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STATISTICAL ESTIMATION OF THE MAXIMUM POSSIBLE EARTHQUAKE MAGNITUDE FOR THE BAIKAL RIFT ZONE

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The Bayesian approach is used to estimate main seismic parameters: M_{max} — maximum possible regional magnitude; λ — seismic-activity rate; and b — slope of the plot for the magnitude frequency law. The suggested method allows one to use catalogs with varying lower magnitude completeness threshold as well as historical catalogs. The quantiles of $M_{max}(T)$ are estimated, where $M_{max}(T)$ is the maximum magnitude of an earthquake that will occur within a future time interval T. Also, the magnitude uncertainties (standard deviations) are established. The method is applied to estimate M_{max} and $M_{max}(T)$ in the Baikal Rift Zone. This estimation gives $M_{max} = 8.07 \pm 0.47$.

Maximum regional magnitude, Bayesian statistics, magnitude frequency law

INTRODUCTION

In past decades, considerable efforts of seismologists have been directed to obtaining the maximum possible magnitude of an earthquake in a seismically active region (see [1–5] as well as [6], where the publications dealing with this problem are reviewed in detail). Confidence intervals for M_{max}, with errors of magnitude measurement ignored, were obtained in [4, 5] on the basis of fiducial approach. The maximum possible regional magnitude M_{max} is one of the most important parameters of seismic regime on which the estimate of seismic hazard depends quite essentially. Because violent earthquakes occur rarely, estimates of M_{max} are very uncertain. In addition, the earthquake magnitudes to be measured depend on a great number of uncontrollable factors: peculiar features of focal mechanism, deviations of the medium structure from a model along propagation of seismic waves, random noise, etc. To take these effects into account, Tinti and Mulargia [7] introduced concepts of apparent (i.e., observed) magnitude ($\overline{M}$) and real magnitude (M). They are related as follows:

$$\overline{M} = M + \varepsilon, \tag{1}$$

where ε is a random quantity which must reflect uncertainty of all above-mentioned factors. Introduction of apparent magnitude had very important consequences. On the one hand, the presence of the random term ε in (1) reduces the accuracy of the estimate of the maximum real magnitude; on the other hand, it smoothes (regularizes) the density of the probability of the observed magnitudes, which leads to positive effects in statistical estimation.

To estimate M_{max}, we propose to use the Bayesian statistics with relation (1) between real and apparent magnitudes. This approach take into account uncertainties in estimates of other parameters of a seismic process (inclination of a plot of earthquake recurrence, b, and earthquake flow intensity λ) and also estimates an uncertainty of M_{max}. Our presentation of the Bayesian statistics is given in [8]. Here we develop this approach and apply it to a seismic regime with periodical components and gaps in a record of earthquakes. We will apply this procedure to estimate M_{max} in the Baikal seismic zone. The Bayesian statistics permits us to estimate not only parameters of a seismic process, M_{max}, b, and λ, but also any function of them. An example is the function of distribution $\Phi_T(x)$ (more exactly, a family of its quantiles) of a random maximum magnitude of earthquakes $M_{max}(T)$ within an arbitrary future interval T. We can take both apparent and real magnitudes. The quantity $M_{max}(T)$ seems to be of even greater practical interest than M_{max}, because many applied problems of seismic hazard are connected with estimation of seismic hazard within a finite interval of time.

Lyubushin et al. [9] have made a spectral analysis of a sequence of earthquakes in the Baikal Rift Zone with the aim of searching for possible recurrences in the seismic regime. Three significant recurrences have been revealed, with periods of about $\tau = 2$, 5, 10, and 40 years. The Bayesian method for estimating $M_{max}(T)$ used in this work takes into account these recurrences.

BAYESIAN STATISTICS IN ESTIMATING PARAMETERS OF A SEISMIC PROCESS

Let there be an earthquake catalog cleared of aftershocks. A method of space-time windows [10] or a more complicated method taking into account the nonspherical character of distribution of aftershocks around the hypocenter of the main shock [11] may be used for this procedure. The earthquake sequence, with aftershocks removed, may be considered Poissonian with a good approximation.

We will denote the function of distribution of a real magnitude of the main shock that occurred at moment t by $F_t(x|\theta)$, where θ is a vector parameter. Further we will restrict ourselves to consideration of the Gutenberg-Richter law [12], depending on two parameters: b, inclination of recurrence plot, and μ, the maximum possible magnitude (M_{max}). This restriction is made for the sake of simplicity. The proposed technique for estimating $\mu = M_{max}$ is appropriate for any other parametric family of distributions, because the Bayesian methods are essentially numerical and do not use any analytical properties of distribution of magnitudes. Thus,

$$F_t(x\,|b,\mu) = \begin{cases} 0, \; x < M_t; \\ (10^{-bM_t} - 10^{-bx})/(10^{-bM_t} - 10^{-b\mu}), \; M_t \le x \le \mu; \\ 1, \; x > \mu; \end{cases} \tag{2}$$

where M_t is the lower threshold of representative record of earthquakes. It can depend on time, thus permitting treatment of time-irregular catalogs of earthquakes, including historical ones. The time dependence of M_t may be considered known, because, as a rule, the researchers themselves choose it with a certain reserve, warranting a safe record of earthquakes with magnitudes $M > M_t$. As the M_t threshold depends on time, the intensity of the Poissonian sequence of main shocks (average number of main shocks in a unit time) $\lambda = \lambda_t$ also depends on time. To describe this dependence, we denote by λ_0 the intensity corresponding to a certain fixed magnitude M_0. Then

$$\lambda_t = \lambda_0(10^{b\mu} - 10^{bM_t})/(10^{b\mu} - 10^{bM_0}). \tag{3}$$

Equation (3) also describes possible no-record periods with corresponding values $M_t \equiv \mu$ and $\lambda \equiv 0$. Suppose, we observe apparent magnitudes of the main shocks

$$M_1 = x_1, M_2 = x_2, ..., M_n = x_n,$$

corresponding to times $t_1, ..., t_n$ within the time interval ($-S$, 0). We denote the vector of observed magnitudes by **x**:

$$\mathbf{x} = (x_1, ..., x_n).$$

In accordance with the Bayesian statistics, the composite vector of the parameters (λ_0, b, μ) will be considered a random vector that has an *a priori* distribution. As usual, we will choose this *a priori* distribution to be uniform within some intervals *a fortiori* containing unknown values of the parameters:

$$\lambda_1 \le \lambda_0 \le \lambda_2;\; b_1 \le b \le b_2;\; \mu_1 \le \mu \le \mu_2. \tag{4}$$

Thus, we have four random quantities λ_0, b, μ, and **x**, the latter being a vector. The first three quantities are unknown (nonobserved); the **x** vector values are known. The *a priori* probability density of the λ_0, b, μ, and **x** quadruple is denoted by $g(u, v, w, \mathbf{z})$. The conventional density of the λ_0, b, μ triple at a given **x** is denoted by $g(u, v, w|\mathbf{x})$, with the conventional variable **x** being separated by a bar from the arguments u, v, and w. According to the Bayesian theorem, we have [13]:

$$g(u,v,w \mid \mathbf{x}) = g(u,v,w,\mathbf{x}) \left\{ \int_{\lambda_1}^{\lambda_2} \int_{b_1}^{b_2} \int_{\mu_1}^{\mu_2} g(\lambda_0, b, \mu, \mathbf{x})\, d\lambda_0\, db\, d\mu \right\}^{-1}. \tag{5}$$

It is more convenient to consider the integrals in (5) numerically. With a knowledge of *a posteriori* density (5), we can calculate the Bayesian estimate h for any function $h(\lambda_0, b, \mu)$ of unknowm parameters λ_0, b, and μ.

$$\tilde{h} = \int_{\lambda_1}^{\lambda_2} \int_{b_1}^{b_2} \int_{\mu_1}^{\mu_2} h(u,v,w)\, g(u,v,w \mid \mathbf{x})\, du\, dv\, dw. \tag{6}$$

Integration in this expression is also performed numerically. In particular, if we choose $h(\lambda_0, b, \mu) \equiv \lambda_0$, $h(\lambda_0, b, \mu) \equiv b$, and $h(\lambda_0, b, \mu) \equiv \mu$, then, using (6), we will obtain Bayesian estimates for λ_0, b, and μ, respectively.

Thus, an explicit expression for the function $g(u, v, w \mid \mathbf{z})$ from (5) remains to be derived under the assumed conditions.

In accordance with (1), the probability density of the apparent magnitude $g_t(x, \mid b, \mu)$ is equal to convolution of the density of real magnitude $f_t(x, \mid b, \mu)$ and density $n_t(x)$ of a random value ε:

$$g_t(x, \mid b, \mu) = \int f_t(x - y \mid b, \mu)\, n_t(y)\, dy. \tag{7}$$

We allow for a possible change in the scatter of the random value ε with time and denote it in density by subscript t: $n_t(x)$. This change is quite natural, at least, because the exactness of magnitude measurement may be significantly increased with time. For example, it is natural to believe that the magnitude determination accuracy in the historical catalogs, where the magnitudes were estimated from macroseismic data, are significantly lower than the magnitude accuracy obtained by instrumental determination, which, in turn, can considerably change as the seismic network develops.

As to the form of probability density $n_t(x)$, the results of numerical experiments show [8] that the form of distribution $n_t(x)$ is of little importance and only the magnitude of the standard deviation of this distribution is important. Therefore, we took the simplest form – uniform distribution within some interval $(-\Delta_t, \Delta_t)$:

$$n_t(x) = \begin{cases} 1/2\Delta_t, & |x| \le \Delta_t; \\ 0, & |x| > \Delta_t. \end{cases} \tag{8}$$

The standard deviation of distribution (8) is equal to $\Delta_t/\sqrt{3}$. Using (7), we have:

$$g_t(x, \mid b, \mu) = \frac{1}{2\Delta_t} \int f_t(x - y \mid b, \mu)\, dy$$

$$= \frac{1}{2\Delta_t} [F_t(x + \Delta_t \mid b, \mu) - F_t(x - \Delta_t \mid b, \mu)]. \tag{9}$$

Thus, (9) gives an explicit expression for the probability density of an apparent magnitude.

Parameter λ_0 will enter the probability density $g(u, v, w, \mathbf{z})$ through the probability $p_n(\lambda_0)$ of an event implying that a Poissonian process with variable intensity λ_t (3) within the observation interval $(-S, 0)$ will give n events at the times $t_1, ..., t_n$. The probability $p_n(\lambda_0)$ is shown [14] to be proportional to the following expression:

$$p_n(\lambda_0) \sim \exp\left\{ -\int_{-S}^{0} \lambda_u\, du \right\} \prod_{k=1}^{n} \lambda_k, \tag{10}$$

where λ_k denotes the intensity λ_t at the time $t = t_k$. Thus, assuming that the magnitudes are distributed independently of the event times, we can, with an accuracy to a multiplier C, write out an explicit expression for the density $g(u, v, w, \mathbf{z})$:

$$g(u,v,w,\mathbf{z}) = C \exp\left\{ -\int_{-S}^{0} \lambda_u\, du \right\} \prod_{k=1}^{n} \lambda_k g_k(x_k \mid b, \mu), \tag{11}$$

where the multiplier $g_k(x_k \mid b, \mu)$ is obtained from (9) at $t = t_k$. Of course, (11) is valid within the parallelepiped defined by inequalities (4), and beyond it $g(u, v, w, \mathbf{z})$ is to equal zero. Now, substituting $g(u, v, w, \mathbf{z})$ into (5) yields the desired *a posteriori* density $g(u, v, w \mid \mathbf{z})$.

It is significant that using (6) we can obtain not only a Bayesian estimate of, e.g., parameter μ but also a Bayesian estimate of its dispersion (this statement is certainly valid for any other parameter or for a function of these parameters). For this purpose, we must take, as function h, the expression

$$h(\lambda_0, b, \mu) = (\mu - \tilde{\mu})^2, \tag{12}$$

where $\tilde{\mu}$ stands for the Bayesian estimate of parameter μ. Substituting (12) into (6) yields the Bayesian estimate of the dispersion of the parameter. As shown by numerical calculations [8], these estimates are close to the corresponding estimates obtained from the equations of maximum likelihood with the use of the bootstrap method.

Now we estimate quantiles of the distribution of $M_{max}(T)$, the maximum magnitude of an earthquake which will occur within a given future interval of time $(0, T)$.

We will not restrict ourselves to the case when the conditions of earthquake record do not change as compared with the presently existing conditions. This corresponds to

$$M_t = \text{const},\ \lambda_t = \text{const},\ 0 \le t \le T. \tag{13}$$

Our procedure permits us to take both apparent and real magnitudes as $M_{max}(T)$. Which of these magnitudes is more adequate depends on a particular practical problem. On the one hand, we can observe only apparent magnitudes. Therefore, if we wish to compare our forecast with some particular results, an apparent magnitude is required to be used. On the other hand, it is natural to believe that real magnitude is more intimately involved with the seismic effect than the apparent one. In this context, a real magnitude is more adequate. Nevertheless, it is significant that all the existing formulas of regression type that express a seismic effect (say, peak acceleration of ground) were derived for apparent magnitudes, whereas for real magnitudes they are required to be derived again.

Taking into account the above, we restrict ourselves to consideration of $M_{max}(T)$ for apparent magnitudes provided that the time function $F_t(x \mid b, \mu)$ in (2) disappears and, therefore, time function of density of apparent magnitude (7) disappears as well. For a Poissonian sequence of main shocks it is easy to derive a formula for the distribution function $\Phi_t(x \mid b, \mu)$ of the maximum magnitude of earthquakes which will occur within a future time interval $(0, T)$ (see [5]):

$$\Phi_t(x \mid b, \mu) = P\{M_{max}(T) < x\} = \frac{\exp[\lambda_0 T G(x \mid b, \mu)] - 1}{\exp[\lambda_0 T] - 1}, \tag{14}$$

where $G(x \mid b, \mu)$ means the function of distribution of apparent magnitude:

$$G(x \mid b, \mu) = \int_{-\infty}^{x} g(u \mid b, \mu)\, du.$$

Function of distribution (14) is valid provided that within the interval $(0, T)$ at least one earthquake will occur (otherwise, it is necessary to define what $M_{max}(T)$ means). In the cases of practical interest, this requirement is fulfilled with a high probability. Quantile x_α of level α of distribution (14) is defined as the root of

$$\Phi_t(x \mid b, \mu) = \alpha,\ 0 < \alpha < 1. \tag{15}$$

Thus, the random value of $M_{max}(T)$ will not exceed quantile x_α with the probability α. The quantile of the level $\alpha = 0.5$ is called median. To specify quantiles at all α, $0 < \alpha < 1$, is to specify functions of distribution. The quantile x_α as the root of (15) depends on parameters b and μ, i.e., is a function of b and μ:

$$x_\alpha = x_\alpha(b, \mu). \tag{16}$$

Taking $x_\alpha(b, \mu)$ as the function $h(\lambda_0, b, \mu)$ and substituting it into (6) yields a Bayesian estimate for quantile $\tilde{x}_\alpha$, and taking $h(\lambda_0, b, \mu) = (x_\alpha - \tilde{x}_\alpha)^2$ yields a Bayesian estimate of the dispersion of estimate $\tilde{x}_\alpha$. Given below are examples of these estimates.

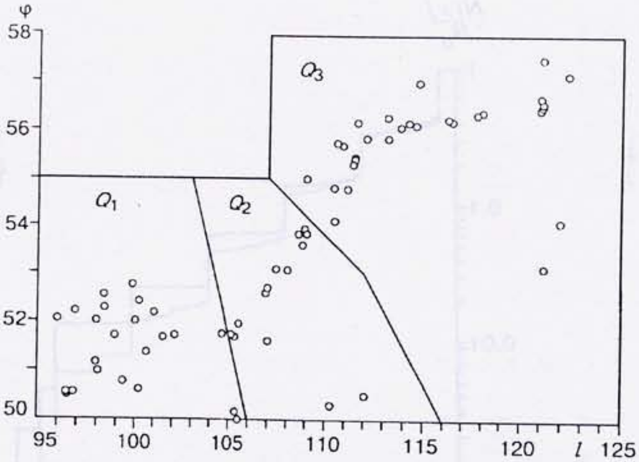

Fig. 1. Epicenters of main shocks with magnitudes M ≥ 5.0 for the period 1952–96 (67 shocks). Three subzones (Q_1, Q_2, and Q_3) of the Baikal Rift Zone are indicated.

ESTIMATES OF $M_{\max}$ AND $M_{\max}(T)$

At first, we consider the estimate of the true maximum possible magnitude $M_{\max}$ in the Baikal Rift Zone on the basis of the catalog of earthquakes for 1952–96. We can suppose that, during this period, the record of earthquakes with apparent magnitudes of $M \geq 5.0$ was reliable and the accuracy of magnitude measurement was virtually kept at the same level. Aftershocks were eliminated from the catalog by means of space-time windows placed at points of main shocks. The window sizes were dependent on magnitude and were chosen in accordance with the recommendations from [10]. A total of 67 main earthquakes with $M \geq 5.0$ occurred for this period. We have chosen the latitude $\varphi = 50°$ and the longitude $l = 95°$ as the southern border of the Baikal Rift Zone, separating it from the North-Mongolian zone, and the western border, respectively. Although there are violent earthquakes (e.g., 23.07.1905: $\varphi = 49.30°$, $l = 96.20°$, $M = 8.2$; 5.01.1967: $\varphi = 48.09°$, $l = 102.90°$, $M = 7.8$) near these borders but beyond the zone under study, we decided to delineate it by these borders in order to have for analysis seismically uniform earthquakes of the Baikal Rift Zone. Calculations have shown, however, that the addition of these violent earthquakes will only insignificantly change the final estimates of the maximum magnitudes: about 0.1–0.2 with a scatter of 0.35–0.45. The location of earthquake epicenters in the region under consideration is shown in Fig. 1. The greatest apparent magnitude $M = 7.8$ was recorded for the 20 April 1989 earthquake: $\varphi = 57.16°$, $l = 122.30°$. A normalized plot of repeatedness (in accumulated form) is given in Fig. 2. The lower magnitude threshold of representative record was chosen to be equal to 5: $M_t \equiv 5.0$. The standard deviation of the random value ε was taken equal to 0.29, which corresponds to the uniform distribution of ε within the magnitude interval (–0.5, 0.5). This interval seems to correspond to a real accuracy of the magnitude estimation in the used catalog of earthquakes.

To choose *a priori* intervals (4), we proceeded as follows. The lower possible threshold of the parameter $M_{\max}$ was the maximum observed magnitude ($M = 7.8$) from which the maximum magnitude error 0.5 was subtracted. As a result, the lower boundary of the *a priori* interval for $M_{\max}$ appeared equal to 7.3. The upper boundary was taken as equal to 9.5, which seems to overlap with a great reserve the possible $M_{\max}$ for this region.

Choosing an *a priori* interval for b, we first established the center of this interval in a rough approximation and then moved right and left for a value equal to three standard deviations of this rough estimate. As such a coarse estimate, we used the estimate of maximum similitude calculation of which implied that $M_{\max}$ is equal to the greatest observed magnitude $M = 7.8$ (see in detail [15]). As a result, we obtained for parameter b the *a priori* interval (0.19, 0.57). Recall that these estimates are obtained for the aftershock-free catalog. This operation is expected to result in some decrease in inclination of the repeatedness plot (see Fig. 2, where a plot of repeatedness of all shocks is given for comparison).

Choosing an *a priori* interval for parameter λ_0, we took, as a coarse approximation of this parameter and its standard deviation, a simple Poissonian model where parameter λ_0 has no relation to other unknown parameters. As a result, the center of the *a priori* interval was estimated at $\lambda_0 = 67/45 = 1.49$ (1/year), and the triple standard deviation appeared to be equal to 0.54. These values led to the *a priori* interval (0.93; 2.03).

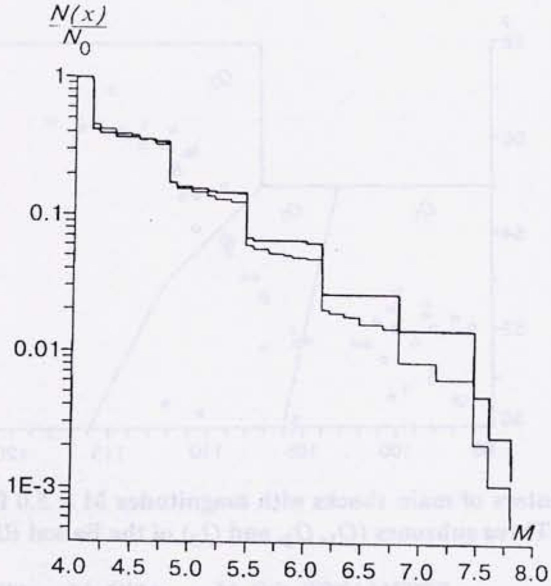

Fig. 2. A normalized curve of earthquake repeatedness in the stacked form for the period 1952–96. ***N(x)*** **is the number of earthquakes with M ≥ x;** N_0 **is a total of earthquakes; thin line — with aftershocks; solid line — without aftershocks.**

Thus, for three unknown parameters, the *a priori* intervals were:

$$0.93 \le \lambda_0 \le 2.03;\; 0.19 \le b \le 0.57;\; 7.3 \le M_{max} \le 9.5.$$

To check that a further increase in these *a priori* intervals will not change essentially the Bayesian estimates, we calculated one additional version of estimates where the *a priori* intervals for parameters λ_0 and b were increased from three to five standard deviations. For parameter M_{max}, the *a priori* interval was left the same, because its values in the primary version were taken with a great reserve, and their further increase seems to bring the value of M_{max} beyond the possible reasonable limits for this parameter. Thus, in the additional version the *a priori* intervals for the parameters were:

$$0.56 \le \lambda_0 \le 2.39;\; 0.10 \le b \le 0.70;\; 7.3 \le M_{max} \le 9.5.$$

The Bayesian estimates for the primary and additional versions of the *a priori* intervals are given in Table 1. We see that the difference in estimates is very small. Thus, we can believe that the choice of the *a priori* intervals in the primary version was made satisfactorily.

The above-described procedure provides the following estimate for the real greatest possible magnitude (hereafter, the discussion concerns the primary version of *a priori* intervals):

$$M_{max} = 8{,}07 \pm 0{,}47. \qquad (17)$$

The estimate itself, 8.07, appeared to be somewhat greater than the maximum observed magnitude 7.8. A quite considerable standard deviation of the estimate indicates that the real M_{max} may actually be more than 8.07. It is natural to believe that the most probable location of a possible devastating earthquake will be around the epicenter of the 20 April 1989 earthquake, although other places are not to be ruled out either. To consider in detail our conclusions on spatial distribution, we subdivided the whole region of the Baikal Rift into three parts: Q_1, Q_2, and Q_3 (see Fig. 1). This subdivision of the region under study naturally led to a decreased number of earthquakes in its separate parts and, hence, to elevated uncertainty in the estimate of M_{max}. Therefore, we had to bring down the lowest magnitude M_l to 4.5. A more detailed subdivision of the territory of the Baikal Rift Zone seems to be nonpermissible because of insufficient number of moderate and violent earthquakes. Our experience hints that the Bayesian estimates of M_{max} give a more or less reliable result when the sample volume n is about n = 50 and more.

As a result of estimation of real M_{max} over separate zones, we have obtained values given in Table 2. We see that the obtained estimates M_{max} over zones are quite high and the accuracy of estimates is low. Given

Table 1

Comparison of Estimated Parameters for Two Versions of *a Priori* Regions (Primary and Additional)

Versions	M_{max}	b	λ	Median		90% quantile	
				$M_{max}(10)$	$M_{max}(50)$	$M_{max}(10)$	$M_{max}(50)$
Primary	8.07	0.41	1.490	7.43	8.00	8.07	8.33
Additional	8.12	0.43	1.485	7.41	8.01	8.08	8.36

Table 2

Estimates of Real Magnitude M_{max}

Region	Observed M_{max}	Estimated M_{max} (real)	n
Q_1	7.6	8.39 ± 0.64	41
Q_2	6.8	7.82 ± 0.90	43
Q_3	7.8	8.41 ± 0.53	61

these estimates, we cannot rule out the possibility of a violent earthquake with $M \geq 8.0$ in each of the regions, but such earthquakes are most probable in Q_1 and Q_3. Perhaps, the lower estimate of M_{max} for Q_2 is explained by greater fragmentation of geological blocks in this zone.

Now we estimate apparent $M_{max}(T)$. For the whole region we have obtained estimates for quantiles $\tilde{x}_\alpha$ shown in Fig. 3 for $T = 10$ and 50 years. The value $T = 50$ years somewhat exceeds the time interval from the catalog we used (45 years) and, therefore, some doubts are possible about the reliability of the estimates obtained. These doubts are somewhat equivalent to the doubts about the adequate description of the law of earthquake repeatedness in the region of most violent events by the Gutenberg-Richter formula [2]. Fortunately,

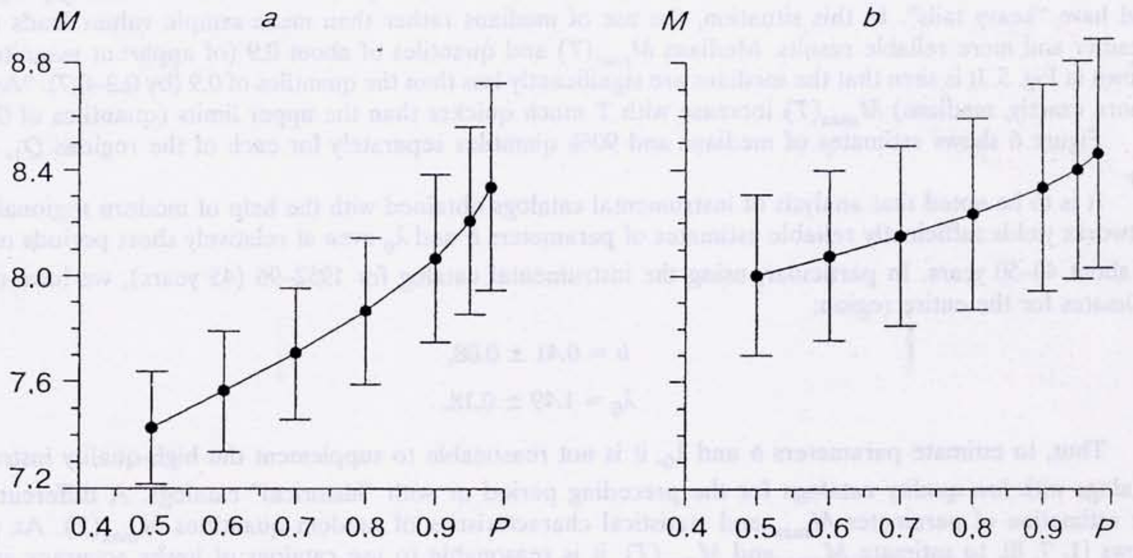

Fig. 3. Estimated quantiles of the predicted random values. *a* – $M_{max}(10)$, *b* – $M_{max}(50)$ (apparent magnitude). From the 1952–96 catalog. Bars show standard deviations. The significance level of *P* quantile is indicated along the axis of abscissae.

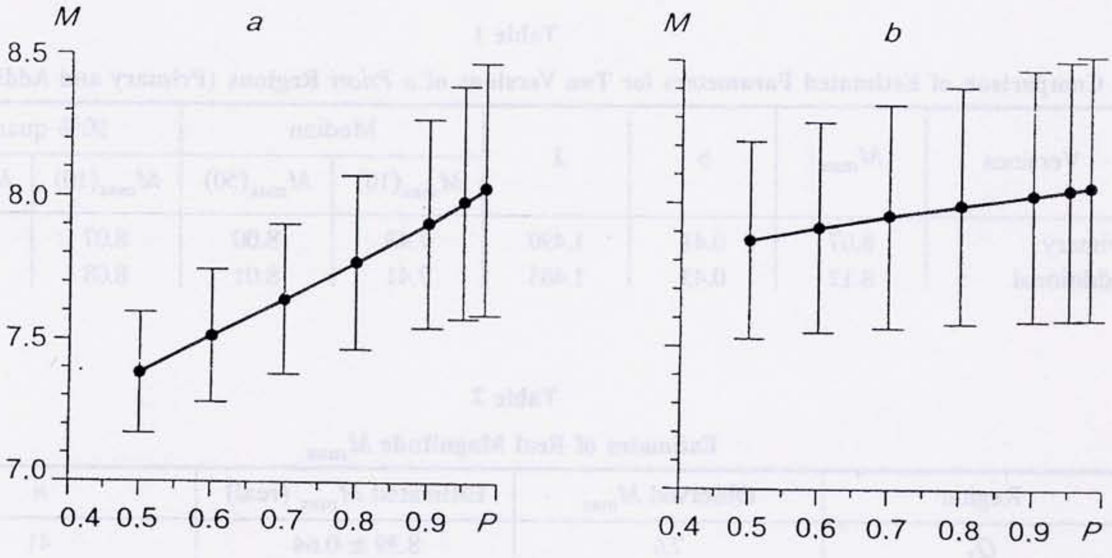

Fig. 4. The same as in Fig. 3 but for real magnitudes.

this conclusion is alleviated by the fact that the random quantity ε we have introduced into (1) smoothes the law of recurrence of apparent magnitudes, which has not a sharp cut-off present in (2). Figure 3 shows that, if we want to estimate $M_{max}(T)$ with high confidence (e.g., with 90% level of confidence), we obtain rather high values: from 8.07 at $T = 10$ years to 8.33 at $T = 50$ years. The accuracy of these estimates is very low: Their standard deviation varies from 0.20 to 0.40. As a rule, these standard deviations are significantly less than those for the estimate of M_{max} (0.47). Quantile $x_{0.9}$ increases with T but slightly. This suggests that the catalog under analysis is quite representative. Figure 4 shows the same plots of quantiles $\tilde{x}_\alpha$ as presented in Fig. 3 but for a real magnitude. We see that the real magnitude is somewhat lower than the apparent one. This difference is better visible for great values of T. Standard deviations for real magnitude are significantly greater: Their values reach 0.45.

The quantile of level 0.9 gives quite a reliable estimate of the parameter (with 90% confidence level). If we want to follow the behavior of the parameter "average", it is natural to consider a median ($\alpha = 0.5$). It is to be noted that in the case under consideration, the distributions, of quantile estimates are strongly asymmetric and have "heavy tails". In this situation, the use of medians rather than mean-sample values leads to much steadier and more reliable results. Medians $M_{max}(T)$ and quantiles of about 0.9 (of apparent magnitude) are shown in Fig. 5. It is seen that the medians are significantly less than the quantiles of 0.9 (by 0.2–0.7). "Averages" (more exactly, medians) $M_{max}(T)$ increase with T much quicker than the upper limits (quantiles of 0.9).

Figure 6 shows estimates of medians and 90% quantiles separately for each of the regions Q_1, Q_2, and Q_3.

It is to be noted that analysis of instrumental catalogs obtained with the help of modern regional seismic networks yields sufficiently reliable estimates of parameters b and λ_0 even at relatively short periods of record of about 40–50 years. In particular, using the instrumental catalog for 1952–96 (45 years), we have obtained estimates for the entire region:

$$b = 0.41 \pm 0.08,$$

$$\lambda_0 = 1.49 \pm 0.18.$$

Thus, to estimate parameters b and λ_0, it is not reasonable to supplement the high-quality instrumental catalogs with low-quality catalogs for the preceding period or with "historical" catalogs. A different case is the estimation of parameter M_{max} and statistical characteristics of random quantities $M_{max}(T)$. As practice shows [1, 7, 8], to estimate M_{max} and $M_{max}(T)$, it is reasonable to use catalogs of lower accuracy and even "historical" catalogs, despite their incompleteness and inaccuracy. Of course, these drawbacks are to be taken into account accordingly. We have considered the version of M_{max} estimation with the use of additional catalogs

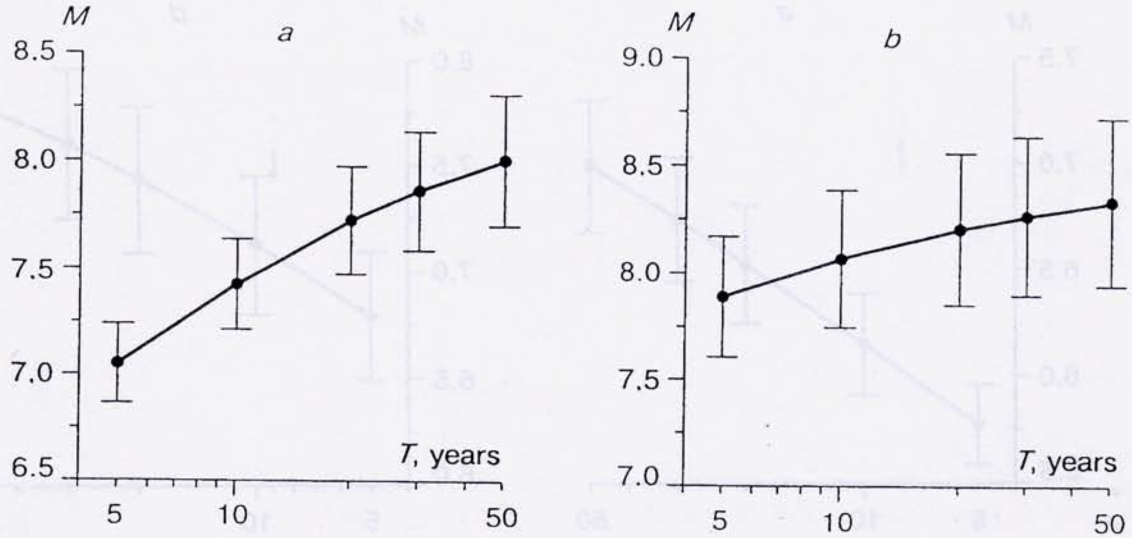

Fig. 5. Dependence of estimates. a – Median and b – quantile of level 0.9 of $M_{max}(T)$ as a function of time (apparent magnitudes). From the 1952–96 catalog. Bars indicate standard deviations.

for the entire Baikal Zone [16] for 1725–1942, parameters b and λ_0 being estimated on the basis of the 1952–96 catalog only.

The lower limit of the representative record of M_t and the interval of variation in error $\varepsilon(\Delta t)$ was specified as follows:

$M_t = 6.0$	$\Delta_t = 1.5$	1725–1900	M_{max}(real) = 8.2
$M_t = 5.5$	$\Delta_t = 1.0$	1901–1942	M_{max}(real) = 7.75
$M_t = 5.0$	$\Delta_t = 0.5$	1952–1996	M_{max}(real) = 8.13

As a result, instead of estimate (17) for the real M_{max}, we have

$$M_{max} = 8.15 \pm 0.38.$$

The maximum magnitude $M = 8.2$ in the historical catalog refers to the 01.02.1725 earthquake; $\varphi = 56.5°$, $l = 118.5°$ (zone Q_3). The addition of historical earthquakes increased the estimate by 0.08 magnitude and decreased the standard deviation of the estimate from 0.47 to 0.38. These changes may be considered insignificant, although a decrease in the standard deviation by 0.1 is quite noticeable. In other cases, however, when the maximum magnitude of the historical catalog considerably exceeds the maximum magnitude of the instrumental catalog, their combination can considerably change the estimated M_{max}. In any case, the addition of the historical catalog decreases the statistical uncertainty of the M_{max} estimation as a function of accuracy and informativeness of this catalog.

As mentioned above, analysis of possible cycles in the seismic regime of the Baikal Rift Zone was given in [9]. In essence, it was the same procedure with the only difference being that for Poissonian processes with a variable intensity $\lambda(t)$, instead of parameter $\lambda_0 T$ (see (14)) the integral

$$\int_0^T \lambda(t)\, dt$$

was used.

Variable intensity was described by the periodical function

$$\lambda(t) = \alpha[1 + p\cos(\omega t + \varphi)],\ p \le 1,$$

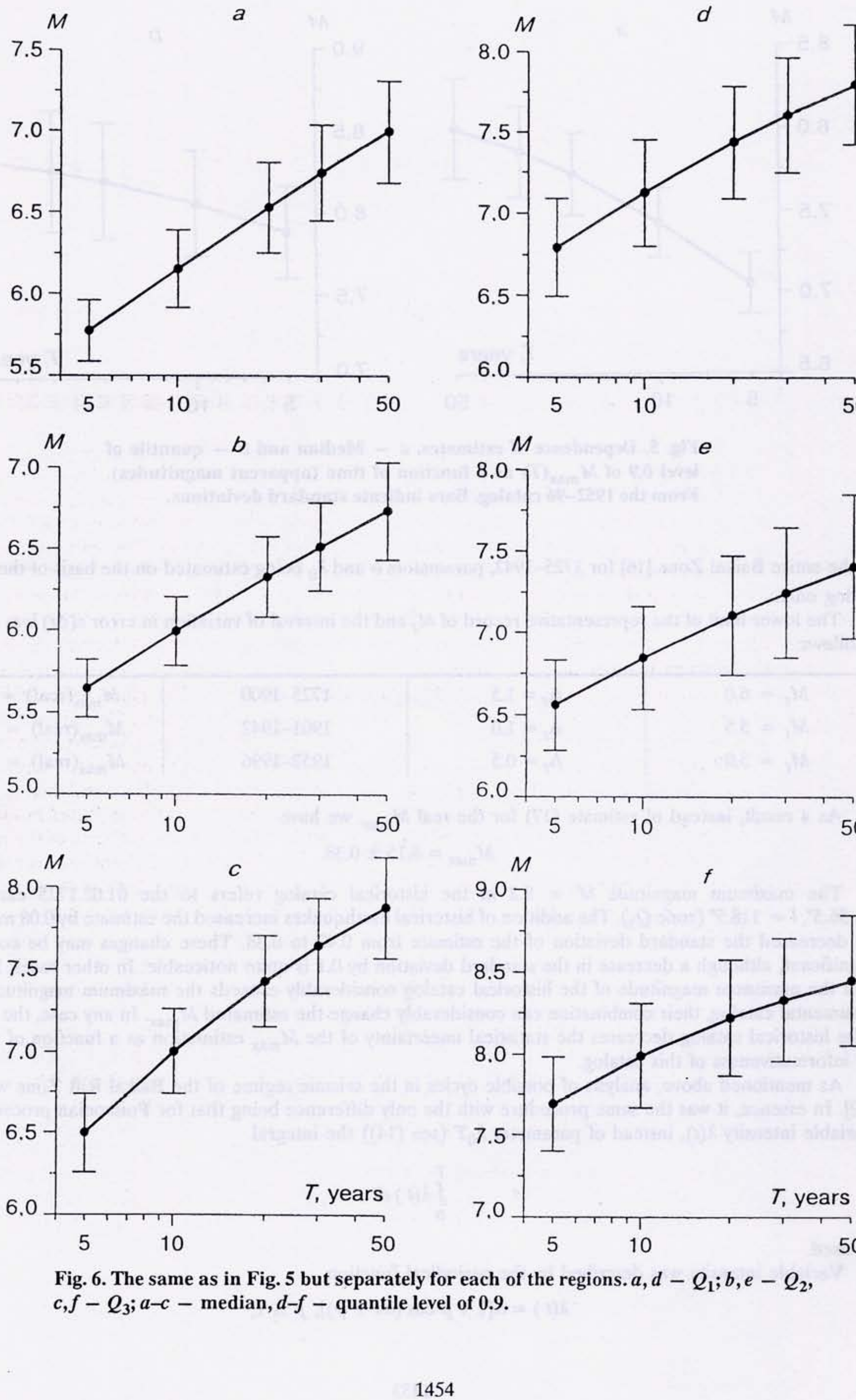

Fig. 6. The same as in Fig. 5 but separately for each of the regions. ***a, d*** **–** Q_1**;** ***b, e*** **–** Q_2**,** ***c, f*** **–** Q_3**;** ***a–c*** **– median,** ***d–f*** **– quantile level of 0.9.**

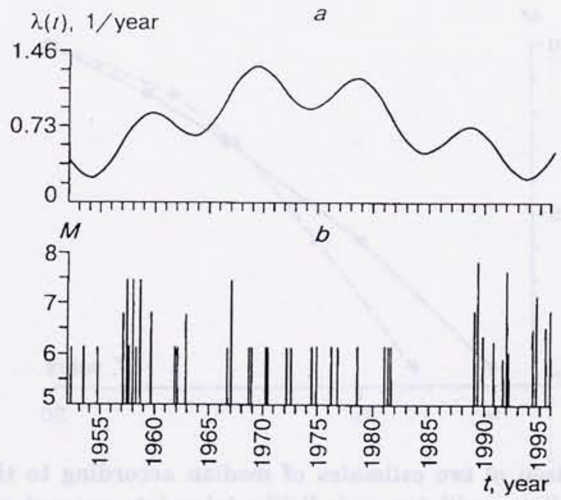

Fig. 7. Intensity λ_t (average number of main shocks with M ≥ 6.0 a year) estimated with two periodicities of seismic regime taken into account (periods of $\tau = 10$ and 40 years).

(for more detail see [9]), whose parameters were estimated by the method of maximum likelihood.

Several periodicities have been recognized, the most significant being those with periods $\tau = 2.5$ and 10.5 years. The 10.5 year periodicity seems to be related to the known periodicity of solar activity and was earlier noticed by a number of authors. The nature of the 2.5 year periodicity remains to be investigated. In addition, one more periodicity seems to exist, with a period of about 40 years. Its significance level is not very high (about 50%), but it is corroborated by the presence of a similar periodicity in the global catalog [9] as well as by the fact that it was mentioned by other authors [17, 18]. It is natural to make an attempt to invoke these periodicities to estimate quantiles $M_{max}(T)$. The Bayesian approach seems to be promising. It should be noted that the periodicity $\tau = 2.5$ years is important only for quantiles $M_{max}(T)$ at $T < 2.5$ years. However, these short-term predictions of $M_{max}(T)$ seem to be of little practical importance. On the other hand, at $T \geq 10$, the periodicity $\tau = 2.5$ years will be considerably averaged and will not exert a significant effect on the increase or decrease in the average number of earthquakes for the prediction period T. The same is true for the periodicity of $\tau = 10$ years in estimation of quantiles $M_{max}(T)$, when $T \geq 20-30$. Therefore, we consider only the case $T = 10$ years, restricting ourselves only to periodicities of 10 and 40 years. Taking into account parameter estimates of these periodicities obtained in [9], we plotted the intensity (Fig. 7), i.e., average number of main shocks in unit time with magnitudes of $M \geq 6.0$. We see that, relative to the constant background of a Poissonian process, the effect of the 10- and 40-year periodicities is quite marked, and their relative amplitudes are equal to 0.56 and 0.85, respectively. However, their effect on the prediction of future magnitudes $M_{max}(10)$ appeared to be insignificant. With these periodicities taken into account, the estimate of the median $M_{max}(10)$ for the nearest 10 years will be somewhat different from what is given a Poissonian process with constant intensity. Figure 8 compares two plots of $M_{max}(T)$ medians of apparent magnitude with the periodicities taken and not taken into account. We see that as T increases they approach one another, and at $T > 10$, the difference between them is insignificant.

DISCUSSION

Now, some words about the seismological sense of M_{max} and $M_{max}(T)$. Parameter M_{max} was introduced by seismologists for the parametric description of the law of earthquake repeatedness (see, e.g., [12, 19–21]). It was necessary to limit from above possible earthquake magnitudes which cannot be arbitrarily large because of natural physical restrictions. Yet the sharp cut-off of the law of earthquake repeatedness is difficult to be grounded physically, because if an earthquake with magnitude M_{max} is possible, one can hardly substantiate that an earthquake with a magnitude, e.g., of $M_{max} + 0.05$ is absolutely impossible. To overcome this difficulty, some authors [20, 21] assume that in the repeatedness law the magnitude can be arbitrarily great but with an infinitesimal probability. Introducing parameter M_{max}, the authors usually do not specify to which time interval it belongs. However, it is one thing with time intervals of about 10^3–10^4 years and quite another matter with

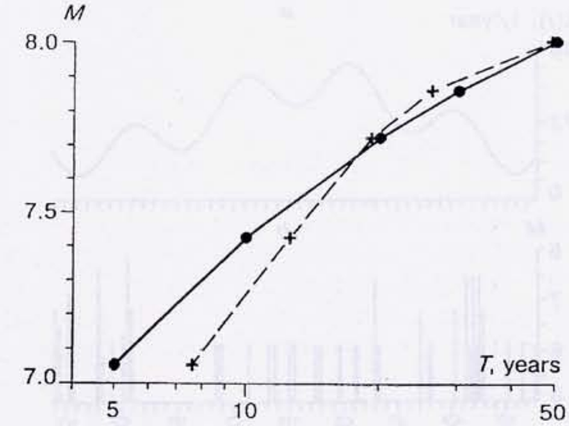

Fig. 8. Comparison of two estimates of median according to the 1952–96 catalog. Dashed line – with two periodicities taken into account, solid line – no periodicities taken into account. Periods of $\tau = 10$ and 40 years.

periods of tens and hundreds of Ma, within which episodes of "fragmentation" and "collision" of entire continents are possible. There is no commonly accepted and logically defensible time interval to which M_{max} values usually defined in various regions belong. One must also bear in mind that magnitude was introduced by Ch. Richter as a rather coarse and simplified means to characterize the earthquake strength not taking into account many characteristics of earthquake foci such as focal mechanism, substance composition and texture of rocks, vibration spectrum, etc. Besides, parameter M_{max} is estimated, as a rule, with great statistical uncertainty, because violent earthquakes occur rarely. Thus, parameter M_{max} must be considered a coarse technical means for description of the law of earthquake repeatedness rather than a parameter of clear physical and geological sense.

On the other hand, it is quite natural to define $M_{max}(T)$ as a random quantity to be observed within a future time interval T. Unlike M_{max}, time interval for $M_{max}(T)$ is explicit. In addition, in practical applications to problems of seismic regionalization and estimation of seismic hazard, $M_{max}(T)$ is more appropriate than M_{max}. Indeed, to calculate seismic hazard, the probability of violent shakes for a period T is required to be known. A possible objection is that M_{max} is necessary for calculation of the most important characteristic of seismic hazard, $A_{max}(T)$, the maximum earthquake-induced acceleration of ground. The situation is, however, different. We have developed a new technique to estimate $A_{max}(T)$ statistically, without direct estimation of M_{max} (for details see [22, 23]). Thus, where possible, $M_{max}(T)$ rather than M_{max} is to be used. In the situations where M_{max} is necessary (e.g., in estimation of characteristics of seismic regime), the problem of more stable and meaningful estimation of decreased probability of greatest magnitude remains to be solved.

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