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Incorporating the Fractal Distribution of Faults as a Measure of Failure Concentration

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The effect of the random spatial distribution of earthquake-generating faults on the estimation of the failure concentration parameter is investigated. It is shown that on the assumption of a fractal distribution of faults the failure concentration parameter is a power function of the size of a space in which this parameter is estimated. The power is defined by the fractal dimension of fault distribution in the space considered. An expression was derived for the critical fault concentration in a region of a given size, based on the stability of two faults in a stress field: $\chi^* \approx 2\,(L/l)^{1-d/3}$, where L is the size of the region, l the mean fault (rupture) length, d the fractal dimension of the fault distribution. The theoretical results were compared to observations in various seismic areas and showed agreement. It is proposed that a calculated fault concentration parameter should be corrected for fault distribution nonuniformity taking into account variations of the fractal dimension of the parameter in different seismic areas and different time periods.

Introduction. The concentration criterion of failure is one of the physically sound failure measures. It is used in seismology for large earthquake prediction and is a very efficient precursor. It is found by estimating of the concentration of earthquake-generating ruptures in a volume of interest through the number and energy of earthquakes that have occurred there.

The mapping procedure consists in covering the study area with a grid of size L cells and computing the rupture concentration parameter in each of them

$$\chi = \mu^{-1/3} / l_{\text{mean}}, \tag{1}$$

where μ is the volumetric density (concentration) of ruptures identified from past earthquakes; $l_{\text{mean}} = (1/n)\sum_{i=1}^{n} l_i$ is the mean rupture length in the cell; n is the number of events in the cell [7]. The quantity $R_{\text{mean}} = \mu^{-1/3}$ has the meaning of the mean inter-rupture distance between the centers of the ruptures. The concentration parameter is known to be estimated over an area or even on a line rather than in a volume [4]. In these cases R_{mean} =

Table 1 Data from earthquake catalogs.

Region	*Magnitude* M_{min} *for energy class* K_{min}	*Time, years*	*Number of events*	*Formulas for magnitude (class) – source size conversion, l im km*
Greece	$M_{min} = 3$	1964–1993	17327	$\log l = 0.440\,M - 1.289$
NE China	$M_{min} = 2$	1970–1994	16383	$\log l = 0.576\,M_L - 2.331$
SW China	$M_{min} = 2$	1970–1993	34839	$\log l = 0.576\,M_L - 2.331$
Turkmenia	$K_{min} = 8.5$	1956–1994	8945	$\log l = 0.244\,K - 2.266$
Kirgizia	$K_{min} = 8.5$	1062–1992	16167	$\log l = 0.244\,K - 2.266$
Kamchatka	$K_{min} = 8.5$	1062–1994	21170	$\log l = 0.244\,K - 2.266$

$=\mu^{-1/r}$, where r takes on the values of 2 and 1, respectively. To make our expression suitable for all cases, we will use the form for:

$$\chi = \mu^{-1/r} / l_{mean}, \tag{2}$$

where $r = 1, 2, 3$ is the relevant dimension.

Applications of the concentration criterion to seismicity in different regions show that different values of the rupture concentration parameter correspond to the critical states of earthquake-generating fault systems [1], [2], [8], [10]. The explanation is that the spatial distribution of ruptures is not uniform. Where the faults are strongly clustered, greater values of the parameter correspond to the critical state compared to the case of uniformly distributed ruptures: the actual distances between the ruptures in a cluster are smaller than the mean distance for the cell. It follows that the critical state is reached at greater mean distances. The effect of spatial nonuniformity in rupture distribution on the concentration parameter can be taken into account by assuming the nonuniformity to obey a certain law. We will consider the fractal character of the spatial distribution of earthquakes, and hence of earthquake-generating faults, as the law in question.

Rupture concentration criterion. Let the size of the study area be $\mathcal{L}$, and the cell in which the concentration parameter will be estimated be of size L. The cell size L is chosen to be much greater than the size of the earthquake-generating faults considered. Let us evaluate the mean χ_L for this a cell.

If the faults are distributed uniformly in space, then the mean number of faults falling in a cell is $\overline{n}_0 = N/m_0$, where $m_0 = (\mathcal{L}/L)^r$ is the number of size L cells covering a region of size $\mathcal{L}$; N is the total number of faults in the region. The density of faults which is now $\mu_0 = (\overline{n}_0/L^r) = (N/\mathcal{L}^r)$ is independent of L, and so is the rupture concentration parameter

$$\chi_L = \frac{\mathcal{L}}{N^{1/r} l_{mean}}.$$

If the faults are not distributed uniformly in space, but form a fractal structure of dimension $d \leq r$, then some of the cells are empty, i.e., they do not contain any faults. According to the meaning of fractal dimension, the number of empty cells is

$$m = (\mathcal{L}/L)^d. \tag{3}$$

The mean number of faults in each nonempty cell is

$$\bar{n} = N/m, \tag{4}$$

the mean density of faults in the cell being

$$\mu = \bar{n}/L^r. \tag{5}$$

The substitution of (3) in (4) and of (4) in (5) gives

$$\mu = \frac{N}{\mathcal{L}^d} L^{d-r}. \tag{6}$$

It follows from (6) that $\mu = \mu_0$ when $d=r$, and $\mu > \mu_0$ when $d<r$, μ increasing with decreasing L.

Substituting (6) in (2), we get

$$\chi_L = \left(\frac{N}{\mathcal{L}^d}\right)^{-1/r} \frac{L^{1-d/r}}{l_{\text{mean}}}. \tag{7}$$

Let χ_{L1} and χ_{L2} be the concentration parameters estimated in cells of size L_1 and L_2, respectively. Assuming l_{mean} to be independent of L, we find from (7):

$$\chi_{L2} = \chi_{L1} \left(\frac{L_2}{L_1}\right)^{1-d/r} \tag{8}$$

The factor $(L_2/L_1)^{1-d/r}$ in (8) incorporates the nonuniformity of the spatial distribution of faults; it describes the functional dependence of the mean fault concentration parameter on the scale of averaging, i.e., on the cell size.

Critical value of the rupture concentration parameter. Viewed from the standpoint of physics, the concentration criterion of failure is a measure of the loss of stability in a set of cracks under stress. The cracks lose stability, grow, and coalesce to form larger fractures, where they are sufficiently close to one another, i.e., where a high concentration of cracks exists in some volume. Supposing that the fractal structure of the spatial distribution of faults (crustal cracks) persists up to the scale of individual cracks, and knowing the critical concentration of ruptures χ^*_{L0} in a region of some size L_0, one can use (8) to calculate the critical concentration in a spatial cell of any size L. In the simplest case of two size l ruptures positioned on a straight line, stability is lost when the distance between the ends of the ruptures is close to l. For this reason the assumption $L_0 = l$ leads to the result $\chi_{L0} \approx 2$ on the average on the average (if the distance between the ends of two size l ruptures is equal to l, then the distance between their centers is $2l$, hence $\chi = 2$). It follows that one should set $\chi^*_l \approx 2$ in the expression which evaluates the critical value of the concentration parameter for ruptures of size l in a spatial cell of size L:

$$\chi^*_L = \chi^*_l (L/l)^{1-d/r}. \tag{9}$$

Comparison with observations. Expression (9) was tested by observations. To do this, earthquake catalogs were used to calculate the critical values of rupture concentration

Table 2 Comparison between (9) and observations.

Region	χ_l^*	d	d_k
Greece	1.6±0.6	1.6±0.3	1.7±0.1
NE China	1.8±0.7	1.1±0.3	1.3±0.2
SW China	2.1±0.5	1.2±0.2	1.52±0.02
Turkmenia	1.7±0.4	1.5±0.2	2.1±0.1
Kirgizia	1.5±0.4	1.3±0.2	1.2±0.1
Kamchatka	1.5±0.4	1.5±0.2	1.9±0.1

χ_L^* for various values of l in spatial cells of varying size L. The method of calculation is described in [1]. The quantity l was set equal to the mean earthquake rupture length deduced from relations between the source size and magnitude. The l value was modified using catalog data for with different thresholds magnitudes. The cell size L was set equal to the cubic root of its volume. Table 1 lists the catalog data used.

The results of this calculation are plotted in Fig. 1. The dependence of χ_L^* on L/l was fitted by (9). The χ_l^* and d parameters were estimated by plotting a robust regression of χ_L^* relative to L/l on a log-log scale. When the errors of the regression coefficients were estimated by Monte Carlo, the uncertainty of the source data was taken into account [3]. Table 2 lists the calculated data and the fractal (correlational) dimensions of earthquake hypocenter sets d_c evaluated independently by the method of the correlation integral.

It is seen in Fig. 1 that the empirical estimates of χ_L^* for different regions are in general agreement with (9). It follows from Table 2 that χ_l^* is indeed close to 2, and that the fractal dimension d differs from the d_c values obtained by an independent method by not more than 30%.

One can see in Fig. 1 that the results for China are different from those for the other regions. It follows from Table 2 that this difference consists in higher values of χ_l^* (by ~ 30%) and lower values of d (by ~ 20%). This might be caused by some regional features. However, a more likely cause was the use of different relations between source dimensions and magnitudes (or energy classes). The world-mean relations [3] were used for all regions except China. The Chinese catalog contains local magnitudes M_L which were converted to M_S magnitudes, using regional relations, for which regional correlations with source dimensions were known. It is difficult to assess the adequacy of the world-mean and regional relations used from the data of the catalogs. Recalling however that source dimension is connected to magnitude by a logarithmic relation of the form $\log l = AM + B$, one can easily derive from (9) that $\Delta d/d \approx \Delta A/A$ and $\Delta\chi_l^*/\chi_l^* \approx -(d/r)\ln 10 \cdot \Delta B$. Therefore, the differences between the regions might well be caused by the very likely 20% errors of A and 0.2−0.3 errors of B.

Conclusion. The above results suggest that the scale factor of the rock failure concentration criterion is of technical rather than physical nature. The fact that the critical rupture concentration parameter is dependent on the size of a space region is not due to the failure mechanism, but arises from the neglect of the fractal character of rupture distribution in it. There is a well-known relation that connects mean values and the scale

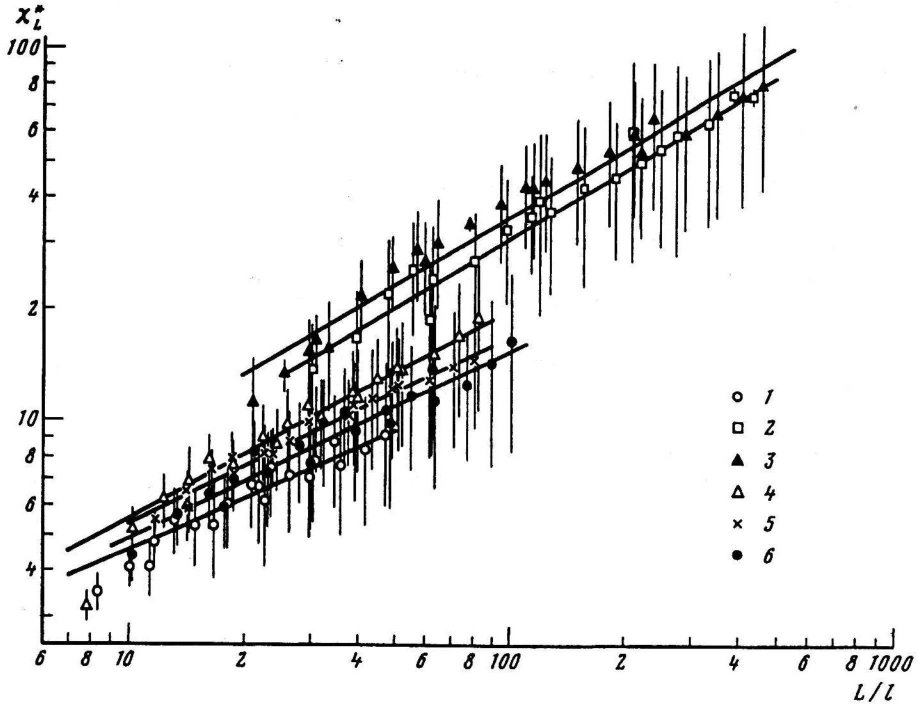

Figure 1 Critical rupture concentration as a function of cell size L and mean rupture length l; vertical bars show the standard deviations. *1* – Greece, *2* – Northeast China, *3* – Southwest China, *4* – Kirgizia, *5* – Turkmenia, *6* – Kamchatka.

of averaging for fractal objects. It is this relation that can to a first approximation account for the observed scale factor of the concentration criterion.

As follows from (9), the observed critical value of the concentration criterion depends, among other things, on the fractal dimension of the earthquake-generating faults involved. The fractal dimension varies in time and space in response to seismicity changes [6], [9]. In this context when concentration parameters are compared between different regions or different time periods, the variation of the fractal dimension of the hypocenter sample should be estimated and used for corrections according to (9).

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