Journal of Hydrology 244 (2001) 97–104
Journal of Hydrology
www.elsevier.com/locate/jhydrol

Short Communication

On noninertia wave versus diffusion wave in flood routing

B.C. Yen*, C.W.-S. Tsai

V.T. Chow Hydrosystems Laboratory, Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

Received 10 April 2000; revised 4 December 2000; accepted 15 December 2000

Abstract

This note intends to address a misnomer in flood routing. The noninertia wave, which ignores both local and convective inertia terms in the momentum equation, is a simplification to the full dynamic wave. While the diffusion wave refers more generally to the wave whose induced disturbance in flow is analogous to the diffusion of particles or heat. For the purpose of clarification, these waves are mathematically formulated and physically interpreted. It is demonstrated that the diffusion wave can be mathematically formulated from different levels of shallow water wave approximations with the assumption that the celerity and coefficient of hydraulic diffusivity are step-wise constants. Both the linear and nonlinear perspectives of waves are discussed. The noninertia wave is demonstrated to be one of the special cases of the diffusion wave. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Diffusion equation; Noninertia wave; Diffusion wave; Flood routing; Open channel flow; Overland flow; Unsteady flow

1. Introduction

Solutions of unsteady flow in channels employ the Saint-Venant equations or their approximations: the kinematic wave, noninertia wave, gravity wave and quasi-steady dynamic wave. These approximations are defined depending on the relative importance of local inertia, convective inertia, pressure gradient, gravity and friction effects involved in the physical mechanisms. The governing equations describing one-dimensional, unsteady, gradually varied open channel flow in prismatic channels can be expressed as (Yen, 1973)

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_r - q_s \tag{1}$$

$$\underset{(a)}{k_t \frac{1}{gA}\frac{\partial Q}{\partial t}} + k_c\left\{ \underset{(b)}{\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)} + \underset{(b')}{\frac{1}{gA}\left[U_{rx}q_r - U_{sx}q_s - (q_r - q_s)\frac{Q}{A}\right]}\right\} + \underset{(c)}{k_p \frac{\partial y}{\partial x}} - k_f[\underset{(d)}{S_0} - \underset{(e)}{S_f}] = 0 \tag{2}$$

where Q is flow discharge; y, flow depth; A, flow cross-sectional area; q_s, net infiltration or seepage lateral outflow rate per unit length along the channel; q_r, net rainfall lateral inflow rate per unit length; U_{rx}, x-component velocity of rain when joining channel flow; U_{sx}, x-component velocity of infiltration or seepage when leaving the channel flow; S_0, channel bed slope; S_f, friction slope; x, longitudinal coordinate; t, time; and k_t, k_c, k_p, k_f are term index integers

* Corresponding author. Tel.: +1-217-333-4934; fax: +1-217-333-0687.
E-mail address: b-yen@uiuc.edu (B.C. Yen).

PII: S0022-1694(00)00422-4

Table 1
Summary of selected previous studies on diffusion wave equation formulated for different levels of wave approximations

Investigators	Study scope	Diffusion wave equation	Remarks
Dooge and Napiorkowski (1987)	Wide rectangular channel Linearized governing equations	$\frac{\partial y'}{\partial t} + c\frac{\partial y'}{\partial x} = D_h \frac{\partial^2 y'}{\partial x^2}$ where $c = 3u_0/2$, $D_h = (1 - 0.25F_0^2)u_0 y_0/2S_0)$. Assume $y = y_0 + y'$ in which y_0 is reference flow depth and y' is perturbed flow depth.	Express the inertia terms as one of other surviving terms of kinematic wave approximation, and formulate the diffusion wave equation form of full Saint-Venant equation
Ponce (1990)	Wide rectangular channel Linearized governing equations	$\frac{\partial y'}{\partial t} + c\frac{\partial y'}{\partial x} = D_h \frac{\partial^2 y'}{\partial x^2}$ where $c = 3u_0/2$, $D_h = [(1 - k_c F_0^2) + 1.5(k_t + k_c)F_0^2 - (9/4)k_t F_0^2](u_0 y_0/2S_0)$ k_t, k_c are integer indices of 0 or 1 corresponding to different levels of wave approximations. Assume $y = y_0 + y'$	Use the parabolic analogy to the hyperbolic Saint-Venant equations, and neglect the third-order terms in the combined equation
Sivapalan et al. (1997)	With lateral flow q_L Prismatic channel with general cross-sectional geometry Nonlinear governing equations	$\frac{\partial Q}{\partial t} + H(Q)\frac{\partial q_L}{\partial t} + c_e\left(\frac{\partial Q}{\partial x} - q_L\right)$ $= \frac{1}{S_0^{1/2}}\frac{\partial}{\partial x}\left[D_h(Q)\left(\frac{\partial Q}{\partial x} - q_L\right)\right]$ where $C_e = C_e\left(Q, \frac{\partial Q}{\partial t}, \frac{\partial Q}{\partial x}\right)$ $D_h(Q) = \frac{q_0}{2S_0^{1/2}} - \frac{u_0}{2gS_0^{1/2}}(c_0 - u_0)^2$ $H(Q) = u_0 c_0/(2gS_0)$ where $c_0 = dQ_0/dA$ is kinematic wave speed, and q_0 is the unit-width discharge for uniform flow	Order of magnitude analysis to full Saint-Venant equations, and derive diffusion wave equation as an approximation to full Saint-Venant equations

of value 0 or 1 depending on the wave approximation considered. For simplicity, the lateral flow rates q_s and q_r and their x-component velocities are assumed constants in this study. In the momentum equation Eq. (2), term (a) denotes local acceleration; term (b) represents convective acceleration; term (b$'$) indicates convective acceleration effect from lateral flow; term (c) is pressure gradient; term (d) denotes channel bed slope, and term (e) is friction slope. Depending on the relative importance of these five terms involved in the physical mechanism, various shallow water wave approximations can be defined accordingly.

(1) Kinematic waves: $k_t = k_c = k_p = 0$ and $k_f = 1$.
(2) Noninertia waves: $k_t = k_c = 0$ and $k_p = k_f = 1$.
(3) Gravity waves: $k_t = k_c = k_p = 1$ and $k_f = 0$.
(4) Quasi-steady dynamic waves: $k_t = 0$ and $k_c = k_p = k_f = 1$.
(5) Dynamic waves: $k_t = k_c = k_p = k_f = 1$.

It has been known that the Saint-Venant equation is not exact (Yen, 1973) but complete in representing characteristics of shallow water wave propagation in a channel. All the approximations, except the kinematic wave can account for the downstream backwater effect of subcritical flow. The case of ignoring local and convective acceleration inertia terms, referred to here as the noninertia wave approximation, has often been improperly called, including by the first writer in his early days (Akan and Yen, 1977), the diffusion wave. Use of the noninertia wave for unsteady flow routing has been increasing in recent years because it is the simplest among the approximations that can account for the downstream backwater effect and yields reasonably good results. The noninertia wave approximation to the full Saint-Venant equations is established by neglecting the two inertia terms in the momentum equation. Conversely, the physical meaning of the diffusion wave is different. In physics, diffusion is the process whereby ionic or molecular constituents move under the influence of their kinetic activity in the direction of their concentration gradient (Freeze and Cherry, 1979). The diffusion wave is defined in such a way that diffusion of the disturbances in flow is analogous to the diffusion of the particles or heat (Chow, 1959). Such a diffusion mechanism can be provided by: (1) the dependence of the flow quantities on the rate of change of flow and/or the flow quantities themselves in kinematic wave process (Lighthill and Whitham, 1955); (2) channel irregularities (Hayami, 1951; Chow, 1959); (3) spatial variations of channel slope, cross-sections or lateral flow (Sutherland and Barnett, 1972); or (4) simply by the presence of pressure gradient, gravity and friction slope terms in the wave propagation process (Keefer and McQuivey, 1974). As shown in previous studies (Ponce, 1990; Sivapalan et al., 1997), the diffusion wave may or may not have an inertia effect while both linear and nonlinear noninertia waves are diffusive.

The first application of the diffusion analogy to flood routing originated from Hayami (1951), followed by Appleby (1954), Cunge (1969), Dooge et al. (1983), Ponce (1990), Rutschmann and Hager (1996), Sivapalan et al. (1997), Bajracharya and Barry (1997) and many others. Singh (1996) gave a detailed description about use of the diffusion wave equation for hydrologic modeling. Most of the diffusion wave equations in the previous investigations were formulated from dropping the two inertia terms from the momentum equation, yielding what is defined here as the noninertia wave. Being aware that the diffusion wave can have inertia effects, Dooge et al. (1983), Ponce (1990) and Sivapalan et al. (1997), among others, formulated the diffusion wave equation from other levels of wave approximations, which are summarized in Table 1.

2. Mathematical formulation

Mathematically, the noninertia wave in a prismatic open channel can be described by combining Eq. (1) with the following dynamic equation:

$$\frac{\partial y}{\partial x} = S_0 - S_\mathrm{f} \tag{3}$$

On the other hand, the diffusion process can be written explicitly in Eq. (4) by assuming that the mass of diffusing substance passing through a given cross-section per unit time is proportional to the concentration gradient.

$$q = -D\frac{\partial \eta}{\partial x} \tag{4}$$

where q is mass flux; D, diffusivity and η, concentration of the particles. In hydrology and hydraulics, diffusion of waves is analogous to diffusion of heat or particles. A generalized hydraulic diffusivity D_h is introduced to describe the relationship between the flow flux and the spatial rate of change of the flow disturbance quantity along the flow direction. This diffusion analogy results in a fundamental differential equation for diffusion waves in rivers, channels, and overland surfaces. Both the diffusion wave equation and the noninertia wave equation can be expressed in terms of flow depth, flow velocity, flow cross-sectional area or discharge as dependent variable. One of the typical differential equations used to describe the diffusion wave can be expressed as

$$\frac{\partial Q}{\partial t} = D_\mathrm{h}\frac{\partial^2 Q}{\partial x^2} \tag{5}$$

or in the form of a convective-diffusion equation as

$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = D_\mathrm{h}\frac{\partial^2 y}{\partial x^2} \tag{6a}$$

$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}\left(D_\mathrm{h}\frac{\partial y}{\partial x}\right) \tag{6b}$$

In Eqs. (5) and (6a), the hydraulic diffusivity is assumed independent of space and time, while in hydraulics and hydrology, depending on the given conditions, the hydraulic diffusivity may be a function of space x and time t as expressed in Eq. (6b).

From the aforementioned physical and mathematical perspectives, the diffusion wave includes, but is not limited to the class of noninertia wave. Under particular assumptions on the physical or mathematical relationships of the variables, it can be demonstrated that the full Saint-Venant equations and their lower levels of wave approximations such as the kinematic wave, noninertia wave, and quasi-steady dynamic wave can all be reformulated into a form of diffusion wave.

2.1. Linear wave perspective

Mathematically, any wave approximation that can be written in a diffusion equation form, assuming that the diffusivity is at least spatial and temporal stepwise quasi-constant, belongs to the class of parabolic partial differential equations. For linearized Saint-Venant equations and lower-level wave approximations, the generalized diffusion wave approximation can also be derived under some simplified assumptions (Dooge and Napiorkowski, 1987; Ponce, 1990). In practice, the ratio of lateral flow of rainfall, seepage or infiltration to the flood-induced discharge is relatively small in Eqs. (1) and (2), i.e. $O(q_s)$ (or $O(q_r)) \ll O(Q') < O(Q_0)$. The x-component velocities of lateral flow of rainfall, seepage, or infiltration are also negligible compared to the flow velocity. Therefore the linearized combination form of the governing equations for unsteady and gradually varying flow in prismatic open channels can be expressed as Eq. (7) assuming $Q = Q_0 + Q'$ and $A = A_0 + A'$.

$$\frac{\partial Q'}{\partial t} + c\frac{\partial Q'}{\partial x} = m\left[(k_p - k_c F_0^2)\frac{\partial^2 Q'}{\partial x^2} - (k_t + k_c)\frac{F_0^2}{u_0}\frac{\partial^2 Q'}{\partial x\,\partial t} - k_t\frac{1}{g y_{h0}}\frac{\partial^2 Q'}{\partial t^2}\right] \tag{7}$$

where the coefficient $m = u_0 y_0/2S_0$; Q_0 is uniform flow discharge; A_0, uniform flow cross-sectional area; Q', perturbed flow discharge; A', perturbed flow cross-sectional area; u_0, uniform flow velocity; y_0, uniform flow depth; F_0, the steady uniform flow Froude number, defined as $F_0 = u_0/\sqrt{g y_{h0}}$; y_h, hydraulic depth defined as $y_h = A/B$; B, surface width; $y_{h0} = A_0/B_0$; and c, the wave celerity, defined as $c = -(\partial S_f/\partial A)/(\partial S_f/\partial Q)$. The celerity depends on the resistance formula used, the channel cross-section geometry, and the area of the flow. For a wide rectangular channel with a constant Chezy resistance coefficient, $c = 3u_0/2$ and with a constant Manning resistance coefficient, $c = 5u_0/3$. For a trapezoidal channel, the wave celerity can be expressed as (Singh, 1996),

$$c = \left[1 + \frac{\alpha}{2}\left(1 - \frac{2\sqrt{1+z^2}\,y_0}{b + 2\sqrt{1+z^2}\,y_0}\,\frac{b + z y_0}{b + 2 z y_0}\right)\right]u_0 \tag{8}$$

for a rectangular channel,

$$c = \left(1 + \frac{\alpha}{2}\,\frac{b}{b + 2y_0}\right)u_0 \tag{9}$$

and for a triangular channel,

$$c = \left(1 + \frac{\alpha}{4}\right)u_0 \tag{10}$$

where b is bottom width; z, side slope; $\alpha = 1$ for Chezy's formula and $\alpha = 4/3$ for Manning's formula.

Differentiating Eq. (7) with respect to x, and similarly, with respect to t and applying the results to Eq. (7), after neglecting the third-order terms, yields

$$\frac{\partial Q'}{\partial t} + c(u_0)\frac{\partial Q'}{\partial x} = \left[(k_p - k_c F_0^2) + (k_t + k_c)\frac{c}{u_0}F_0^2 - k_t\left(\frac{c}{u_0}\right)^2 F_0^2\right] m\frac{\partial^2 Q'}{\partial x^2} \tag{11}$$

Eq. (11) gives a mathematical description of the linearized shallow water waves with the assumption that higher-order effects are ignored. For the dynamic wave where $k_t = k_c = k_p = k_f = 1$, Eq. (11) can be simplified to

$$\frac{\partial Q'}{\partial t} + c(u_0)\frac{\partial Q'}{\partial x} = \left[1 - \left(1 - 2\frac{c}{u_0} + \frac{c^2}{u_0^2}\right)F_0^2\right]m\frac{\partial^2 Q'}{\partial x^2} \quad (12)$$

from which the hydraulic diffusivity coefficient is $D_h = [1 - (1 - 2(c/u_0) + c^2/u_0^2)F_0^2]m$. Special cases such as noninertia wave and quasi-steady dynamic wave can be derived from Eq. (11). For the quasi-steady dynamic wave where $k_t = 0$ and $k_c = k_p = k_f = 1$,

$$\frac{\partial Q'}{\partial t} + c(u_0)\frac{\partial Q'}{\partial x} = \left[1 - \left(1 - \frac{c}{u_0}\right)F_0^2\right]m\frac{\partial^2 Q'}{\partial x^2} \quad (13)$$

Thus the hydraulic diffusivity coefficient is $D_h = [1 - (1 - c/u_0)F_0^2]m$. For the noninertia wave where $k_t = k_c = 0$, and $k_p = k_f = 1$,

$$\frac{\partial Q'}{\partial t} + c(u_0)\frac{\partial Q'}{\partial x} = m\frac{\partial^2 Q'}{\partial x^2} \quad (14)$$

hence the hydraulic diffusivity coefficient is $D_h = m$. For the kinematic wave where $k_t = k_c = k_p = 0$ and $k_f = 1$,

$$\frac{\partial Q'}{\partial t} + c(u_0)\frac{\partial Q'}{\partial x} = 0 \quad (15)$$

Thus the hydraulic diffusivity coefficient is $D_h = 0$. Eqs. (11)–(15) represent the generalized diffusion wave equations formulated for different levels of wave approximations ranging from the dynamic wave to the kinematic wave. These equations describe the generalized diffusion wave in a prismatic open channel with arbitrary cross-sectional geometry. For the diffusion wave in a wide rectangular and prismatic channel, the wave celerity and hydraulic diffusivity coefficient can be simplified to those by Ponce (1990).

2.2. Nonlinear wave perspective

Eqs. (1) and (2) can be further combined to give a generalized nonlinear diffusion wave. The term S_f is usually expressed using the Chezy or Manning formulas, respectively.

$$Q = CR^{1/2}S_f^{1/2}A \quad (16)$$

$$Q = \frac{K_n}{n}R^{2/3}S_f^{1/2}A \quad (17)$$

where R is the hydraulic radius; C, Chezy's resistance coefficient; n, Manning's resistance coefficient; K_n, a constant depending on the measurement units used (Yen, 1992). Incorporating Eq. (16) or (17) with Eqs. (1) and (2), the generalized nonlinear diffusion wave can be derived in the following form:

$$\frac{\partial A}{\partial t} = \frac{\partial}{\partial x}\left(D_h\frac{\partial A}{\partial x}\right) + K \quad (18)$$

where the hydraulic diffusivity D_h allows the spatial variation of flow quantities and K is a source term such as the lateral flow. The hydraulic diffusivity differs for various wave approximations. Eq. (2) can be rearranged to better illustrate the friction slope term from different wave perspectives.

$$S_f = -\frac{1}{gA}\left[\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)\right] - \frac{1}{gA}\left[U_{rx}q_r - U_{sx}q_s - (q_r - q_s)\frac{Q}{A}\right] - \frac{\partial y}{\partial x} + S_0 \quad (19)$$

For the dynamic wave approximation, the friction slope can be expressed using Eq. (19). Substitution of Eq. (19) into the Manning or Chezy formula yields

$$Q = -D_h\frac{\partial A}{\partial x} \quad (20)$$

For Manning's formula the hydraulic diffusivity coefficient is expressed as

$$D_h = \frac{K_n R^{2/3} A}{n}\left\{\frac{|(\partial Q/\partial t) + Q\,\partial(Q/A)/\partial x + [U_{rx}q_r - U_{sx}q_s - (q_r - q_s)Q/A] + gA(\partial y/\partial x) - gAS_0|}{gA(\partial A/\partial x)^2}\right\}^{1/2} \quad (21)$$

Similarly if Chezy's formula is used,

$$D_h = CR^{1/2}A\left\{\frac{|(\partial Q/\partial t) + Q\,\partial(Q/A)/\partial x + [U_{rx}q_r - U_{sx}q_s - (q_r - q_s)Q/A] + gA(\partial y/\partial x) - gAS_0|}{gA(\partial A/\partial x)^2}\right\}^{1/2} \quad (22)$$

Therefore, the generalized diffusion wave equation, Eq. (18), can be derived from the dynamic wave approximation with the hydraulic diffusivity given in Eqs. (21) or (22) in which the source is $q_r - q_s$.

Similarly, the generalized diffusion wave equation can be derived for the quasi-steady dynamic wave and noninertia approximations. The results are given in Table 2.

In flood routing, the hydraulic diffusivity can be treated as approximately constant for each temporal or spatial step. Therefore, generalized diffusion wave equations can be derived for different levels of wave approximations accordingly.

2.3. Other perspectives

In addition to the aforementioned mathematical formulation of the generalized linear or nonlinear diffusion wave, the diffusion wave equation can also be constructed under different physical assumptions and expressed in terms of flow depth, flow velocity or discharge as the dependent variable. The subject of diffusion wave has been widely discussed in previous studies. Wave equations from previous studies have been developed based on particular assumptions and have appeared in the form of convective-diffusion equations, whether or not the inertia effect was included.

2.3.1. Kinematic wave

Lighthill and Whitham (1955) demonstrated that dependence of the flow quantities (such as flow depth or velocity) on the rate of change of flow and/or the flow quantities themselves can introduce diffusion effects to the kinematic waves. This dependent

Table 2
Diffusion wave equations for different wave approximations of flow in open channels with prismatic arbitrary cross-section

A. Linear wave perspective	$\frac{\partial Q'}{\partial t} + c\frac{\partial Q'}{\partial x} = D_h\frac{\partial^2 Q'}{\partial x^2},\ Q = Q_0 + Q'$
Linearized noninertia wave	$c = -(\partial S_f/\partial A)/(\partial S_f/\partial Q)$ and $D_h = u_0y_0/2S_0$
Linearized quasi-steady dynamic wave	$c = -(\partial S_f/\partial A)/(\partial S_f/\partial Q)$ and $D_h = [1 - (1 - (c/u_0))F_0^2]u_0y_0/2S_0$
Linearized dynamic wave	$c = -(\partial S_f/\partial A)/(\partial S_f/\partial Q)$ and $D_h = [1 - (1 - (2c/u_0 + (c^2/u_0^2))F_0^2]u_0y_0/2S_0$
B. Nonlinear wave perspective	$\frac{\partial A}{\partial t} = \frac{\partial}{\partial x}(D_h\frac{\partial A}{\partial x}) + q_r - q_s$
Noninertia wave	$D_h = \zeta\left\{\frac{\|(\partial y/\partial x) - S_0\|}{(\partial A/\partial x)^2}\right\}^{1/2}$
Quasi-steady dynamic wave	$D_h = \zeta\left\{\frac{\|Q\,\partial(Q/A)/\partial x + [U_{rx}q_r - U_{sx}q_s - (q_r - q_s)Q/A] + gA(\partial y/\partial x) - gAS_0\|}{gA(\partial A/\partial x)^2}\right\}^{1/2}$
Dynamic wave	$D_h = \zeta\left\{\frac{\|(\partial Q/\partial t) + Q\,\partial(Q/A)/\partial x + [U_{rx}q_r - U_{sx}q_s - (q_r - q_s)Q/A] + gA(\partial y/\partial x) - gAS_0\|}{gA(\partial A/\partial x)^2}\right\}^{1/2}$
C. Other perspectives	(a) $\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = D_h\frac{\partial^2 y}{\partial x^2}$ (b) $\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = -D_h\frac{\partial^2 Q}{\partial x\,\partial t}$
Noninertia wave Eq. (a)	$c = (\alpha + 2)u/2$ and $D_h = \omega y^{(\alpha+2)/2}/2S_0^{1/2}$ (for wide rectangular channels)
Kinematic wave Eq. (b)	$c = \partial Q/\partial A$ and $D_h = c\,\partial A/\partial Q_x$

Table 3
Convective-diffusion wave equation of different wave approximations for wide open channels

Wave approximation	Governing equation	Celerity, c	Hydraulic diffusivity, D_h
Kinematic wave (Lighthill and Whitham, 1955)	$\frac{\partial q}{\partial t} + c\frac{\partial q}{\partial x} = D_h \frac{\partial^2 q}{\partial x\, \partial t}$, q is flow discharge per unit width	$c = \partial q/\partial y$	$D_h = -c\, \partial y/\partial q_x$
Kinematic wave	$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = D_h \frac{\partial^2 y}{\partial x^2}$	$c = (\alpha + 2)u/2$	$D_h = 0$
Linearized kinematic wave (Weinmann and Laurenson, 1972)	$\frac{\partial y'}{\partial t} + c\frac{\partial y'}{\partial x} = D_h \frac{\partial^2 y'}{\partial x^2}$, $y = y_0 + y'$	$c = (\alpha + 2)u_0/2$	$D_h = 0$
Noninertia wave	$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = D_h \frac{\partial^2 y}{\partial x^2}$	$c = (\alpha + 2)u/2$	$D_h = \frac{\alpha + 2}{2}\omega y^{(\alpha+2)/2}/2S_0^{1/2}$
Linearized noninertia wave (Ponce, 1990)	$\frac{\partial y'}{\partial t} + c\frac{\partial y'}{\partial x} = D_h \frac{\partial^2 y'}{\partial x^2}$, $y = y_0 + y'$	$c = (\alpha + 2)u_0/2$	$D_h = u_0 y_0/2S_0$
Linearized quasi-steady dynamic wave (Ponce, 1990)	$\frac{\partial y'}{\partial t} + c\frac{\partial y'}{\partial x} = D_h \frac{\partial^2 y'}{\partial x^2}$, $y = y_0 + y'$	$c = (\alpha + 2)u_0/2$	$D_h = (1 + \frac{\alpha}{2}F_0^2)u_0 y_0/2S_0$
Linearized dynamic wave (Ponce, 1990)	$\frac{\partial y'}{\partial t} + c\frac{\partial y'}{\partial x} = D_h \frac{\partial^2 y'}{\partial x^2}$, $y = y_0 + y'$	$c = (\alpha + 2)u_0/2$	$D_h = (1 - \frac{\alpha^2}{4}F_0^2)u_0 y_0/2S_0$

function relationship, when combined with the continuity equation, can be expressed as

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = -D_h \frac{\partial^2 Q}{\partial x\, \partial t} \tag{23}$$

From Eq. (23) it is shown mathematically, with the assumption of the dependence relationship, that the kinematic wave approximation can be re-expressed as a convective-diffusion equation with $c = \partial Q/\partial A$ and $D_h = c\, \partial A/\partial Q_x$ where $Q_x = \partial Q/\partial x$.

2.3.2. Noninertia wave

The noninertia wave approximation can be treated as a special case of the diffusion wave. For the noninertia wave in a wide rectangular open channel, the governing equations can be rearranged into the form of Eq. (6a) with the wave celerity

$$c = \frac{\alpha + 2}{2}\omega[S_0 - (\partial y/\partial x)]^{1/2} y^{\alpha/2} \approx \frac{\alpha + 2}{2}\omega S_0^{1/2} y^{\alpha/2} = \frac{\alpha + 2}{2}u \tag{24}$$

and the hydraulic conductivity

$$D_h = \frac{\alpha + 2}{2}\omega y^{(\alpha+2)/2}/2[S_0 - (\partial y/\partial x)]^{1/2} \approx \frac{\alpha + 2}{2}\omega y^{(\alpha+2)/2}/2S_0^{1/2} \tag{25}$$

The constant $\omega = C$ corresponds to Chezy's formula or $\omega = K_n/n$ for Manning's formula. The value of α equals 1 if Chezy's formula is used, and 4/3 if Manning's formula is used.

3. Concluding remarks

It is demonstrated from the above formulations that the diffusion wave, from both physical and mathematical perspectives, includes but is not limited to the noninertia wave. The results are summarized in Tables 2 and 3. A generalized diffusion wave equation, from both linear and nonlinear perspectives, can be formulated from different levels of wave approximations under the assumption that the wave celerity and hydraulic diffusivity are step-wise constants. On the other hand, the noninertia wave is specifically defined from physics as a simplification to the full dynamic wave, where the inertia terms are considered insignificant compared with the pressure gradient, gravity and friction slope terms. Differences among the various wave approximations resulting from their physical mechanisms and mathematical structures are reflected in the wave celerity and coefficient of hydraulic diffusivity. For the purpose of clarification and to avoid confusion, the 'noninertia wave' is a more appropriate name to describe specifically the approximate wave model in flood routing whose inertia terms are omitted in the momentum equation.

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