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Relationship between seismic velocity and heat production: comparison of two sets of data and test of validity

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ABSTRACT

An empirical relationship between compressional wave velocity (V_p) and radiogenic heat generation (A) is frequently used to estimate heat production distributions in the continental crust. Two data sets from Rybach and Buntebarth [1] and Kern and Siegesmund [2] have been re-evaluated and made comparable. Bulk density (ρ) was included as a further parameter to investigate the relationships v_p–A, ρ–v_p, and ρ–A. Statistical analysis shows reasonably high and comparable regression coefficients for the first two relationships, but much less agreement for the ρ–A relationship.

1. Introduction

The knowledge of the distribution of heat producing radioelements is indispensable for any geothermal model of the continental crust. Unfortunately, this distribution is poorly constrained and thus subject to much speculation. Most current heat production models are based on a presumed layered structure and the petrological composition of individual strata, supplemented by laboratory data on the radioelement content of corresponding crustal rocks. The linear relationship between surface heat flow and near-surface heat production yielded a simple step function [3] and/or exponential model [4], for the crustal radioactivity. However, the validity of these models for greater crustal depths could not yet be adequately tested and may be limited only to the uppermost part of the crust [5–7].

Potential progress to improve the knowledge of crustal radioactivity at greater depths, which are inaccessible for direct sampling, may be made by an empirical relationship between seismic velocity (v_p) and heat production (A). The relationship, first proposed on the basis of literature data (measurements of A and v_p on different samples [8,9]) was subsequently established by measurements on the same specimens and the v_p–A relationship was integrated into a more general systematics that included bulk density and the specific rock parameter k-value (cation packing index) [1]. The k-value is closely related to the mineralogical constitution and thus characterizes the rock type in question. The v_p–A relationship originally proposed for igneous rocks was later extended also to metamorphic rocks [10]. To show the effect of the geological age two formulae were derived describing Precambrian and Phanerozoic rocks [11]. To consider the varying pressure and temperature conditions at depth, a correction function was proposed to make the laboratory and in-situ conditions comparable [11–13]. In addition to laboratory-established relationships, other seismic velocity—heat production relationships were proposed [14–16] based on combined evaluations of surface heat flow, the Moho heat flow and assessed heat contributions of a seismically defined multi-layered crust. Rybach and Buntebarth's formulae for the conversion of seismic velocity into heat production [11] provided a very useful tool to forecast crustal heat sources and were used for calculations of the deep temperature distribution along several geotraverses, estimation of the Moho heat flow [12,17], and assessment of the general features of the radioelement distribution in the continental crust [7].

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The v_p–A relationship was established empirically and is based on statistically processed laboratory data on rock samples that are, regardless of their great number, subject to limitations of their representativeness. Some critical comments on the general validity of the relationship were raised by Fountain [18] and followed by discussions by Rybach and Buntebarth [19] and by Fountain [20]. The argument that v_p is a physical parameter reflecting the properties of major rock-forming minerals whereas A, in addition to the contribution of the major minerals, strongly reflects trace amounts of radioelements fixed in accessory minerals neglects the fact that considerable proportions of the radioelements can also be present in the intergranular space [21,22].

In any case, all efforts to extend the data base on the relationship are welcome. Defining a clear relationship between the two parameters is difficult and before a satisfactory solution is found, maximum efforts must be focussed on more data covering all respective rock parameters and rock types. Statistical evaluation and data analysis may then contribute to formulating some ideas about the crustal formation and its evolution.

2. Data base

Recently Kern and Siegesmund [2] published a new series of 41 rock samples of crustal rocks and tested the empirical velocity-density-heat production systematics proposed by Rybach and Buntebarth [1]. They claimed that their data did not convincingly fit any of the earlier proposed relationships. We have reconsidered the original Rybach and Bunteberth's data [1], compared them individually with the new results, and contrary to Kern and Siegesmund's conclusion we believe that there is a reasonably good confirmation of the original findings.

The k-value, which is the number of cations per molar volume, normalized by Avogadro's number, present in the mineral structure, is believed to be a parameter to characterize a given rock quantitatively [23]. Rybach and Buntebarth [1] investigated 99 crystalline rocks that were sampled to cover the widest possible range in k-value. This suite of rocks included silicic types like rhyolites ($n = 5$) and granites (16), granodiorites (11), tonalites (8), diorites (7), and basic and ultrabasic types like gabbros (6), amphibolites (6), hornblendites (3), pyroxenites (4), peridotites (4), serpentinites (6) and also numerous miscellaneous rocks (23). Density (ρ), seismic velocity (v_p), radiogenic heat production (A) and cation fraction (k) were determined experimentally on the individual samples and the interrelations between the mean values of the properties of all types were established by regression analysis [1,11].

The relationship between seismic velocity and heat production is one of the dependences within complex systematics which was formulated in a matrix representation [1]:

$$\begin{vmatrix} \ln A \\ v_p \\ \rho \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} k$$

which includes six individual relationships, three of them linear: v_p–k, ρ–k, ρ–v_p, and three exponential: A–v_p, A–k, and A–ρ (i.e., linear for ln A). As seismic velocity is pressure dependent, some of the coefficients u_{ij} also depend on pressure and the pressure used during the laboratory experiment must be stated. Rybach and Buntebarth [1] reported data obtained at 50 MPa, later they presented eleven additional results measured also at 100 and 200 MPa [11].

All rocks originally reported [1] were Phanerozoic rocks from Variscan and Alpine realms of Western Europe. To characterize the age dependence of heat production a few (eleven) additional results for Precambrian rocks were added and a

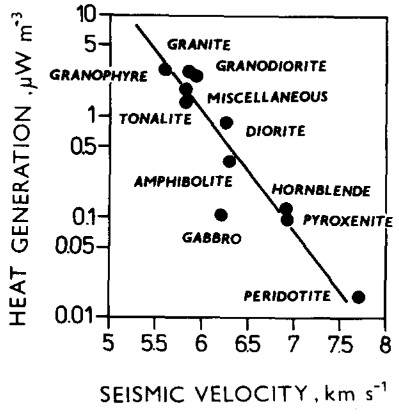

Fig. 1. The $A(v_p)$ relationship ln $A = 16.5 - 2.74\ v_p$ (at 50 MPa).

TABLE 1

Rock samples investigated: rock type and sampling locality, modal composition, k-value, density (at surface conditions), compressional wave velocity (v_p) at 50 MPa pressure and room temperature, and heat generation (A)

Rock type/ sampling locality	Modal composition (vol.%)	k-value (10^{-2} mole/cm^3)	Density (g/cm^3)	v_p (km/s)	A (μW/m^3)
Acidic rocks					
Rhyolites					
Vico Morcote/CH	29qu, 46kf, 23plg, 2bio	4.66	2.51	5.51	2.77
Figino/CH	38qu, 40kf, 19plg, 3bio	4.50	2.55	5.67	2.79
Figino/CH	33qu, 43kf, 21plg, 2phl, 1or	4.74	2.56	–	3.12
Ciona/CH	18qu, 41kf, 26plg, 14chl + bio, 1or	4.97	2.70	5.68	2.37
Wieden/FRG	17qu, 33kf, 24plg, 12chl, 12bio, 2or	5.01	2.55	5.43	2.94
Granites					
Maloja/CH	48qu, 10kf, 9plg, 30phl, 2cc, 1zr	5.14	2.69	5.93	2.05
Alpbruch/FRG	41qu, 23kf, 26plg, 7ms, 2bio, 1or	5.40	2.61	5.84	0.806
Alpbruch/FRG	42qu, 28kf, 23plg, 4ms, 1bio, 2or	4.73	2.62	5.80	0.892
Tiefenstein/FRG	24qu, 23kf, 37plg, 16bio	5.05	2.66	5.98	3.96
Hag/FRG	21qu, 30kf, 30plg, 19chl + bio	5.05	2.65	5.98	2.79
Happach/FRG	27qu, 25kf, 37plg, 11bio	4.88	2.60	5.66	3.37
Bernina-Suot/CH	44qu, 22kf, 25plg, 6ms, 3or	4.78	2.60	5.92	3.33
Bernina-Suot/CH	33qu, 25kf, 28plg, 11phl + chl + stp, 3or	4.93	2.59	5.98	4.65
Vicosoprano/CH	38qu, 27kf, 25plg, 9bio, 1or	4.80	2.62	5.59	3.28
Vicosoprano/CH	35qu, 20kf, 33plg, 12bio	4.86	2.62	5.88	3.13
Alp Gueglia/CH	32qu, 58kf, 4plg, 5chl + bio, 1or	4.66	2.63	6.00	2.70
Alp Gueglia/CH	49qu, 20kf, 25plg, 6chl + bio	4.80	2.68	5.91	2.42
Alp Gueglia/CH	27qu, 25kf, 36plg, 8chl + bio, 4ms + phl	5.08	2.66	–	3.03
Wiesental/FRG	30qu, 30kf, 29plg, 11bio	4.90	2.62	6.04	1.80
Todtmoos/FRG	18qu, 38kf, 28plg, 11chl + bio, 2ms, 3or	5.07	2.57	5.52	3.54
Pontresina/CH	40qu, 38kf, 17plg, 5chl + bio	4.67	2.61	5.75	3.45
Granodiorites					
Hag/FRG	27qu, 15kf, 50plg, 8chl + bio	4.96	2.86	5.83	4.41
Tiefenstein/FRG	23qu, 21kf, 41plg, 15bio	4.95	2.67	–	3.79
Bernina-Suot/CH	35qu, 7kf, 38plg, 17bio, 3ms	5.18	2.72	5.66	1.98
Novate/I	43qu, 15kf, 35plg, 1bio, 6ms	4.82	2.61	5.78	1.21
Ponte Tresa/CH	23qu, 10kf, 47plg, 18chl + bio, 2or	5.22	2.67	5.51	1.30
Pontresina/CH	35qu, 17kf, 35plg, 12bio, 1hbl	5.00	2.73	6.30	2.07
Pontresina/CH	22qu, 19kf, 43plg, 14bio, 2hbl	5.10	2.74	6.12	2.87
Vicosoprano/CH	28qu, 15kf, 49plg, 8bio	4.88	2.64	6.01	3.63
Vicosoprano/CH	21qu, 16kf, 37plg, 26bio	5.09	2.71	5.94	3.74
Val di Campo/CH	23qu, 16kf, 45plg, 16bio	5.10	2.67	5.84	0.628
Ponte Tresa/CH	21qu, 7kf, 44plg, 23chl + bio, 3ms, 2or	5.40	2.71	5.85	1.37
Tonalites					
Alp Gueglia/CH	33qu, 8kf, 43plg, 16chl + bio	5.05	2.69	6.00	1.84
Sorico/I	37qu, 40plg, 11bio, 6hbl, 6ep	5.09	2.79	5.64	1.37
Dascio/I	29qu, 36plg, 21chl + bio, 10hbl, 4ep	5.31	2.79	–	0.981
Spinida/I	23qu, 29plg, 21bio, 16hbl, 9ep	5.38	2.82	6.18	2.35
Pontresina/CH	27qu, 1kf, 45plg, 18chl, 7ep, 2or	5.33	2.77	5.67	1.61
Pontresina/CH	19qu, 54plg, 26bio + chl, 1akt	5.46	2.77	6.02	0.820
Val di Campo/CH	16qu, 58plg, 14bio, 11hbl, 1ep	5.23	2.75	5.76	0.978
Val di Campo/CH	34qu, 48plg, 11bio, 6ms, 1or	5.09	2.67	5.96	1.88
Diorites/quartzdiorites					
Val di Campo/CH	2qu, 51plg, 30hbl, 7chl, 6ep, 4or	5.63	2.90	6.12	0.472
Val di Campo/CH	6qu, 50plg, 33hbl, 4bio, 7ep	5.54	2.84	6.31	0.676
Val di Campo/CH	55qu + plg, 10hbl, 35bio	5.60	2.80	5.84	1.32
Tiefenstein/FRG	76plg, 23hbl, 1ep	5.53	2.74	–	1.18
Pontresina/CH	5qu, 56plg, 31hbl, 7bio, 1or	5.56	2.98	6.57	0.819
Hag/FRG	7qu, 67plg, 13hbl, 11chl + bio, 2or	5.34	2.83	6.41	0.989
Pontresina/CH	12qu, 52plg, 10chl, 25hbl, 1opx	5.63	2.92	6.20	0.681

TABLE 1 (continued)

Rock type/ sampling locality	Modal composition (vol.%)	k-value (10^{-2} mole/cm^3)	Density (g/cm^3)	v_p (km/s)	A (μW/m^3)
Basic rocks					
Gabbros					
Marmorera/CH	44plg, 30act, 21dps, 5or	5.65	2.86	5.70	0.129
Marmorera/CH	43plg, 29act, 25dps, 3or	5.55	2.94	–	0.0659
Marmorera/CH	29plg, 22act, 36dps, 9chl, 4or	5.59	2.92	5.05	0.0715
Pontresima/CH	45plg, 44hbl, 8bio, 3or	5.86	3.06	6.83	0.345
Finero/I	26plg, 27act, 25dps, 8gt, 14or	6.00	3.07	6.30	0.0073
Finero/I	37plg, 4gt, 6srp, 35hbl, 17px	5.82	2.97	7.06	0.0144
Amphibolites					
Val di Campo/CH	14plg + qu, 10cc, 7ep, 2or, 32hbl(g), 35hbl(b)	5.98	2.93	6.09	0.386
Val di Campo/CH	2qu, 34plg, 59hbl, 1ep, 4or	5.80	2.96	6.66	0.351
Val di Campo/CH	30plg, 9ep, 50hbl(g), 11 hbl(b)	5.93	3.02	6.52	0.410
Val di Campo/CH	8qu, 26plg, 48hbl, 10ep, 8aph	5.85	3.03	6.32	0.701
Tiefenstein/FRG	39plg, 58hbl, 1ep, 2or	5.80	2.99	5.92	0.198
Goldenhof/FRG	37plg, 62hbl, 1qu + or	5.88	2.89	–	0.154
Pyroxenites					
Finero/I	16hbl, 26srp, 53px, 5or	6.68	3.10	6.82	0.104
Ultrabasic rocks					
Hornblendites					
Novate/I	hbl + bio (composition in % difficult to determine)	6.23	2.92	6.03	0.295
Finero/I	62hbl, 23opx, 10dps, 5gt	6.14	3.08	7.23	0.0048
Chiavenna/I	90hbl (groundmass difficult to analyze)	6.23	2.98	7.48	0.0687
Pyroxenites					
Finero/I	13hbl, 56opx, 15srp, 10tc, 6or	6.69	3.07	7.02	0.0874
Finero/I	26srp, 60opx, 10tc, 4or	6.84	3.06	7.48	0.0212
Finero/I	20srp, 75opx, 3tc, 2or	6.70	3.05	7.47	0.0265
Peridotites					
Finero/I	2srp, 28px, 65ol, 3tc, 2or	6.57	3.23	7.99	0.0257
Finero/I	93ol, 7or	6.81	3.29	7.80	0.0019
Finero/I	95ol, 5or	6.81	3.27	7.77	0.0068
Finero/I	12srp, 73ol, 7tc, 6hbl, 2or	6.58	3.16	7.72	0.0182
Serpentinites					
Finero/I	68srp, 13opx, 12tc, 7or	7.69	2.75	7.44	0.0070
Finero/I	53srp, 13tc, 34or(mg)	7.50	2.67	6.36	0.0450
Marmorera/CH	76srp, 1tc, 23or(mg)	7.86	2.64	–	0.0056
Marmorera/CH	92srp, 8or	8.10	2.66	5.72	0.0072
Sils/CH	98srp, 2or	8.19	2.67	6.23	0.0072
Todtmoos/FRG	50srp, 33ol, 3gt, 12or	7.38	2.73	6.70	0.0007
Miscellaneous					
Glassy porphyrite (Ciona/CH)	3qu, 15plg, 11chl, 67gl, 4or	5.50	2.63	5.70	2.30
Chlorite–porphyrite (Carona/CH)	4qu, 50plg, 28chl, 18or	6.00	2.67	5.97	1.13
Quartz–Latite (Vico Morcote/CH)	9qu, 29kf, 46plg, 9chl, 7or(mg)	5.18	2.65	5.64	1.66
Phenodacite (Wieden/FRG)	17qu, 24plg, 14bio, 45gl	5.25	2.55	5.52	2.85
Biodacite (Silvaplana/CH)	38qu, 20plg, 9chl + bio, 29phl, 4cc + or	5.49	2.70	5.62	3.22

TABLE 1 (continued)

Rock type/ sampling locality	Modal composition (vol.%)	k-value (10^{-2} mole/cm^3)	Density (g/cm^3)	v_p (km/s)	A (μW/m^3)
Miscellaneous					
Muscodacite (Silvaplana/CH)	37qu, 14plg, 6chl + bio, 12cc, 26ms, 5or	5.49	2.69	5.70	2.41
Quartzite (Maloja/CH)	60qu, 36ms, 4zr	5.19	2.70	5.39	2.09
Aplite (Happach/FRG)	43qu, 42kf, 12plg, 3sp	4.65	2.58	5.78	0.475
Kersantite (Tiefenstein/FRG)	8qu, 51plg, 32bio, 6act, 3or	5.30	2.64	5.08	4.30
Kersantite (Tiefenstein/FRG)	qu + plg + bio + or(mg) (composition in % not given)	5.45	2.51	4.65	3.74
Kersantite (Todtmoos/FRG)	53plg, 19bio, 13cc, 15or(mg)	5.50	2.64	5.83	2.57
Kersantite (Todtmoos/FRG)	49plg, 18bio, 14cc, 19or(mg)	5.55	2.61	5.26	2.56
Prasinite (Mulegns/CH)	srp + chl (composition in % difficult to determine)	5.05	2.97	6.42	0.0497
Prasinite (Marmorera/CH)	qu + act (composition of groundmass could not be determined)	5.05	2.88	6.08	0.0855
Prasinite (Marmorera/CH)	cc, chl, act + px, plg + kf + qu	5.05	2.95	6.23	0.103
Spilite (Marmorera/CH)	plg, px, qu + kf, chl + act	5.05	2.90	6.70	0.119
Spilite (Marmorera/CH)	plg, px, qu + kf, chl + act	5.05	2.89	–	0.0503
Spilite (Marmorera/CH)	cc, chl, act, px, qu + kf + plg	5.05	2.91	7.16	0.0195
Silexite (Val di Campo/CH)	64qu, 30plg, 4ms, 2or	4.68	2.64	5.72	1.73
Quartzmonzonite (Tiefenstein/FRG)	13qu, 36kf, 38plg, 11bio + or, 2cc	5.01	2.61	5.80	3.82
Phenodacite (Figino/I)	qu, plg, kf	–	2.59	5.58	2.72
Aplite (Pontresina/CH)	45qu + 45kf + 10plg	4.55	2.61	5.67	3.36
Pegmatite (Prata/I)	59qu, 21kf, 18plg, 2bio	4.59	2.60	5.68	2.62

Abbreviations: act = actinolite, aph = antophyllite, bio = biotite, cc = calcite, chl = chlorite, dps = diopside, ep = epidote, gl = glass, gt = garnet, hbl = hornblende, hbl(g) = hornblende(green), hbl(b) = hornblende(brown), kf = kfeldspar, ms = muscovite, ol = olivine, or = ore mineral(s), or(m) = magnetite, phl = phlogopite, plg = plagioclase, px = pyroxene, opx = orthopyroxene, qu = quartz, sp = spinel, stp = stilpnomelane, srp = serpentine, tc = talc, zr = zirkon. CH: Switzerland, FRG: W. Germany, I: Italy.

correspondingly modified formula for ln $A = f(v_p)$ was given later [11].

In the original work [1], the individual data has been grouped, first, according to main rock types before the regression coefficients were assessed. With the exception of serpentinites, all three calculated correlation coefficients were better than 0.9; namely ρ–k: 0.982; v_p–k: 0.933; A–k = −0.935. The other three equations were derived, since they are not independent, from the above matrix formulation. Figure 1 shows the grouped A–v_p data, which revealed the originally published relationship: ln $A = 16.5 - 2.74\ v_p$ (at 50 MPa pressure) [1]. The miscellaneous group generally fits the corresponding regression lines, however, the individual data within this group exhibited large deviations in some cases. As probably not quite typical for the crust, the miscellaneous group was not included in the regression calculations. Further, it is not anticipated that monomineralic rocks like quartzites or anorthosites will follow the v_p–A relationship.

As only mean values for the individual rock types were given in the previous publications [1,11], the complete list of all measured parameters for all rock types investigated is given here (Table 1). The value of v_p given for each specimen is based on several determinations in order to minimize the effect of compositional and textural inhomogeneity.

3. Data processing

Recently Kern and Siegesmund [2] investigated a set of 41 rock samples and tested the relationship between v_p and A proposed by Rybach and Buntebarth [11]. They concluded that their data did not convincingly fit with previous results. We have tried to re-evaluate the above comparison and we suggest that the disagreement between both data sets is not severe.

To be able to compare both sets of data, several modifications were necessary. As the results reported by Kern and Siegesmund [2] were based on the regression analysis applied directly on the individual samples, we also used individual data as given in Table 1 rather than the grouped and averaged values used earlier [1]. The major problems, however, are the different laboratory conditions; Rybach and Buntebarth [1] obtained their v_p-data at 50 MPa pressure, while Kern and Siegesmund [2] reported data at 200 MPa. The latter experimental conditions are thus much closer to what happens in nature. Rybach and Buntebarth [1] took the effect of increasing pressure into consideration and complemented their formulae by additional relationships at 100 and 200 MPa, but unfortunately these latter relationships were only based on a few determinations. For practical applications of the v_p–A in geothermal modelling Cermak and Rybach [7] and Cermak et al. [13] proposed correction functions to account for the pressure and temperature effects on v_p,their approach was based on reviewing numerous laboratory investigations on all types of rocks [24,25]. This correction technique has been used now; Fig. 2 shows the reported values of the pressure derivative of seismic velocity (dv_p/dP) as a function of pressure (P). This function was simplified as $dv_p/dP = c/(P + d)$, with the least-squares fitted coefficients: $c = 0.135$ km s^{-1}. $d = 25$ MPa. After integration the corresponding correction term was expressed: $v_p(20\,°\text{C},\ P_1) = v_p(20\,°\text{C},\ P_2) + c[\ln(P_1 + d) - \ln(P_2 + d)]$. For the pressure increase from e.g. 50 MPa to 200 MPa the correction amounts to -0.148 km s^{-1}. The slope of the regression line in the graph $A(v_p)$ does not change, the correction affects only the intercept value.

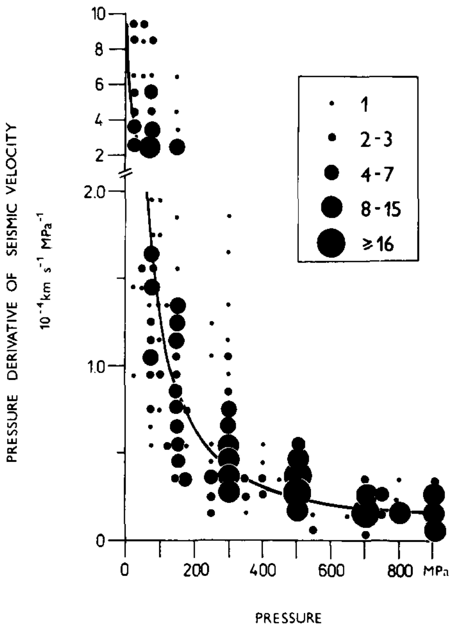

Fig. 2. Pressure derivative of seismic velocity, dv_p/dP, as a function of pressure. The size of dots corresponds to the number of observations, the curve shown was least-squares fitted to experimental material compiled by Gebrande [24].

Table 2 summarizes the calculated coefficients of the regression analysis for four groups of data, and for all three combinations of the parameters investigated (density, heat production, and seismic velocity). The k-value was calculated differently; Kern and Siegesmund [2] did not treat hydrogen in the OH group as a cation, contrary to Rybach and Buntebarth [1]. The two sets of k-values are therefore not directly comparable.

Group a includes all samples originally reported [1]. As all parameters were not always measured for all rock types, the actual number of pairs used for testing the respective relationship may be smaller in some cases (compare Tables 1 and 2).

TABLE 2

Calculated values of regression coefficients a and b, their standard errors, value of correlation coefficient r, and the integral probability $i.p.$ of several sets of crustal rocks investigated by Rybach and Buntebarth [1,11] and Kern and Siegesmund [2] (see text)

Item	Relationship	N	$a \pm \Delta a$	$b \pm \Delta b$	r	$i.p.$
1a	$\ln A = a + bv_p$	89	13.92 ± 1.50	-2.38 ± 0.24	-0.72	~ 0
1b		51	6.44 ± 2.91	-1.06 ± 0.49	-0.30	0.035
1c		10	9.60 ± 5.06	-1.81 ± 0.68	-0.68	0.029
1d		41	5.65 ± 2.29	-0.96 ± 0.35	-0.40	0.009
2a	$\ln A = a + bp$	99	21.16 ± 2.52	-7.85 ± 0.91	-0.68	~ 0
2b		60	18.89 ± 2.11	-6.83 ± 0.76	-0.76	~ 0
2c		10	25.36 ± 8.28	-9.38 ± 2.66	-0.78	0.008
2d		41	5.99 ± 2.71	-2.27 ± 0.93	-0.36	0.019
3a	$p = a + bv_p$	99	1.40 ± 0.12	0.22 ± 0.02	0.79	~ 0
3b		53	1.17 ± 0.27	0.27 ± 0.04	0.64	~ 0
3c		10	1.85 ± 0.37	0.17 ± 0.05	0.77	0.009
3d		41	1.49 ± 0.33	0.22 ± 0.05	0.57	~ 0

Note: As the number of samples in the individual groups is rather limited, the error bounds of the correlation coefficient cannot be given, the integral probability ($i.p$) expresses the probability of $r = 0$, i.e. the probability that no correlation exists.

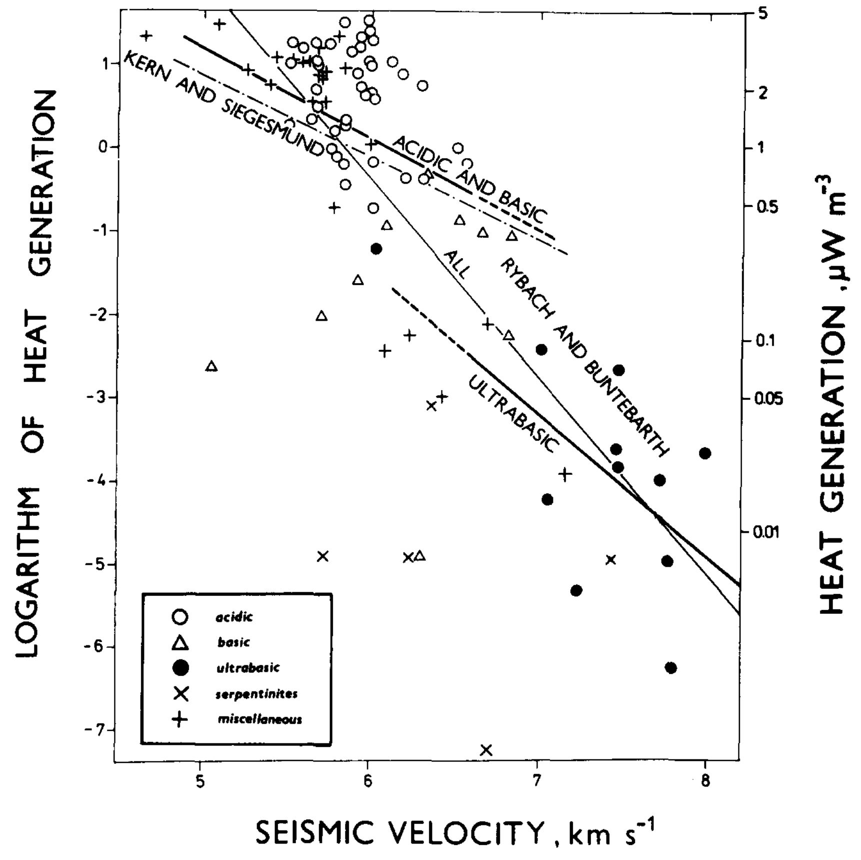

Fig. 3. Compressional wave velocity versus heat generation for individual rock samples reported by Rybach and Buntebarth [1], together with calculated relationships for selected rock groups. The dot-dashed line indicates the relationship corresponding to the data presented by Kern and Siegesmund [2].

Group b includes rock samples that represent a subset of the former group and covers acidic and basic rocks only: rhyolites, granites, granodiorites, tonalites, diorites, gabbros, amphibolites, and one pyroxenite.

Group c represents ten rock samples, another subset of group a and includes ultrabasic rocks only: hornblendites, pyroxenites and peridotites. Pyroxenites appear in both groups of basic (b) and ultrabasic (c) rocks according to their modal composition, see Table 1.

Group d is identical, with the 41 rock samples reported by Kern and Siegesmund [2].

Serpentinites and miscellaneous rocks were included only in group a, but not in group b or c. Group d represents mostly metasedimentary rocks, partly also igneous rocks and only three other rock types (one serpentinite, one peridotite and one dunite) and resembles thus more or less our group b. Groups b and d cover rock types typical of the major part of the continental crust, for which the relationship v_p–A was originally proposed. Ultrabasic rocks may well describe the lowermost crust and upper mantle but should not be compared with the data reported by Kern and Siegesmund [2].

4. Results

With the above subdivision, the regression coefficients for group b and d are quite similar and no significant differences exist (Table 2). The regression coefficients were calculated for the data as given by the individual authors, that is for pres-

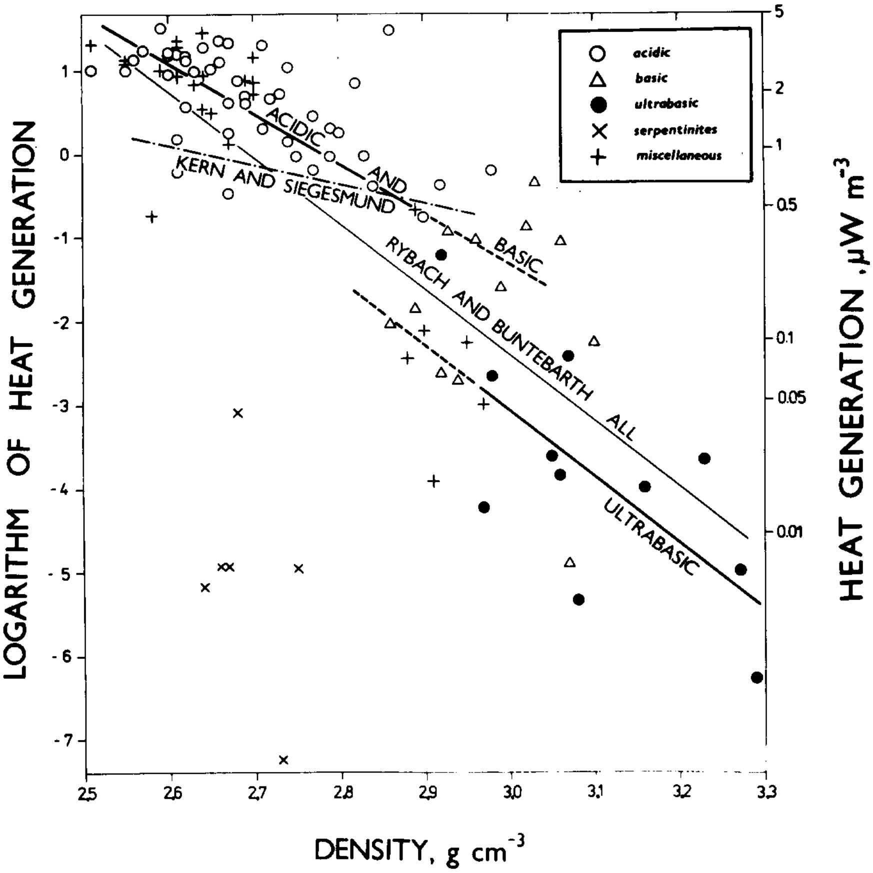

Fig. 4. Density versus heat generation for individual rock samples reported by Rybach and Buntebarth [1], together with calculated relationships for selected rock groups. The dot–dashed line indicates the relationship corresponding to the data presented by Kern and Siegesmund [2].

sures of 50 MPa and 200 MPa, respectively. The application of the pressure correction to Rybach and Buntebarth's [1]) 50 MPa data to meet Kern and Siegesmund's [2] 200 MPa data gives a modified relationship: $\ln A = 6.60 - 1.06\ v_p$ for group b, increasing the intercept, but having little effect on the slope value.

$A - v_p$: regression coefficients depend on the type of rock, but for both sets of data [1,2] there is good agreement between groups b and d, that is for typical crustal rocks (Fig. 3).

$A - \rho$: for the data reported by Rybach and Buntebarth [1] there is good correlation for all groups, depending little on the rock type. For Kern and Siegesmund's [2] data there is practically no correlation at all, which may have been the reason for their statement of a non-existing agreement (Fig. 4).

$\rho - v_p$: high correlation coefficients with no dependence on the type of rock, both sets of data show good agreement.

5. Conclusions

In the present work we tried to prove that a careful subdivision of the original data [1] made correlation coefficients and calculated regression lines of both data sets [1,2] comparable and that no principle disagreement exists. The increase of seismic velocity with decreasing heat production corresponding to the increase of basicity with depths is a general phenomenon. This tendency is documented by the solid line marked "ALL" in Figs. 3 and 4 and may serve for a rough estimate of heat production at depth by converting the observed seismic velocities. Other relationships corresponding to groups b and c may better suit the conditions of the upper/middle crust or lower crust/upper mantle.

Nevertheless many more laboratory investigations of characteristic rocks are necessary to obtain a sound experimental background and to help in creating firm empirical relationships between the individual parameters. The data on samples collected at the earth's surface up to now must be supplemented by future measurements on xenoliths, preferably covering a significant range in depth of origin. In this paper we are not only trying to give further evidence of the applicability of the $v_p - A$ relationship, but we also want to stress the necessity of comparing the proper sets of rocks in any testing attempt.

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