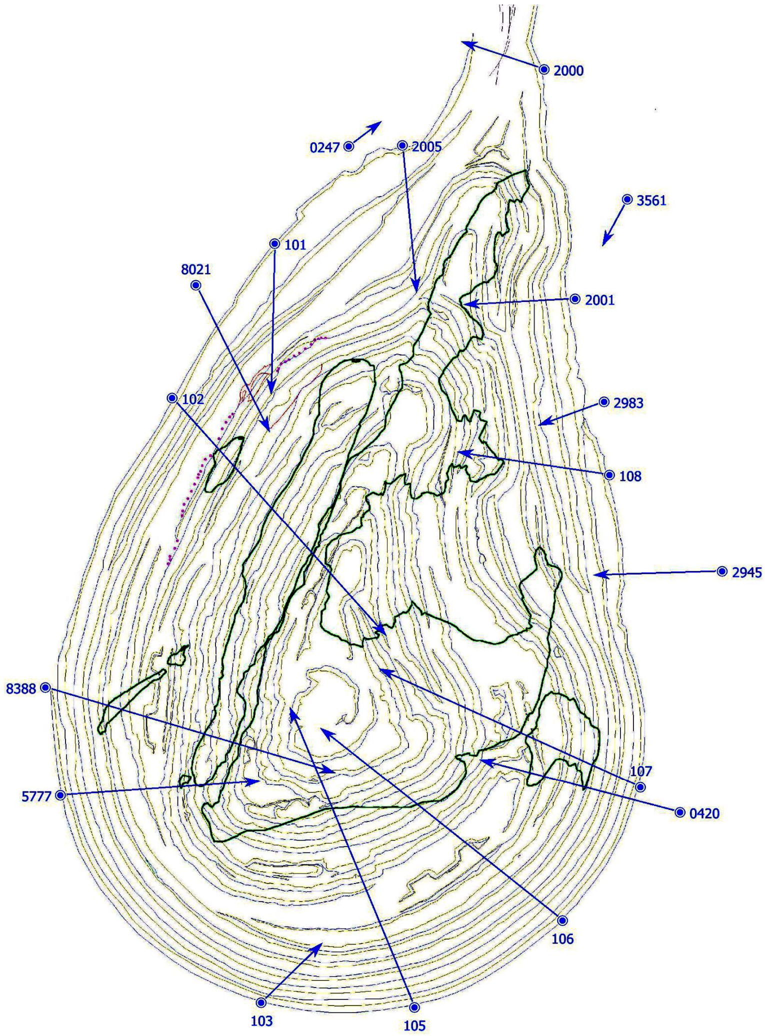

Fig.1. Horizontal vectors of modern geodynamic movements in the proximity of an open-pit mine in 2006-2017

Data on parameters of rock mass integral displacement can only be obtained with direct geodetic methods using Global Navigation Satellite System (GNSS) and conventional (tacheometers and levels) geodesy [14, 16, 17, 19-23]. At the foundation of this method lie multiple monitoring measurements of special benchmark displacement, which include points of the State Geodetic Network, Survey Control Network and survey stations.

Comparison of initial and predetermined spatial coordinates of benchmarks obtained during monitoring permits mathematical modeling of both displacement vectors and principal strain field. Territorial scale of geodetic measurements varies from tens and hundreds of km to several meters [10, 14, 20, 23]. Visualization examples of displacements and deformations are presented in Fig.1 and 2 for adjacent rock mass of Kiembayevsky mining plant.

By means of further pooling of displacement data, the main clusters of deformed structure blocks are identified, and their boundaries are determined. As a result of geomechanical modeling, theoretical and actual models of deformed rock mass disrupted by mining operations are compared; parameters of not only secondary, but also initial strain field are defined.

Methodology. To solve the problems associated with geomechanical forecasts and localization of intensive deformation regions, it is necessary to define not only displacement vectors and deformation tensors, but also a differential characteristic of the vector field – divergence.

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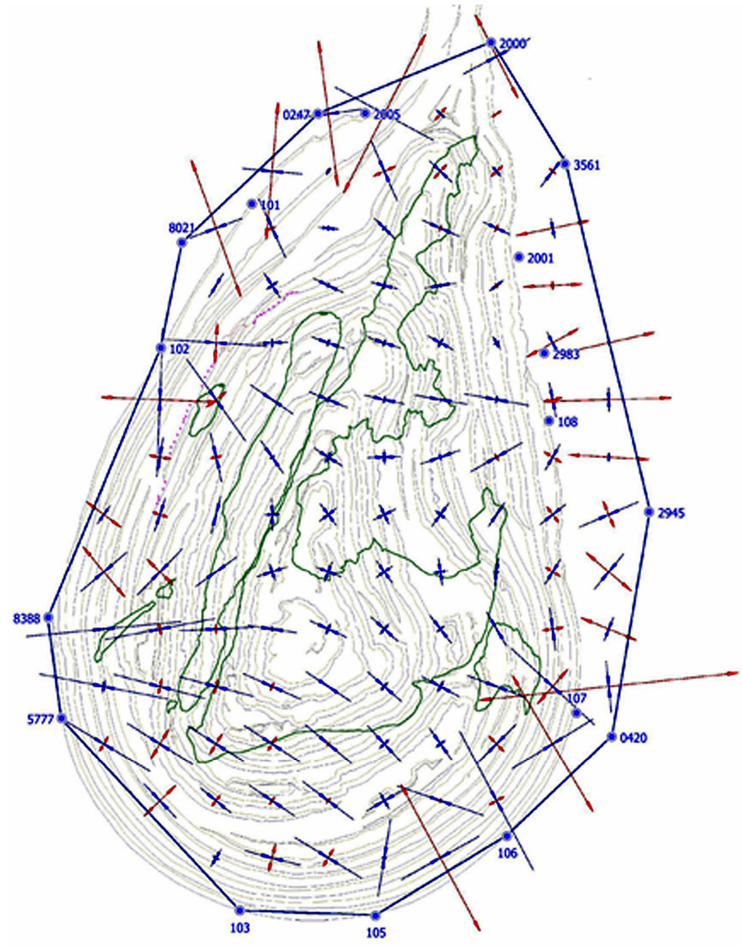

Fig.2. Tensors of horizontal deformations, caused by modern geodynamic movements in the open-pit mine in 2006-2017

Divergence is one of the frequently used characteristics of the vector field, represented by a single value (scalar) associated with a particular point. The vector field as a whole is described with a scalar divergence field, which reflects changes in the vector value in the nearest proximity of the point in question in all directions.

Divergence is a volume derivative of the vector field. In mathematical notation divergence can be defined as follows:

$$\operatorname{div} F = \lim_{V \to \infty} \frac{F_F}{V}, \qquad (1)$$

where F_F – flux of the vector field F through a spherical surface with area S, confining volume V.

In the general case it can be any region with area S and volume V. However, the entire region must be located in an infinitesimally small area in the proximity of a point in question. Thus, divergence (1) must be a local operation. In physical terms, divergence of a vector field characterizes a spatial point from the viewpoint whether it is a source or a sink. This interpretation can be exemplified by a lake with a 2D vector field of horizontal water motion. Positive divergence of the current velocities field will come from the springs rising from the bottom of the lake, the negative – from underwater sinks with water migrating from the reservoir:

div $F < 0$ – the point is a sink;

div $F = 0$ – no sinks or sources, or they compensate one another;

div $F > 0$ – the point is a source.

Calculation of divergence reflects location of peaks and valleys of the gradient pattern (directions of the steepest descent). The peaks have a positive divergence, the valleys – a negative one.

Mathematical field theory [1, 3] can be used not only to describe current flow in liquids and gases, but also to study substance motion in the mantle and the Earth core, to examine deformations in flowing rock formations of sedimentary cover and in rock masses under the influence of regional metamorphism in the depth of the Earth crust. Complex layer deformations in the gneisses developed under significant flow of the rock masses, which can be comprehended basing on mathematical theory of liquid flows. This same theory can be used to examine the role of magmatic melt in the development of tectonic processes [13, 16, 19, 22, 23], volcanicity [6, 7], as well as research on modern displacement of large deformed masses of near-surface structures of the Earth crust.

To implement formula (1) under real conditions of discrete coordinatization of the Earth crust and its 3D displacement it is necessary to introduce the terms of vector field flux and vector tube.

In the vector field one can divide a certain closed or open surface Σ into small elements with areas $d\sigma$. For each element exists a unit normal vector $\vec{n}$ and a vector of this field $\vec{V}$ (average for the element).

After surface integration we obtain total flux of the vector field:

$$Q = \iint_{\Sigma} (\vec{n}\vec{V}) d\sigma. \qquad (2)$$

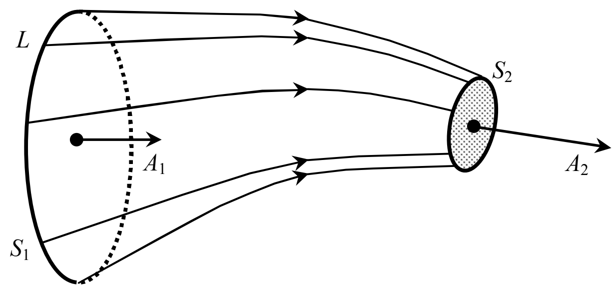

Fig.3. Vector tube, bound by field lines, input cross-section S_1 and output cross-section S_2

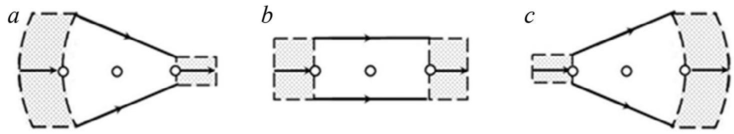

Fig.4. Some possible values of divergence: *a* – div $F < 0$; *b* – div $F = 0$; *c* – div $F > 0$

For vector $\vec{V}$, reflecting substance flow rate (e.g., plastic rocks), the Q value calculated using formula (2) represents the volume of substance, flowing through selected surface Σ per unit time.

An important term in vector field theory is the vector tube. It can be defined when a random closed loop L in each point is bound by a field line (Fig.3).

A difference between output cross-section S_2 and input cross-section S_1 of the vector tube, if it is small-sized and its input and output vectors are equal, allows to quantify divergence value. The sign of divergence value is conceptually shown in Fig.4.

Discussion. Suggested by the authors estimation of divergence for vector field points is associated with the possibility to use discrete geodetic measurements of displacement vectors only on the surface of the are in question. It is proposed to reconstruct the vector field, required for further calculations, using polynomial models. E.g., spatial vector field for each point with known coordinates x, y, z and displacement u_x, u_y, u_z can be defined by the following polynomials:

$$\left.\begin{aligned} u_x &= a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + a_6 z^2 + a_7 xy + a_8 xz + a_9 yz; \\ u_y &= b_0 + b_1 x + b_2 y + b_3 z + b_4 x^2 + b_5 y^2 + b_6 z^2 + b_7 xy + b_8 xz + b_9 yz; \\ u_z &= c_0 + c_1 x + c_2 y + c_3 z + c_4 x^2 + c_5 y^2 + c_6 z^2 + c_7 xy + c_8 xz + c_9 yz. \end{aligned}\right\} \qquad (3)$$

The order of polynomials (3), approximating displacement vector field, can be selected judging from the actual configuration of geodetic surveillance network, e.g. on the industrial polygons in the regions of mining operations, as well as on expected polygons in earthquake-prone and volcanic zones. In most cases, divergence can be estimated using second order polynomials.

Then polynomial coefficients a, b, c are calculated by solving the general system of such polynomials, formulated for every point of surveillance geodetic network. Obtained equations of the vector field allow to estimate displacement vectors in any point, to form small-size vector tubes and use their input and output cross-sections to calculate divergence value.

This is the essence of the proposed method. Let us review it in greater detail, including the options of its algorithmization. Let us assume that after two cycles of geodetic surveillance over certain territory the values and directions of spatial displacement have been identified. These values are results of high-precision leveling, as well as mathematical and statistical processing by global navigation systems. Obtained vector pattern allows to construct a vector tube in the proximity of examined geodetic point A (Fig.5).

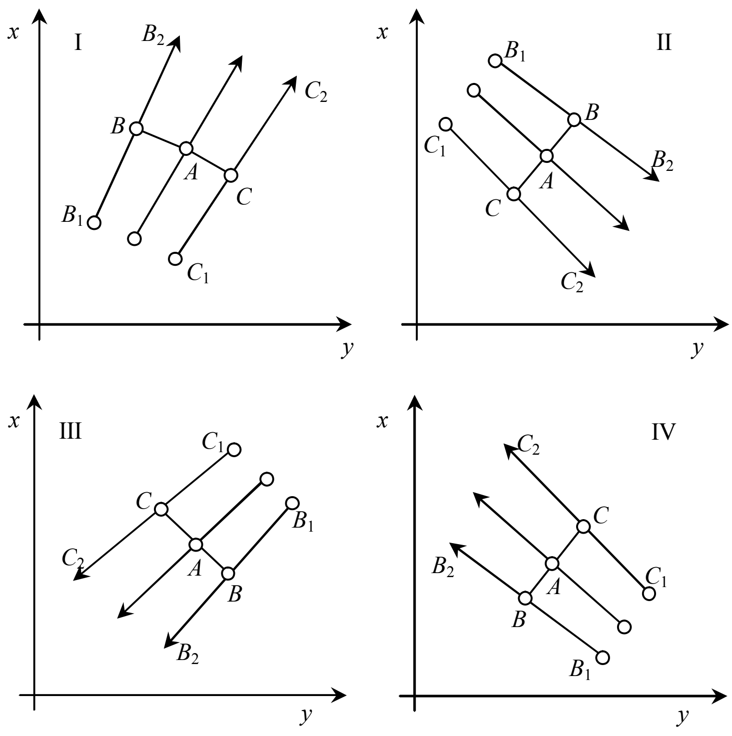

Fig.5. Directions of vectors in I-IV quarters of a geodetic rectangular coordinate system

In Fig.5 the vector that passes through point A (x_A, y_A, z_A) is a result of comparison between coordinate estimations Δx_A, Δy_A , Δz_A of two measurement cycles (not necessarily consequent ones). Then let us proceed to the implementation of our algorithmization proposal. Let us find coordinates of points B and C, perpendicular (in space) to the measured vector to the left and to the right from it:

$$\left.\begin{aligned} x_B = x_A + \frac{\Delta y_A}{100};\ x_C = x_A - \frac{\Delta y_A}{100};\\ y_B = y_A - \frac{\Delta x_A}{100};\ y_C = y_A + \frac{\Delta x_A}{100}.\end{aligned}\right\} \quad (4)$$

The distance from the point A is determined using expert method, taking into account average distance between the points of geodetic network; for actual industrial polygons it can reach several meters. In formulas (4), e.g., the relation of distance to the increment of displacement coordinates is 100 times lower than their average value in the area – 1 %.

Once we know the coordinates of points B and C, polynomial models of vector field (3) are used to estimate vectors passing through them. Once the vectors are estimated, it is easy to calculate coordinates of their tails B_1, C_1 and heads B_2, C_2, and then using these values to define components of vector displacement in point B (Δx_B , Δy_B, Δz_B) and in point C (Δx_C , Δy_C, Δz_C). For the quarter I (see Fig.3) the algorithmization formulas are:

$$\left.\begin{aligned} x_{B_1} = x_B - \frac{\Delta x_B}{2};\ y_{B_1} = y_B - \frac{\Delta y_B}{2};\ z_{B_1} = -\frac{\Delta z_B}{2};\\ x_{B_2} = x_B + \frac{\Delta x_B}{2};\ y_{B_2} = y_B + \frac{\Delta x_A}{2};\ z_{B_2} = \frac{\Delta z_B}{2};\end{aligned}\right\}$$

$$\left.\begin{aligned} x_{C_1} = x_C - \frac{\Delta x_C}{2};\ y_{C_1} = y_C - \frac{\Delta y_C}{2};\ z_{C_1} = -\frac{\Delta z_C}{2};\\ x_{C_2} = x_C + \frac{\Delta x_C}{2};\ y_{C_2} = y_C + \frac{\Delta x_C}{2};\ z_{C_2} = \frac{\Delta z_C}{2}.\end{aligned}\right\}$$

Obtained head and tail coordinates of vectors B and C allow to calculate the distance between them at the entrance of a vector tube d_{en} and its exit d_{ex}:

$$\left.\begin{aligned} d_{\text{en}} = \sqrt{(x_{B_1} - x_{C_1})^2 + (y_{B_1} - y_{C_1})^2 + (z_{B_1} - z_{C_1})^2};\\ d_{\text{ex}} = \sqrt{(x_{B_2} - x_{C_2})^2 + (y_{B_2} - y_{C_2})^2 + (z_{B_2} - z_{C_2})^2}.\end{aligned}\right\}$$

Then using these results one can estimate the areas of output cross-section S_2 and input cross-section S_1 of the vector tube under the assumption that they are round:

$$S_1 = \pi\left(\frac{d_{\text{en}}}{2}\right)^2 \text{ and } S_2 = \pi\left(\frac{d^{\text{ex}}}{2}\right)^2.$$

Data on areas of input and output cross-sections, as well as displacement vectors allow to calculate divergence of the vector field in the selected point of coordinate estimations on the surface. The authors suggest the following simplified formula for point A:

$$\operatorname{div} A = (S_2 - S_1)\left|\vec{V}_A\right|.$$

Conclusion. Subsurface layer of the Earth crust, as well as the entire planet in general, is a very complex system. Understanding of structural elements of this system, their size and hierarchy is closely associated with the solution of inverse problems using certain geophysical and geodetic data. One must rely on the results of applied mathematics, elasticity theory, methods of mathematical modeling [3, 8, 15]. The paper describes a method to estimate divergence in the subsurface Earth crust basing on the results of discrete geodetic measurements performed with a specified time interval. Estimation of divergence distribution over the area in question allows to obtain a more objective picture of the geodynamic process of the specific mineral deposit and to adjust its environment conditions (boundaries, mechanical characteristics etc.), which increases forecast accuracy and the quality of project decisions and plans on the mining plants.

The method and algorithms have been tested in the process of real-life surveillance measurements in Tashtagolsky iron-ore deposit [5], Kiyembayevsky mining plant and other deposits.

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