Trace element fractionation during modal and nonmodal dynamic melting and open-system melting: A mathematical treatment

Article *in* Geochimica et Cosmochimica Acta · June 1998

DOI: 10.1016/S0016-7037(98)00115-X

CITATIONS
107

READS
207

1 author:

Haibo Zou
Auburn University
112 PUBLICATIONS **3,753** CITATIONS

SEE PROFILE

Pergamon

Geochimica et Cosmochimica Acta, Vol. 62, No. 11, pp. 1937–1945, 1998

0016-7037/98 $19.00 + .00

PII S0016-7037(98)00115-X

Trace element fractionation during modal and nonmodal dynamic melting and open-system melting: A mathematical treatment

HAIBO ZOU

Division of Isotope Geochemistry, National High Magnetic Field Laboratory, and Department of Geology, Florida State University, Tallahassee, Florida 32306, USA

(*Received July* 30, 1997; *accepted in revised form February* 13, 1998)

Abstract—Dynamic partial melting model has recently drawn considerable attention because this model may explain the fractionation of some strongly incompatible nuclides in the uranium decay series (McKenzie, 1985; Beattie, 1993) and may account for ultra-depleted melt inclusions in olivine grains from mid-ocean ridge basalts (Sobolev and Shimizu, 1993) and Icelandic picrites (Eggins, 1992). There are three subsystems for dynamic melting model: the residual melt, the extracted melt, and the residual solid. This paper systematically derives consistent equations for the residual melt, the extracted melt, and the residual solid in the context of both modal and nonmodal dynamic melting models. Previous available equations are also evaluated. The keys for the derivation or evaluation of equations for dynamic melting models are (1) the exact relationship between the melting rate and the melt extraction rate, (2) the exact relationship between the total melting degree and the fraction of extracted melt relative to the initial amount of source before melting, and (3) clear concepts of physical parameters.

In addition, the equation for the residual melt during open-system (nonmodal dynamic) melting (Ozawa and Shimizu, 1995) with material influx in the melting region is corrected and an equation for the extracted melt in the context of open-system melting is proposed here. Since melting and melt extraction processes are often coupled with slab-derived material influx in the melting region in the arc environments, these two new equations for the residual melt and the extracted melt during open-system melting are very useful in modeling arc magmatism. *Copyright © 1998 Elsevier Science Ltd*

1. INTRODUCTION

Mantle partial melting is a fundamental process for the differentiation and evolution of the Earth. Modeling of partial melting using trace element concentrations is often required to understand the melt generation and segregation process and to interpret the chemical composition of primary melts. There are three general models: batch, fractional (Schilling and Winchester, 1967; Gast, 1968; Shaw, 1970), and dynamic melting (Langmuir et al., 1977; McKenzie, 1985). Another model is called continuous melting (Williams and Gill, 1989; Albaréde, 1995) or critical melting (Maaløe, 1982; Sobolev and Shimizu, 1992). Although continuous melting is commonly distinguished from dynamic melting in that in the former an excess melt is removed from a static column whereas in the latter the entire melting region migrates and new fertile material is added to the column, the real difference between them is only the aggregation time required to produce the magmas. The difference in aggregation time certainly affects the activity of a short-lived radioactive nuclide in magmas, however, it will not affect the concentration of stable trace elements (Williams and Gill, 1989). Therefore, although continuous melting and dynamic melting appear different conceptually, for the purpose of mathematical treatment of stable trace element fractionation, they are mathematically identical.

Among the three general models, the batch melting model assumes that melt remains in equilibrium with the solid throughout the melting event whereas the fractional melting model assumes that (1) the melt is removed from the initial source as it is formed, (2) only the last drop of melt is in equilibrium with the residue, and (3) there is no residual melt. Dynamic melting involves the retention of a critical fraction of melt in the mantle residue. During dynamic melting, when the melt mass fraction in the residue (or the mass porosity of the residue, Ψ) is less than the critical value for melt separation (or the critical mass porosity of the residue, Φ), there is no melt extraction (as in batch melting); when the melt fraction in the residue is greater than Φ, any infinitesimal excess melt will be extracted from the matrix. The difference between the three basic models can be illustrated in the mass porosity of the melting residue (Ψ) vs. the total partial melting degree (F) diagram (Fig. 1). For batch melting, the mass porosity is equal to the partial melting degree until extraction of melt begins, that is, $\Psi = F$ before melt extraction begins and $\Psi = 0$ after melt extraction takes place; for (perfect) fractional melting, $\Psi = 0$ during the whole melting process; for dynamic melting, $\Psi = F$ when $F \leq \Phi$; $\Psi = \Phi$ when $F \geq \Phi$.

As our understanding of the physics of the melting process has evolved, it is generally agreed that dynamic melting is a more realistic scenario when considering fusion during decompression (Richter and McKenzie, 1984; Ribe, 1988; Williams and Gill, 1989; Hèmond et al., 1994; Zou and Zindler, 1996). Dynamic melting model is consistent with extreme depletion of incompatible elements in clinopyroxenes from abyssal peridotites (Johnson et al., 1990) and oceanic cumulates (Ross and Elthon, 1993), uranium-series disequilibrium data of mid-ocean ridge basalts (MORBs) and oceanic island basalts (McKenzie, 1985; Beattie, 1993; Richarson and McKenzie, 1994; Chabaux and Allègre, 1994), and major element (iron) data of MORBs (Langmuir et al., 1992). Clear evidence of dynamic melting processes has recently been found in the form of ultra-depleted

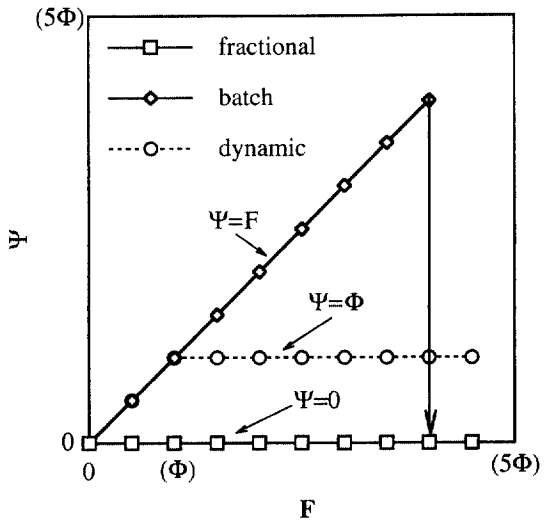

Fig. 1. Mass porosity of melting residue (Ψ) vs. the total partial melting degree (F) diagram. The long heavy arrow shows that the mass porosity for batch melting drops to zero after melt extraction begins.

melt inclusions in olivine grains from MORBs (Sobolev and Shimizu, 1993) and Icelandic picrites (Eggins, 1992).

A dynamic melting model is consisted of three subsystems: the residual solid undergoing partial melting, the residual melt which remains in equilibrium with the residual solid, and the extracted melt which is formed by continuous extraction of the residual melt and is isolated from the residual solid. As for the extracted melt, only its last drop (increment) is in equilibrium with the residual solid and has the same composition as the residual melt. A dynamic melting process keeping a constant fraction of each solid phase and a constant bulk distribution coefficient is called modal dynamic melting and nonmodal dynamic melting, otherwise. As for modal dynamic melting, the two published equations for residual melt (McKenzie, 1985; Albarède, 1995) differ from each other and the reason has not been investigated. As for nonmodal dynamic melting, only the equations for residual melt and extracted melt are available (Pedersen and Hertogen, 1990; Sobolev and Shimizu, 1992) and the derivation processes have not been shown or evaluated. Applications of all these equations for both dynamic melting models have been found in recent literature (e.g., Williams and Gill, 1989; Eggins, 1992; Sobolev and Shimizu, 1993; Gurenko and Chaussidon, 1995; Feigenson et al., 1996; Zou and Zindler, 1996; Arndt et al., 1997; Caroff et al., 1997). Considering the importance of the dynamic melting model and some inconsistency or incompleteness of previous equations, it is the main purpose of this paper to systematically derive consistent equations for the residual melt, the extracted melt, and the residual solid in terms of both dynamic melting models. In addition, the equations for open-system melting are also investigated here.

2. MODAL DYNAMIC MELTING

2.1. Derivation

During partial melting, the conservation law which is concerned with the concentration of a trace element in the solid and melt is given by McKenzie (1984) as

$$(1-\phi)\rho_s\frac{\partial C_s}{\partial t}+\rho_f\phi\frac{\partial C_f}{\partial t}+(1-\phi)\rho_s\mathbf{V}\cdot\nabla C_s+\rho_f\phi\mathbf{v}\cdot\nabla C_f = \rho_f\nabla\cdot\phi K\nabla C_f+(C_s-C_f)M \quad (1)$$

where C_S and C_f are the concentrations of an element in the solid and melt, respectively; ρ_S is the density of solid (3300 kg/m^3), ρ_f is the density of melt (2800 kg/m^3), ϕ is the volume porosity of the mantle, V is the moving velocity of the solid, v is the moving velocity of the residual melt, K is the diffusion coefficient, M is the melting rate (the rate of mass transfer /unit volume from the residual solid to the residual melt, and its dimension is mass/volume/time) (complete list of symbols is summarized in Appendix).

If the solid and the melt are in equilibrium ($C_S = D_0C_f$, where D_0 is the bulk solid/liquid distribution coefficient) and C_S and C_f are functions of time only (i. e., setting the space variables ∇C_S and ∇C_f to be zero), Eqn. 1 can be simplified as

$$[(1-\phi)\rho_S D_0+\rho_f\phi]\frac{\partial C_f}{\partial t}=(D_0-1)C_fM \quad (2)$$

For dynamic melting, the melting rate (M) is constant in the upwelling material, and the melt volume fraction (ϕ) in equilibrium with the matrix remains constant (as the critical volume porosity) because additional melt is drained into a melt channel (the details of the model are shown in Fig. 3 in McKenzie, 1985). In this case, both M and ϕ are independent of time, and Eqn. 2 with initial condition $C_f(0) = C_f^0$ (the concentration of the first drop of extracted melt when melt fraction in the residue reaches to its critical mass porosity) has analytical solution:

$$C_f=C_f^0\exp\left[\frac{(D_0-1)M}{\rho_f\phi+\rho_S(1-\phi)D_0}t\right] \quad (3)$$

Equations 1–3 are the same as those in McKenzie (1985), but the following steps are necessary to derive the exact equation for C_f. Let X be the mass fraction of extracted melt relative to the total amount of melt and solid in the residue before melt extraction begins which is also the initial amount of the solid source before melting. The mass fraction of the residue relative to the initial amount of the source is thus $(1 - X)$, and the specific variation rate of the residue is $1/(1 - X)\ d(1 - X)/dt$. Let M_e be the melt extraction rate which is defined as the rate of mass transfer/unit volume from the residue to the extracted melt and has the same dimension with the melting rate (M). The density of the residue (the mass of the residue per unit volume) is $[\rho_f\phi + \rho_S(1 - \phi)]$ and thus $M_e/[\rho_f\phi + \rho_S(1 - \phi)]$ is the melt extraction percentage per unit time from the residue. Therefore, we have

$$\frac{1}{1-X}\frac{d(1-X)}{dt}=-\frac{M_e}{\rho_f\phi+\rho_S(1-\phi)} \quad (4)$$

The solution to Eqn. 4 with initial condition $X(0) = 0$ is

$$X=1-\exp\left[\frac{-M_e}{\rho_f\phi+\rho_S(1-\phi)}t\right] \quad (5)$$

Substitution of Eqn. 5 into Eqn. 3 yields

$$C_f=C_f^0(1-X)^{G(1-D_0)M/M_e} \quad (6)$$

where

$$G = \frac{\rho_f\phi + \rho_S(1-\phi)}{\rho_f\phi + \rho_S(1-\phi)D_0} \tag{7}$$

We need to find the relationship between C_f^0 and C_0 (initial source concentration before melting) and that between M and M_e. The constant ϕ in the melting matrix requires constant melt/solid mass ratio in the residue (residual solid + residual melt) during dynamic melting:

$$\frac{\rho_f\phi + Mt - M_e t}{\rho_S(1-\phi) - Mt} = \frac{\rho_f\phi}{\rho_S(1-\phi)} \tag{8}$$

and, therefore

$$\frac{M}{M_e} = \frac{\rho_S(1-\phi)}{\rho_f\phi + \rho_s(1-\phi)} = f_s \tag{9}$$

where f_S is the mass fraction of the residual solid relative to the whole residue. The relationship between C_f^0 and C_0 can be obtained from mass balance when the melting degree reaches the critical mass porosity (Φ)

$$C_f^0\Phi + (1-\Phi)D_0C_f^0 = C_0 \tag{10}$$

where

$$\Phi = \frac{\rho_f\phi}{\rho_f\phi + \rho_S(1-\phi)} \tag{11}$$

is the critical mass porosity of the residue. The solution to Eqn. 10 is

$$C_f^0 = C_0\frac{1}{\Phi + (1-\Phi)D_0} \tag{12}$$

Substitute Eqns. 7, 9, 11, and 12 into Eqn. 6 and we have

$$C_f = C_0\frac{1}{\Phi + (1-\Phi)D_0}(1-X)^{\{1/[\Phi+(1-\Phi)D_0]\}-1} \tag{13}$$

For the extracted dynamic melt, the concentration can be obtained by integrating Eqn. 13 from 0 to X and then dividing by X,

$$\overline{C}_L = \frac{1}{X}\int_0^X C_f(x)dx = \frac{1}{X}C_0\left\{1-(1-X)^{1/[\Phi+(1-\Phi)D_0]}\right\} \tag{14}$$

The residual solid concentration in equilibrium with the residual melt is

$$C_S = D_0C_f = C_0\frac{D_0}{\Phi + (1-\Phi)D_0}(1-X)^{\{1/[\Phi+(1-\Phi)D_0]\}-1} \tag{15}$$

Then the concentration in the residue (residual solid + residual melt) is

$$C_{res} = C_f\Phi + C_s(1-\Phi) = C_0(1-X)^{\{1/[\Phi+(1-\Phi)D_0]\}-1} \tag{16}$$

If a set of equations is correct for modal dynamic melting, the whole system (the residual melt, the extracted melt, and the residual solid) must satisfy mass balance requirements. It can be easily shown that, for a given element in the whole system during modal dynamic melting, Eqns. 13–15 satisfy the following mass balance requirement,

$$X\overline{C}_L + (1-X)[C_f\Phi + C_S(1-\Phi)] = C_0 \tag{17}$$

2.2. Comparison of the Equation for C_f

By substituting Eqn. 11 (the relationship between Φ and ϕ) into Eqn. 13, we have

$$C_f = \frac{\rho_f\phi + \rho_S(1-\phi)}{\rho_f\phi + \rho_S(1-\phi)D_0}C_0(1-X)^{[\rho_S(1-\phi)(1-D_0)]/[\rho_f\phi+\rho_S(1-\phi)D_0]} \tag{18}$$

which would be the same as the equation of Albaréde (1995) obtained from a different approach except that the term $\rho_s\phi$ in the power part of Eqn. 9.3.23 of Albaréde (1995) (or Eqn. 3 in Feigenson et al., 1996) should be replaced by $\rho_f\phi$.

The concentration of a trace element in the residual melt of the modal dynamic melting model is given by McKenzie (1985) as

$$C_f = GC_0(1-X)^{G(1-D_0)} \tag{19}$$

Substitute Eqn. 7 into Eqn. 19 and we obtain

$$C_f = \frac{\rho_f\phi + \rho_S(1-\phi)}{\rho_f\phi + \rho_s(1-\phi)D_0}C_0(1-X)^{[\rho_f\phi+\rho_s(1-\phi)](1-D_0)/[\rho_f\phi+\rho_s(1-\phi)D_0]} \tag{20}$$

which is different from Eqn. 18 in the power part. The derivation method here is similar to that in McKenzie (1985). The difference is that $M/M_e = f_S$ is used here while $M = M_e$ is assumed in McKenzie (1985). It can be seen that if $M = M_e$, Eqn. 8 can not be satisfied. In addition, if $M = M_e$, the porosity of the residue actually increases (Fig. 2); to keep an constant mass porosity, a small extra amount of residual melt has to be extracted during melting. When $M = M_e$, we can not obtain Eqn. 3 from Eqn. 2 because in this case porosity is also a function of time and Eqn. 2 has to be solved numerically.

According to Eqn. 19, the concentration of the element in the extracted dynamic melt is

$$\overline{C}_L = \frac{1}{X}GC_0\frac{1-(1-X)^{G(1-D_0)+1}}{G(1-D_0)+1} \tag{21}$$

and the concentration in the residual solid is

$$C_s = D_0C_f = D_0GC_0(1-X)^{G(1-D_0)} \tag{22}$$

It can be easily shown that Eqns. 19, 21, and 22 do not satisfy the mass balance Eqn. 17 for the whole system. However, by replacing $G(1-D_0)$ with $G(1-D_0)f_S$ in Eqns. 19, 21, and 22, they will become Eqns. 13, 14, and 15, respectively. The difference between Eqn. 13 and Eqn. 19 and that between Eqn. 14 and 21 and that between Eqn. 15 and 22 are all small if f_S is close to unity, which is generally the case. Since the volume porosity of the residual mantle is usually less than 1% for the formation of basalts (Riley and Kohlstedt, 1991; LaTourrette et al., 1993), the mass fraction of solid in the residue f_S is between 0.992 to 1 (see Eqn. 11) . Therefore, previous modeling using Eqns. 19, 21, and 22 is still a good approximation.

3. NONMODAL DYNAMIC MELTING

3.1. Derivation

During partial melting both clinopyroxene and garnet, the major repositories of incompatible trace elements in peridotites, will play a diminishing roles as residual phases because they are consumed preferentially by partial melting. As a result, the

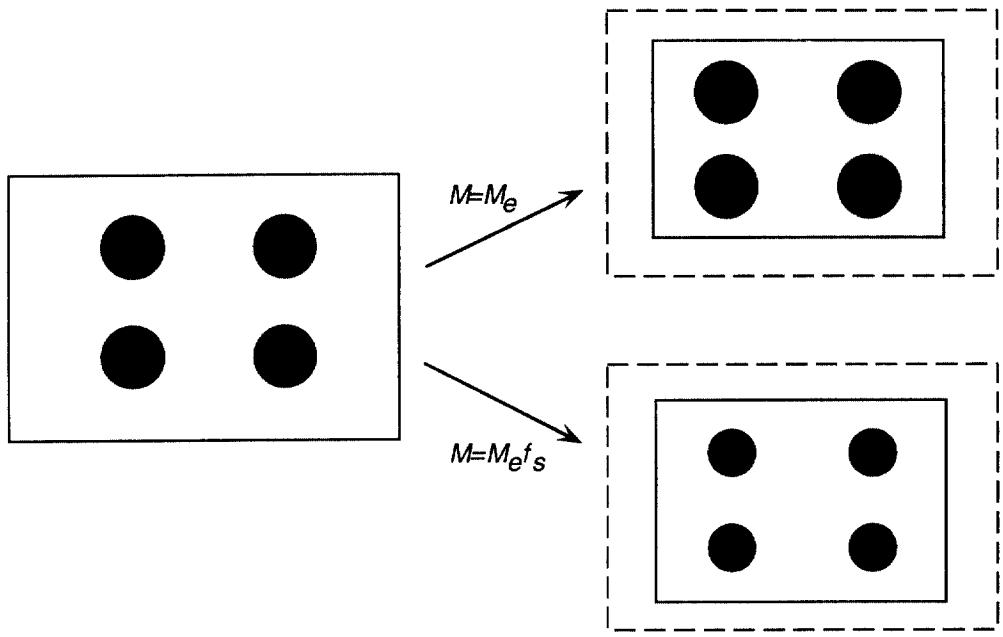

Fig. 2. A cartoon showing that if the melting extraction rate (M_e) is equal to the melting rate (M), the porosity of the residue actually increases; to keep a constant porosity, the melting extraction rate is actually greater than the melting rate (i.e., $M/M_e = f_S$). The solid circles represents the amount of residual melt. The solid rectangle on the left-hand side represents the total amount of melt and solid before melt extraction begins, which is equal to the total amount of solid before melting. The two dashed rectangles on the right-hand side also indicate the total amount of solid and melt before melt extraction begins for comparison. The two smaller solid rectangles inside two dashed rectangles represent the amount of residue after a certain amount of melt extraction.

bulk distribution coefficients of many trace elements are likely to change as melting progresses and the nonmodal dynamic partial melting might take place.

The relationship between the concentration in the residue and the mass fraction of extracted melt relative to the initial source is given by Albaréde (1995) as

$$d\ln C_{res} = \frac{1 - D_i}{D_i} d\ln(1 - X) \tag{23}$$

where D_i is the ratio of the concentration of an element in the residue to the concentration of the element in the last increment of extracted melt. To solve this differential equation, we need to express D_i as a function of X.

For dynamic melting, D_i is the effective distribution coefficient (Albaréde, 1976) between the residue and the last increment of melt that is extracted, we have

$$D_i = \Phi + (1 - \Phi) D_{s/f} \tag{24}$$

because the last drop of extracted melt has the same composition with the residual melt and thus the melt proportion Φ has a partition coefficient of 1. $D_{s/f}$ is the bulk distribution coefficient during partial melting. Although there is not a unique nonmodal melting model to describe the variation of $D_{s/f}$, the model of Shaw (1970) has been widely accepted and is thus used here. The variation of $D_{S/f}$ as a function of total melting degree (F) is given by Shaw (1970) as

$$D_{s/f} = \frac{D_0 - FP}{1 - F} \tag{25}$$

where D_0 is the initial bulk distribution coefficient of the source and P is a constant whose value is determined by the proportions in which the minerals contribute to the melt.

Combining Eqn. 24 and Eqn. 25, we have

$$D_i = \Phi + (1 - \Phi) \frac{D_0 - FP}{1 - F} \tag{26}$$

The key in the derivation is the exact relationship between F and X. During partial melting, when the total partial melting degree is F and the extracted melt fraction relative to the initial source is X, the amount of residual melt is $(F - X)$ and the amount of residual solid is $(1 - F)$. The constant mass porosity requires constant mass ratio of residual melt over residual solid in the residue

$$\frac{F - X}{1 - F} = \frac{\Phi}{1 - \Phi} \tag{27}$$

therefore, the relationship between F and X during partial melting is

$$F = \Phi + (1 - \Phi) X \tag{28}$$

Equation 28 suggests that additional melting degree beyond Φ, which is $(F - \Phi)$, is equal to the product of the total fraction of extracted melt (X) and the fraction of residual solid in the residue $(1 - \Phi)$ in order to keep constant critical mass porosity.

The following relationship between F and X has been assumed (e.g., Eggins, 1992) for dynamic melting

$$F = X + \Phi \tag{29}$$

which does not satisfy the requirement of Eqn. 27.

Substituting Eqn. 28 into Eqn. 26, we obtain D_i as a function of X

$$D_i = \frac{[D_0 + \Phi(1 - P)] - X[P + \Phi(1 - P)]}{1 - X} \tag{30}$$

Substituting Eqn. 30 into Eqn. 23, we get

$$\frac{d\ln C_{res}}{d\ln(1 - X)} = \frac{1 - X}{[D_0 + \Phi(1 - P)] - X[P + \Phi(1 - P)]} - 1 \tag{31}$$

The solution to Eqn. 31 with initial condition $C_{res}(0) = C_0$ is

$$C_{res} = C_0 \frac{1}{1 - X}\left\{1 - \frac{X[P + \Phi(1 - P)]}{D_0 + \Phi(1 - P)}\right\}^{1/[\Phi + (1-\Phi)P]} \tag{32}$$

The concentration of residual melt is the same as the concentration of the last increment of extracted melt,

$$C_f = \frac{C_{res}}{D_i} = \frac{1}{D_0 + \Phi(1 - P)} \times C_0\left\{1 - \frac{X[P + \Phi(1 - P)]}{D_0 + \Phi(1 - P)}\right\}^{\{1/[\Phi + (1-\Phi)P]\} - 1} \tag{33}$$

For the extracted melt, we have

$$\overline{C_L} = \frac{1}{X}\int_0^X C_f(x)dx = \frac{C_0}{X}\left\{1 - \left[1 - \frac{X[P + \Phi(1 - P)]}{D_0 + \Phi(1 - P)}\right]^{1/[\Phi + (1-\Phi)P]}\right\} \tag{34}$$

For the residual solid,

$$C_S = C_f D_{Sf} = \frac{D_0 - [\Phi + (1 - \Phi)X]P}{1 - [\Phi + (1 - \Phi)X]} \frac{1}{D_0 + \Phi(1 - P)} \times C_0\left\{1 - \frac{X[P + \Phi(1 - P)]}{D_0 + \Phi(1 - P)}\right\}^{\{1/[\Phi + (1-\Phi)P]\} - 1} \tag{35}$$

Equations 32, 33, and 35 satisfy the following relationship in the residue

$$\Phi C_f + (1 - \Phi)C_S = C_{res} \tag{36}$$

If $P = D_0$, Eqns. 33–35 become Eqns. 13–15, respectively, for modal dynamic melting. If $\Phi = 0$ (i.e., fractional melting), Eqns. 33–35 become those for nonmodal fractional melting model of Shaw (1970). It is easy to show that Eqns. 33–35 satisfy the mass balance Eqn. 17 for the whole system.

It is noted that the relationship between X and F is also important to distinguish the dynamic melting model from both the batch melting and the fractional melting models (Fig. 3). For batch melting, $X = 0$ before melt extraction begins and $X = F$ after melt extraction takes place; for fractional melting, $X = F$; and for dynamic melting, $X = 0$, when $F \leq \Phi$ and, according to Eqn. 28,

$$X = \frac{1}{1 - \Phi}F - \frac{\Phi}{1 - \Phi}, \text{ when } F > \Phi. \tag{37}$$

The slope in the X vs. F diagram for dynamic melting is $1/(1-\Phi)$ and is greater than 1 because $0 < \Phi < 1$.

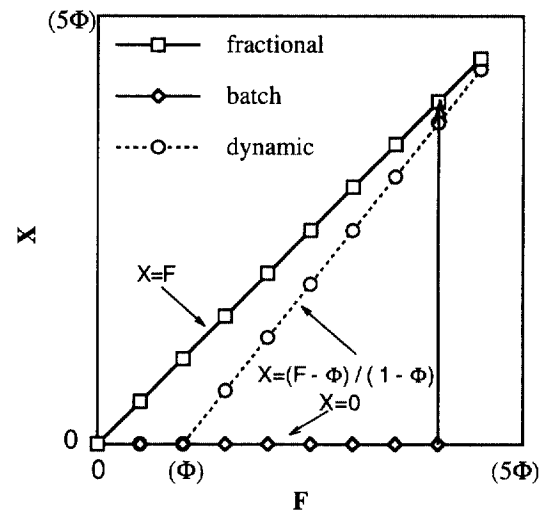

Fig. 3. The fraction of extracted melt relative to the initial solid before melting starts (X) vs. the total partial melting degree (F) diagram. The long heavy arrow shows that the mass fraction of extracted melt (X) for batch melting increases from zero to F after melt extraction begins.

3.2. Comparison of the Equations for C_f and $\overline{C_L}$

The concentrations in the residual melt and in the extracted melt are given by Sobolev and Shimizu (1992) as

$$C_f = \frac{1}{D_0 + (1 - P)\frac{\alpha}{\alpha + 1}} C_0\left[\frac{(D_0 + \alpha) - (P + \alpha)F}{(D_0 + \alpha) - (P + \alpha)\frac{\alpha}{\alpha + 1}}\right]^{(1-P)/(P+\alpha)} \tag{38}$$

$$\overline{C_L} = \frac{(D_0 + \alpha) - (P + \alpha)\frac{\alpha}{\alpha + 1}}{\left(F - \frac{\alpha}{\alpha + 1}\right)\left[D_0 + (1 - P)\frac{\alpha}{\alpha + 1}\right](\alpha + 1)} C_0 \times \left\{1 - \left[\frac{(D_0 + \alpha) - (P + \alpha)F}{(D_0 + \alpha) - (P + \alpha)\frac{\alpha}{\alpha + 1}}\right]^{(\alpha+1)/(P+\alpha)}\right\} \tag{39}$$

where F is the total melting degree and α is the mass ratio of the residual melt over residual solid in the residue. Φ and α are related by the following relationship according to definitions,

$$\Phi = \alpha/(\alpha + 1) \tag{40}$$

The key to compare Eqns. 38–39 with Eqns. 33–34 is the exact relationship between X and F. Using Eqn. 28 derived here for the relationship between X and F, Eqns. 38–39 will become Eqns. 33–34. Note that the term

$$\left[(D_0 + \alpha) - (P + \alpha)\frac{\alpha}{\alpha + 1}\right] \Big/ \left\{\left(F - \frac{\alpha}{\alpha + 1}\right) \times \left[D_0 + (1 - P)\frac{\alpha}{\alpha + 1}\right](\alpha + 1)\right\}$$

in Eqn. 39 is simply equal to $1/X$ in Eqn. 34. However, if we use Eqn. 29 for the relationship between X and F, Eqns. 38–39 would be different from Eqns. 33–34.

The concentrations in the residual melt (C_f) and in the extracted melt ($\overline{C_L}$) for nonmodal dynamic melting are given by Pedersen and Hertogen (1990) as

$$C_f = C_0 \frac{1+Z}{(1+Z)D_0 + Z(1-P)}\left(1 - \frac{P+Z}{(1+Z)D_0 + Z(1-P)}X\right)^{[(1+Z)/(P+Z)]-1} \quad (41)$$

$$\overline{C_L} = C_0 \frac{1}{X}\left[1 - \left(1 - \frac{P+Z}{(1+Z)D_0 + Z(1-P)}X\right)^{(1+Z)/(P+Z)}\right] \quad (42)$$

Z is the fraction of melt trapped in the residue. The key to compare Eqns. 41–42 with Eqns. 33–34 is the meaning of Z. When Z is the mass ratio of the residual melt over the residual solid, that is $Z = \alpha$, Eqns. 41–42 become Eqns. 33–34 in this paper (note that if we use $Z = \Phi$, the two sets of equations would not be equivalent).

In summary, due to different derivation methods, C_f and $\overline{C_L}$ in the equations of Sobolev and Shimizu (1992) are expressed as a function of F and α; C_f and $\overline{C_L}$ in the equations of Pedersen and Hertogen (1990) are expressed as a function of X and α; and C_f and $\overline{C_L}$ in the equations of this paper are expressed as a function of X and Φ. All three sets of equations for the residual melt and the extracted melt are sound. Possible advantages of expressing C_f and $\overline{C_L}$ as a function of X and Φ in this paper are (1) the equations are simpler; (2) Φ is a conventional parameter compared with α; and (3) it is easy to use mass balance calculation in the residue (Eqn. 36) and in the whole system (Eqn. 17). In addition to giving simpler equations for the residual melt and the extracted melt, this paper also presents the equation for the residual solid and the whole residue in the context of nonmodal dynamic melting, which is helpful for modeling residual peridotites and clinopyroxenes (Zou, 1997).

4. OPEN-SYSTEM MELTING WITH MATERIAL INFLUX

Nonmodal dynamic melting only considers melting and melt extraction (output) processes. A more complex system includes material influx (input) in the melting region, which might be the case in the arc environment. Ozawa and Shimizu (1995) proposed an open-system melting model and gave the equation for the residual melt. By setting the mass influx rate (β) to be zero in Eqn. A3 of Ozawa and Shimizu (1995), we can obtain the equation for residual melt during nonmodal dynamic melting as

$$C_f = \frac{C_0}{D_0 + \frac{\alpha}{\alpha+1}(1-P)}\left[\frac{D_0 + \alpha - F(P+\alpha)}{D_0 + \frac{\alpha}{\alpha+1}(1-P)}\right]^{(1-P)/(P+\alpha)} \quad (43)$$

which is different from Eqn. 38, suggesting that their Eqn. A3 is not correct. The solution to their open-system melting model

$$\frac{dC_f}{dF} = \frac{C_f(P-\beta-1) + C_A\beta}{\alpha + D_0 - (\alpha+P)F} \quad (44)$$

with initial condition

$$C_f^0 = \frac{C_0 + C_A\beta F_C}{D_0 + F_C(1+\beta-P)} \quad (45)$$

should be

$$C_f = \frac{1}{P-\beta-1}\left\{-C_A\beta + \frac{C_A\beta D_0 + C_0(P-\beta-1)}{D_0 + F_C(1+\beta-P)} \times \left[\frac{D_0 + \alpha - F(P+\alpha)}{D_0 + \alpha - F_C(P+\alpha)}\right]^{(1+\beta-P)/(P+\alpha)}\right\} \quad (46)$$

which becomes Eqn. 38 for the residual melt during nonmodal dynamic melting when $\beta = 0$. C_A is the concentration of the element in influxing material and $F_C = \alpha/(\alpha + \beta + 1)$. The relationship between C_f and $\overline{C_L}$ for open-system melting model is

$$\overline{C_L} = \frac{1}{F - F_C}\int_{F_C}^{F} C_f(f)\,df \quad (47)$$

therefore, combining Eqn. 46 with Eqn. 47, we can obtain the equation for the extracted melt during open-system melting as

$$\overline{C_L} = \frac{1}{F-F_C} \times \frac{1}{P-\beta-1}\left\{C_A\beta(F_C - F) + \left\{\frac{[C_A\beta D_0 + C_0(P-\beta-1)][D_0 + \alpha - F_C(\alpha+P)]}{[D_0 + F_C(1+\beta-P)](\alpha+\beta+1)} \times \left\{1 - \left[\frac{D_0 + \alpha - F(\alpha+P)}{D_0 + \alpha - F_C(\alpha+P)}\right]^{[(1+\beta-P)/(\alpha+P)]+1}\right\}\right\}\right\} \quad (48)$$

which becomes Eqn. 39 for the extracted melt during nonmodal dynamic melting when $\beta = 0$. Equations 46 and 48 are very useful in modeling arc magmatism because melting and melt extraction processes in the arc environments are often coupled with slab-derived material influx in the melting region. Since the open-system melting model involves more parameters, successful modeling using Eqns. 46 and 48 requires that at least part of related parameters are reasonably well-constrained. This requirement might be satisfied for some well-studied magmatic arcs.

5. DISCUSSION AND SUMMARY

All the equations for stable trace elements derived here are suitable for dynamic melting models with constant critical mass porosity, where the melt fraction in chemical equilibrium with the solid residue remains constant as melting proceeds, and any melt that is produced in excess of this critical porosity is extracted into a network of veins or dykes in which the melt does not re-equilibrium with the residual solid. However, the critical melt porosity might change during melting (in the range of 0 to F). This additional complexity can be approximately quantified by simply extending the single-porosity melting calculation to a multiple-porosity model using step-wise numerical method. For a more rigorous approach, the relationship between the critical porosity and time (or F, or X) must be known or assumed and the related differential equations have to

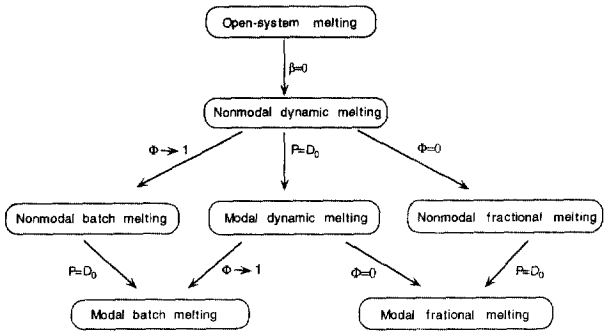

Fig. 4. A cartoon showing the relationships between different melting models.

be solved by numerical methods. Although Spiegelman and Elliott (1993) have used variable porosity in a melting column to study the uranium-series isotope disequilibrium in young lavas during melt percolation at chemical equilibrium, their Eqn. 10 for radioactive nuclides simply becomes modal batch melting equation for stable trace elements (Ribe, 1985; Spiegelman and Elliott, 1993).

The derivation of all the equations here does not take into account of the microscopic process of chemical diffusion within individual mineral grains, otherwise the related differential equations can not be solved analytically. Qin (1992) and Iwamori (1993) have theoretically modeled the chemical diffusion process that controls the disequilibrium melting. Both studies made the following assumptions: (1) the solid is composed of spherical grains where the surface of each grain is in chemical equilibrium with the melt phase but the interior can only equilibrate with the melt by chemical diffusion, and (2) bulk solid/melt partition coefficients are constant during melting. Qin (1992) also ignored the density difference between solid and melt for simplicity. It has been concluded that the principle effect of disequilibrium melting controlled by solid state diffusion is simply to lower the incompatibility of an incompatible element (i.e., to raise the effective partition coefficient of the incompatible element).

Hofmann and Hart (1978) and Hart (1993) have given quantitative arguments to show that trace element equilibration between melt and crystalline phases is highly likely during mantle melting except for extreme situations. When solid grain size is very large and diffusivities are very small and mantle upwelling is very rapid, disequilibrium melting may be of significance. Even in this case, if higher partition coefficients for incompatible elements than equilibrium values are used, the treatment using equilibrium melting will still be a good approximation, as has been suggested by Qin (1993).

In summary, this paper systematically derives the equations for the residual melt, the extracted melt, and the residual solid in the context of both modal and nonmodal dynamic melting models and evaluates previous available equations. In addition, the equations for residual melt and the extracted melt for open-system melting model with material influx in the melting region are also provided. The relationships among sets of consistent equations for modal and nonmodal dynamic melting, open-system melting, and the classic modal and nonmodal batch melting and modal and nonmodal fractional melting can be shown in Fig. 4.

The general equations for modal and nonmodal dynamic melting models are summarized in Table 1. The corresponding equations for classic fractional melting and batch melting models of Shaw (1970) are also included for comparison. In general, the equations for residual melts (Eqns. 13, 33, 46) are suitable for modeling the trace element concentrations in inclusions trapped in olivine while the equations for extracted melts (Eqns. 14, 34, 48) are suitable for modeling trace element concentrations in primary basalts. To model the variation of trace elements in the whole residue, we need to use Eqn. 16 or Eqn. 32.

Acknowledgements—Dr. Alex Halliday and an anonymous reviewer provided very constructive reviews that considerably improved the quality of the paper. I also thank Dr. Alan Zindler for his support and Dr. Karl Turekian for his speedy handling of this manuscript. Drs. Jean-Guy Schilling, Marc Feigenson, and Yongjun Su reviewed an early version of the manuscript before its submission to GCA and are gratefully acknowledged. Dr. Jean-Guy Schilling in particular called my attention to two publications. I am also grateful to numerous

Table 1. Summary of the equations for three general melting models

models		C_f	C_s	$\overline{C}_L$
dynamic melting	modal	$\frac{1}{\Phi+(1-\Phi)D_0}C_0(1-X)^{\{1/[\Phi+(1-\Phi)D_0]\}-1}$	D_0C_f	$\frac{1}{X}C_0\left[1-(1-X)^{1/[\Phi+(1-\Phi)D_0]}\right]$
	nonmodal	$\frac{1}{D_0+\Phi(1-P)}C_0\left\{1-\frac{X[P+\Phi(1-P)]}{D_0+\Phi(1-P)}\right\}^{\{1/[\Phi+(1-\Phi)P]\}-1}$	$D_{S/f}C_f$	$\frac{C_0}{X}\left\{1-\left[1-\frac{X[P+\Phi(1-P)]}{D_0+\Phi(1-P)}\right]^{1/[\Phi+(1-\Phi)P]}\right\}$
fractional melting	modal	$\frac{1}{D_0}C_0(1-X)^{(1/D_0)-1}$	D_0C_f	$\frac{1}{X}C_0\left[1-(1-X)^{1/D_0}\right]$
	nonmodal	$\frac{1}{D_0}C_0\left\{1-\frac{XP}{D_0}\right\}^{(1/P)-1}$	$\frac{D_0-XP}{1-X}C_f$	$\frac{C_0}{X}\left\{1-\left[1-\frac{XP}{D_0}\right]^{1/P}\right\}$
batch melting	modal	$\frac{C_0}{D_0+F(1-D_0)}$	D_0C_f	C_f
	nonmodal	$\frac{C_0}{D_0+F(1-P)}$	$\frac{D_0-FP}{1-F}C_f$	C_f

Notations are the same as those in the text. The equations for fractional melting and batch melting models are from Shaw (1970). Note that when $\Phi = 0$, the equations for dynamic melting become the corresponding equations for fractional melting; when $P = D_0$, the equations for nonmodal melting become the corresponding equations for modal melting. As for dynamic melting, the concentration in the residue (C_{res}) is related to the concentration in the residual solid and the concentration in the residual melt by $C_{res} = C_f\Phi + C_s(1-\Phi)$, which is equivalent to Eqn. 16 for modal dynamic melting or Eqn. 32 for nonmodal dynamic melting; but for fractional melting and batch melting, $C_{res} = C_s$.

participants of 1997 Fall AGU meeting for their discussions. The derivation of the equations in this paper is built upon previous studies listed in the reference, particularly the works of Shaw (1970), McKenzie (1985), Albarède (1995), and Ozawa and Shimizu (1995).

REFERENCES

Albaréde F. (1976) Some trace element relationships amongst liquid and solid phases in the course of the fractional crystallization of magmas. *Geochim. Cosmochim. Acta* **40,** 667–673.

Albarède F. (1995) *Introduction to Geochemical Modeling*. Cambridge Univ. Press.

Arndt N. T., Kerr A. C., and Tarney J. (1997) Dynamic melting in plume heads: The formation of Gorgona komatiites and basalts. *Earth Planet. Sci. Lett.* **146,** 289–301.

Beattie P. (1993) Uranium-thorium disequilibria and partitioning of melt of garnet peridotite. *Nature* **363,** 63–65.

Caroff M., Maury R. C., Guille G., and Cotten J. (1997) Partial melting below Tubuai (Austral Islands, French Polynesia). *Contrib. Mineral. Petrol.* **127,** 369–382.

Chabaux F. and Allègre C. J. (1994) ^{238}U-^{230}Th-^{226}Ra disequilibria in volcanics: A new insight into melting conditions. *Earth Planet. Sci. Lett.* **126,** 61–74.

Eggins S. M.(1992) Petrogenesis of Hawaiian tholeiites: 2, aspects of dynamic melt segregation. *Contrib. Mineral. Petrol.* **110,** 398–410.

Feigenson M., Patino L., and Carr M. J. (1996) Constraints on partial melting by rare earth element variations in Mauna Kea basalts. *J. Geophys. Res.* **101,** 11815–11829.

Gast P. W. (1968) Trace element fractionations and the origin of tholeiitic and alkaline magma types. *Geochim. Cosmochim. Acta* **32,** 1057–1086.

Gurenko A. A. and Chaussidon M. (1995) Enriched and depleted primitive melts included in olivine from Icelandic tholeiites: Origin by continuous melting of a single mantle column. *Geochim. Cosmochim. Acta* **59,** 2905–2917.

Hart S. R. (1993) Equilibration during mantle melting: A fractal tree model. *Proc. Natl. Acad. Sci. USA* **90,** 11914–11918.

Hèmond C., Hofmann A. W., Heusser G., Condomines M., Raczek I., and Rhodes J. M. (1994) U-Th-Ra systematics in Kilauea and Mauna Loa basalts, Hawaii. *Chem. Geol.* **116,** 163–180.

Hofmann A. W. and Hart S. R. (1978) An assessment of local and regional isotopic equilibrium in the mantle. *Earth Planet. Sci. Lett.* **38,** 44–62.

Iwamori H. (1993) Dynamic disequilibrium melting model with porous flow and diffusion-controlled chemical equilibration. *Earth Planet. Sci. Lett.* **114,** 301–313.

Johnson K. T. M., Dick H. J. B., and Shimizu N. (1990) Melting of the oceanic upper mantle: An ion microprobe study of diopsides in abyssal peridotites. *J. Geophys. Res.* **95,** 2661–2678.

Langmuir C. H., Bender J. F., Bence A. E., Hanson G. N., and Taylor S. R. (1977) Petrogenesis of basalts from the FAMOUS-area, Mid-Atlantic ridge. *Earth Planet. Sci. Lett.* **36,** 133–156.

Langmuir C. H., Klein E. M., and Plank T. (1992) Petrological systematics of mid-ocean ridge basalts: Constraints on melt generation beneath ocean ridges. In *Mantle Flow and Melt Generation at Mid-ocean Ridges* (ed. J. Phipps Morgan et al.), pp. 183–280. AGU.

LaTourrette T. Z., Kennedy A. K., and Wasserburg G. J. (1993) Thorium-uranium fractionation by garnet: evidence for a deep source and rapid rise of oceanic basalts. *Science* **261,** 739–742.

Maaløe S. (1982) Geochemical aspects of permeability controlled partial melting and fractional crystallization. *Geochim. Cosmochim. Acta* **46,** 43–57.

McKenzie D. P. (1984) The generation and compaction of partially molten rock. *J. Petrol.* **25,** 713–765.

McKenzie D. P. (1985) ^{230}Th-^{238}U disequilibrium and the melting processes beneath ridge axes. *Earth Planet. Sci. Lett.* **72,** 149–157.

Ozawa K. and Shimizu N. (1995) Open-system melting in the upper mantle: Constraints from the Hayachine-Miyamori ophiolite, northeastern Japan. *J. Geophys. Res.* **100,** 22315–22335.

Pedersen R. B. and Hertogen J. (1990) Magmatic evolution of the Karmoy Ophiolite Complex, SW Norway: relationship between MORB-IAT-boninitic-calc-alkaline and alkaline magmatism. *Contrib. Mineral. Petrol.* **104,** 277–293.

Qin Z. (1992) Disequilibrium partial melting model and its implications for trace element fractionations during mantle melting. *Earth Planet. Sci. Lett.* **112,** 75–90.

Qin Z. (1993) Dynamics of melt generation beneath mid-ocean ridge axes: Theoretical analysis based on ^{238}U-^{230}Th-^{226}Ra and ^{235}U-^{231}Pa disequilibria. *Geochim. Cosmochim. Acta* **57,** 1629–1634.

Ribe N. M. (1985) The generation and composition of partial melts in the earth's mantle. *Earth Planet. Sci. Lett.* **73,** 361–376.

Ribe N. M.(1988) Dynamical geochemistry of the Hawaiian plume. *Earth Planet. Sci. Lett.* **88,** 37–46.

Richardson C. and McKenzie D. (1994) Radioactive disequilibria from 2D models of melt generation by plume and ridges. *Earth Planet. Sci. Lett.* **128,** 425–437.

Richter F. M. and McKenzie D. (1984) Dynamic models for melt segregation from a deformable matrix. *J. Geol.* **92,** 729–740.

Riley G. N. and Kohlstedt D. L. (1991) Kinetics of melt migration in upper mantle-type rocks. *Earth Planet. Sci. Lett.* **105,** 500–521.

Ross K. and Elthon D. (1993) Cumulates from strongly depleted mid-ocean-ridge basalt. *Nature* **365,** 826–829.

Schilling J. G. and Winchester (1967) Rare-earth fractionation and magmatic processes. In *Mantle of the Earth and Terrestrial Planets* (ed. S. K. Runcorn), pp. 267–283. Interscience Publ.

Shaw D. M. (1970) Trace element fractionation during anatexis. *Geochim. Cosmochim. Acta* **34,** 237–243.

Sobolev A. V. and Shimizu N. (1992) Superdepleted melts and ocean mantle permeability. *Dokl. Ross. Akad. Nauk* **326,** 354–360.

Sobolev A. V. and Shimizu N. (1993) Ultra-depleted primary melt included in an olivine from the Mid-Atlantic Ridge. *Nature* **363,** 151–154.

Spiegelman M. and Elliott T. (1993) Consequences of melt transport for uranium series disequilibrium in young lavas. *Earth Planet. Sci. Lett.* **118,** 1–20.

Williams R. W. and Gill J. B. (1989) Effect of partial melting on the uranium decay series. *Geochim. Cosmochim. Acta* **53,** 1607–1619.

Zou H. (1997) Inversion of partial melting through residual peridotites or clinopyroxenes. *Geochim. Cosmochim. Acta* **61,** 4571–4582.

Zou H. and Zindler A. (1996) Constraints on the degree of dynamic partial melting and source composition using concentration ratios in magma. *Geochim. Cosmochim. Acta* **60,** 711–717.

APPENDIX

List of Symbols

C_j	the concentration of a trace element in the residual melt, or the concentration of the incremental melt
C_S	the concentration of a trace element in the residual solid
C_{res}	the concentration of a trace element in the residue (residual solid + residual melt)
C_0	the concentration of a trace element in the source before melting
$\overline{C_f}$	the concentration of a trace element in the extracted melt
C_f^0	the concentration of the residual melt when the melting degree reaches its critical value, or the concentration of the first drop of the extracted melt
ϕ	the critical volume porosity of the residue
Φ	the critical mass porosity of the residue
Ψ	the mass porosity of the residue
D_0	bulk distribution coefficient of the source
f_S	defined in Eqn. 8, the mass fraction of residual solid relative to the residue when the volume porosity of the residue reaches its critical value ϕ.
M	melting rate, the rate of mass transfer/unit volume from the residual solid to the residual melt (mass/volume/time)
M_e	melt extraction rate, the rate of mass transfer/unit volume from the residue to the extracted melt (mass/volume/time)
X	the fraction of extracted melt relative to the initial amount of solid source before melting
F	the total partial melting degree of the source
G	defined in Eqn. 7

ρ_f density of melt
ρ_S density of solid
K diffusion coefficient
$D_{s/f}$ bulk solid-melt distribution coefficient during nonmodal partial melting
D_i effective distribution coefficient
α the mass ratio of the residual melt over the residual solid in the residue
β mass influx rate (influxing mass fraction of the solid mass divided by degree of melting)
F_C defined as $\alpha/(\alpha + \beta + 1)$
C_A the concentration of a trace element in influxing material