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Effective middle surface of lithosphere

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Abstract

Representation of the lithosphere by an equivalent elastic plate is a common method in Earth sciences, when the mechanical behaviour of the lithosphere is investigated. The equivalent plate is determined by two parameters: thickness and configuration of the middle surface, named as effective elastic thickness (EET) and effective middle surface (EMS) of the lithosphere. EET is related to the flexural deformation of the lithosphere to vertical loading while EMS controls the lithosphere's response to lateral force variations. EET has been well investigated, whereas EMS remains 'in the shadow' in geophysics. The present paper proposes a mathematical formulation for the EMS allowing to calculate it *theoretically*, from a stress–strain distribution. The equilibrium equation of the equivalent elastic plate is derived from the general rheologically independent equilibrium equation for the lithosphere. It contains a member proportional to the EMS curvature, which describes pre-existing flexure of the equivalent elastic plate. It must be included in flexural calculations with non-zero in-plane forces, because it is an integral part of the equilibrium. EMS (like EET) depends on the lithosphere's structure, constitution and thermal state. Contrasting with EET, bending and mechanical layering do not substantially affect EMS. Temperature exerts the strongest influence on EMS: change in thermal regime may shift EMS vertically by 50 km. Possible deflection of EMS, due to variations of other parameters, is usually lower than 10 km. Tectonically, EMS reveals through stress-induced vertical movements. Their amplitude may be detectable even under the action of moderate intraplate force. The most pronounced effect of EMS variation is expected at continental rifts and orogens.

Keywords: lithosphere; elastic properties; mechanical properties; stress; strain

1. Introduction

It is now well accepted that the mechanical response of the lithosphere to applied horizontal and vertical loading can be modelled, at least in a first-order approximation, as the response of a thin elastic plate, named equivalent elastic plate. In general, the mechanical properties of an elastic plate are determined by its elastic modulus, thickness and configuration of a middle surface. Thickness of the equivalent elastic plate is called effective elastic thickness (EET) of the lithosphere. Let us call middle surface of the equivalent plate, the *effective middle surface* (EMS) of the lithosphere.

EET characterises the flexural response of the lithosphere to vertical loading. Based on geophysical observations, EET estimates were performed by fitting the computed response of the equivalent plate

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and observed response of the lithosphere to the given vertical loading (e.g., [1,2]). EET can also be estimated *theoretically*, by computing depth-dependent rheological structure of the lithosphere inferred from lithospheric structure, thermal state, stress regime and laboratory-derived rheological constitutive equations [3–5].

Configuration of the EMS determines the flexural response of the lithosphere to in-plane forces variations. Attempts to explain vertical movements of the Earth surface as a response to variations of the imposed horizontal forces have been undertaken a long time ago (see [6] for a review). During the past two decades, this mechanism was considered to model intraplate folding (in the Indian Ocean [7,8], Australia [9], Canada [10], Asia [11,12] and over the whole Eurasia territory [13]), as an additional factor of sedimentary basin subsidence [14–16] and as one of the causes of the third-order fluctuations in the sea-level record [17–19].

However, quantitative description of EMS has not yet been proposed. Ways to describe pre-existing lithosphere flexure adopted in some modelling studies [14–21] were based on qualitative arguments. The neutral surface of the lithosphere [3,4,22] cannot be exploited as EMS when the in-plane force is not zero, as discussed below.

This study provides a mathematical formulation for the EMS. First, the equilibrium of the lithosphere with realistic rheology is considered. The equilibrium equation is transformed into a form similar to the thin elastic plate equation. This allows deriving analytical expressions of EMS/EET. The obtained expressions are used to investigate the EMS behaviour, dependent on the crustal structure, thermal regime, imposed in-plane force and plate curvature. Finally, the amplitude of EMS deflection, needed to induce detectable vertical movements under typical plate-tectonic forces, is estimated.

2. Representation of the lithosphere by an equivalent elastic plate

Mathematically, replacing the lithosphere by an equivalent elastic plate consists of reducing the equation describing flexure of the lithosphere to a form which is similar to the elastic plate equation.

Let us consider the thin plate approximation of 2D (or cylindrical) bending of the lithosphere, i.e. plane-strain state, and deal with deviatoric components of stress/strain, assuming the average components can be excluded. Let us assume that the x-axis is horizontal, the z-axis is vertical, the compressive and extensional stress and strain are respectively positive and negative. The variables and parameters used in this study are presented in Table 1.

The rheologically independent equilibrium equation will first be considered. Then, it will be transformed into a form similar to the elastic thin-plane equation. The analysis will be applied to the cases of non-layered and layered lithosphere.

2.1. Rheologically independent equilibrium equations

Let us consider the balance of forces and moments applied to a small (in the horizontal direction) element of the lithosphere bounded by vertical planes (Fig. 1). An element is in equilibrium when applied forces and moments are balanced. The forces acting on the element are: loading on the upper (p_1) and lower (p_2) surfaces of the element, stresses acting from the side of neighbouring elements and applied to the lateral surfaces ($\sigma(x, z)$ and $\sigma(x + \mathrm{d}x, z)$) and volume gravitational loading ($\rho(x, z) \cdot g$). The loads on the upper and lower surfaces of the element are supposed to be enough even and we will neglect their variations over the considered small interval.

Let us express the forces and moments applied to a vertical section of the lithosphere as:

$$\begin{cases} N_{xx}(x) = \displaystyle\int_{z_1(x)}^{z_2(x)} \sigma_{xx}(x, z)\,\mathrm{d}z \\ N_{xz}(x) = \displaystyle\int_{z_1(x)}^{z_2(x)} \sigma_{xz}(x, z)\,\mathrm{d}z \end{cases} \tag{1}$$

$$\begin{cases} M_{xx}(x) = \displaystyle\int_{z_1(x)}^{z_2(x)} \sigma_{xx}(x, z)\,z\,\mathrm{d}z \\ M_{xz}(x) = \displaystyle\int_{z_1(x)}^{z_2(x)} \sigma_{xz}(x, z)\,x\,\mathrm{d}z = x \cdot N_{xz}(x) \end{cases} \tag{2}$$

where moments are determined with respect to the coordinate origin, $z_1(x)$ and $z_2(x)$ are the upper and lower boundaries of the element. Force balance

Table 1
Parameters and variables used in equations

Designation	Equation	Description
x		horizontal coordinate
z		vertical coordinate in Eulerian coordinate system (CS) fixed in space
z'	$= z - w(x)$	vertical coordinate in CS linked with the upper surface of lithosphere
z_1, z_2, z'_1, z'_2		upper (1) and lower (2) bounds of the mechanically competent lithosphere
$z_{1i} \ldots z_{Ki}, z'_{1i} \ldots z'_{Ki}$		boundaries of the mechanically competent lithospheric layers in mechanically layered lithosphere
z_r, z_{r1}		reference surfaces coordinates in CS z
z'_r, z'_{r1}		reference surfaces coordinates in CS z'
w		surface deflection in CS z due to the flexure
w_m	17, 25, 28	EMS coordinate in CS z'
w_n	21	neutral surface coordinate in CS z'
$\sigma_{xx}(\sigma), \sigma_{zz}$		normal stresses
σ_{zx}, σ_{xz}		shearing stresses
$\varepsilon_{xx}(\varepsilon)$	10	normal strain
$N_{xx}(N), N_{xz}$	1	lateral force and transversal force
$M_{xx}(M)$	3	moment of σ_{xx} relative to origin of CS z
p_1, p_2		loading applied to the upper and to the lower surface of lithosphere
q_{in}	4	integral weight of lithospheric column
q	$p_{1z} + q_{in} + p_{2z}$	total vertical loading
ρ		density
G		gravity acceleration
ν		Poisson's coefficient
E		Young's modulus
$\langle E \rangle$	20	averaged along lithosphere Young modulus
E^*, E^*_ν	9	effective elastic modulus
I_r, I_m	13, 15	first moment of effective elastic modulus distribution with respect to the reference surfaces z'_r and w_m
$D_{r,r1}, D_m$	14, 18, 25, 28	second moment of effective elastic modulus distribution, flexural rigidity
T_e	19	effective elastic thickness

equation in the x- and z-directions are:

$$\begin{cases} dN_{xx} + p_{1x}dx + p_{2x}\,dx = 0 \\ dN_{xz} + (p_{1z} + q_{in} + p_{2z}) \cdot dx = 0 \end{cases} \tag{3}$$

where q_{in} is the integral weight of a lithospheric column:

$$q_{in}(x) = \int_{z_1(x)}^{z_2(x)} \rho(x, z)\,dz \tag{4}$$

The equation of moments balance is:

$$dM_{xx} + dM_{xz} + (p_{1z} + q_{in} + p_{2z}) \cdot x \cdot dx + (p_{1x} \cdot z_1 + p_{2x} \cdot z_2) \cdot dx = 0 \tag{5}$$

Differentiating and substituting the expressions of M_{xz} in Eq. 2 and dN_{xz} in Eq. 3 lead to a rheologically independent equilibrium equation (e.g. [23]):

$$\begin{aligned} \frac{d^2 M_{xx}}{dx^2} &= p_{1z} + q_{in} + p_{2z} - \frac{d}{dx}(p_{1x} \cdot z_1 + p_{2x} \cdot z_2) \\ &= q + \frac{d}{dx}\left[(z_2 - z_1)\frac{p_{1x} - p_{2x}}{2} - \frac{z_1 + z_2}{2}(p_{1x} + p_{2x})\right] \end{aligned} \tag{6}$$

where moment M_{xx} is determined with respect to the coordinate origin and q is the total vertical loading. Below we will consider only the xx components of tensors and vectors and xx subscripts will be dropped out.

2.2. Mechanically competent lithosphere

Further simplification consists of moment transformations, which will explicitly represent the plate

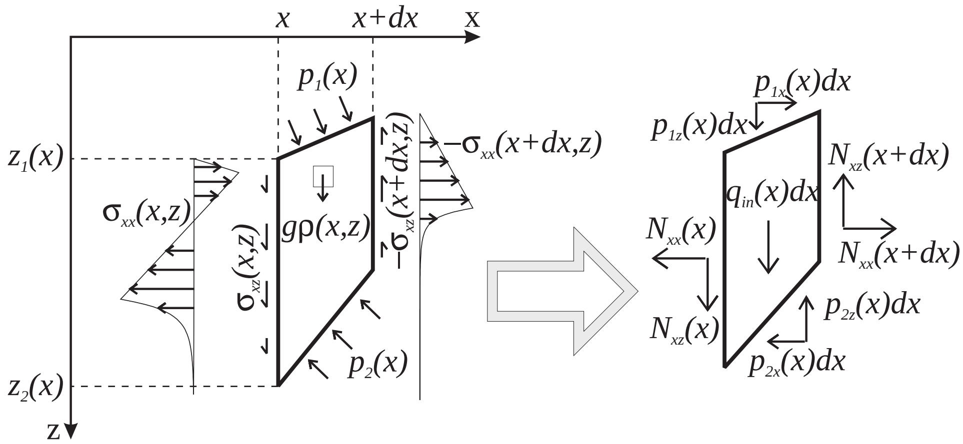

Fig. 1. Scheme showing the forces acting on a small element of lithosphere.

response to the applied moment (parametrized through deflection) and the intrinsic mechanical properties of the lithosphere (such as flexural rigidity), characterising the link between moment and response.

The moment expression can be rewritten as:

$$M = \int_{z_1}^{z_2} \sigma z \,\mathrm{d}z = \int_{z_1}^{z_2} \sigma z_\mathrm{r} \,\mathrm{d}z + \int_{z_1}^{z_2} \sigma (z - z_\mathrm{r}) \,\mathrm{d}z$$
$$= z_\mathrm{r} \int_{z_1}^{z_2} \sigma \,\mathrm{d}z + \int_{z_1}^{z_2} \sigma (z - z_\mathrm{r}) \,\mathrm{d}z \quad (7)$$

where $z_\mathrm{r}(x)$ is the vertical coordinate of some arbitrary point. The first term of the right-hand side of Eq. 7 is moment (with respect to the coordinate origin) of the total applied horizontal force as if it is applied to the z_r-point; the second term is moment with respect to the z_r-point.

Now, let us turn into the material coordinate system $z' = z - w(x)$ of each x, with coordinate origin at the point $z = w(x)$, where $w(x)$ is the flexural deflection (deflection due to flexural deformation) of the upper surface of the lithosphere. The moment expression takes the form:

$$M = (z'_\mathrm{r} + w) \cdot N + \int_{z'_1}^{z'_2} \sigma (z' - z'_\mathrm{r}) \,\mathrm{d}z' \quad (8)$$

Let us introduce an effective elastic modulus (E^*) as:

$$E^* = \frac{\sigma(z')}{\varepsilon(z')} \cdot (1 - \nu^2) = E^*_\nu \cdot (1 - \nu^2) \quad (9)$$

where $\varepsilon(z)$ is total strain consisting of elastic and inelastic parts, ν is the Poisson ratio, the term $(1 - \nu^2)$ accounts for the plane-strain state. The designation E^*_ν is introduced for more compact writing. In the perfectly elastic case, the effective modulus is equal to Young's modulus. The thin-plate theory is based on Kirchhoff's assumptions, which can be expressed as a linear dependence of the strain on plate curvature ($\mathrm{d}^2 w/\mathrm{d}x^2$) and depth (e.g. [24]):

$$\varepsilon(z') = \varepsilon(z'_{\mathrm{r}1}) + \mathrm{d}^2 w/\mathrm{d}x^2 \cdot (z' - z'_{\mathrm{r}1}) \quad (10)$$

where $z'_{\mathrm{r}1}$ is coordinate of some arbitrary point in the lithosphere and $\varepsilon(z'_{\mathrm{r}1})$ is strain at this point. Substituting the last two equations (Eqs. 9 and 10) into the moment expression (Eq. 8):

$$M = (z'_\mathrm{r} + w) \cdot N + \int_{z'_1}^{z'_2} E^*_\nu \left(\varepsilon(z'_{\mathrm{r}1}) + \frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \cdot (z' - z'_{\mathrm{r}1}) \right) (z' - z'_\mathrm{r}) \,\mathrm{d}z'$$
$$= (z'_\mathrm{r} + w) \cdot N + \varepsilon(z'_{\mathrm{r}1}) \int_{z'_1}^{z'_2} E^*_\nu (z' - z'_\mathrm{r}) \,\mathrm{d}z'$$
$$+ \frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \cdot \int_{z'_1}^{z'_2} E^*_\nu (z' - z'_{\mathrm{r}1})(z' - z'_\mathrm{r}) \,\mathrm{d}z' \quad (11)$$

and Eq. 11 into the equilibrium equation (Eq. 6), we obtain an equilibrium equation in terms of deflection (w):

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[D_{\mathrm{r,r1}}\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + N\cdot(z'_{\mathrm{r}}+w) + \varepsilon(z'_{\mathrm{r1}})\cdot I_{\mathrm{r}}\right]$$
$$= q + \frac{\mathrm{d}}{\mathrm{d}x}\left[(z'_2 - z'_1)\cdot\frac{p_{1x}+p_{2x}}{2}\right]$$
$$- \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{z'_1+z'_2+2w}{2}\cdot\frac{\mathrm{d}N}{\mathrm{d}x}\right] \tag{12}$$

with

$$I_{\mathrm{r}} = \int_{z'_1}^{z'_2} E^*_\nu (z' - z'_{\mathrm{r}})\,\mathrm{d}z' \tag{13}$$

$$D_{\mathrm{r,r1}} = \int_{z'_1}^{z'_2} E^*_\nu (z' - z'_{\mathrm{r1}})(z' - z'_{\mathrm{r}})\,\mathrm{d}z' \tag{14}$$

The two parameters z_{r} and z_{r1} can be arbitrarily chosen. The freedom of their choice can be used to simplify the expression. In the case of a mechanically competent lithosphere, for the sake of simplicity we choose $z'_{\mathrm{r1}} = z'_{\mathrm{r}}$.

We further choose z'_{r} (further designated by w_{m}) in order that the following equation is satisfied:

$$I_{\mathrm{m}} = \int_{z'_1}^{z'_2} E^*_\nu (z' - w_{\mathrm{m}})\,\mathrm{d}z' = 0 \tag{15}$$

In this case, the corresponding term of the equilibrium Eq. 12 becomes zero:

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(D_{\mathrm{m}}\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) + \frac{\mathrm{d}}{\mathrm{d}x}\left(N\frac{\mathrm{d}w}{\mathrm{d}x}\right) + \frac{\mathrm{d}}{\mathrm{d}x}\left(N\frac{\mathrm{d}w_{\mathrm{m}}}{\mathrm{d}x}\right)$$
$$= q + \frac{\mathrm{d}}{\mathrm{d}x}\left[(z'_2 - z'_1)\cdot\frac{p_{1x}+p_{2x}}{2}\right]$$
$$- \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{z'_1+z'_2-2w_{\mathrm{m}}}{2}\cdot\frac{\mathrm{d}N}{\mathrm{d}x}\right] \tag{16}$$

Eq. 16 is similar to the classical equation for a thin elastic plate (e.g. [25]), where the last term in the left-hand part describes pre-existing flexure of the middle surface of the elastic plate. Therefore, w_{m} represents the middle surface of the equivalent plate, i.e. the effective middle surface (EMS). The right-hand part of Eq. 16 is not considered in this paper. The value of $w_{\mathrm{m}}(x)$ is equal to:

$$w_{\mathrm{m}} = \frac{\int_{z'_1}^{z'_2} E^*_\nu z\,\mathrm{d}z}{\int_{z'_1}^{z'_2} E^*_\nu\,\mathrm{d}z} \tag{17}$$

Expression of the flexural rigidity (Eq. 15) is similar to the pure elastic case, except for the use of the effective elastic modulus instead of the Young modulus and the effective middle surface instead of the geometrically middle surface:

$$D_{\mathrm{m}} = \int_{z'_1}^{z'_2} E^*_\nu (z' - w_{\mathrm{m}})^2\,\mathrm{d}z' \tag{18}$$

The effective elastic thickness (EET) is derived from the flexural rigidity by the usual way (e.g. [26]):

$$T_{\mathrm{e}} = \left(\frac{12(1-\nu^2)D}{\langle E\rangle}\right)^{1/3} \tag{19}$$

where $\langle E\rangle$ is some representative Young modulus (for example: averaged along the vertical section):

$$\langle E\rangle = \frac{1}{(z'_2 - z'_1)}\cdot\int_{z'_1}^{z'_2} E\,\mathrm{d}z' \tag{20}$$

EET is only used as a visual representation of the flexural rigidity.

2.3. Neutral surface and effective middle surface

The general equilibrium equation in terms of deflection (Eq. 12) differs from the common elastic plate equation by the member $\varepsilon(z'_{\mathrm{r1}})\cdot I_{\mathrm{r}}$, which only vanishes in two cases: $I_{\mathrm{r}} = 0$ or $\varepsilon(z'_{\mathrm{r1}}) = 0$. The corresponding reference surfaces are EMS and neutral surface (NS, w_{n}), i.e. surface of zero deformation ($\varepsilon(w_{\mathrm{n}}) = 0$). All other choices of reference surface give an equilibrium equation different from the standard equation for the thin elastic plate. As this equation is used in all *observation-based* determinations of EET, the comparison of *observed* and *theoretical* EET is only possible when either EMS or NS is used in theoretical calculations of the moment.

As the flexural rigidity and therefore EET, depends on the choice of the reference surface (Eq. 15), EET determined with respect to EMS and NS are different. When the horizontal force is equal to zero ($P = 0$), NS and EMS coincide (compare with Eq. 16):

$$P = \int_{z'_1}^{z'_2} E^*_\nu\left(\varepsilon(w_{\mathrm{n}}) + \frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\cdot(z' - w_{\mathrm{n}})\right)\mathrm{d}z'$$
$$= \frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\cdot\int_{z'_1}^{z'_2} E^*_\nu (z' - w_{\mathrm{n}})\,\mathrm{d}z' = 0 \tag{21}$$

For non-zero force, it is impossible to determine the flexural rigidity (EET) without knowledge of the reference surface as they both participate in the equilibrium equation. Thus, in the case of non-zero in-plane force, one can invoke two flexural rigidities: one determined with respect to EMS and another with respect to NS, and they cannot be discriminated on the basis of observations.

But, the use of NS as a reference surface under non-zero horizontal force is limited due to the asymptotic behaviour in the area of low plate curvatures. Fig. 2 shows the difference between EMS and NS on one particular example, which was calculated using the model described in Appendix A. NS depends on the plate curvature being much stronger than EMS, and becoming asymptotically indefinite. Dependence on plate curvature of EET determined with respect to NS is stronger as compared with EET determined with respect to EMS. EMS and NS almost coincide in the area of large $\mathrm{d}^2w/\mathrm{d}x^2$. This arises when flexural stresses dominate on the intraplate stresses and this case generally corresponds to the case of zero horizontal force.

2.4. Mechanically layered lithosphere

In the previous sections we have analysed the mechanically competent lithosphere. Let us consider a mechanically layered lithosphere consisting of N layers with boundaries z_{1i} and z_{2i} numbered from top ($i = 1$) to bottom ($i = N$) (z_{2i} is not necessary equal to z_{1i+1}, because a purely viscous sublayer can be present between the considered layers). Each layer is supposed to be mechanically competent. The layering means that shear stresses on layer boundaries vanish. This results in independent bending of each layer. For simplicity, we suggest that flexural curvatures of each sublayer are equal. The expression for total strain becomes:

$$\varepsilon(z') = \varepsilon(z'_{\mathrm{r}i}) + \mathrm{d}^2w/\mathrm{d}x^2 \cdot (z' - z'_{\mathrm{r}i}),$$
$$\text{for } z'_{1i} \le z' \le z'_{2i}, \quad i = 1\ldots N \tag{22}$$

where $z'_{\mathrm{r}i}$ is some arbitrary point belonging to the ith layer. Derivation of the 'layered' equations will not be described in detail, as it repeats the non-layered one. Only guidelines are presented.

It is possible to use one of two approaches: either the 'multi-plate' one or the 'single-plate' one. In the first case, each layer is treated separately and the final expressions are summed up. The equilibrium equation has the form:

$$\sum_{i=1}^{N} \frac{\mathrm{d}^2 M_i}{\mathrm{d}x^2} = q + \frac{\mathrm{d}m_i}{\mathrm{d}x} \tag{23}$$

where $m_i = p_{1ix} \cdot z_{1i} + p_{2ix} \cdot z_{2i}$.

After transformations for each sublayer and final summation, one obtains:

$$\begin{cases} \dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\left[\sum_{i=1}^{N} D_i \cdot \dfrac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right] \\ \qquad + \dfrac{\mathrm{d}}{\mathrm{d}x}\left[\sum_{i=1}^{N} N_i \dfrac{\mathrm{d}(w_{mi} + w)}{\mathrm{d}x}\right] = q + \dfrac{\mathrm{d}m'_i}{\mathrm{d}x} \\ \sum_{i=1}^{N} N_i = N, \quad N_i = \displaystyle\int_{z'_{1i}}^{z'_{2i}} \sigma \,\mathrm{d}z' \end{cases} \tag{24}$$

Then, the flexural rigidity and w_{m} of a layered lithosphere in the framework of a 'multi-plate' approach take the form:

$$\begin{cases} D = \sum_{i=1}^{N} D_i, & D_i = \displaystyle\int_{z'_{1,i}}^{z'_{2,i}} E^*_\nu (z - w_{\mathrm{m}i})^2 \mathrm{d}z \\ w_{\mathrm{m}} = \dfrac{\sum_{i=1}^{N} w_{\mathrm{m}i} \cdot N_i}{N}, & w_{\mathrm{m}i} = \dfrac{\displaystyle\int_{z'_{1,i}}^{z'_{1,i}} E^*_\nu z' \,\mathrm{d}z'}{\displaystyle\int_{z'_{1,i}}^{z'_{2,i}} E^*_\nu \,\mathrm{d}z'} \end{cases} \tag{25}$$

In the 'single-plate' approach, the lithosphere is treated as a single plate but with a more complex internal stress and strain distribution (Eq. 23) than in the non-layered case (Eq. 10). The expression of the moment is:

$$M = (w_{\mathrm{m}} + w)P + \sum_{i=1}^{N} \varepsilon(z'_{ri}) \int_{z'_{1,i}}^{z'_{2,i}} E^*_\nu (z - w_{\mathrm{m}}) \,\mathrm{d}z$$
$$+ \frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \cdot \sum_{i=1}^{N} \int_{z'_{1i}}^{z'_{2i}} E^*_\nu (z - w_{\mathrm{m}})(z - z'_{ri}) \,\mathrm{d}z. \tag{26}$$

It can be simplified by the appropriate choice of w_{m} and z'_{ri}:

$$\sum_{i=1}^{N} \varepsilon(z'_{\mathrm{r}i}) \int_{z'_{1,i}}^{z'_{2,i}} E^*_\nu (z' - w_{\mathrm{m}}) \,\mathrm{d}z' = 0 \tag{27}$$

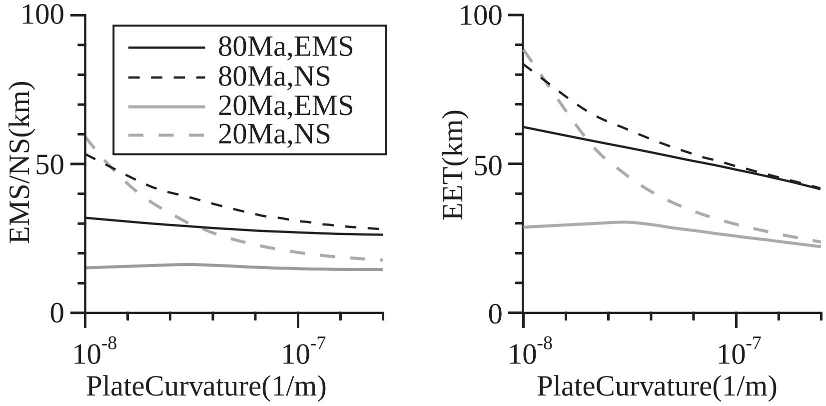

Fig. 2. Dependence on plate curvature of the location of EMS and NS and values of EET, calculated relative to EMS/NS, for an oceanic lithosphere of 20 Ma (grey) and 80 Ma (black) of age. Applied longitudinal force is $P = 2 \cdot 10^{12}$ (compressional). NS (neutral surface) and corresponding EET are shown by the dashed lines. EMS (effective middle surface) and corresponding EET are shown by the solid lines.

In particular, assuming that z'_{ri} coincides with the median surface of each sublayer equates $\varepsilon(z'_{ri})$ in the sublayers and simplifies the expression of w_m. Finally, we have:

$$\begin{cases} D = \sum_{i=1}^{N} \int_{z'_{1,i}}^{z'_{2,i}} E^*_\nu (z' - w_m) \left(z' - \frac{z'_{1,i} + z'_{2,i}}{2} \right) dz' \\ w_m = \frac{\sum_{i=1}^{N} \int_{z'_{1,i}}^{z'_{2,i}} E^*_\nu z' \, dz'}{\sum_{i=1}^{N} \int_{z'_{1,i}}^{z'_{2,i}} E^*_\nu \, dz'} \end{cases} \quad (28)$$

Note that the expression for w_m is the same as in the non-layered case. These two approaches only differ by the way mathematical transformations are made, final expressions Eq. 25 and Eq. 28 being equivalent. Either can be used for applications.

2.5. Some comments on the derived equations

Note that EMS (Eq. 18) does not correspond to any real surface in the lithosphere. Its flexure does not result from flexural lithospheric deformation. Therefore, the thin-plate constraint which requires the deflection to be small compared to EET, is not applicable to EMS. Deflection of EMS can be quite large, even more than for EET.

The presented equations are rheologically independent and therefore valid for any particular rheology. The only constraint used is the thin-plate approximation (Eqs. 10 and 23). To calculate the effective flexural rigidity and position of EMS by means of Eqs. 18 and 19, and Eqs. 25 and 28, one has to know the actual stress distribution (to derive the effective elastic modulus, Eq. 9). It could be obtained in the framework of some particular modelling techniques like the strength-envelope one (e.g. [3–5]) or the visco-elastic (e.g. [27,28]) one, for instance. An example is presented in Appendix A.

3. Control on EET and EMS

The parameters influencing EET are well investigated [4]. EET is controlled by the lithosphere's structure and composition, rheological properties of constitutive rocks, thermal state, in-plane forces and plate curvature. One can expect that EMS be influenced by the same factors because of some similarity between EET and EMS expressions. Let us consider EMS and EET in comparison. To investigate the behaviour of EMS a simple model is adopted allowing the calculation of lithospheric stresses in the framework of a strength-envelope technique (Appendix A). Some results are presented in Figs. 3–7.

Mechanical properties of the oceanic lithosphere (Fig. 3) are mainly controlled by its thermal age. The oceanic lithosphere is mechanically competent. In the absence of any significant flexure, EET can

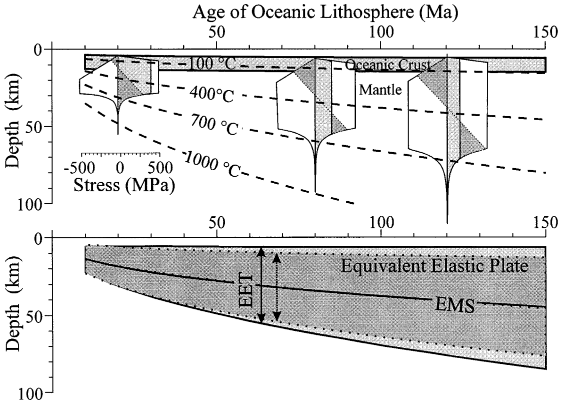

Fig. 3. Thermal structure (upper plot) of oceanic lithosphere; equivalent elastic plate (lower plot) in dependence on thermal age. Some stress-distributions are shown on the upper plot. Light grey/solid lines: $P = 5 \cdot 10^{12}$ N/m, $w'' = 0$; dark grey/dotted lines: $P = 5 \cdot 10^{12}$ N/m, $w'' = 10^{-7}$ m^{-1}.

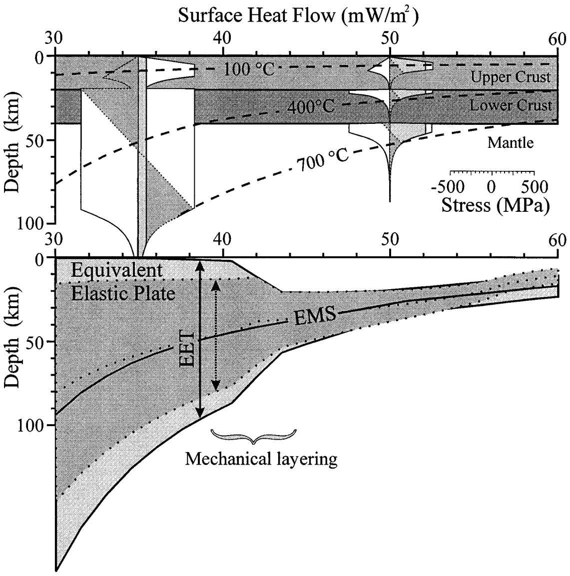

Fig. 4. Dependence of the equivalent elastic plate (lower plot) on the thermal state for the continental lithosphere. Crustal structure and thermal state are shown in the upper plot. Thermal structure is derived by the inversion of the surface heat flow (Appendix A). Some stress-distributions are shown in the upper plot. Light grey/solid lines: $P = 5 \cdot 10^{12}$ N/m, $w'' = 0$; dark grey/dotted lines: $P = 5 \cdot 10^{12}$ N/m, $w'' = 10^{-7}$ m^{-1}.

be approximately determined as the depth of an isotherm (~700ºC) and EMS is located at the depth of about 0.5 EET. Bending significantly reduces oceanic EET (Figs. 2 and 3), but does not substantially affect EMS. As determinations of oceanic EET are performed in the context of the bended lithosphere, most of the *observed* EET values are lower than non-bended EET and usually fall in the depth range of the 450–600ºC isotherms [1].

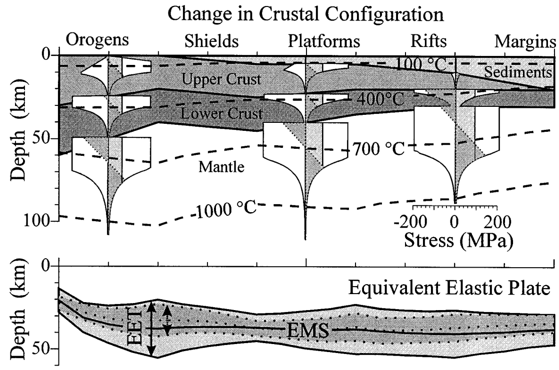

Fig. 5. Dependence of equivalent elastic plate (lower plot) on the change in crustal configuration of the continental lithosphere. Crustal structure and thermal state are shown in the upper plot. Thermal structure is derived from the surface heat flow $q = 40$ mW/m^2, assumed constant for all considered crustal configurations. Selected stress-distributions are shown in the upper plot. Light grey/solid lines: $P = 5 \cdot 10^{12}$ N/m, $w'' = 0$; dark grey/dotted lines: $P = 5 \cdot 10^{12}$ N/m, $w'' = 10^{-7}$ m^{-1}.

The strongest influence on continental EMS/EET is exerted by the thermal state of the continental lithosphere (Fig. 4). Deflection of EMS induced by variations of the thermal regime may exceed 50 km. EET cannot be associated with a specific isotherm (even for zero flexure and uniform crustal structure and composition) due to layering, which induces an additional reduction of EET. In contrast to EET, EMS is not substantially influenced by mechanical layering.

The dependence of EMS on crustal structure under a given surface heat flow (Fig. 5) is weak. EMS variations can be generally constrained within 10 km, excepting the case of a very thick crust. The dependence of EMS and EET on the applied horizontal force (Fig. 6) is weak for typical plate-tectonic forces and prior to a whole lithospheric failure [27]. EET does not depend on the curvature of plate flexure until some threshold value is reached. Above this threshold the curvature rapidly decreases (Fig. 7). In contrast to EET, EMS does not depend on flexural curvature in the whole range of possible plate flexures.

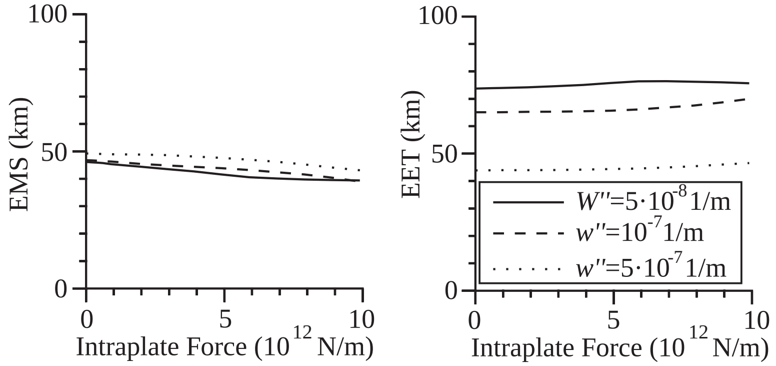

Fig. 6. Dependence on the axial force of EMS and EET for the continental lithosphere (with $20 + 20 = 40$ km of crustal thickness) and three values of plate curvature. Thermal structure is derived from the surface heat flow $q = 40$ mW/m^2.

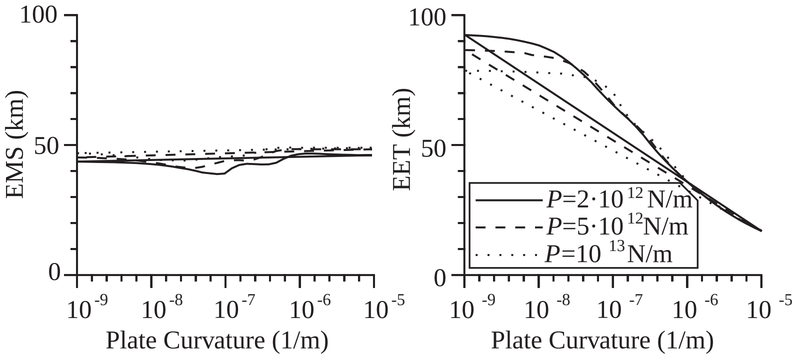

Fig. 7. Dependence on the plate curvature of EMS and EET for the continental lithosphere (with $20 + 20 = 40$ km of crustal thickness) and three values of the axial force. Thermal structure is derived from the surface heat flow $q = 40$ mW/m^2. Tectonic structures genetically related to the lithospheric flexure are extended from the low-amplitude folds (with about 100 m maximal deflection and more than 100 km EET) to the foredeeps and trenches (with 10 km maximal deflection and about 5–10 km EET). Values of plate curvatures range from 10^{-10} to 10^{-5} m^{-1}, with the most typical values being about 10^{-8}–10^{-7} m^{-1}.

In principle, all controls can be subdivided into 'imposed', like in-plane forces and plate curvature, and 'intrinsic', representing the internal state of the lithosphere like lithospheric temperature and structure. EMS does not strongly depend on the imposed factors. Therefore, it can be considered as an intrinsic characteristic of the lithosphere. Being computed for some 'typical' values of lateral force and flexural curvature, it does not significantly fluctuate when controls are changing.

In the literature, flexure of the middle surface of the equivalent elastic plate is often discussed in terms of 'pre-existing deformation of lithosphere'. Indeed, it is not a real deformation but something similar to some pre-existing deformation from the viewpoint of equations. The major factor influencing EMS is the thermal regime, EMS being significantly shifted when the thermal state is changing, even in the absence of mechanical deformation.

4. Lithospheric response to horizontal loading

Mechanically, there are only three ways for horizontal forces to induce vertical movements: (1) elastic deformation, (2) plastic buckling due to stability loss and (3) amplification of pre-existing deflections of EMS. Buckling occurs when imposed horizontal loading exceeds the critical (or 'buckling') value. It has been recognized that elastic buckling of the lithosphere is impossible [6], because, first, it requires too large forces, which cannot be found in the framework of plate tectonics, and second, needed stresses would exceed the yield strength of the constitutive rocks (see [6] for a review). Plastic buckling is not subject to the second constraint, but nevertheless requires large tectonic forces, which are difficult to find in nature. Thus the only way by which plate-tectonic forces can induce vertical movements is an amplification of pre-existing deflections of EMS. In this case, any lateral force induces vertical movements but their amplitudes may be too low to be observable.

The response of the lithosphere to vertical surface loading is in general better studied than response to in-plane forces. Let us introduce an equivalent vertical loading $P\mathrm{d}^2 w_\mathrm{m}/\mathrm{d}x^2 = \rho g h_\mathrm{eq}$ to estimate the response to vertical loading through the response to horizontal loading. EMS curvature $\mathrm{d}^2 w_\mathrm{m}/\mathrm{d}x^2$ equal to $\sim 3 \cdot 10^{-7}$ to 10^{-6} m^{-1} is equivalent to about 100 m of topographic loading (for an in-plane force P equal to $\sim 2 \cdot 10^{12}$ to 10^{13} N/m). This load can be accepted as the lower threshold to obtain detectable vertical movements. Thus, influence of the EMS configuration on vertical movements is expected when its curvature exceeds 10^{-6} m^{-1}.

The amplitude of w_m can be evaluated assuming its sinusoidal form: $w_\mathrm{m} = w_\mathrm{m,max} \cdot \sin(2\pi x/l)$ (l is the characteristic wavelength). Then, the equivalent loading is estimated as $\rho g h_\mathrm{eq} \leq P \cdot (2\pi/l)^2 \cdot w_{m,\mathrm{max}}$. For a characteristic size of lithospheric inhomogeneity of about 600 km, one finds that 100 m of topographical loading is equivalent to 2.5–10 km of EMS deflection and 1 km of topographic loading (i.e. 'sinusoidal' orogen of 1 km height and 300 km width) is equivalent to 25–100 km of EMS deflection (for an in-plane force $P = 2 \cdot 10^{12}$ to 10^{13} N/m). Such values of EMS deflection are quite realistic as shown in the previous section. Thus, vertical movements, as a result of in-plane force variation imposed on pre-existing EMS deflection, are not exotic, but rather common in nature.

Three major conditions should be satisfied to induce such tectonic movements with noticeable amplitude: (1) large amplitude of EMS deflection, (2) adequate deflection wavelength in combination with (3) high applied in-plane forces. Therefore, the largest effect should be expected for rifts and orogens with adjacent foreland basins. Passive margin subsidence is also potentially affected by this mechanism [14–18]. It is possible that low-amplitude (50–200 m) lithospheric folds, observed over the whole Eurasian territory [13], with a spacing close to the characteristic wavelength of the lithosphere, can be explained by response of inhomogeneous lithosphere to imposed horizontal forces. It is also possible that this mechanism played some role during recent subsidence of marginal seas, such as the North Sea [16], the Barents Sea, the Black Sea and the South Caspian Sea [29]. Detailed modelling accounting for actual lithospheric structure, thermal and stress regimes of these regions, is necessary to further discuss this question.

5. Conclusions

In order to define the mechanically equivalent elastic plate, one has to determine its flexural rigidity (or EET) and middle surface (effective middle surface, EMS). The EET determines the response of the lithosphere to vertical loading, while configuration of the EMS controls the reaction of the lithosphere to horizontal forces. Mechanical equilibrium of the equivalent plate is described by Eq. 17. Deflection of the EMS participates in the equilibrium equation as a pre-existing flexure of the equivalent elastic plate. The location of EMS and value of EET can be derived from stress distribution, calculated on the basis of a realistic rheology by Eqs. 18 and 19 and Eqs. 25 and 28 in the case of non-layered and layered lithosphere, respectively.

Dependence of EET on different parameters has been well investigated in the literature, and this study has mainly focused on EMS. Location of EMS depends on the same factors, which influence the EET value [3]: lithospheric structure and constitution, thermal state, applied force and plate curvature. The EET and EMS responses to change of controlling factors are similar, excluding the response to bending and mechanical layering, which both significantly reduce EET and do not substantially influence EMS. Temperature exerts the strongest influence on EMS. EMS deflection due to change in thermal regime may exceed 50 km. Possible deflection of EMS due to variation of other parameters is usually lower than 10 km.

The response of the lithosphere to horizontal forces strongly depends on EMS configuration. Even moderate plate-tectonic forces could induce detectable vertical movements (several hundred metres) in the presence of rheological (mainly thermal) inhomogeneities of an appropriate size. This factor may be the first-order control in some environments, especially in areas of high tectonic stresses with an inhomogeneous structure and thermal regime, e.g. rifts, orogens and foreland basins, passive and active margins.

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Appendix A. An algorithm to calculate EET and EMS

To compute EET and EMS one has to determine lithospheric stresses which are strongly controlled by temperature.

Temperature of the oceanic lithosphere is derived from its age by the expression [26]:

$$T(z,\ \text{age}) = T(z_1) \cdot \Phi\left(\frac{z}{2\sqrt{k \cdot \text{age}}}\right) \tag{A1}$$

The continental lithosphere is assumed to consist of four compositional layers: sediments, upper crust, lower crust and mantle. Temperature of the continental lithosphere is derived by inversion of the surface heat flow:

$$T(z) = T(z_1) + \int_{z_1}^{z} \frac{1}{k}\left[q(z_1) - \int_{\zeta_1}^{\zeta} A(\xi)\,\mathrm{d}\xi\right]\mathrm{d}\zeta \tag{A2}$$

where k is a heat conductivity, A is heat production. Subscript 1 denotes the coordinate of the upper surface of the lithosphere. Heat production is supposed to be exponentially distributed in the crust. The adopted numerical values are displayed in Table 2.

Stresses are defined in the framework of a yield–strength envelope approach [30]. Elastic stresses are defined by:

$$\sigma = \frac{E}{1-\nu^2}\varepsilon \tag{A3}$$

These stresses must satisfy the following two constraints: they should not exceed (1) the brittle yielding limit, given by the Byerlee law [31] (with $\mu = 0.75$, $\lambda = 0.35$):

$$\sigma_{\text{yield}} = \begin{cases} -0.5\rho g z & \text{for the extension} \\ 2\rho g z & \text{for the compression} \end{cases} \tag{A4}$$

Table 2
Values of the parameters, used in the calculations

Parameter		Sediments	Oceanic crust	Upper crust	Lower crust	Mantle	Meas. unit
Density	ρ	2500	2950	2700	2900	3300	kg/m^3
Young's modulus	E	–	70	70	70	90	GPa
Poisson's coeff.	ν	0.25	0.25	0.25	0.25	0.25	–
Heat conductivity	k	2.0	3.1	2.7	3.0	3.5	W m^{-1}K^{-1}
Heat capacity	C_p	1050	1050	1050	1050	1050	J kg^{-1}K^{-1}
Heat production	A_0	1.4	0	2.0	2.0	0	10^{-6} W/m^3
Decay rate	h_r	–	–	9	9	–	10^3 m
Representative mineralogy		No strength	Maryland diabase	Simpson quartzite	Quartz diorite	Olivine	–
			[32]	[32]	[32]	[33]	–
Power factor	N	–	3.05	2.72	2.4	3.6	0
Activation energy	E_p	–	276	134	212	530	10^3 J/mole
Pre-exp. factor	A_p	–	$3.16 \cdot 10^{-20}$	$6.03 \cdot 10^{-24}$	$1.26 \cdot 10^{-16}$	$7.2 \cdot 10^{-18}$	1 s per Pa^N
Activation energy	E_{plb}	–	–	–	–	535	10^3 J/mole
Pre-exp. factor	A_{plb}	–	–	–	–	$5.7 \cdot 10^{11}$	1/s
Flow stress at 0K	σ_{plb}	–	–	–	–	8.5	GPa

and (2) the stresses necessary to maintain the given strain rate of ductile flow [30–33]:

$$\sigma_{\text{yield}} = \text{sign}(\dot{\varepsilon}) \begin{cases} \left[|\dot{\varepsilon}| A_{\text{p}} \exp\left(\dfrac{E_{\text{p}}}{RT}\right)\right]^{1/N} & \sigma_{\text{yield}} \leq 200\ \text{MPa} \\ \sigma_{\text{plb}} \left[1 - \sqrt{\dfrac{RT}{E_{\text{plb}}} \ln\left(\dfrac{A_{\text{plb}}}{|\dot{\varepsilon}|}\right)}\right] & \sigma_{\text{yield}} > 200\ \text{MPa} \end{cases} \tag{A5}$$

Here ε is deviatoric strain, $\dot{\varepsilon}$ is strain rate of the ductile flow, σ is deviatoric stress, E is Young's modulus, ν is Poisson's ratio, R is universal gas constant, T is absolute temperature in degree Kelvin, A_{p}, E_{p}, A_{plb}, E_{plb}, σ_{plb} are material parameters. The adopted values of the parameters are displayed in Table 2. Strain is derived from Eqs. 10 and 22, where $\varepsilon(z_{\text{r}})$ was derived from the given horizontal force (i.e. it was matched to equalise the integral of lithospheric stresses (Eq. 1) and the given horizontal force value). Total lithosphere strength (TLS) under the given strain rate is defined as the maximum possible force, and is equal to the integral of yielding stresses [31].

One important difference of the used algorithm and the common strength-envelope technique is the adopted strain rate values. Usually strain rate values of 10^{-14}–10^{-16} s^{-1} are used, on the basis of observations in tectonically active areas, such as rifts and orogenic belts. But in stable areas, strain rates are much slower. Most of the deformation is concentrated in narrow weak zones (rifts and orogenic belts), whereas broad, stronger areas remain almost undeformed. From a rheological viewpoint, if the elastic layer is present in the lithosphere, the only mechanism to produce non-zero strain rate is the stress redistribution due to viscous relaxation in the lower ductile part of the lithosphere [27]. But such redistribution cannot produce large strain rates. The calculations in the framework of the visco-elastic model show that after some short initial time (less than 0.1 Ma), a strain rate of about 10^{-19}–10^{-17} s^{-1} is established in the lithosphere with non-zero elastic core [34,35]. Stability of the continents over time periods of more than 1000 Ma provides evidence for this fact. For these reasons, the strain rate value $\dot{\varepsilon} = 10^{-18}$ s^{-1} was used by default. If the applied force exceeds TLS for this strain rate, then the strain rate value was fitted to equalise TLS and the applied force, i.e. the strain rate was derived from the imposed force. More accurate justification of the adopted strain rate will be considered elsewhere (Ershov and Stephenson, Visco-plastic and visco-elastic models of lithosphere: a comparison, in preparation; see also [35]). Anyway, a change in strain rate can only shift absolute values of EET and EMS, but will not significantly affect their variations in response to change of thermal regime, crustal thickness, etc.

If strain and stress distributions are known, one can compute the effective elastic modulus (Eq. 9). EET and EMS are determined through the effective elastic modulus by means of Eqs. 18–21 and 26.

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