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The age of the inner core

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Abstract

The energy conservation law, when applied to the Earth's core and integrated between the onset of the crystallization of the inner core and the present time, gives an equation for the age of the inner core. In this equation, all the terms can be expressed theoretically and, given values and uncertainties of all relevant physical parameters, the age of the inner core can be obtained as a function of the heat flux at the core–mantle boundary and the concentrations in radioactive elements. It is found that in absence of radioactive elements in the core, the age of the inner core cannot exceed 2.5 Ga and is most likely around 1 Ga. In addition, to have an inner core as old as the Earth, concentrations in radioactive elements needed in the core are too high to be acceptable on geochemical grounds.

Keywords: core; inner core; thermal evolution

1. Introduction

The ages of the Earth and of mantle–core differentiation are now precisely known [1] by the use of radiogenic methods on both meteorite and mantle samples. However, as no sample of the Earth's core is available, no such possibility exists to date the onset of the formation of the inner core, and the only way to find its age is by a Kelvin-type method [2,3]: modeling core evolution in order to match all available constraints.

In a recent paper [4] we claimed that if the Earth's inner core was older than 1.7 Ga it would be larger than it is. This conclusion was based on a cooling model for the core and is dependent on several assumptions concerning parameter values and cooling processes. However, this conclusion was in general agreement with previous studies [5,6] with the exception of the global cooling model by Mollett [7]. The main difference between his model and ours is our assumption of no radiogenic heating in the Earth's core, based on the partition coefficient values of Oversby and Ringwood [8]. If this assumption was relaxed, a smaller part of the heat flux at the core–mantle boundary (CMB) would correspond to the cooling of core and the inner core growth would then be slower, allowing a greater age of the inner core with the same constraint of its present size.

The goal of this paper is to show that the age of the inner core is the solution of an equation involving intrinsic expressions which can be solved analytically in the zero radiogenic heat case and numerically if we allow some radioactive elements. Also, this allows derivation of an estimate

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of the uncertainty on the age of the inner core and an investigation of its sensitivity on the less well known relevant physical parameters.

2. Radial profiles

The computation of the heat balance of the core requires the definition of profiles for the average temperature, density and gravity. Convective motions are assumed to transport both heat and light elements in the whole outer core which is then taken as hydrostatic, well mixed and adiabatic:

$$\nabla P = -\rho g \tag{1}$$

$$\nabla \xi = 0 \tag{2}$$

$$T_{\mathrm{ad}}(r,t) = T_{\mathrm{s}}[c(t)]\,\exp\left(-\int_{c(t)}^{r}\frac{\alpha g(u)}{C_P}\mathrm{d}u\right) \tag{3}$$

where P is the pressure, ξ is the concentration in light elements, g is the gravity, $T_{\mathrm{s}}[c(t)]$ is the solidification temperature at the radius of the inner core $c(t)$, α is the expansion coefficient and C_P is the heat capacity at constant pressure. The logarithmic equation of state of Poirier and Tarantola [9] is used:

$$P = K_0\frac{\rho}{\rho_0}\ln\frac{\rho}{\rho_0} \tag{4}$$

with ρ_0 and K_0 the density and incompressibility at zero pressure, respectively. Using this equation of state and the hydrostatic balance (Eq. 1) leads to:

$$\rho = \rho_0\,\exp\left[\sqrt{\left(\log\frac{\rho_{\mathrm{c}}}{\rho_0}+1\right)^2-\frac{2\rho_0}{K_0}\int_0^r g\mathrm{d}r'}-1\right] \tag{5}$$

where ρ_{c} is the density at the center. In order to integrate this expression, one needs to know the gravity profile, which itself is given by:

$$g(r) = \frac{4\pi G}{r^2}\int_0^r \rho(u)u^2\mathrm{d}u \tag{6}$$

with G the gravitational constant. There is no way to solve this coupled problem exactly and we chose to develop both profiles to the third order in radius. The coefficients of these developments are obtained by an iterative procedure described in Appendix A. The resulting two profiles are:

$$\rho = \rho_{\mathrm{c}}\,exp\left(-\frac{r^2}{L^2}\right) \tag{7}$$

$$g(r) = \frac{4\pi}{3}G\rho_{\mathrm{c}}r\left(1-\frac{3r^2}{5L^2}\right) \tag{8}$$

L being a length scale for the compression given by:

$$L = \sqrt{\frac{3K_0\left(\log\frac{\rho_{\mathrm{c}}}{\rho_0}+1\right)}{2\pi G\rho_0\rho_{\mathrm{c}}}} = 7400\pm150\ \mathrm{km} \tag{9}$$

The numerical value of L given here is chosen to give a good fit of Eq. 7 to the PREM density values in the outer core. The density jump at the inner core boundary (ICB) $\Delta\rho$ is added to the density profile in the whole inner core to fit the PREM inner core values. This extra density modifies the gravity to give:

$$g(r) = \frac{4\pi}{3}G\rho_{\mathrm{c}}r\left[1-\frac{3}{5}\left(\frac{r}{L}\right)^2\right]+\frac{4\pi}{3}G\Delta\rho\begin{cases} r \text{ if } 0\leq r\leq c \\ \frac{c^3}{r^2} \text{ if } r>c \end{cases} \tag{10}$$

The correction to the density profile due to the extra term in the gravity profile is of the order $\Delta\rho/\rho_{\mathrm{c}}\times r^2/L^2 \ll r^4/L^4 < 0.05$ and is neglected (see Table 1).

Assuming that α/C_P is uniform, the adiabatic temperature profile can also be obtained by direct integration of Eq. 3 to give, to the same order:

$$T_{\mathrm{ad}}(r,0) = T_{\mathrm{s}}(c)\,\exp\left(\frac{c^2-r^2}{D^2}\right) \tag{11}$$

with the adiabatic height $D = \sqrt{3C_P/2\pi\alpha\rho_{\mathrm{c}}G} = 8830\pm1000$ km (Table 1) close to the numerical value of L. Note that, as in the case of the density profile (Eq. 7), the terms involving $\Delta\rho$ in the gravity give a negligible contribution to the temper-

Table 1
Parameter values

Parameter	Value
Gravitational constant[a], G	$6.6873 \pm 0.0094 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
Core radius[b], b	3480 ± 5 km
Present inner core radius[b], c_f	1221 ± 1 km
Density at the center[c], ρ_c	$12.5 \pm 0.55 \times 10^3$ kg m^{-3}
Density jump at ICB[d], $\Delta\rho$	500 ± 100 kg m^{-3}
Specific heat[e], C_P	860 ± 86 J kg^{-1} K^{-1}
Thermal expansion coefficient[f], α	$6.3 \pm 0.6 \times 10^{-6}$ K^{-1}
Entropy of crystallization[f], ΔS	118 ± 12 J kg^{-1} K^{-1}
Temperature of solidification at the center[f], T_{s0}	5270 ± 500 K
Grüneisen parameter[e], γ	1.3 ± 0.2
^{235}U half life[g], $\tau_{^{235}\mathrm{U}}$	$7.0381 \pm 0.0048 \times 10^8$ yr
^{238}U half life[g], $\tau_{^{238}\mathrm{U}}$	$4.4683 \pm 0.0024 \times 10^9$ yr
^{40}K half life[g], $\tau_{^{40}\mathrm{K}}$	$1.2511 \pm 0.002 \times 10^9$ yr
^{232}Th half life[g], $\tau_{^{232}\mathrm{Th}}$	$1.401 \pm 0.008 \times 10^{10}$ yr
^{235}U heat production[h], $H_{^{235}\mathrm{U}}$	5.687×10^{-4} W kg^{-1}
^{238}U heat production[h], $H_{^{238}\mathrm{U}}$	9.465×10^{-5} W kg^{-1}
^{40}K heat production[h], $H_{^{40}\mathrm{K}}$	1.917×10^{-5} W kg^{-1}
^{232}Th heat production[h], $H_{^{232}\mathrm{Th}}$	2.638×10^{-5} W kg^{-1}

[a] From [43].
[b] From PREM [44] with a reasonable estimate for CMB and ICB topography.
[c] From PREM with 5% uncertainty [45] and after subtraction of the density jump at the ICB.
[d] From PREM after subtraction of up to 1.7% density change upon freezing [46,47].
[e] From [28] with 10% uncertainty (see text for a discussion about the choice of Grüneisen parameter).
[f] From [46].
[g] From [48].
[h] From [49].

ature profile. The solidification temperature is obtained from Lindeman's law of melting ([10], p.132):

$$\frac{\partial \log T_s}{\partial \log \rho} = 2\left(\gamma - \frac{1}{3}\right) \qquad (12)$$

with γ the Grüneisen parameter, giving (see Appendix A):

$$T_s(r) = T_{s0} \exp\left[-2\left(1 - \frac{1}{3\gamma}\right)\frac{r^2}{D^2}\right] \qquad (13)$$

T_{s0} is the solidification temperature of core material at the pressure of the center of the Earth.

3. Heat balance of the core

The energy balance in the core [4,5,11–13] states that the total heat flux across the CMB $Q(t)$ has to be equilibrated by the sum of four terms: secular cooling, latent heat, gravitational heat due to inner core differentiation and radiogenic heat if present. This equation can be integrated from the onset of the inner core ($t = -a$) to present ($t = 0$) to give an equation involving total energies:

$$\int_{-a}^{0} Q(t)\mathrm{d}t = \mathcal{C} + \mathcal{L} + \mathcal{G} + \mathcal{H} \qquad (14)$$

with $\mathcal{C}$, $\mathcal{L}$, $\mathcal{G}$ and $\mathcal{H}$, the total secular cooling, latent, gravitational and radioactive energies released since the onset of the crystallization of the inner core, given by:

$$\mathcal{C} = \frac{4\pi}{3}\rho_c b^3 C_P T_{s0} \frac{c_f^2}{D^2}\left(1 - \frac{2}{3\gamma}\right) \qquad (15)$$

$$\mathcal{L} = M_{ic}\Delta S T_{s0} \qquad (16)$$

$$\mathcal{G} = \frac{8\pi^2}{15} G \Delta\rho\, \rho_c b^2 c_f^3 \left[1 - (c_f/b)^2\right] \quad (17)$$

$$\mathcal{H} = M_N \sum_e C_e H_e \frac{\tau_e}{\log 2} \left[\exp\left(\frac{a \log 2}{\tau_e}\right) - 1\right] \quad (18)$$

ρ is the mean density of the core, $\Delta\rho$ is the part of the density jump across the ICB which is due to compositional change, c_f is the present inner core radius, ΔS is the entropy of solidification (the latent heat being $L = T\Delta S$), b is the radius of the core, M_{ic} is the mass of the present inner core, C_e is the massic concentration in radioactive element e, H_e and τ_e being the heat release and half life associated, respectively.

For the sake of clarity, we only give here the expression for the secular cooling term (Eq. 15) as a development limited to the first non-zero term in powers of b/D and c_f/D of the total expression. However, the total expression obtained in Appendix B is used when computing numerical results. This expression is obtained from the only assumptions of a hydrostatic balance, an adiabatic temperature profile in the whole core, as presented in Section 2.

The approximation of an adiabatic inner core can be proved to be very good (see Appendix B1). However, the error coming from this approximation is taken into account in the forthcoming uncertainty computation. Another approximation we have done is that the adiabat holds throughout the liquid core even if a low heat flux across the CMB imposes a sub-adiabatic temperature gradient at the top of the core, a possibility which cannot be ruled out [4]. However, we have shown [4] that this situation, although affecting the thermal evolution of the core, would not have an important effect on the growth of the inner core, hence on its age.

The latent energy released by inner core crystallization ($\mathcal{L}$, Eq. 16) has been computed with the reasonable approximation of a constant density and solidification temperature between $r = 0$ and $r = c_f$. This assumption could easily be relaxed (see Appendix B3) but this would give an unnecessarily complicated expression.

The total gravitational energy coming from the change of density and thereby gravity has to be separated into two parts: one due to rapid changes in the mass distribution associated with convective motions and one due to the slow adjustment of the core to its cooling. The former is converted in Joule heating and is the part to be included in the heat balance (see Eq. 44 and its limited development Eq. 17). The latter does not drive any motion and, as stated by Buffett et al. [14], is totally converted in compressional energy and a negligible adiabatic heating (see Appendix C). Therefore, it does not enter the heat budget of the core. Stacey and Stacey [15] recently produced an estimate of the total gravitational energy released during core cooling history and also computed that part of this energy stored as compressional energy. The value they obtain for the compressional energy is much smaller than the gravitational energy coming from core cooling, in contradiction with the argument of Buffett et al. [14]. However, they did not compute the sensitivity of their numerical results to the choice of input parameters. These energies are subject to large uncertainty since they are very small differences between very large energies. We then believe that their numerical result is not sufficient to dismiss the simple physical argument of Buffett et al. [14].

The expression for the gravitational energy $\mathcal{G}$ given in Eq. 17 is, as in the case of the secular cooling term, only the first term in the development of the more complete result given in Appendix C. The complete result is however used when dealing with numerical values.

The only term involving the age of the inner core in the RHS of Eq. 14 is the radioactive energy term $\mathcal{H}$ given by Eq. 18. In the past, only ^{40}K was considered, but the small difference in the Th/U ratios of the chondrites and mantle allows in the core a small fraction of ^{238}U, ^{235}U and ^{232}Th (Bourdon, personal communication). A general form for the total radioactive energy is then used, the concentrations in all the elements being considered as free parameters in this study.

The precise knowledge of the history of the heat flux at the CMB, $Q(t)$, requires modeling of coupled mantle and core thermal evolution. This has been done by several authors using parameterized convection models for the mantle [6,7,16,17] based on the relation between the Rayleigh number and the non-dimensional heat flux obtained in experiments of convection at high Rayleigh number. This relation, of the form $Nu \propto Ra^{\beta}$ (Ra is the Rayleigh number and Nu is the Nusselt number or the non-dimensional heat flux), is dictated by boundary layer theory and well satisfied in many configurations ([18], and references therein). The dynamics of the system being controlled mostly by radioactive internal heat generation, the heat flux at the surface decreases with time following a exp $(-t/\tau)$ form and global energy conservation of the mantle leads to the same kind of time evolution for the heat flux at the CMB. This is what all the cooling models of the Earth based on the parameterized approach give [6,7,16,17].

Other authors proposed to model heat transfer in the mantle by separating the contributions of different geodynamical objects, namely plates, plumes and thermals. Stacey and Loper [19] proposed a thermal history model of the Earth based on the assumption that the core is totally cooled by hot plumes departing from the D″ layer. Their parameterization is based on different models of the D″ layer as the source of the plumes at the origin of hotspots [20–22] and the heat flux obtained is, after a sharp increase, fairly constant and low (1.7 TW). However, this is an unavoidable result of assuming that 'the inner core has been present and growing for most of the Earth's history' [19]. Relaxing this constraint and using the same parameterization Davies [23] obtained a heat flux that, after the same sharp increase, is decreasing with time with an exponential shape. He also recognized the existence of other modes of cooling of the core, principally the arrival of cold down-welling currents (plates) on the CMB. This mode actually dominates the heat flux at the bottom in a fluid between isothermal surfaces which is, in addition, volumetrically heated [18,24], as is the mantle by radioactive elements and secular cooling. This mode of core cooling is proportional to the surface heat flux which is controlled by the heating rate of the mantle, decreasing exponentially with time.

The sharp increase in the CMB heat flux at the beginning of the models by Davies [23] and Stacey and Loper [19] comes from the assumption of an initial thermal equilibrium between the core and mantle. This initial stage being very short it will not affect the age of the inner core as long as this age is smaller than about 4 Ga, which will turn out to be the case.

Following all these lines of evidences, the heat flux at the CMB can be assumed to vary exponentially with time, $Q(t) = Q_{\mathrm{p}} \exp(-t/\tau_Q)$, the present heat Q_{p}, and the characteristic time τ_Q being then the free parameters. The constant heat flux case is also considered as a limit ($\tau_Q = \infty$) of this evolution function. This type of time variation agrees well with global thermal evolution models but any other form of $Q(t)$ could be used. The linearly varying heat flux previously considered [4] has also been investigated here, resulting in similar values for the age of the inner core. Of course, this type of idealized time evolution has to be interpreted as the average evolution, the short (~ 100 Ma) fluctuations inherent to high Rayleigh number convection [18] being neglected. The equation for the age of the inner core a is then:

$$Q_{\mathrm{p}}\tau_Q\left[\exp\left(\frac{a}{\tau_Q}\right)-1\right]-M_N\sum_e C_e H_e\frac{\tau_e}{\log 2}\left[\exp\left(\frac{a\log 2}{\tau_e}\right)-1\right]=E_{\mathrm{tot}}(c_{\mathrm{f}}) \quad (19)$$

in which the total energy $E_{\mathrm{tot}}(c_{\mathrm{f}}) = \mathcal{C} + \mathcal{L} + \mathcal{G}$ released by cooling of the core since the onset of the crystallization of the inner core does depend neither on the free parameters Q_{p}, τ_Q and C_e nor on the age of the inner core itself.

4. Parameter values, uncertainties and resulting age of the inner core

We see that the age of the inner core is the solution of Eq. 19 involving several exponentials, which can be solved analytically in the case of zero concentration of radioactive elements and numerically otherwise. All the parameters are given in Table 1 with estimated uncertainties and the

Table 2
Total energies

Energy	Value
Gravitational, $\mathcal{G}$	4.1 ± 1.0
Latent, $\mathcal{L}$	6.0 ± 1.6
Cooling, $\mathcal{C}$	7.7 ± 5.0
Total, $E_{tot} = \mathcal{G} + \mathcal{L} + \mathcal{C}$	17.81 ± 7.83

Computed total cooling energies and uncertainties in units of 10^{28} J. The uncertainties comprise the formal uncertainties on the parameter values and those coming from the various assumptions made (see text).

resulting total energies of cooling are given in Table 2.

There are several and much different estimates for the present heat flux at the CMB, ranging from 3 to 10 TW [5,25–27]. We will then explore this whole range of possibilities. If there is no radioactive element in the core, then the age obtained for the inner core is shown on Fig. 1 as well as the uncertainty resulting from the uncertainties prescribed on the various parameters. On all the figures, the present heat flux at the CMB Q_p is on the abscissa and the initial ($t = t_0 = -4.5$ Ga) heat flux Q_0 is on the ordinate. Q_0 is related to τ_Q by $Q_0 = Q_p \exp(t_0/\tau_Q)$. We can see that an age between 600 Ma and 1.8 Ga is obtained with less than 50% uncertainty.

There are two kinds of uncertainties involved in that problem: those attached to the estimates of the different parameters (Table 1) and those due to simplifying assumptions. If the first ones are well estimated (which is not always an easy task), their effect on the results can readily be computed from differentiation of Eq. 19. In order to ensure that we do not underestimate this uncertainty, the highest acceptable values of the uncertainties on all parameters were chosen. We see that, for each chosen value for the two parameters defining the heat flux at the CMB (Q_p, Q_0), the uncertainty on the age of the inner core is less than 50%. Of course, the highest uncertainty lies with the heat flux at the CMB, then chosen to be a free parameter.

The Grüneisen parameter that enters in Lindeman's law of melting (Eq. 12) is the vibrational parameter ([10]). Two other definitions for this parameter are often used: the thermodynamical Grüneisen parameter and the Slater Grüneisen parameter. These three definitions can be proved to be equivalent if Debye theory holds [10]. The thermodynamical Grüneisen parameter, estimated by Stacey [28], is $\gamma_{th} = 1.38$. On the other hand, the Slater Grüneisen parameter can be estimated from PREM by using the approximation $\gamma_{Sl} \simeq -1/6+1/2(\Delta K/\Delta P)$, giving a value of 1.1 at ICB. A satisfactory agreement between these two estimates can be obtained by affecting a reasonable uncertainty of 0.2 to each of them, giving support to Debye theory in the core. We then chose to use the thermodynamical definition since it allows to relate the gradient of solidification temperature to the adiabatic gradient. Consequently, Stacey's value [28] is used. This value is also consistent with a

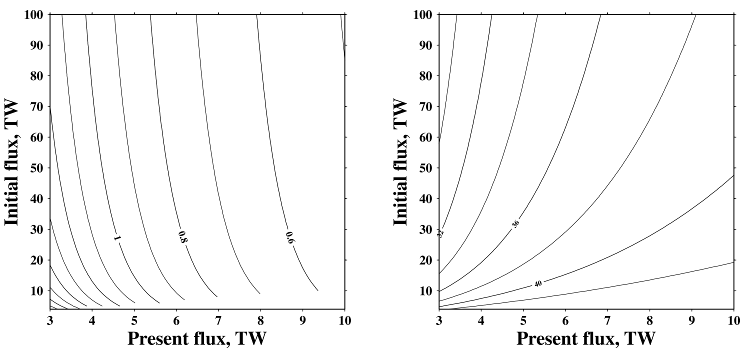

Fig. 1. Age of the inner core in Ga (left panel) and % uncertainty (right panel) as a function of the parameters entering the relation between the heat flux at the CMB and the time (present heat flux, Q_p, on the abscissa, and initial heat flux, Q_0, on the ordinate) if there is no radioactive element in the core. Initial flux is the heat flux just after core differentiation, $t = -4.5$ Ga. Equivalently, τ_Q (see text) varies between 1.3 and 47 Ga.

recent experimental determination of the vibrational Grüneisen parameter [29].

Several assumptions were made and it is necessary to estimate the errors they imply. First, the inner core was assumed to be adiabatic. In a previous study [4], we solved the conduction problem in the inner core and obtained temperature gradients always sub-adiabatic so that our present value of the total cooling energy for the inner core is slightly underestimated. An overestimate can be obtained by assuming a perfectly conducting inner core [5] so that the corresponding cooling energy to be added is $C_{ic} = 2/5 C_P M_{ic} T_s(c_f) c_f^2/D^2$ (see Appendix B2), M_{ic} being the mass of the present inner core. This energy amounts to about 2.5×10^{27} J and is then a small contribution to the total uncertainty of the cooling energy (Table 2).

Another simplification in the computation comes from neglecting the possible presence of a conduction shell at the top of the core, but it was shown to have no detectable effect on the inner core growth [4]. The assumption that the solidification temperature does not change with the gradual enrichment in light elements accompanying the inner core growth is well justified: the present inner core is 3% of the total volume of the core and the increase in light element concentration in the outer core due to the inner core growth is less than 5% of its present value. This means a temperature variation much lower than the prescribed uncertainty on T_{s0}.

The lowest value we accept for the present heat flux across the CMB, 3 TW, is the estimated total heat coming from hotspots [25,26]. If one believes that hotspots are surface expressions of mantle plumes originating at the CMB, this value can indeed be taken as a lower bound of the heat flux at the CMB since in volumetrically heated convection (as the mantle is) a large part of the heat flux at the bottom boundary is due to cold matter going down and can be significantly non-null even if hot plumes are totally absent [18,24]. Also, in convection with internal heating, many hot plumes start from the lower boundary but do not make their way up to the surface, because of the sub-adiabatic temperature gradient in the central region. This means that in the mantle many hot plumes will not produce any surface expression even without taking into account phase transitions or the lithospheric filter [30].

This estimated heat out of the hotspots was taken by Stacey [27] as the actual heat flux at the CMB to infer a maximum possible value for the thermal conductivity of core material, arguing that the heat flux conducted down the adiabatic temperature gradient cannot exceed the heat flux at the CMB. We think that this possibility needs not to be excluded [4] and a 3 TW CMB heat flux can be accepted even if independent estimates of the thermal conductivity lead to a heat flux down the adiabat about twice this value [4,31]. However, even though the thermal conductivity of the core is a badly known and much discussed parameter, it does not directly appear in the problem of the age of the inner core.

5. Effects of radioactive elements

The possibility of radioactive elements in the core was previously rejected because the most likely candidate was ^{40}K and partition coefficients were not favorable [8]. However, this view might be altered depending on the exact scenario for core differentiation [32,33] and other radioactive elements might also enter the core (^{238}U, ^{235}U, ^{232}Th, [34]). Finding the exact values of the concentrations for these elements requires understanding accretion and differentiation processes and measuring the partition coefficients at the correct conditions of temperature, pressure, oxidation and composition of the alloy forming the core. If one knew the concentration of all radioactive elements in the core, it would be possible to compute the age of the inner core for each possible heat flux history, from Eq. 19. As an example, using the maximum acceptable difference in Th/U between mantle and chondrites, one can obtain maximum values of concentration of U and Th in the core as 5 ppb and 0.17 ppb (B. Bourdon, pers. commun.) and solving numerically Eq. 19 using these values for the only three isotopes concerned, we obtain the age given in Fig. 2. As anticipated, the age obtained is greater than without radioactive elements but not by a large amount.

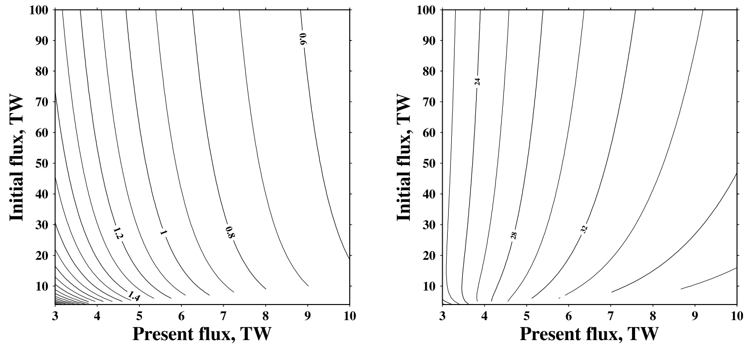

Fig. 2. Age of the inner core and % uncertainty as a function of the heat flux parameters in the case of a 5 ppb U concentration and a 0.17 ppb Th concentration (see text).

The same problem may be treated the other way around: if, by any means, the age of the inner core was known, it would be possible to compute the required concentration in radioactive elements compatible with it, depending on the heat flux history. For example, Hale [35] claimed that a sharp increase of the magnetic field is apparent in paleomagnetic records at 2.7 Ga and that it can be attributed to the onset of the inner core. Although the suggestion is interesting, it relies too much on a single value of paleointensity at 2.5 Ga and needs more quantitative results to be confirmed. However, as an exercise, we can use that value in Eq. 19 and compute the needed radioactive element contents of the core. To that end we have to fix the concentration ratios, for example to bulk Earth values [36], and parameterize all concentrations by the uranium concentration. In the previous computation, we needed a maximum value for the concentrations in radioactive elements which were obtained by maximizing the possible difference between Th/U in chondrites and in the mantle. Here, as this difference is small, we assume it is actually zero, which means that core segregation does not fractionate these radioactive elements. Resulting from this assumption, Fig. 3 gives the values of concentration in uranium needed to have a 2.7 Ga old inner core for the same heat flux histories as previously used, as well as the concentration needed to have an inner core as old as the Earth itself.

We can see that a 2.7 Ga old inner core could be accepted if we account for all the uncertainties but the values obtained for a 4.5 Ga old inner core are unrealistically high. The minimum uranium concentration needed to have an inner core as old as the Earth is about 10 ppb, which corresponds to a present heat production of 3 TW. This value is of the same order than the one used by Mollett [7] without any geochemical con-

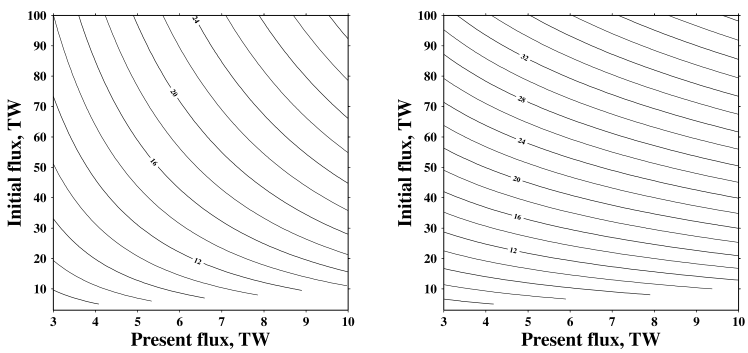

Fig. 3. Concentration (ppb) of uranium in the core in order to have an inner core 2.7 Ga old (left panel) or 4.5 Ga old (right panel) as a function of the heat flux parameters. Th and K are present with ratios Th/U = 4 and K/U = 10^4.

straints. To our knowledge, this value would not be accepted by most geochemists.

6. Conclusion

Accepting the usual assumption of no radioactive elements in the core, it is seen that, for reasonable CMB heat flux histories and taking into account accepted uncertainties in all the relevant parameters, the age of the inner core cannot exceed 2.5 Ga and this extreme value is obtained for an extremely low heat flux (3 TW) during all inner core's life and assuming that all uncertainties act in the same way. A more acceptable value would be around 1 ± 0.5 Ga. If radioactive elements are present in the core, the age of the inner core could be extended to a value of 3 Ga but it seems unrealistic to extend it to the age of the Earth's magnetic field, known to be at least 3.8 Ga [37].

In contrast with a widespread opinion, inclusion of radiogenic heat in Kelvin's calculation of the age of the Earth would not have changed the result [38] as much as would have the inclusion of convective transport or equivalently a greater conductivity in deep Earth than at the surface [39,40]. In the problem of the age of the inner core, the radiogenic content is a more important parameter and this points out the necessity of constraining it more precisely. The effect of the inner core radius on magnetic observables currently studied [41] could also be used to find evidence of the onset of the crystallization of the inner core in paleomagnetic records, thus helping to constrain the core geochemistry. The question of the age of the inner core builds a bridge between geochemistry, paleomagnetism, accretion and thermal evolution models.

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A. Reference state of the core

The gravity is first formally written as:

$$g(r)=\frac{4\pi}{3}G\rho_c r\left(1+\frac{r}{a_1}+\frac{r^2}{a_2}\right) \tag{20}$$

the coefficients a_1 and a_2 being unknown. This expression is used to compute the density profile from Eq. 5 which in turn can be developed to give:

$$\rho=\rho_c\exp\left[-\frac{r^2}{L^2}\left(1+\frac{2r}{3a_1}+\frac{r^2}{2}\left(\frac{1}{\left(\log\frac{\rho_c}{\rho_0}+1\right)L^2}+\frac{1}{a_2}\right)\right)\right] \tag{21}$$

where L, given in Eq. 9, is a length scale for the compressibility. The gravity can then be obtained by integration of Eq. 6 and after development:

$$g(r)=\frac{4\pi}{3}G\rho_c r\left(1-\frac{3r^2}{5L^2}-\frac{r^3}{3L^2a_1}\right) \tag{22}$$

Identification of this expression with Eq. 20 shows that $1/a_1=0$ and $a_2=-5L^2/3$. This implies that the order 4 which is included in Eq. 22 is actually zero and that the next correction is of order 5. This procedure can be performed to get the development to any order but we think that the third order is enough, the following order being about 2% of the leading order for the density. This gives the expressions in Eqs. 6 and 7).

B. Computation of the secular cooling

B1. Totally adiabatic core

We first assume the core to be adiabatic, in its solid part as well as in its liquid part and the error due to this approximation for the inner core will be estimated. The total energy released by core cooling between the onset of the inner core ($t=-a$) and present ($t=0$) is:

$$\mathcal{C} = -\int_{-a}^{0}\int_{0}^{b} 4\pi r^2 \rho C_P \frac{\partial T}{\partial t} \mathrm{d}r\mathrm{d}t \tag{23}$$

Neglecting secular density variation in the cooling energy (its gravitational effect appears in $\mathcal{G}$) and assuming C_P to be constant leads to:

$$\mathcal{C} = -\int_{0}^{b} 4\pi r^2 \rho C_P \int_{-a}^{0} \frac{\partial T}{\partial t} \mathrm{d}t\mathrm{d}r =$$

$$\int_{0}^{b} 4\pi r^2 \rho C_P [T_{\mathrm{ad}}(r,-a) - T_{\mathrm{ad}}(r,0)]\mathrm{d}r \tag{24}$$

where the temperature profiles at the onset of the inner core $T_{\mathrm{ad}}(r,-a)$ and at present $T_{\mathrm{ad}}(r,0)$ are given by Eq. 11:

$$T_{\mathrm{ad}}(r,-a) = T_{\mathrm{s0}} \exp\left(\frac{-r^2}{D^2}\right) \tag{25}$$

$$T_{\mathrm{ad}}(r,0) = T_{\mathrm{s}}(c_{\mathrm{f}}) \exp\left(\frac{c_{\mathrm{f}}^2 - r^2}{D^2}\right) \tag{26}$$

The total energy which has been released by going from profile (Eq. 25) to profile (Eq. 26) is:

$$\mathcal{C} = 4\pi C_P \left[T_{\mathrm{s0}} - T_{\mathrm{s}}(c_{\mathrm{f}}) \exp\left(\frac{c_{\mathrm{f}}^2}{D^2}\right)\right]$$

$$\int_{0}^{b} \rho r^2 \exp\left(\frac{-r^2}{D^2}\right)\mathrm{d}r \tag{27}$$

Using the density profile of Eq. 7, this equation can be integrated to give:

$$\mathcal{C} = 2\pi\rho_{\mathrm{c}} C_P T_{\mathrm{s0}} H^3 \left[1 - \exp\left[-\left(1-\frac{2}{3\gamma}\right)\frac{c_{\mathrm{f}}^2}{D^2}\right]\right] \times$$

$$\left[\frac{\sqrt{\pi}}{2}\mathrm{erf}\left(\frac{b}{H}\right) - \frac{b}{H}\exp\left(-\frac{b^2}{H^2}\right)\right] \tag{28}$$

with $H = LD/\sqrt{L^2+D^2}$. $T_{\mathrm{s}}(c_{\mathrm{f}})$ has been replaced by using Eq. 13, obtained from Lindeman's law (Eq. 12) after transformation of the density derivative to a radius derivative:

$$\frac{\mathrm{d}T_{\mathrm{s}}}{\mathrm{d}r} = \frac{\mathrm{d}P}{\mathrm{d}r}\frac{\mathrm{d}\rho}{\mathrm{d}P}\frac{\mathrm{d}T_{\mathrm{s}}}{\mathrm{d}\rho} = -\rho g \frac{\rho}{K_{\mathrm{S}}} 2\left(\gamma - \frac{1}{3}\right)\frac{T_{\mathrm{s}}}{\rho} =$$

$$-2\left(\gamma - \frac{1}{3}\right)\frac{\rho g}{K_{\mathrm{S}}} T_{\mathrm{s}} \tag{29}$$

where the hydrostatic balance and the definition of the isentropic incompressibility parameter K_{S} have been used. Using now the following identity for the thermodynamic Grüneisen parameter:

$$\gamma = \frac{\alpha K_{\mathrm{S}}}{\rho C_P} \tag{30}$$

leads to Eq. 13.

Eq. 28 can be developed in powers of b/H and c_{f}/D ($D = 6500$ km with parameters in Table 1) to give:

$$\mathcal{C} = \frac{4\pi}{3}\rho_{\mathrm{c}} b^3 C_P T_{\mathrm{s0}} \frac{c_{\mathrm{f}}^2}{D^2}\left(1 - \frac{2}{3\gamma}\right)\left(1 + O\left(\frac{b^2}{H^2}, \frac{c_{\mathrm{f}}^2}{D^2}\right)\right) \tag{31}$$

Although Eq. 31 is simpler and more elegant, the total Eq. 28 was used in the computation of $\mathcal{C}$ to avoid unnecessary errors. Eq. 31 was however useful to estimate the uncertainty on $\mathcal{C}$.

B2. Perfectly conducting inner core

In a previous study [4], solving the conduction equation in the inner core proved that it was always sub-adiabatic. The preceding calculation gives then a lower bound of the inner core cooling energy. An upper bound can be obtained by assuming a perfectly conducting inner core [5] leading to a uniform temperature in the core. To Eq. 28, the heat corresponding to the area between the present adiabat in the inner core and the constant profile $T = T_{\mathrm{s}}(c_{\mathrm{f}})$ has to be added:

$$\mathcal{C}_{\mathrm{ic}} = \int_{0}^{c_{\mathrm{f}}} 4\pi r^2 \rho C_P [T_{\mathrm{ad}}(c_{\mathrm{f}}, r) - T_{\mathrm{s}}(c_{\mathrm{f}})]\mathrm{d}r \tag{32}$$

$$= 4\pi\rho C_P T_{\mathrm{s}}(c_{\mathrm{f}}) \int_{0}^{c_{\mathrm{f}}} r^2 \left[\exp\left(\frac{c_{\mathrm{f}}^2 - r^2}{D^2}\right) - 1\right]\mathrm{d}r \tag{33}$$

$$= 4\pi\rho C_P T_s(c_f)\left[\exp\left(\frac{c_f^2}{D^2}\right)\int_0^{c_f} r^2 \exp\left(\frac{-r^2}{D^2}\right)\mathrm{d}r - \frac{c_f^3}{3}\right] \quad (34)$$

the approximation of a uniform density in the inner core being totally justified. Again, this expression can be computed and developed in powers of c_f/D, giving:

$$\mathcal{C}_{ic} = M_g C_P T_s(c_f)\frac{2}{5}\frac{c_f^2}{D^2} \quad (35)$$

M_g being the mass of the inner core.

B3. Latent heat

The latent heat released at time t is:

$$Q_L(t) = 4\pi c^2 \rho(t)\Delta S T_s(c(t))\frac{\mathrm{d}c}{\mathrm{d}t} \quad (36)$$

$\rho(t)$ being the density of the newly formed inner core material. Integrating this expression between the onset of the inner core and present gives the total latent energy released:

$$\mathcal{L} = 4\pi\Delta S\int_0^{c_f} c^2 \rho(c) T_s(c(t))\mathrm{d}c \quad (37)$$

We then see that the same integral as in Eq. 27 has to be computed but, in the present case, the integral is only running between 0 and c_f and we know that a limited development is totally justified. This means assuming both constant density and solidification temperature, leading to the expression of Eq. 16.

C. Gravitational energy

The global gravitational energy released by formation of the core is:

$$E_G = 4\pi\int_0^b g\rho r^3 \mathrm{d}r \quad (38)$$

and this quantity has to be computed for the core at the onset of the inner core and at present. The present density profile is:

$$\rho = \rho_c \exp\left(-\frac{r^2}{L^2}\right) + \Delta\rho \begin{cases} 1 \text{ if } 0 \leq r \leq c_f \\ 0 \text{ if } r > c_f \end{cases} \quad (39)$$

and the corresponding gravity is given in Eq. 10. At the onset of the inner core, the light elements were present in the whole core instead of the outer core only at present. The initial density profile was then:

$$\rho_i = \rho_c \exp\left(-\frac{r^2}{L^2}\right) + \Delta\rho_i \quad (40)$$

the initial excess density being computed by conservation of mass:

$$\frac{4\pi}{3}c_f^3\Delta\rho = \frac{4\pi}{3}b^3\Delta\rho_i \quad (41)$$

The resulting gravity profile is:

$$g_i(r) = \frac{4\pi}{3}G\rho_c r\left[1 - \frac{3}{5}\left(\frac{r}{L}\right)^2\right] + \frac{4\pi}{3}G\Delta\rho_i r \quad (42)$$

Obviously, the additional terms proportional to $\Delta\rho$ in the gravity profiles modify the density profiles by use of Eq. 5. But the corrections to the density profiles given here (Eqs. 39 and 40) are of order $(\Delta\rho/\rho_c)(r^2/L^2) \ll r^4/L^4$ and can be neglected.

The density also changes with time due to core cooling but as argued by Buffett et al. [14] this part of the gravitational energy is retained in the core as compressional energy and does not enter the heat balance, as well as the work of the pressure forces acting on the CMB. Then, we only consider the changes mentioned in the above equations and the volume change of the core is not taken into account. The change of density distribution accompanying inner core chemical differentiation induces a change in gravity and pressure. The adiabatic heating hence produced:

$$E_{ad} = 4\pi\int_0^b \alpha T\Delta P r^2 \mathrm{d}r \quad (43)$$

is, to first order (the pressure change is computed by integration of the hydrostatic equilibrium down from the CMB where it is equal to zero, taking no effect of the mantle), equal to 6.5×10^{26} J, two orders of magnitude smaller than the gravitational heat (Table 2), and is then neglected.

The total gravitational energy in the core can be computed from Eq. 38 both at present and at the onset of the inner core and the difference gives the energy entering the heat balance of the core:

$$\mathcal{G} = 4\pi^2 G\Delta\rho\,\rho_c L^5 \left[\frac{\sqrt{\pi}}{2}\mathrm{erf}\left(\frac{c_f}{L}\right) - \frac{c_f}{L}\mathrm{e}^{-\frac{c_f^2}{L^2}} - \frac{c_f^3}{b^3}\left(\frac{\sqrt{\pi}}{2}\mathrm{erf}\left(\frac{b}{L}\right) - \frac{b}{L}\mathrm{e}^{-\frac{b^2}{L^2}}\right) - \frac{4}{5}\frac{b^2 c_f^3}{L^5}\left(\frac{1}{3} - \frac{1}{7}\frac{b^2}{L^2}\left(1 + \frac{c_f^2}{b^2}\right)\right)\left(1 - \frac{c_f^2}{b^2}\right) - \frac{4}{15}\frac{\Delta\rho}{\rho_c}\frac{c_f^5}{L^5}\left(1 - \frac{c_f}{b}\right)\right] \quad (44)$$

In this energy, the term involving $\Delta\rho^2$ is clearly negligible. In the limit of infinite L (incompressible core) Eq. 17 is obtained and is the same as the expression obtained by Loper [42].

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