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Historical eclipses and the variability of the Earth's rotation

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Abstract

Ancient Babylonian clay tablets buried for centuries beneath the sands of the desert are part of an extensive historical archive which contains vital information about the rotation of the Earth. Many eclipse records are preserved from a variety of cultures, and using these seemingly crude ancient and medieval observations, variations in the Earth's rotation can be traced back over the past 2500 years. The tidal torque exerted by the Moon (and to a lesser extent, the Sun) is the predominant mechanism in reducing the Earth's rate of spin. It is known that by this mechanism, the length of the day is increasing by 2.3 ms per century. From analyses of the discordance between observations and calculations of eclipses, the average measured increase in the length of the day is 1.8 ms per century, from which it is concluded that besides the tidal contribution there is another long-term component acting to *decrease* the length of the day by 0.5 ms per century. This component, which is thought to result from the decrease of the Earth's oblateness following the last ice age, is consistent with recent measurements made by artificial satellites. Besides these secular trends, there is clear evidence of fluctuations of the length of the day on a timescale of centuries, with an amplitude of 4 ms which is similar to that of the decade fluctuations. © 2001 Elsevier Science Ltd. All rights reserved.

1. Historical background

1.1. Tidal friction: the pursuit of a small effect

The rate of rotation of the Earth varies by approximately 1 part in 10^8 on timescales ranging from months to years. Until the advent of crystal-controlled clocks in the 1930s, and later, atomic clocks in the 1950s, no man-made clock was accurate enough to measure these variations. Indeed, astronomical observations of the successive meridian passages of the 'fixed' stars were used to

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regulate the observatory clocks, hence removing any evidence there might have been of variability. Nevertheless, the existence of fluctuations in the Earth's rotation was proved beyond doubt from astronomical observations alone, before the introduction of crystal-controlled clocks.

As early as the 18th century several writers speculated that tidal friction and other mechanisms might cause variations in the Earth's rate of rotation. In 1754, the following problem was set by the Royal Academy of Sciences at Berlin: "Whether the Earth in its rotation round its axis, by which it brings about the alternation of day and night, has undergone any change since the first period of origin. What may be the cause of this, and what can make us certain of it?" (Ley, 1968). The problem was tackled by the philosopher Immanuel Kant, who reasoned that the combined attraction of the Moon and Sun, by moving the waters of the Earth in a westerly direction, would oppose the diurnal motion, thus gradually reducing the rate of rotation.

Later in the 18th century, Sir William Herschel considered the question of changes in rotation on an observational basis. In 1781, he remarked that no astronomer had investigated this problem, because of "the difficulty of finding a proper standard by which to measure it by; since it is itself used as the standard by which we measure all the other motions." He concluded that, "It is not altogether impossible but that inequalities may exist in the (diurnal) motion which, in an age where observations are carried to such a degree of refinement, may be of some consequence" (Herschel, 1781).

The first actual evidence that the rotation might not be uniform was the continued failure of successive theories of the Moon to predict correctly its observed position. The main problem was that the observed position, and hence motion, was measured using a unit of time derived from the Earth's rate of rotation. Since this unit was actually increasing in size with time, this gave an apparent acceleration in the observed motion of the Moon. During the 18th and 19th centuries, much effort went into the determination of this apparent acceleration of the Moon using observations of ancient eclipses—a technique pioneered by Edmond Halley (1695).

In addition to the secular departure of the Moon from its theory, further variations of an irregular character were found in the observations in the 19th century. Attempts were made to represent these fluctuations by empirical terms added to the purely gravitational theory, but these invariably failed to represent successive observations. After a very thorough investigation of all the observations available to him towards the end of the 19th century, Newcomb (1882), the Director of the US Naval Observatory, surmised: "Now it is most remarkable that observations of transits of Mercury agree with those of the Moon, and with those of the first satellite of Jupiter, in indicating that this apparent inequality (in the Moon's motion) was in part at least due to the Earth's rotation."

The regular orbits of Mercury around the Sun and Io around Jupiter define their own celestial clocks, which are independent of the Earth's rotational clock. If the Earth's clock is running slow, the observed positions of bodies in the Solar System will appear to be ahead of their positions predicted by their gravitational theories. The amount they are ahead will be proportional to the angular speed in their orbits. This is the reason for Newcomb's remarks on the motions of Mercury (as determined from transits over the face of the Sun) and the first satellite of Jupiter (as determined from eclipses by the shadow of Jupiter). The data available to Newcomb were not quite accurate enough to confirm his suspicions. The matter was settled conclusively in 1939 by the Astronomer Royal of the day, Sir Harold Spencer Jones (1939), who carried out a similar investigation to that of Newcomb. Spencer Jones had the advantages of 50 more years of data, and the fact that, fortuitously, the Earth's rate of rotation varied by 8 parts in 10^8 in the period 1870–1910, rather than the more typical 1–2 parts. He found a strong correlation between the angular fluctuations in the positions of the Sun, Moon, Mercury and Venus, which was a clear indication of variations in the Earth's rate of rotation. Besides this he found an apparent acceleration of the Sun which was interpreted as a real deceleration of the Earth's spin due to tides. These results were obtained from telescopic observations made over a period of about 200 years.

Kant's hypothesis about tidal friction was correct. The Moon, and to a lesser extent, the Sun, raise tides on the oceans and solid body of the Earth, which due to friction (almost wholly in the oceans), reduce the rate of rotation. Due to the inelastic response of the Earth's tides, the tidal bulge can be thought of as being carried ahead of the the sub-lunar point. This misalignment produces a torque on the Moon, which, in accordance with Kepler's Third Law, drives the Moon away from the Earth and reduces its angular velocity. Angular momentum is conserved in this tidal interaction: the Moon gains orbital angular momentum, whilst the Earth loses rotational angular momentum.

2. Long-term trends

2.1. The value of eclipses

As atomic clocks have been in existence for only 45 years, it is not possible to measure by this means the actual long-term changes in the Earth's rotation on a timescale of centuries. However, it is possible to measure the tidal friction component by indirect methods from about 30 years of contemporary data on the Moon and artificial satellites. The direct measurement of the long-term changes in the Earth's rotation are masked by the shorter-term decade fluctuations which cannot be removed from the observational record because no precise physical model of their cause exists. We have to turn to the historical observations of eclipses to measure the underlying long-term trends.

The value of eclipses to the subject of the Earth's rotation arises from the following factors:

1. Eclipses, particularly total solar eclipses, are awesome events, and they have been recorded in the historical records of several ancient and medieval civilizations. They cover a time span of about 2500 years.
2. Given the theories of the motions of the Sun (in fact, the Earth's reflected orbit) and the Moon, it is possible to predict when and where these eclipses should have been seen. These predictions are made on the assumption that the Earth's spin is constant. The discrepancy between the predictions and observations can be ascribed to the Earth's variable rate of rotation, after allowing for accidental errors in the observations themselves.

The problem is not quite as straightforward as stated in the second point, because in the theories of their motions neither the Earth, nor the Moon, can be treated as rigid bodies. Their tidal distortion and inelastic responses complicate the dynamics of the situation. As we stated earlier, the Moon experiences an angular orbital deceleration, and the Earth a deceleration in spin under this tidal interaction. Until the advent of the space-age, no-one was sure how large this tidal interaction was, although Spencer Jones (1939) obtained a good estimate from astronomical observations without the use of eclipses. The effects of these two accelerations—in the Moon's orbit and the Earth's spin—were highly correlated in the analyses of eclipses: a change of the acceleration in one producing an almost equal and opposite effect in the other.

This correlation vitiated all the attempts by astronomers from Halley to Muller and Stephenson (1975) to derive reliable results from eclipses for the secular change in the rate of rotation of the Earth. This was further complicated by the inclusion of what were subsequently shown to be largely unreliable observations in the analyses. Two British astronomers, Philip Cowell and John Fotheringham made important progress at the beginning of the 20th century in the analyses of eclipses. Their work was aided by the improvements in the theory of the Moon's motion, culminating in the monumental treatise of Ernest W. Brown (1919). Cowell's (1905) detection of an apparent acceleration of the Sun, although based on rather flimsy historical evidence, was a pioneering step. Fotheringham (1920) improved upon Cowell and derived a value for the apparent acceleration of the Sun which, interpreted as a deceleration in the Earth's rotation, corresponds to an increase of 2 ms per century (ms/cy) in the length of the day. In the 1970s, Robert Newton (1970, 1972b) further advanced the subject by adding more data to the earlier work. Concurrently, modern historical scholarship was bringing to light more records of eclipses, especially from ancient Babylon and China.

Meanwhile, more work was in progress to try to measure, reliably, the tidal acceleration of the Moon, using data other than eclipses. In 1975 the first author of this article updated Spencer Jones' work on the transits of Mercury and lunar occultations, and derived the result $-26 \pm 2''/\mathrm{cy}^2$ (Morrison and Ward, 1975). This was later confirmed by results from observations of near-Earth artificial satellites whose orbits underwent changes due to the tidally distorted figure of the Earth (Christodoulidis et al., 1988). Laser-ranging to the Moon also produced very reliable results for the Moon's tidal acceleration in the range -25.8 to $-26.0''/\mathrm{cy}^2$ (Williams et al., 1992).

As there are good reasons for supposing that tidal friction has not changed significantly over the past 2500 years (Lambeck, 1980), the way was then open to make full use of the eclipse data by fixing the Moon's tidal acceleration at $-26''/\mathrm{cy}^2$ and solving for the other unknown: the variability of the Earth's rotation. The rest of this article outlines the considerable advances in this subject made in recent years by the authors of this article.

2.2. Time-scales and the tidal change in the length of the day

The time-scale based on the variable rate of rotation of the Earth is known as universal time (UT), and that on a fixed rate as terrestrial time (TT). The difference between these time-scales over the past 400 years is shown in Fig. 1. The overall trend is parabolic, resulting from the double integral of the acceleration of the rotation of the Earth. The choice by astronomers of the two arbitrary constants of integration results in the apex of the parabola lying near AD 1820 and the value of the ordinate being zero at AD 1900. The actual behaviour of TT–UT departs from a parabola because of erratic changes in the rotation of the Earth on a timescale of decades.

After the introduction in 1955 of the atomic time-scale, TAI, the difference TT–UT is derived from TAI and astronomical observations, with a fixed offset of 32.184 s. Before 1955, TT–UT is derived from astronomical observations and celestial mechanics, as described in this paper. The difference in time, TT–UT, between the predicted and observed time of an astronomical event, such as an eclipse, is the observable and is hereafter referred to as the *discrepancy in time*. The

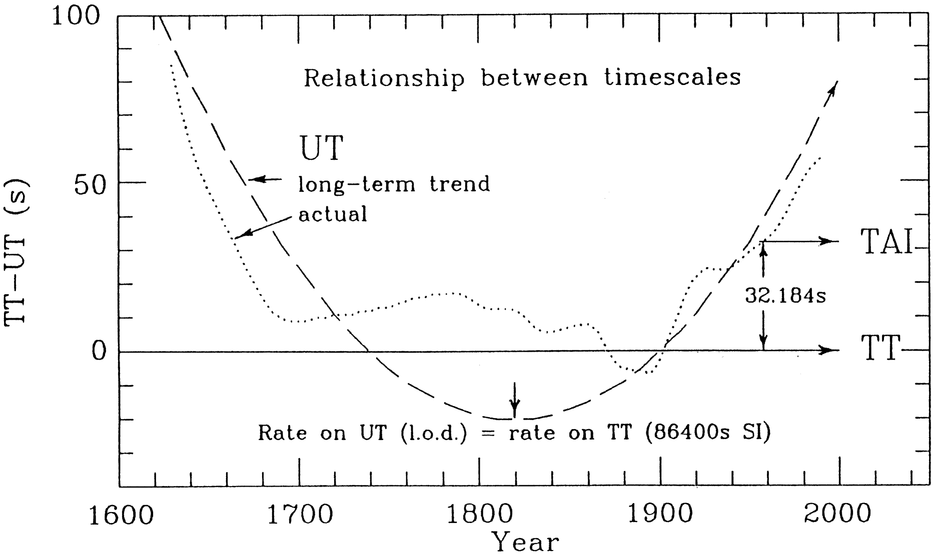

Fig. 1. Diagrammatical representation of the relationship between UT, TT and TAI (see text for details).

change in the length of the day from the standard of 86400 s SI—hereafter contracted to lod—is obtained by differentiation with respect to time. Differentiation again gives the coefficient of acceleration, or the rate of change in the lod.

The gain in orbital angular momentum by the Moon (and a negligibly small amount by the Sun) due to the tidal interaction in the Earth–Moon–Sun system, requires a loss of rotational angular momentum by the Earth. From the results mentioned in the previous section, the acceleration in the rotation of the Earth due to tidal friction is found to be $-6.15 \pm 0.37 \times 10^{-22}$ rad/s^2, which is equivalent to a rate of increase of $+2.3 \pm 0.1$ ms/cy in the lod (Stephenson and Morrison, 1995). The earliest observations that we discuss in this article date from 700 BC. This is a time interval of more than 25 centuries from AD 1820. The average increase in the lod due to tides over that interval is $\frac{1}{2} \times 2.3 \times 25 \approx 29$ ms. As almost a million days have elapsed, the cumulative 'error' in a clock keeping pace with the Earth's rotation over 25 centuries is about 8 h. Therefore, when we compare the predicted and observed times of eclipses in the earliest period, we expect the *discrepancy in time* to be about 8 h. However, we will show from a discussion of many historical eclipses that the discrepancy is not as great as that based on tidal friction alone and that the observational record is physically more complicated than this.

3. Historical observations of eclipses of the Sun and Moon

The study of historical eclipses involves a considerable amount of detective work. A wide variety of sources must be consulted: Babylonian clay tablets, Chinese printed books, and European and Arabic printed books and manuscripts. Each individual eclipse record needs to be carefully translated and assessed. For example, the exact place of observation must be established and the date must be converted to the modern Western calendar. Most early calendars were lunar, and are, therefore, far from ideal for counting days. However, when the date is expressed in terms of the Julian (or later the Gregorian) calendar it is a simple matter to deduce the exact number of days which have elapsed between an eclipse and the reference epoch of AD 1820. Where a time measurement is recorded, it is necessary to convert this to local mean time on the 24-h clock reckoned from midnight. If no time is recorded, then a fairly careful description is required. To establish totality in the case of a solar eclipse, a clear statement that the Sun completely disappeared is essential. Fortunately, many ancient and medieval records satisfy these criteria, but many more do not. A clear distinction must be made before an observation is analysed. For detailed translations and investigations of numerous ancient and medieval eclipse records, see the recent book by one of the present authors (Stephenson, 1997).

Eclipses of the Sun and Moon can be timed with the unaided eye to within a minute or two without difficulty, and this is certainly good enough for our present purpose. However, this precision is not attained in historical observations, mainly because of the difficulty of measuring time with crude instruments. Nevertheless, we shall show that they are still more than adequate for our purpose.

Fortunately, in the particular case of a total solar eclipse, it is not necessary to know the time of day when the eclipse occurred because the path of totality is narrow and this in itself fixes the position of the Earth, as illustrated in Fig. 2. The uncertainty of the observation is the projected width of the belt of totality parallel to the equator. All we require to know is the date and place at which the eclipse was reported to have been total. However, it is crucial that the eclipse was indeed total, and not just nearly so. Otherwise, the uncertainty of the observation would be much greater. We refer to these observations of total solar eclipses as untimed events because their utility is independent of timing and relies on the geometrical circumstances of the eclipses. For most other eclipses, we require the time of day, and we refer to these as timed events. The untimed and timed events give us two independent sets of data with which to investigate variations in the lod over the past 2500 years.

Many reliable records of timed and untimed observations of eclipses were made in the following four civilizations: ancient Babylon; ancient and medieval China; the medieval Arab world, and ancient and medieval Europe. We give an outline of the sources of the observations and also discuss their reliability.

3.1. Babylon

In the more ancient period, the most useful observations are nearly all from the city of Babylon. Astronomers were active at the site from around 750 BC to at least AD 75. Their observations were systematically recorded on astronomical diaries—clay tablets inscribed with a cuneiform script. Predictions of solar eclipses and other events are recorded in almanacs. Most of the extant tablets are very fragmentary, and it is estimated that today only about 10 per cent of the original archive has come to light. Nevertheless, the surviving texts still form a substantial archive. Nearly all of the extant texts—numbering some 2000—are in the British Museum, having been recovered at the site of Babylon more than a century ago. (Sachs, 1974; Stephenson and Walker, 1985). Most of the surviving texts range in date from about 700 to 50 BC. In particular, the fate of virtually the whole of the last century or so of observations is unknown. The latest known observation—of a solar eclipse—is from 10 BC (Steele, 2000), although a very few almanacs date from the first century AD—the most recent in AD 75 (Sachs, 1976).

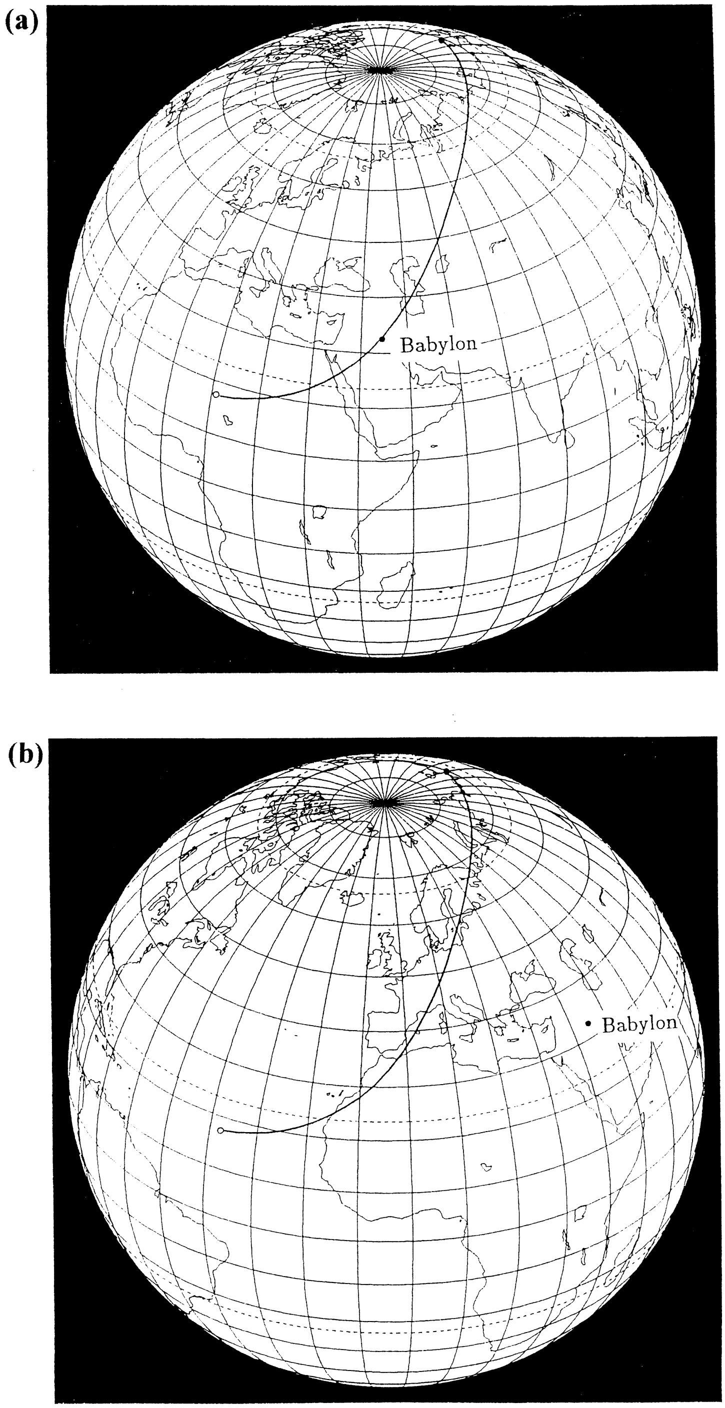

Fig. 2. (a) Observed path of the total solar eclipse of 136 BC passing over Babylon, and (b) the computed path based on the assumption that the length of day (lod) has been constant. The difference in longitude is 48.8°, which is equivalent to 3.25 h.

Both solar and lunar eclipses were of equal interest to the Babylonian astronomers although by chance there is a marked excess of lunar records in the extant texts. It was the practice of the Babylonian astronomers to time the various phases of eclipses relative to sunrise or sunset, depending on which was nearer. The standard unit of time adopted was the *uš*, being equal to $\frac{1}{360}$ of the day and night and thus 4 min. Since this unit was the interval required for the celestial sphere to turn through 1 degree, it is customary to translate *uš* directly as time–degree (contracted to deg, here). The times of the oldest observations (before about 600 BC) are quoted to the nearest 10 *us*, but by about 550 BC all times were estimated to the nearest 1 *us*. The primary source of eclipse observations is the astronomical diaries; these contain a wide variety of lunar and planetary data and originally covered 6 or 7 months of observation. After about 300 BC, information from the various diaries was selectively abstracted by scribes, who used it to compile secondary texts: e.g. lists of eclipse observations. Despite the loss of so many astronomical diaries, especially before about 400 BC, these secondary texts have proved to be a vital source of supplementary data. In all, we have been able to assemble approximately 150 timings of individual eclipse phases from the Babylonian records (Huber, 1973; Sachs and Hunger, 1989, 1990, 1996).

The operation of the Babylonian calendar is well understood. Hence conversion of dates to the Julian calendar can be effected fairly readily. Conversion tables are available for the period from 626 BC onwards (Parker and Dubberstein, 1956). Before 311 BC, the Babylonians counted years from the accession of each successive king. However, commencing with the reign of Seleucus I (the Seleucid Era), a continuous year count was adopted. A luni-solar calendar was employed at all periods. Most years contained 12 months (each of 29 or 30 days), but every 2 or 3 years an intercalary 13th month was added so that the new year always began around the vernal equinox. Since the month began with the first visibility of the young crescent Moon, lunar eclipses tended to occur around the 14th day and solar eclipses around the 28th. When the various Babylonian dates are converted to the Julian calendar, there is usually exact accord with the date according to modern computations; errors seldom exceed a single day.

Throughout the period covered by the texts, Babylonian astronomers systematically timed the interval between the onset of an eclipse and sunrise or sunset (whichever was nearer). Probably some kind of clepsydra (water clock) was used for timing the various contacts.

Illustrative examples of Babylonian observations are provided by two accounts of the total solar eclipse of 15 Apr 136 BC which are preserved on separate Babylonian tablets. Both tablets are somewhat damaged. A secondary text (BM 34034) asserts that the eclipse was total, while an astronomical diary (BM 45745) provides additional descriptive details. H. Hunger (1994 and preprint) has translated both texts. The following is a composite translation based on the renderings by Hunger:

"Year 175 (Seleucid), month XII_2, day 29, solar eclipse beginning at 24 deg. after sunrise. When it began on the south-west side, in 18 deg daytime in the morning it became entirely total [....] Venus, Mercury and the 'Normal Stars' were visible; Jupiter and Mars, which were in their period of disappearance, became visible in its eclipse [....]. It threw off (the shadow) from south-west to north-east. (Time interval of) 35 deg. for onset, maximal phase and clearing. In its eclipse, the north wind which [....] was [set towards the west (?) blew ....]". [BM 34034, Rev. 24–28; BM 45745, Rev. 13′–15.

Month XII_2 denotes the intercalary 13th month. The date corresponds exactly to 15 Apr in 136 BC. It is clear from the combined records that Babylon lay within the path of totality. Several stars and as many as four planets (Mercury, Venus, Mars and Jupiter) became visible. In particular, as the account emphasises, both Mars and Jupiter had already set heliacally. Apart from the description of the total phase, three timings—the start of the eclipse, onset of totality and the end of the eclipse—are preserved.

In order for the eclipse to have been total at Babylon, the *discrepancy in time* at this date must have been between 3.11 and 3.37 h (see Fig. 2). This range is closely supported by the results determined from the measured timings of the individual phases: respectively 3.50, 3.36 and 3.40 h.

For other eclipses, some of the time intervals measured relative to sunrise or sunset are extremely short. In these examples, clock drift—possibly rather serious for a primitive clepsydra when operating over several hours—would be minimal. Thus the solar eclipse of 26 September 322 BC was reported as beginning only 3 deg. before sunset, leading to a value for the *discrepancy in time* of 3.93 h at this date.

3.2. Chinese observations

The earliest Chinese records of eclipses date from some time before 1000 BC. These are found on the oracle bones of the Shang Dynasty. Unfortunately it is not possible to derive an unambiguous date for any of these very ancient observations. Hence they are of dubious value in studying the Earth's past rotation. The oldest reliable records—all of *solar* eclipses—are cited in an early state chronicle. This work, entitled the *Chunqiu* ("Spring and Autumn Annals"), lists 36 solar eclipses between 720 and 480 BC. Although eclipse records are relatively sparse during the subsequent Warring States Period (which began around 480 BC), they are very numerous at all periods after the unification of the empire (221 BC). The main sources of these later observations are the officially compiled dynastic histories. We have extracted some further material from chronicles and historical compendia—notably the great *Wenxian Tungkao* ("Comprehensive history of civilization") compiled by Ma Duanlin around AD 1300. We have also made reference to the extensive list of eclipses and other celestial observations in Chinese history compiled by Beijing Observatory (1988).

Although the original records for virtually all of the observations which we have used no longer survive, printed versions are readily available. This results from the very early discovery of printing in China (7th century AD). Regrettably, it seems likely that in many cases only summaries of the original observations are preserved. Lunar eclipses—which were regarded as of less astrological significance than their solar counterparts—attracted little attention in China before the 5th century AD. However, from this period onwards the two phenomena are recorded with comparable frequency.

In common with the Babylonians, the Chinese adopted a luni-solar calendar. A continuous scheme for numbering years was never followed: instead, years were counted from the accession of each ruler, although from around 100 BC onwards reign-periods (subdivisions of a reign) were also regularly used. Years normally began roughly midway between the winter solstice and the vernal equinox. Although most years contained 12 months (each of 29 or 30 days), a 13th month was inserted every 2 or 3 years to keep the calendar in step with the seasons. However, unlike the Babylonians, the Chinese did not begin the month with the first visibility of the crescent Moon; instead true conjunction between the Moon and Sun—as represented, for example, by a solar eclipse—was chosen. One of the most important innovations in the Chinese calendar was the adoption of a continuous day count. From well before 1000 BC days were systematically expressed in terms of a 60-day cycle and this continued in use throughout Chinese history. This scheme enables Chinese dates to be converted to the Julian or Gregorian calendar both readily and accurately. Convenient calendar tables have been produced by Hsueh and Ou-yang (1956).

Many total and nearly total solar eclipses are recorded in Chinese history, commencing with that of 16 July in 709 BC. Some of these records are extremely brief, but in other cases a detailed description is given, as in the following examples:

19 June in 28 BC (capital Ch'angan: modern Xi'an): "He-ping reign-period, first year, 4th month, day *jihai* [36], the last day of the month. The Sun was eclipsed; it was not complete and like a hook. A moment later it was almost exhausted; it was not much different from a total eclipse. It was 6 deg. in *Dongjing* (lunar lodge). When the eclipse first began, it started from the south-west" [*Hanshu*, chaps. 27c and 97b].

AD 1275 25 June (capital Linan: modern Hangzhou): "Deyou reign period, 1st year, month VI, day *gengzi* [37], the first day of the month. The Sun was eclipsed; it was total. The sky and Earth were in darkness. People could not be distinguished within a foot. The chickens and ducks returned to roost. (It lasted) from the hour *si* (9–11 h) to the hour *wu* (11–13 h); then it regained its brightness" [*Songshi*, chapter 67].

In the above examples, each cyclical day number is given in square brackets after the name of the appropriate day. For the eclipse of 28 BC to be partial at Ch'angan, as the record implies, a value for the *discrepancy in time* of either <2.25 h or >2.65 h is required. The latter range is in much better accord with other roughly contemporaneous observations. It is clear that the eclipse of AD 1275 was total at Linan. By this late date, the *discrepancy in time* had considerably decreased and the indicated range is between -0.19 h and $+0.36$ h.

Most measured times of eclipses are preserved from two distinct periods: AD 400–600 and AD 1000–1300. Solar timings are consistently expressed in terms of a fixed unit: double hours and marks (the latter unit being equivalent to $\frac{1}{100}$ day, or 0.24 h). Although in the later period, the same units are usually used for lunar eclipses, in the earlier period a variable unit—night watches—were employed instead. The interval from dusk to dawn was divided into five equal watches, each of which was subdivided into five equal sections, usually known as "calls" (after the periodic calls of night watchmen). The average length of a "call" was about 0.4 h. A clepsydra (water clock) was used for measuring the various times. When measuring night-watches and their subdvisions, this device was adjusted for seasonal variations in the lengths of the units as necessary (Needham et al., 1986).

The following example of a Chinese record of a lunar eclipse illustrates the use of the night watches:

AD 434 4/5 September (capital Jiankang: modern Nanjing): "Yuanjia reign period, 11th year, 7th month, 16th day, full Moon. The Moon was eclipsed. The calculated time was the hour of *mao* (5–7 h). The Moon (actually) began to be eclipsed at the second call of the fourth watch, in the initial half of the hour of *chou* (1–2 h). The eclipse was total at the fourth call. The Moon was at the end of the 15th degree of *Yingshi* (lunar lodge)" [*Songshu*, chapter 12].

The measured times of first and second contact yield results for the *discrepancy in time* of respectively 0.49 and 0.78 h. The discord between these values may well be largely due to the significant rounding errors.

3.3. Arab dominions

Before AD 800 observations from the Arab world are rare and of little utility. However, over the subsequent 200 years or so, many eclipses of both Sun and Moon were carefully timed by Muslim astronomers—notably in Baghdad and Cairo. Most of these are recorded in an astronomical treatise (or *zij*) compiled by the great Cairo astronomer Ibn Yunus who died in AD 1009. Only a single manuscript containing these observations now survives (in Leiden University Library).

Between about AD 800 and 1500, Arab chronicles recorded both solar and lunar eclipses in qualitative terms. In particular, there are several graphic descriptions of total solar eclipses. Arab chronicles cover much the same period as their European counterparts. However, published Arabic chronicles (or, for that matter *zijes*) are fairly sparse. Most still exist only in manuscript form. Probably only a small proportion of the available eclipse and other astronomical records have so far been consulted.

Dates in both chronicles and *zijes* are normally given in terms of the Islamic lunar calendar. On this system, every year contains 12 lunar months, each of 29 or 30 days. Since the Islamic year falls short of the tropical year by about 11 days, the start of the year continually retrogrades, making a full cycle of the seasons every 33 years or so. Years are numbered from the *Hijrah*, the migration of Muhammad from Mecca to Medina in AD 622. Tables which allow the rapid conversion of dates to the Julian or Gregorian calendar have been produced by Freeman-Grenville (1977). Alternatively a suitable algorithm may be readily devised (see, for example, Newton, 1972a).

Times in chronicles are usually only crudely expressed, but the measurements made by Arab astronomers in *zijes* are much more precise. In general, the astronomers preferred to determine eclipse times indirectly by measuring the altitudes of the Sun, Moon or a selected bright star, usually to the nearest degree. These measurements were afterwards reduced to local time with the aid of an astrolabe or tables.

We give below two examples of eclipse records from the medieval Arab world. The first of these is an account of a total solar eclipse recorded by a chronicler and the second a report of a partial lunar eclipse observed by an astronomer.

In AD 912 a total solar eclipse was observed in al-Andalus (Arab Spain) on a date corresponding to 17 June. It was recorded by the Cordoba annalist Ibn Hayyan in his chronicle of the reign of the Umayyad Caliph 'Abd Allah b. Muhammad in Cordoba.

"(299 AH). In this year the Sun was eclipsed and all of it disappeared on the 4th day of the week (Wednesday) when one night remained to the completion of (the month of) Shawwal. The stars appeared and darkness covered the Earth. Thinking it was sunset, most of the people prayed the *Maghrib* (Sunset) Prayer. Afterwards the darkness cleared and the Sun then reappeared for half an hour and then set" (Ibn Hayyan, *al-Muqtabis fi Tarikh al-Andalus*, III, 147).

For the eclipse to have been total at Cordoba, then capital of Arab Spain, a value for the *discrepancy in time* of between 0.24 and 0.72 h is required. For results near the lower end of this range, the Sun would have set totally eclipsed. However, for values of the *discrepancy in time* near the upper end of the range, the computed interval between the end of totality and sunset would be about 0.25 h. Although the recorded interval of "half an hour" is only a rough estimate, a result for the *discrepancy in time* nearer 0.72 h than 0.24 h is clearly indicated.

In AD 981 a group of astronomers met in Cairo to observe the partial lunar eclipse on the morning of 22 April. Ibn Yunus was among this group and he reported the eclipse as follows:

"This lunar eclipse was in the month of Shawwal in the year 370 of *al-Hijrah*. We gathered to observe this eclipse at al-Qarafa (a district of Cairo) in the Mosque of Ibn Nasr al-Maghribi. We perceived the beginning of this eclipse when the altitude of the Moon was approximately 21 deg. About one-quarter of the Moon's diameter was eclipsed. The Moon cleared completely when about $\frac{1}{4}$ hour remained to sunrise" (Ibn Yunus, *al-Zij al-Kabir al-Hakimi*).

The altitude measurement at first contact and the estimate of time at last contact lead to fairly self-consistent results for the *discrepancy in time* at this date: respectively 0.56 and 0.65 h. We have assembled 72 Arab observations of solar and lunar eclipses from both chronicles and *zijes*.

3.4. Europe

Ancient European records of eclipses are mainly found in the Greek and Roman Classics (historical, poetical and philosophical works, etc.). These cover the period from about 650 BC to AD 500. An extensive compilation of this material is due to Ginzel (1918). In the Classics there are no careful measurements of time, so that the numerous observations of lunar eclipses are of negligible value in Earth rotation studies. Although the many allusions to solar eclipses are often of considerable historical interest, the date and place of observation are often questionable. Furthermore, only rarely can it be ascertained whether the Sun was completely obscured or whether a portion of the solar disc remained visible even at maximal phase.

Ptolemy's *'E Mathematike Syntaxis* or *Almagest* contains a number of time measurements of lunar eclipses made by Greek astronomers—mainly at Alexandria—between 201 BC and AD 125. These form a useful supplement to the much more numerous Babylonian observations.

Ptolemy's account of the partial lunar eclipse of 1 May in 174 BC is fairly typical. Following the date, which he quotes accurately on the Egyptian solar calendar, he cites the following details:

"From the beginning of the 8th hour until the end of the 10th in Alexandria there was an eclipse of the Moon. It reached a maximum obscuration of 7 digits (=7 twelfths) from the north..." (*Almagest*, VI, 5).

Comparison of the above measured times with their computed equivalents (based on the assumption of a fixed length of day) leads to remarkably self-consistent values for the *discrepancy in time* of respectively 3.06 and 2.99 h.

After AD 125 very few records of eclipses are found in European sources until around AD 800. The intervening period was very much the "Dark Ages" in Europe. However, between about AD 800 and 1500, numerous qualitative descriptions of both solar and lunar eclipses are preserved in chronicles of towns and monasteries. Vast numbers of medieval chronicles have been published in their original language (usually Latin). In particular, the multi-volume compilations of these works by editors such as Muratori (1723–) and Pertz (1826–) are a rich source of eclipse and other astronomical records. Extensive searches through these and other compilations by Celoria (1877a and b) and Ginzel (1884a,b, 1918) uncovered numerous records of solar eclipses. These papers—and also the book by Newton (1972b)—provide a valuable secondary source of data.

Dates are usually expressed in terms of the Julian Calendar, with its Kalends, Nones and Ides. Seasonal hours (12 to the day and 12 to the night) were in common usage, noon occurring at the 6th hour of the day. However, chroniclers tended to express times rather informally—usually to the nearest hour. Because of the lack of measured times, the numerous medieval records of lunar eclipses and small partial solar eclipses are valueless for investigating the

Earth's past rotation. However, the many careful descriptions of total solar eclipses provide a major source of information on changes in the length of the day in the past. Since monasteries and towns were scattered over a wide area of Europe, there were numerous separate centres of observation. Frequently, the same eclipse was recorded independently at a wide variety of locations. This situation is without parallel in other medieval or ancient cultures. For example, for the eclipse of 3 June AD 1239 there are at least eight separate accounts of totality from southern Europe.

An observation of the total solar eclipse of 2 August AD 1133 is unusual in describing the reappearance of the Sun after totality in some detail. This account, from Augsburg, may be translated as follows.

"(1133). Duke Frederick.... set fire to the town of Augsburg and killed many of its citizens..... An eclipse of the Sun occurred on the 4th day before the Nones of August (2 August) at midday for about 1 h, such as is not seen in a thousand years. Eventually the whole sky was dark like night, and stars were seen over almost the whole sky. At length the Sun, emerging from the darkness, appeared like a star, afterwards in the form of a new Moon; finally it assumed its original form" (*Honorii Augustodensis: Summa Totius et Imagine Mundi*; Pertz (1826–), 10, 131).

The chronicler Honorius, who was then presbyter at Augsburg, may well have witnessed the eclipse himself. The eclipse would only have been total at Augsburg for values of the *discrepancy in time* of between −0.02 and +0.32 h. However, combining this observation with a further account of totality for the same eclipse from the monastery of Reichersberg, limits the value of the *discrepancy in time* to between +0.15 and +0.32 h at this date.

The most important eclipse observations in the centuries immediately preceding the invention of the telescope are by the Jesuit astronomer Christopher Clavius (1593). Whilst in Coimbra (Portugal) he observed a total solar eclipse in AD 1560. Seven years later, at Rome, he observed another great eclipse in which the Moon, although centrally placed on the solar disc, was a little too far from the Earth to hide the Sun completely. Clavius' account of the latter event may be translated as follows:

"The other eclipse I saw in Rome in the year 1567, also about midday, in which although the Moon was placed between my sight and the Sun, it did not obscure the whole Sun as previously. But (a thing which perhaps never occurred at any other time), a certain narrow circle was left on the Moon surrounding it on all sides" (Clavius, 1593).

The "narrow circle" which Clavius described was almost certainly the inner corona. Making careful allowance for the irregular profile of the lunar limb, Stephenson et al. (1997) demonstrated that only values of the *discrepancy in time* in the narrow range from 145 s to 165 s satisfy Clavius' critical observation in AD 1567.

4. Analysis of the eclipses: geophysical results

The two independent datasets of untimed and timed eclipses collected in Table 1 are analysed separately for changes in the Earth's rotation over the past 2500 years. The analysis follows closely that in our previous paper (Stephenson and Morrison, 1995), but with the inclusion of more data.

Table 1
Numbers of eclipse observations

Source	Untimed	Timed
Babylon	32	152
China	35	121
Arab	13	59
Europe	26	11

4.1. Untimed data

Each untimed total solar eclipse gives a range of values with sharp boundaries for the difference between the predicted and observed longitudes of the path of totality. This difference is dependent on the width of the path and its angle relative to the equator, as can be seen from Fig. 2. Differences in longitude, converted to units of time, are equivalent to the discrepancy in time between the standard clock and the Earth's clock. In the case of Fig. 2, the difference in longitude is 48.8° which is equivalent to a *discrepancy in time* of 3.25 h, with a maximum range of 15 min (here termed solution space) resulting from the width of the path at Babylon. The solution spaces

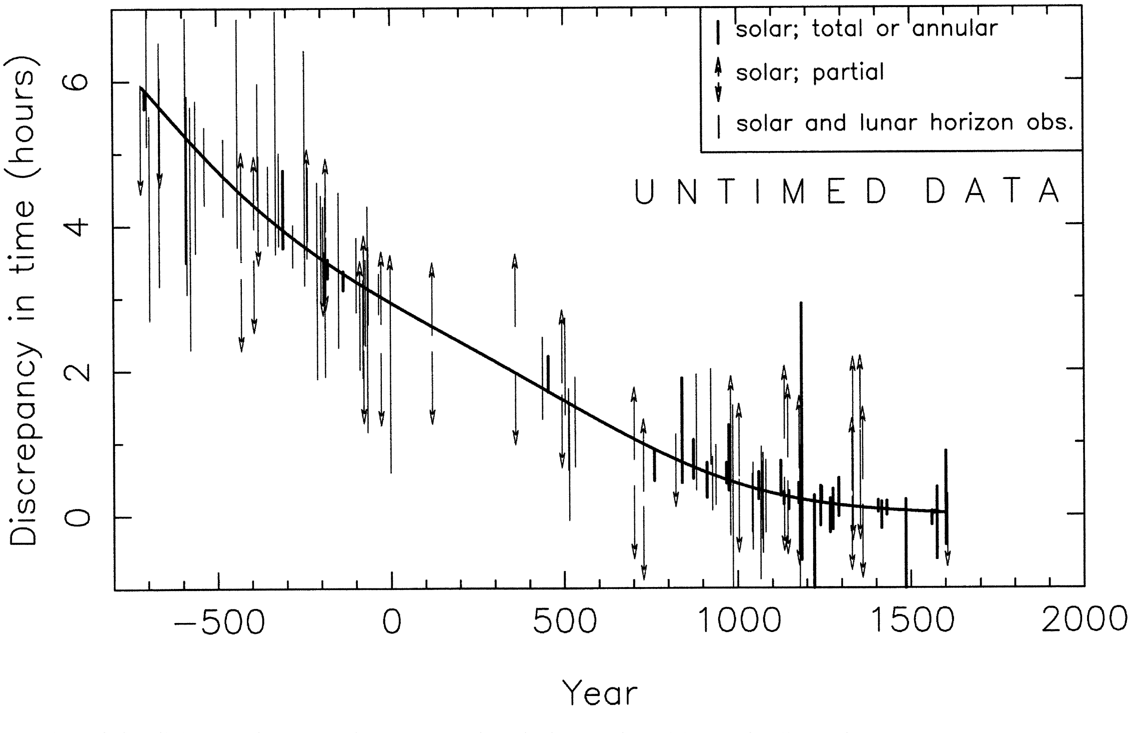

Fig. 3. Plot of the difference between the computed and observed positions of eclipses from −719 to +1605, converted to time. The curve was fitted by cubic splines to the data in Figs. 3 and 4. Note that the vertical lines are not error bars, but solution space, anywhere in which the actual value is equally likely to lie. Arrowheads denote that the solution space extends several hours in that direction.

are plotted as vertical lines in Fig. 3. Twenty more Chinese observations have been added to the dataset of our 1995 paper.

Where an eclipse was reported as being not quite total, the solution space lies on either side of that for totality. These are plotted as bars with arrowheads. One of these two sections of the solution space can sometimes be discarded because it is redundant. Even though the solution space is almost unbounded in the direction of the arrowhead, the other end is sharp and often produces an effective limit.

Observations that the Moon (or rarely the Sun) rose or set while eclipsed, produce similar types of solution space as total or near-total solar eclipses, but with wider boundaries. Only those with more critical limits were plotted in our 1995 paper. Here, we plot all the observations in Fig. 3 (labelled solar and lunar horizon obs.).

4.2. Timed data

The discrepancies in time between the predicted and observed times of solar and lunar eclipses are plotted as discrete points in Fig. 4. They are subject to considerable error, of course, and estimates of these can be made from the vertical scatter of the points. About 30 more Babylonian and 40 Chinese observations have been added to the dataset analysed in our 1995 paper.

After the introduction of the telescope in the early part of the 17th century, the timing of thousands of occultations of stars by the Moon produces much higher resolution, and the discrete

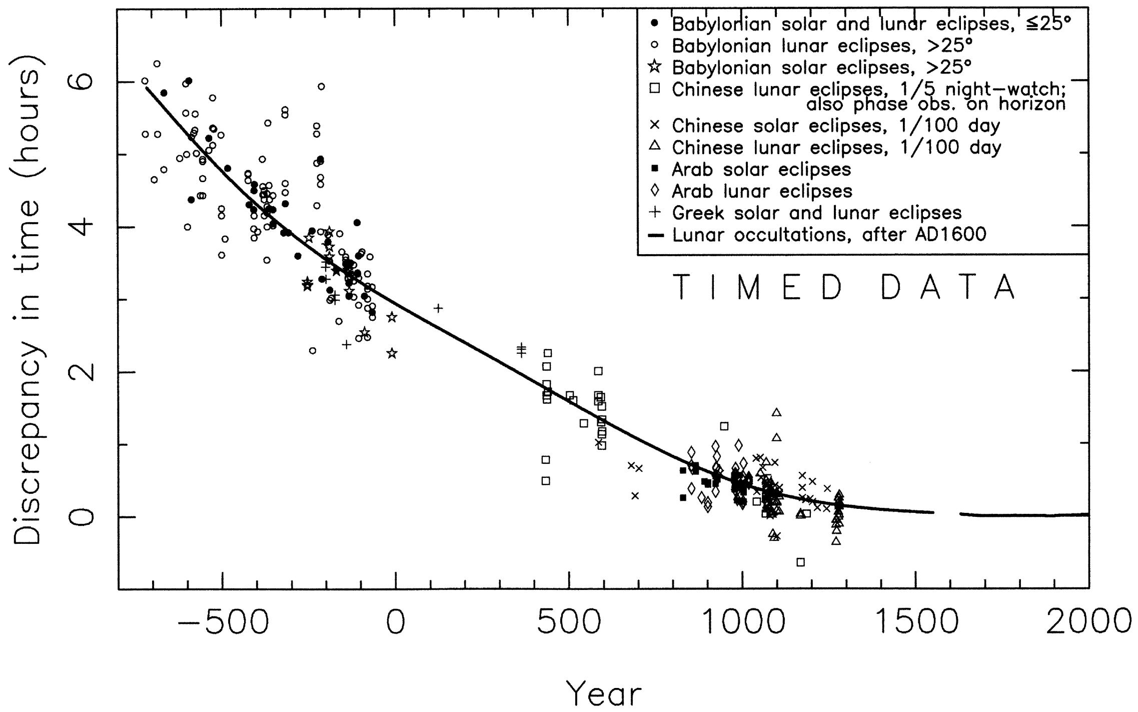

Fig. 4. Plot of the difference between the computed and observed times of eclipses from −720 to +1279. The curve before 1600 was fitted by cubic splines to the data in Figs. 3 and 4. The curve after 1600 was derived mainly from lunar occultations and its uncertainty is less than its width.

points lie close to a narrow, continuous curve. This is shown as a continuous line in Fig. 4. The errors on this curve range from a few tens of seconds in the 17th century to less than 1 s by the 19th century, and are thus negligible on the scale of Fig. 4.

4.3. Curve-fitting to the raw data

It is clear from Figs. 3 and 4 that the data follow a very similar trend. The simplest model that one might propose to fit these data is a parabola with its apex at zero around the epoch +1820 (see Section 2.2), corresponding to one or a combination of constant torques producing an acceleration in the Earth's rotation.

Whilst in general a parabola is a good fit to most of the data, it does not satisfy the very reliable boundary conditions of several eclipses in Fig. 3. These relate to total or annular eclipses in +454, +761, +1133, +1147, +1221 and +1267, and the partial eclipse of +1178. The historical accounts of these eclipses are all very reliable. Also, the median of the timed data around +1000 lies significantly below the parabola. See our 1995 paper for a discussion of this point. For these reasons, we reject a purely parabolic solution.

Curve–fitting by cubic splines with knots at the epochs −200, +300, +1100, +1700 and +1990 is found to be the best approach, conversant with the principle of economy of degrees of freedom and having regard to the smoothness of the record after +1600. The positioning of the knots is not critical, but their frequency is. Too many knots permit the curve to fluctuate unjustifiably. The resulting cubic spline curve is plotted in Figs. 3 and 4 as a continuous curve. It satisfies all the constraints in Fig. 3 imposed by the limits of the untimed solar eclipses, including all those of the 54 (less reliable) lunar eclipses, except for −382, −239, +923 and +1067. The main difference from our 1995 paper is the addition of the comparatively early Chinese total solar eclipse of −708 which sets a tighter constraint on the solution at that epoch. This produces a steeper slope there, and this is supported by the increased dataset of timed Babylonian observations. A rediscussion of the +1567 solar eclipse (Stephenson et al., 1997) also leads to tighter constraints on the solution at that epoch than in our 1995 paper.

4.4. Changes in lod

By taking the first derivative along the cubic spline curve in Figs. 3 and 4 we derive the change in the lod. This is plotted in Fig. 5, together with the fluctuations in the lod from AD 1700 to the present taken from the occultation observations which are discussed in Stephenson and Morrison (1984). This shows an average trend of $+1.8\pm0.1$ ms/cy in the lod which is equivalent to an acceleration of $(-4.8\pm0.2)\times10^{-22}$ rad/s^2 in the Earth's rotation.

The observed trend of +1.8 ms/cy is obviously at variance with the predicted trend of +2.3 ms/cy due to tidal friction alone. Clearly, there is another component acting in the opposite sense which decreases the lod by -0.5 ± 0.1 ms/cy. This non-tidal acceleration may be associated with the rate of change in the Earth's oblateness attributed to viscous rebound of the solid Earth from the decrease in load on the polar caps following the last deglaciation (Peltier and Wu, 1983; Pirazzoli, 1991). From an analysis of the acceleration of the node of the orbit of near-Earth satellites (Cheng et al. 1989), a present-day fractional rate of change of the Earth's second zonal harmonic

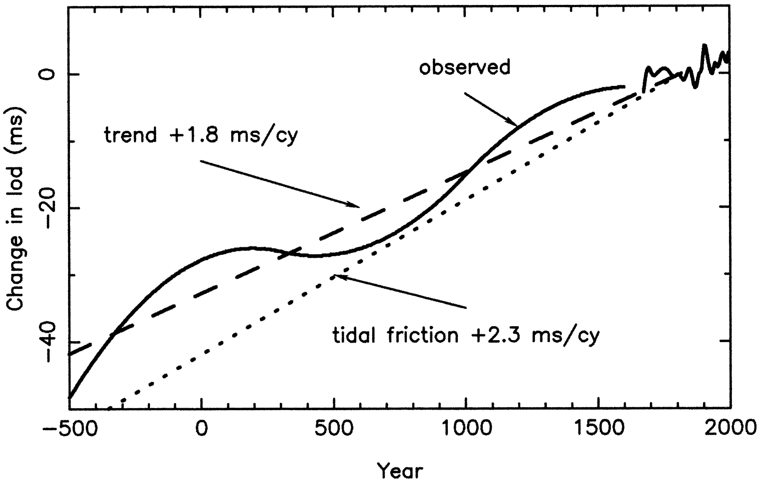

Fig. 5. Plot of the changes in the length of day (lod) −500 to + 1996 obtained by taking the first time-derivative along the curves shown in Figs. 2 and 3. The change due to tidal friction is + 2.3 ms/cy. The high-frequency changes in the lod after 1700 are taken from Stephenson and Morrison (1984).

J_2 of $(-2.5\pm0.3)\times10^{-11}$/year has been derived, which implies an acceleration in the Earth's rotation equivalent to a rate of change in the lod of -0.44 ± 0.05 ms/cy. This is consistent to within the errors of measurement with our result from eclipses of -0.5 ± 0.1 ms/cy, assuming an exponential rate of decay of J_2 with a relaxation time of not less than 4000 years.

Decade fluctuations revealed after the introduction of the telescope are no doubt present on a similar scale throughout the entire period of this investigation, but the integral of these fluctuations is too small to be detected in the pre-telescopic results. All that can be resolved is the long-term envelope of these fluctuations (see Fig. 5) which, in common with the decade fluctuations, probably have their origin in core-mantle coupling (Lambeck, 1980).

5. Conclusion

The coincidence in size of the apparent diameters of the Moon and Sun produces a startling phenomenon at the surface of the Earth which has held Man in awe. In several civilizations he has recorded these events on clay, parchment or paper, and some have survived to the present time. These records are the only way known to us at present of measuring the actual changes—as distinct from that due to tidal friction—in the Earth's rotation over the course of recorded history.

The results that we have obtained from two independent datasets for the non-tidal component of the Earth's rotation serve as a constraint on contemporary geophysical models, such as that of post-glacial uplift. More records of eclipses are probably waiting to be unearthed, and these might help fill in the gaps in Figs. 3 and 4.

References

Beijing Observatory (Ed.), 1988. Zhongguo Gudai Tianxiang Jilu Zongji (A Union Table of Ancient Chinese Records of Celestial Phenomena). Kexue Jishi Chubanshe, Kiangxu.

Brown, E.W., 1919. Tables of the Motion of the Moon. Yale University Press, New Haven.

Celoria, G., 1977a. Sull 'Eclissi Solare Totale del 3 Giugno 1239. Memorie del Reale Istituto Lombardo di Scienze e Letteri, Classe di Scienze Matematiche e Naturali 13, 275–300.

Celoria, G., 1877b. Sull 'Eclissi Solari Totali del 3 Giugno 1239 e del 6 Ottobre 1241. Memorie del Reale Istituto Lombardo di Scienze e Letteri, Classe di Scienze Matematiche e Naturali 13, 367–382.

Cheng, M.K., Eanes, R.J., Shum, C.K., Schutz, B.E., Tapley, B.D., 1989. Temporal variation in low degree zonal harmonics from Starlette orbit analysis. Geophys. Res. Lett. 16 (5), 393–396.

Christodoulidis, D.C., Smith, D.E., Williamson, R.G., Klosko, S.M., 1988. Observed tidal braking in the Earth/Moon/Sun system. J. Geophys. R. 93, 6216–6236.

Clavius, C., 1593. Sphaeram Ioannis de Sacro Bosco Commentarius. Sumptibus Fratrum de Gabiano, Lyons.

Cowell, P.H., 1905. Mon. Not. R. Astr. Soc. 66, 3–5.

Fotheringham, J.K., 1920. A solution of ancient eclipses of the Sun. Mon. Not. R. Astr. Soc. 81, 104–126.

Freeman-Grenville, G.S.P., 1977. The Muslim and Christian Calendars, 2nd Edition. Oxford University Press, Oxford.

Ginzel, F.K., 1884a. Astronomische Untersuchungen uber Finsternisse, II. Sitzungsberichte der Kaiserlichen Akademie des Wissenschaften, Wien. Math. maturwiss. Kl. 88, 629–755.

Ginzel, F.K., 1884b. Astronomische Untersuchungen uber Finsternisse, III. Sitzungsberichte der Kaiserlichen Akademie des Wissenschaften, Wien. Math. Naturwiss. Kl. 89, 491–559.

Ginzel, F.K., 1918. Beitrage zur Kenntnis der Historische Sonnenfinsternisse und zur Frage Ihrer Verwendbarkeit. Abhandlungen der Königlichen Akademie der Wissenschaften, Berlin. Phys. Math. Kl. 4, 3–46.

Halley, E., 1695. Phil. Trans. R. Soc. Lond. 19, 106–175.

Herschel, W., 1781. Phil. Trans. R. Soc. Lond. 71, 115–121.

Hsueh, C.-s., Ou-yang, I., 1956. A Sino-western Calendar for Two Thousand Years. San-lien Shu-tien, Beijing.

Huber, P.J., 1973. Babylonian Eclipse Observations: 750 BC to 0 (unpublished manuscript).

Lambeck, K., 1980. The Earth's Variable Rotation. Cambridge University Press, Cambridge.

Ley, W., 1968. Kant's Cosmogony. Greenwood Publishing Company, New York.

Morrison, L.V., Ward, C.G., 1975. An analysis of the transits of Mercury: 1677–1973. Mon. Not. R. Astr. Soc. 173, 183–206.

Muller, P.M., Stephenson, F.R., 1975. In: Rosenberg, G.D., Runcorn, S.K. (Eds.), Growth Rhythms and the History of the Earth's Rotation. John Wiley & Sons, London, pp. 459–534.

Muratori, S. (Ed.), 1723. Rerum Italicarum Scriptores, 25 Vols. Milan.

Needham, J., Wang, L., Price, D. de S., 1986. Heavenly Clockwork. Cambridge University Press, Cambridge.

Newcomb, S., 1882. Discussion and results of observations on transits of Mercury, from 1677 to 1881. It Astr. Pap. Am. Eph. 1, 465.

Newton, R.R., 1970. Ancient Astronomical Observations and the Accelerations of the Earth and Moon. Johns Hopkins University Press, Baltimore.

Newton, R.R., 1972a. The Earth's acceleration as deduced from al-Biruni's solar data. It Mem. R. Astr. Soc. 76, 99–128.

Newton, R.R., 1972b. Medieval Chronicles and the Rotation of the Earth. Johns Hopkins University Press, Baltimore.

Parker, R.A., Dubberstein, W.H., 1956. Babylonian Chronology: 626 BC–AD 75. Brown University Press, Providence, RI.

Peltier, W.R., Wu, P., 1983. History of the Earth's rotation. Geophys. Res. Lett. 10, 181–184.

Pertz, G.H. (Ed.), 1826. Monumenta Germaniae Historica, Scriptores, 32 Vols. Hanover: Hahn (Reprinted Kraus Reprint Corpn., Stuttgart, 1963).

Pirazzoli, P.A., 1991. In: Sabadini, R., Lambeck, K., Boschi, E. (Eds.), Glacial Isostacy, Sea-level and Mantle Rheology. Kluwer, Dordrecht, pp. 259–270.

Sachs, A.J., 1974. Babylonian observational astronomy. Phil. Trans. R. Soc. Lond. A 276, 43–50.

Sachs, A.J., 1976. The latest datable cuneiform tablets. In: Eichler, B.L. (Ed.), Cuneiform Studies in Honor of Samuel

Noah Kramer, Alter Orient und Altes Testament Sonderreihe, vol. 25. Butzon und Berker Kevelaer, Neukirchen-Vluyn.

Sachs, A.J., Hunger, H. 1989, 1990, 1996. Astronomical Diaries and Related Texts from Babylon, Vols. I II and III. Osterreichischen Akademie der Wissenschaften, Wien.

Spences Jones, H., 1939. The rotation of the Earth and the secular accelerations of the Sun, Moon and Planets. Mon. Not. R. Astr. Soc. 99, 541–558.

Steele, J.M., in press. The latest datable astronomical observation from Babylon. In: Millard, A.R. (Ed.). Archaeological Sciences 97.

Stephenson, F.R., 1997. Historical Eclipses and Earth's Rotation. Cambridge University Press, Cambridge.

Stephenson, F.R., Jones, J.E., Morrison, L.V., 1997. The solar eclipse observed by Clavius in AD 1567. A&A 322, 347–351.

Stephenson, F.R., Morrison, L.V., 1984. Long-term changes in the rotation of the Earth: 700 BC to AD 1980. Phil. Trans. R. Soc. Lond. Ser. A 313, 47–70.

Stephenson, F.R., Morrison, L.V., 1995. Long-term fluctuations in the Earth's rotation: 700 BC—AD 1990. Phil. Trans. R. Soc. Lond. Ser. A 351, 165–202.

Stephenson, F.R., Walker, C.B.F., 1985. Halley's Comet in History. British Museum Publications, London.

Williams, J.G., Newhall, X.X., Dickey, J.O., 1992. Diurnal and semidiurnal tidal contributions to lunar secular acceleration. Eos. Trans. Am. Geophys. Un. 73 (43), 126.