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Depth solution from borehole and travel time data using three-variable hypersurface splines

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Abstract

A new methodology, three-variable hypersurface splines, is presented for depth solution of geological surfaces in a certain exploration field based on the integration of elevation data, obtained from drilling, and travel time data, obtained from high-resolution seismic exploration. It is based on two-variable surface splines and could be used to generate a three-variable transformation function so that the travel time data could be globally inverted to the elevations of geological surfaces for a whole exploration field. In this procedure, no velocity parameters are used, so the selection of a suitable velocity background model is not necessary in the area with complex geological bodies. Another characteristic of this method is the ability to solve the discordance between travel time data and elevation data which always occurs while using conventional interpretation method and ensure the depths of geological surfaces transformed from seismic data full coinciding with those from original drilling data.

Keywords: Hypersurface splines; Inversion; Elevation; Travel time; Drilling data

1. Introduction

In geological exploration, one of the major concerns is to infer the depths (or elevations) of geological surfaces (or beds) and concealed geological structures (Tan and Yu, 1995; Han and Yu, 1996; Boehm et al., 1996). In order to determine the depths of geological surfaces and hidden structures in a certain exploration field, different kinds of data, including those obtained by geological, geophysical and remote sensing methods etc., should be comprehensively used, for the geological bodies are very complex (Tan and Yu, 1995). For example, to determine depths of coal beams, it is needed that integration of drilling data and travel time data of reflex wave of coal beds, obtained from seismic exploration with a high resolution.

Integration of drilling data and travel time data is essentially a three-variable data inversion, i.e. in the light of the relationship between x, y, z and t to find the function $z = f(x, y, t)$. Here, x and y

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represent coordinates, z is the depth of a geological surface, t is the travel time, i.e. the time a reflex wave needs for the vertical two-way travelling between a ground surface and a geological surface. In conventional seismic processing, the measured data, $D=D(x, y, t)$, is mapped into the image, $I=I(x, y, z)$, where the causality of the wave field comprising D and a suitable velocity background model are used. Due to the complexity of the geological bodies in some areas, however, velocity parameters vary greatly in both horizontal and vertical direction. So it is difficult to get a suitable velocity background model for the whole area. On the other hand, some borehole data in such areas are usually available, which should all be used in a complete inversion model. In this study, an inversion function of three-variable supersurface splines without velocity parameters is proposed, which directly inverts travel time data to depth of the geological surface. According to this function, i.e. three-variable supersurface splines based on two-variable surface splines, all the inferred depth data could be coincide with the actual depth at the corresponding borehole. For an exploration area with detail borehole data, more precious depth data could be obtained using this method. Another characteristic of this method is the ability to solve the discordance between travel time data and elevation data which always occurs while using conventional interpretation method. Traditionally, the inversion is only one-variable dependent, i.e. to find the function $z=f(t)$. In general, of course, travel time increases with the increase of the depth. However, it does not mean that the travel time is a monotone increasing or decreasing function dependent on the depths of geological surfaces. Travel time of the reflex wave varies with the stratigraphic structure or rock physical parameters, which is related to the position. Therefore, all the three arguments (x, y, t) should be under consideration in order to solve the discordance between travel time data and depth data in the whole region. And three-variable supersurface splines is such a tool with this function.

The paper is organized as follows. Section 2 provides the mathematical models of two-variable surface spline function and three-variable surface spline function. Subsequently, the description of the procedure is given, and finally a case study will be presented.

2. Mathematical models

2.1. Two-variable surface splines

Two-variable surface spline function, which has an elegant theory in a Hilbert space setting (Franke, 1982), is a mathematical tool for interpolating a function of two variables. It is based upon the small deflection equation of an infinite plate and was developed originally for interpolating wing deflections and computing slopes for aeroelastic calculations (Harder and Desmarais, 1972;). It is also called the thin plate splines, minimum curvature splines, and biharmonic splines (Franke, 1982; Enriquez et al., 1983; Sandwell, 1987; Powell, 1987; Dyn, 1987; Dyn et al., 1989; Watson, 1992). Its main advantage as a surface spline function is that (a) the coordinates of the known points need not be located in a rectangular array, (b) it uses a natural boundary condition and does not require any information about the boundary derivatives, (c) it may be differentiated to find slopes and (d) its interpolating results are usually agreement with geological situation (Harder and Desmarais, 1972; Yu, 1987). In geology, it could be used for interpolating a geological surface, e.g. coal seam surface from drilling data and its first and second partial derivatives can be used to structural analysis, especially for recognition of the concealed structures (Yu, 1987; Tan and Yu, 1995; Han and Yu, 1996).

In a certain exploration field, if the coordinates x, y and the elevation z of some points $(x_i, y_i, z_i, = 1, 2, \ldots, N)$ on a geological surface are known from drilling data, its surface spline function could be written as (Yu, 1987; Han and Yu, 1996):

$$z(x,y) = a_0 + a_1 x + a_2 y + \sum_{i=l}^{N} F_i r_i^2 \ln\left(r_i^2 + \varepsilon\right) \tag{1}$$

where $r_i^2 = (x - x_i)^2 + (y - y_i)^2$ and the parameter ε is a small quantity, which is usually taken to be between 10^{-2} and 10^{-6}, depending on the degree of the curvature variation of the surface. The a_is and F_is are $N+3$ coefficients to be determined such that the surface splines include the N data values z_i at their locations (x_i, y_i) and satisfies the smooth condition, i.e. the first and second derivatives of the function exist everywhere.

The $N+3$ unknowns are determined from:

$$\begin{cases} z_j = a_0 + a_1 x_j + a_2 y_j + \sum_{i=l}^{N} F_i r_{ij}^2 \ln\left(r_{ij}^2 + \varepsilon\right) + C_j F_j \quad \text{for } j = 1, \dots, N \\ \sum_{i=l}^{N} F_i = 0 \\ \sum_{i=l}^{N} x_i F_i = 0 \\ \sum_{i=l}^{N} y_i F_i = 0 \end{cases} \tag{2}$$

where $r_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2$. The coefficients C_j, which have units of length squared, are equal to $16\pi D/K_j$, where D is the plate rigidity and K_j is the spring constant associated with the jth point.

In interpolating geological surfaces, the control points are sample locations with known x, y coordinates at which some quantity of research interest, z, has been observed. The quantity might be the elevation of each location above the datum, the fluid speed at various locations in cross sections of rivers, the depth to bedrock, or travel time of reflected waves in seismic exploration, etc. For $C_j = 0 (K_j = \infty)$, the function $z(x_j, y_j)$ equals to z_j, the quantity observed.

In matrix form, Eq. (2) is written:

$$\mathbf{AX} = \mathbf{B} \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} C_1 & r_{12}^2 \ln(r_{12}^2 + \varepsilon) & \cdots & r_{1,N-1}^2 \ln(r_{1,N-1}^2 + \varepsilon) & r_{1N}^2 \ln(r_{1N}^2 + \varepsilon) & 1 & x_1 & y_1 \\ r_{12}^2 \ln(r_{12}^2 + \varepsilon) & C_2 & \cdots & r_{2,N-1}^2 \ln(r_{2,N-1}^2 + \varepsilon) & r_{2N}^2 \ln(r_{2N}^2 + \varepsilon) & 1 & x_2 & y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{1,N-1}^2 \ln(r_{1,N-1}^2 + \varepsilon) & r_{2,N-1}^2 \ln(r_{2,N-1}^2 + \varepsilon) & \cdots & C_{N-1} & r_{N-1,N}^2 \ln(r_{N-1,N}^2 + \varepsilon) & 1 & x_{N-1} & y_{N-1} \\ r_{1N}^2 \ln(r_{1N}^2 + \varepsilon) & r_{2N}^2 \ln(r_{2N}^2 + \varepsilon) & \cdots & r_{N-1,N}^2 \ln(r_{N-1,N}^2 + \varepsilon) & C_N & 1 & x_N & y_N \\ 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \cdots & x_{N-1} & x_N & 0 & 0 & 0 \\ y_1 & y_2 & \cdots & y_{N-1} & y_N & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{X} = (F_1, F_2, \dots, F_N, a_0, a_1, a_2)^T$$

$$\mathbf{B} = (z_1, z_2, \dots, z_N, 0,0,0)^T$$

Matrix **A** is a symmetric matrix. In the geological applications, C_j equals to zero generally, so the elements of the main diagonal of matrix **A** equal to zero. According to the Householder transformation algorithm, the matrix solutions can be obtained stably. Therefore, from Eq. (1), the elevations of the geological surface in a whole area can be calculated and provided on a uniform grid of points.

2.2. Three-variable hypersurface splines

The two-variable surface splines can be extended the three-variable hypersurface splines which describes the quantity relationship between z and three independent variables and is essentially a nonlinear mapping function. This means that all arguments used in the three-variable hypersurface splines have lost its physical significance and stand for a fuzzy coordinates, i.e. they have only value significance and their calibrations or dimensions can not be considered. In order to get a more accurate solution, of course, these variables should be on the same quantity scale.

Based on the two-variable surface splines, three-variable hypersurface splines can be deduced. Its expression is described as:

$$z(x,y,t) = a_0 + a_1 x + a_2 y + a_3 t + \sum_{i=l}^{N} F_i r_i^2 \ln\left(r_i^2 + \varepsilon\right) \tag{4}$$

where z is the elevation of a point on a geological surface , x and y are the coordinates, t, the third arguments denotes the travel time of reflected waves and $r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (t - t_i)^2$.

The $N + 4$ unknowns are determined from:

$$\begin{cases} z_j = a_0 + a_1 x_j + a_2 y_j + a_3 t_j + \sum_{i=l}^{N} F_i r_{ij}^2 \ln\left(r_{ij}^2 + \varepsilon\right) + c_j F_j \quad \text{for } j = 1,\ldots,N \\ \sum_{i=l}^{N} F_i = 0 \\ \sum_{i=l}^{N} x_i F_i = 0 \\ \sum_{i=l}^{N} y_i F_i = 0 \\ \sum_{i=l}^{N} t_i F_i = 0 \end{cases} \tag{5}$$

where $r_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (t_i - t_j)^2$.

In matrix form, Eq. (5) can be written as:

$$\mathbf{AX} = \mathbf{B} \tag{6}$$

where

$$\begin{bmatrix} C_1 & r_{12}^2 \ln\left(r_{12}^2 + \varepsilon\right) & \cdots & r_{1,N-1}^2 \ln\left(r_{1,N-1}^2 + \varepsilon\right) & r_{1N}^2 \ln\left(r_{1N}^2 + \varepsilon\right) & 1 & x_1 & y_1 & t_1 \\ r_{12}^2 \ln\left(r_{12}^2 + \varepsilon\right) & C_2 & \cdots & r_{2,N-1}^2 \ln\left(r_{2,N-1}^2 + \varepsilon\right) & r_{2N}^2 \ln\left(r_{2N}^2 + \varepsilon\right) & 1 & x_2 & y_2 & t_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{1,N-1}^2 \ln\left(r_{1,N-1}^2 + \varepsilon\right) & r_{2,N-1}^2 \ln\left(r_{2,N-1}^2 + \varepsilon\right) & \cdots & C_{N-1} & r_{N-1,N}^2 \ln\left(r_{N-1,N}^2 + \varepsilon\right) & 1 & x_{N-1} & y_{N-1} & t_{N-1} \\ r_{1N}^2 \ln\left(r_{1N}^2 + \varepsilon\right) & r_{2N}^2 \ln\left(r_{2N}^2 + \varepsilon\right) & \cdots & r_{N-1,N}^2 \ln\left(r_{N-1,N}^2 + \varepsilon\right) & C_N & 1 & x_N & y_N & t_N \\ 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 0 & 0 \\ x_1 & x_2 & \cdots & x_{N-1} & x_N & 0 & 0 & 0 & 0 \\ y_1 & y_2 & \cdots & y_{N-1} & y_N & 0 & 0 & 0 & 0 \\ t_1 & t_2 & \cdots & t_{N-1} & t_N & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{X} = (F_1, F_2, \ldots, F_N, a_0, a_1, a_2, a_3)^T$$

$$\mathbf{B} = (z_1, z_2, \ldots, z_N, 0,0,0,0)^T$$

As in the two-variable surface splines, the matrix solutions can also be obtained stably by using the Householder transformation algorithm. And from Eq. (4), the elevations of the geological surface in a whole area can be calculated and provided on a uniform grid of points.

3. Procedure

The detailed procedure for depth inversion based on integration of coordinates (x, y) and elevation data from drilling (z) and travel time data (t) of reflected arrivals using three-variable hypersurface splines is described as follows. In order to make these variables be on the same quantity scale, the relative coordinates should be employed and generally speaking, the calibration of x, y, and z could be meter and the calibration of travel time could be millisecond.

3.1. Getting original travel time data of geological surface interest on a uniform grid of points

If the original data are in raster format and meet the density need for the study, they could be directly

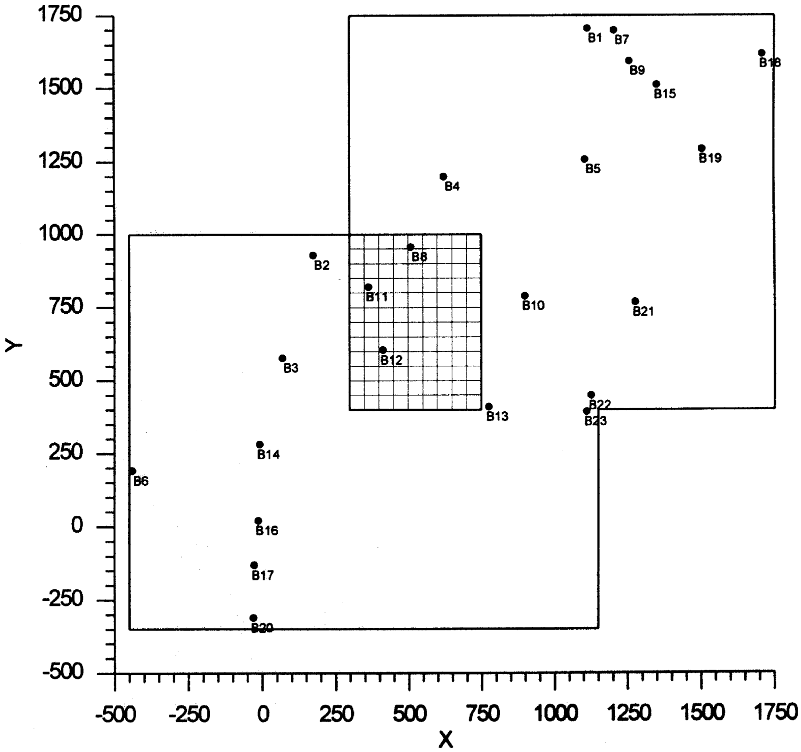

Fig. 1. Location of the exploration area, the example area and boreholes in this study in relative coordinates. The grid area is the example area and dots indicate positions of 23 boreholes used in case study. The coordinates are expressed in meters.

used for the spatial calculation and 3D surface plot. Otherwise, the original scattered data should first be interpolated to raster form with specific spacing or density, using 2D interpolation method like the two-variable surface splines.

3.2. Obtaining the positioning (spatial) data of geological surface interest from drilling data in the same region

From drilling data, the x and y coordinates and depth z $(x_i,\ y_i,\ z_i,\ i=1,2,\ldots,N)$ of some points on geological surface interest can be obtained easily. These data establishes the basis of inversion.

3.3. Selecting the corresponding travel time data at specific points

According to Eq. (4), both the positioning data and the travel time data are needed for the establishment of the transform function. So a set of specific borehole at which both the spatial and travel time data are available should be picked out. Then a set of

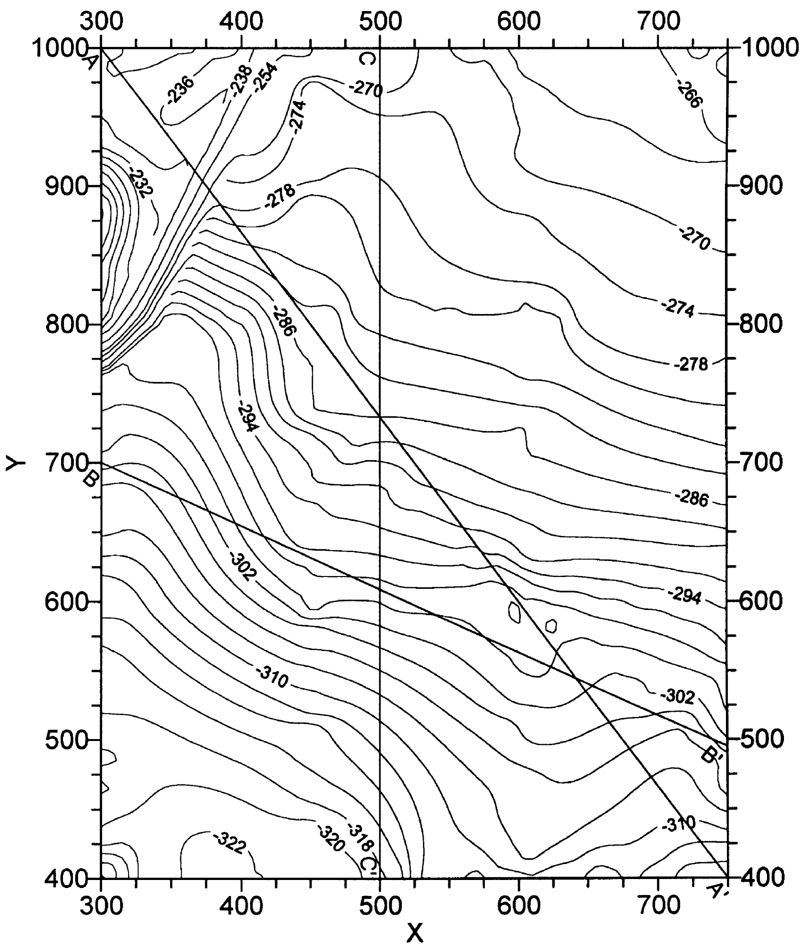

Fig. 2. Contour map of travel time of reflex waves of coal beam floors based on a 5-m spacing grid in the example area. The coordinates are expressed in meters and the travel time in milliseconds. A–A′, B–B′ and C–C′ are locations of the profiles used for comparison in Fig. 5.

constrained data $(x_i, y_i, z_i, t_i, i=1,2,\ldots,N)$ for inversion has been formed.

3.4. Calculating the coefficients of inversion function

Based on the constrained data, the $N+4$ coefficients in Eq. (4) can be obtained stably. So the inversion function, i.e. three-variable hypersurface splines will then be generated.

3.5. Providing elevations of each point using three-variable inversion function

According to Eq. (4), three-variable hypersurface splines, the elevations of the geological surface interest in a whole area can be calculated using the x, y and t values from step 1 and provided on a uniform grid of points.

4. A case study

4.1. Data

As an example, elevation data and travel time data of coal beam floors from a certain exploration field were gathered. Fig. 1 shows the location of the exploration area, in which data of 23 boreholes are collected. The drilling data $(x_i, y_i, z_i, i=1,2,\ldots,23)$ accurately represent the depths (elevations) of a coal beam floor at these points$(x_i, y_i, i=1,2,\ldots,23)$. The example area for the three-variable hypersurface spline inversion in this study is shown in grid form in Fig. 1. The travel time data which have been turned to negative for the comparison to depth data and given in millisecond are interpolated on a uniform grid of points using original scatter data, which represent the time a reflex wave needs for the vertical two-way travelling between the ground surface and the coal beam floor at these points, and point

Table 1
Values $(x, y, t$ and $z)$ of the coal beam floors of the 23 boreholes used in the case study

Name	x (m)	y (m)	t (ms)	z (m)
B1	1113.50	1704.98	−217	−165.90
B2	176.88	928.98	−221	−166.56
B3	71.35	578.01	−242	−180.52
B4	622.67	1199.44	−248	−214.94
B5	1104.92	1258.1	−257	−231.37
B6	−441.04	191.25	−261	−218.14
B7	1203.64	1698.54	−262	−238.92
B8	509.16	956.79	−273	−242.28
B9	1256.28	1592.45	−276	−254.09
B10	899.12	788.60	−277	−253.13
B11	365.71	819.83	−292	−273.67
B12	414.52	604.89	−303	−299.70
B13	775.59	410.05	−313	−316.48
B14	−7.68	281.95	−328	−335.61
B15	1350.79	1510.81	−340	−335.89
B16	−14.60	21.69	−353	−394.19
B17	−28.14	−130.49	−375	−437.76
B18	1708.55	1616.78	−438	−530.95
B19	1505.02	1292.91	−447	−553.61
B20	−31.07	−311.26	−486	−579.61
B21	1275.98	767.41	−502	−638.86
B22	1125.67	448.61	−516	−686.95
B23	1110.40	393.75	−528	−700.00

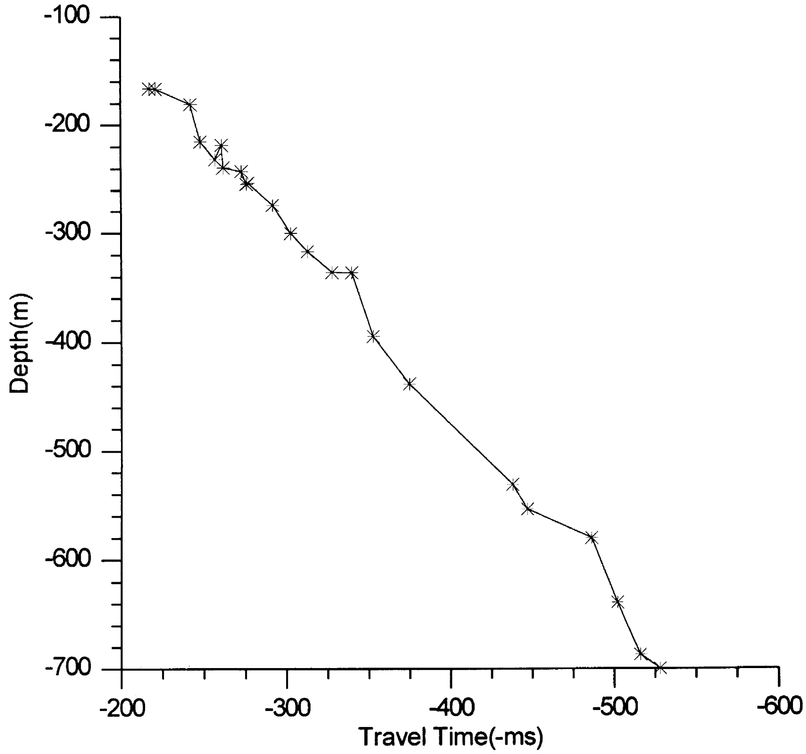

Fig. 3. Relationship between travel time and elevation of coal beam floors for different boreholes in the exploration area.

intervals in both x and y direction are 5 m. The contour map of the travel time in this example area is illustrated in Fig. 2. Meanwhile, the travel time data at these points $(t_i,\ i=1,2,\ldots,23)$ are also picked out from the original travel time data set. Table 1 shows these x, y, t and z values of the 23 boreholes. Fig. 3 graphically illustrates the relationship between the travel time and the floor elevation. From Fig. 3, it can be seen that there exists a local inverse, namely not all increase of travel time corresponds with increase of depths of coal beam floors in this exploration area. In other words, this function is not a monotone increasing or decreasing function.

4.2. Calculation and results

Twenty seven coefficients of the three-varible hypersurface splines were calculated from data of Table 1 through solving Eq. (5). Table 2 shows these

Table 2
Coefficients of three-varible hypersurface splines obtained in the case study *

-0.4242358	$-3.860097\times 10e-2$	$-2.557075\times 10e-2$	1.61926
$1.022926\times 10e-4$	$-4.736221\times 10e-5$	$9.206857\times 10e-5$	$-4.118957\times 10e-5$
$-2.706428\times 10e-5$	$9.554975\times 10e-6$	$1.288419\times 10e-4$	$4.462745\times 10e-5$
$-1.280468\times 10e-4$	$4.76869\times 10e-5$	$.292411\times 10e-6$	$-8.310532\times 10e-5$
$6.769381\times 10e-5$	$-3.827784\times 10e-5$	$2.752756\times 10e-4$	$5.551866\times 10e-5$
$-1.583648\times 10e-4$	$8.596873\times 10e-6$	$-1.171519\times 10e-4$	$9.252073\times 10e-5$
$6.36063\times 10e-5$	$-3.036528\times 10e-4$	$2.039178\times 10e-4$	

*Arranged in $a_0, a_1, a_2, a_3, F_1, F_2, \ldots, F_{23}$ order (from left to right).

coefficients arranged in a_0, a_1, a_2, a_3, F_1, $F_2,\ldots,$ F_{23} order, which can be used for the interpolation of all points in the whole exploration area. In this study, only those in the example area are inverted, its value of z can be calculated and provided on a uniform grid of points after the values of x, y, t are called in Eq. (4). As a floor-depth contour map of the coal beds, Fig. 4 illustrates the calculation results mentioned above, which covers the same area as that in Fig. 2.

Comparison of Fig. 4 to Fig. 2 indicates that the relationship between travel time data and depth values are dependent on arguments x, y in the example area. To further illustrate the spatial-related characteristic of this function, three profiles, A–A′, B–B′ and C–C′, at the same location are selected in both Figs. 2 and 4. Fig. 5 illustrates the comparison results using the profiles data, in which dashed lines represent the travel time, while the solid lines, the depth. As shown in profiles A–A′ and C–C′ in Fig. 5, the relationship between the floor elevation and the travel time indeed is neither linear nor monotonous. Although the travel time data at both end-points of profile B–B′ are the same (−303 ms),

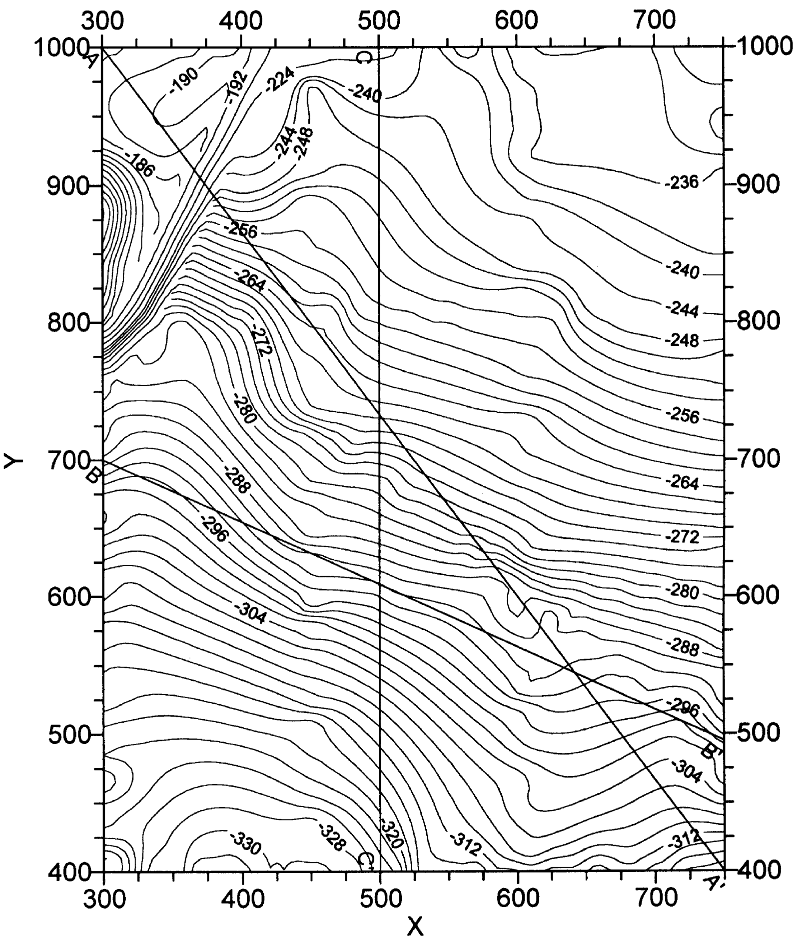

Fig. 4. Contour map of coal beam floors based on a 5-m spacing grid in the example area. The data was obtained by three-variable hypersurface splines. The coordinates and depth units are expressed in meters. A–A′, B–B′ and C–C′ are locations of the profiles used for comparison in Fig. 5.

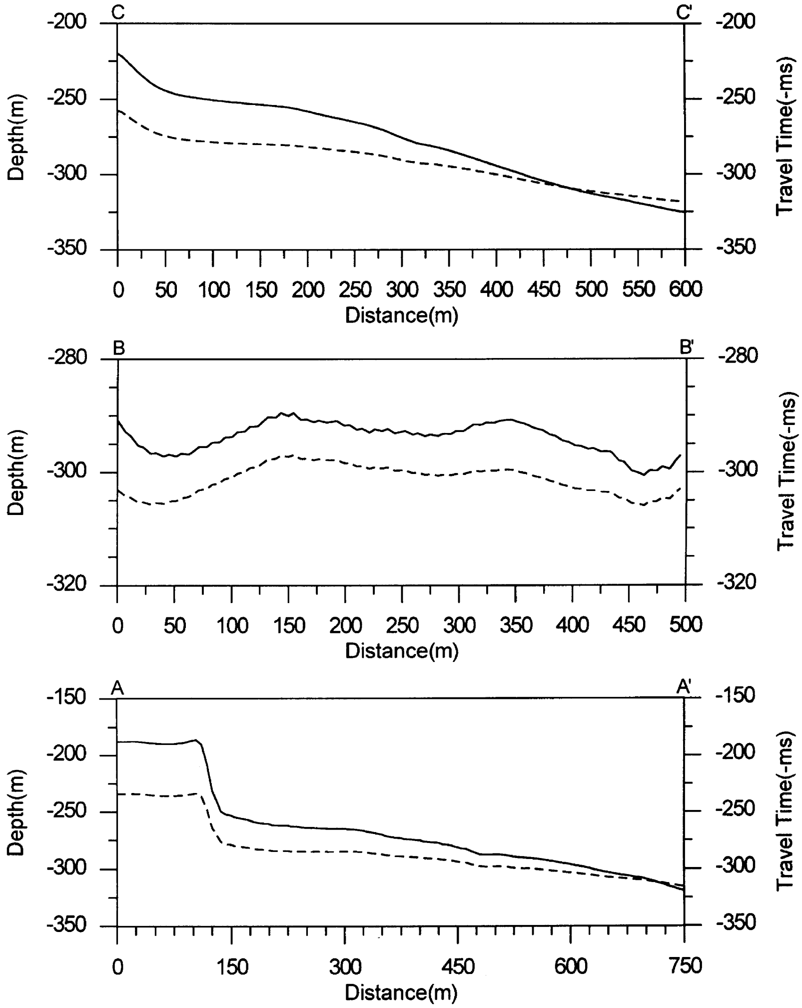

Fig. 5. Comparison of travel time (dashed lines) and depth (solid lines) of coal beam floors obtained by three-variable hypersurface splines in the three profiles, A–A′, B–B′ and C–C′, the locations of which are labeled in both Figs. 2 and 4.

the depth values at the same points have difference up to 6 m (see Fig. 5).

5. Conclusions

The relationship between travel time data and depth values are dependent on spatial coordinates x, y in the inversion area and the change of the depths (elevations) of geological surface is not a monotone function of the travel time of reflex wave, so a nonlinear mapping the measured data, $D = D(x, y, t)$ into the image, $I = I(x, y, z)$ is needed in depth solving based on travel time data and borehole data. The three-variable hypersurface splines which is developed based on the two-variable surface splines is a suitable mapping function for this purpose. Employing this apparatus, the travel time data could be globally inverted to the elevations of geological surfaces for a whole exploration field. In brief, one characteristic of this inverse function is that no velocity parameters are used, so the difficulty in selection of a suitable velocity background model for some areas with complex geological structures could be avoided. Another characteristic of this method is that it can easily solve the discordance between travel time data and depth data which always occurs when conventional method for data solving is used.

The present inversion scheme is very fast and ease to implement. Therefore, it can be employed for fast depth solving in a seismic exploration area with detail borehole data. Moreover, it is clear that the inversion problem involving more arguments could also be solved if the surface splines are extended to a multi-variable (four or more variables) spline function.

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