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Estimation of the Oil Reserves Requirement to Meet a Given Production Level—Mathematical Modeling

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The problem of estimating annual oil reserves demand needed to maintain some given production level may be of real importance for big oil companies. The existing mathematical models of oil production permit the calculation or estimation of the daily or annual production from the reservoir of given dimensions with given properties. Nevertheless, the existing models do not answer many questions important for reservoir management, and, first of all, estimating the requirements in annual and total reserves increase. The authors developed an analytical model for predicting the reserves requirement to ensure a given production rate from a specific formation or region. The model is based on the data of annual rate of oil production measured in fractions of the field reserves. In modeling, we assumed as given the productive quality of reserves, that is, production rate from the unit of the proved reserves. As input function the so-called production deficit is considered, namely, a function describing the difference between the total annual planned production level and annual planned production from the already discovered fields. The model uses approximate continuous analytical description of discrete input data. Governing intergral equations of material balance relating the reserves requirement, production deficit, and productive quality of reserves are solved by application of the Laplace transformation. The solution permits calculation of the annual reserves increase to meet a given production rate specified by an analytical expression (polynomial or exponential) with 3–5 free parameters. The parameters are to be appraised from a prescribed production curve by a best-fit technique. Prediction horizon may be about 10 to 20 years.

KEY WORDS: Reserves planning; Laplace transform; oil exploration and exploitation; production demand.

INTRODUCTION

The estimation of the need in explored (proved) oil reserves to ensure a given production rate is an important problem for oil companies in assessment of the prospects of oil-bearing formations and regions and in management of oil reserves. The existing methods of calculation and estimation of annual oil production for a given field or stratum, based on the data on the properties of reserves, and using hydrodynamic models, are reliable and well developed. On the other side, there exist no accepted reliable method for solving the inverse problem: how much proved reserves must one have, and how much must be increase of proved reserves in a year to ensure a given annual production schedule?

In this paper, a mathematical model is proposed for the estimation of quantity of oil reserves needed to put annually into development to perform a given production schedule. The model is based on the material balance equation connecting the annual production level with the rate of putting reserves into production. In the model, the data of the production rate (more exactly annual rate of oil production, measured in fractions of the field reserves) are used as primary, and rate of input of the reserves is considered as a function of time to be calculated. The presented model

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develops the ideas proposed previously by Ryzhik and Feygin (1972) and Zheltov (1971).

In modeling, we assume as given the data on the productive quality (productivity) of reserves, by which the economically optimal production rate is known as a function of time, from the unit of the proved reserves in development (e.g., 1 million bbl)

In the course of modeling, we consider the current values of production rate, reserves, even the number of wells, as continuous, rather than discrete, functions of time. Such approach, described in more detail, is convenient for our modeling purposes.

BASIC EQUATIONS

We begin by deducing the basic equations from the material balance equation, stating the interdependence between the production and development of reserves in the discrete form.

We assume that in some initial year, which we denote $k = 0$ (k is considered as number of years), reserves of some oil reservoir begin to be put into production. First, we characterize the reservoir production in the kth year from the beginning, $Q(k)$. Let $v(i)$ be the volume of reserves ("ith element"), put into production in some ith year ($0 < i < k$). At the kth year, the time (number of years), which passed from the beginning of production of the ith element ("age of the element") will be equal to $j = k - i$. Our fundamental assumption is that the production from the ith element in the kth year from the beginning, $\Delta Q_{i,k}$, can be described as

$$\Delta Q_{i,k} = v(i)\varphi(k - i) \tag{1}$$

where $\varphi(k - i)$ is a known function of the "age of the element," $j = k - i$. $\varphi(j)$ is equal to the fraction of the initial reserves of the element, $v(i)$, produced in the year number j from the putting of the element into production. The distribution of the values $\varphi(j)$ by year depends on the quality of reserves and on the rate of their development. If the total time of production from the element is K years, we have the relation

$$\sum_{j=1}^{K} \varphi(j) = 1 \tag{2}$$

indicating that in K years from the beginning of production, all the reserves of the element will be extracted.

Summing the production from all the elements of a given oil reservoir, put into production from the year $i = 1$ to the year $i = k$, we obtain

$$Q_k = \sum_{i=1}^{k} \Delta Q_{i,k} = \sum_{i=1}^{k} v(i)\varphi(k - 1) \tag{3}$$

In this paper, for solving problems of reserves and production predictions, we use continuous models, which are more appropriate for application of analytical methods. To transform material balance Equation (3) from discrete into continuous form, we must change from discrete variables, depending on the number of years, i, k, j, to the continuous, depending on continuous time, and apply the smoothed curves, instead of stepwise variables used in discrete approach.

We note that in Equations (1–3) the equality of the time interval to 1 year is not substantial; they hold at every value of the time interval. Let us explicitly introduce the elementary time interval, Δt (equal to 1 year in the previous discussion), and time variables $t = \Delta tn$ and $\tau = \Delta ti$. If we assume that $t \gg \Delta t$ and $\tau \gg \Delta t$, we may change in (3) summation to integration, which gives

$$Q(t) = \int_0^t v(t)\varphi(t - \tau)d\tau \tag{4}$$

where we may interpret the variable t as the total production time and τ as the time when the reserves of the element, equal to $v(\tau)d\tau$, are put into production. Then, $t - \tau$ is the time interval of production from the element at the moment t. The function $\varphi(t - \tau)$ can be interpreted as part of the reserves of the element, $v(\tau)$, produced in the time interval from $t - \tau$ to $t - \tau + d\tau$.

In terms of (4), our main problem is to determine the function $v(t)$, describing the annual demand in new reserves, for a given annual "production deficit" $Q(t)$, at given reservoir conditions, presented by the function $\varphi(t)$.

Transforming (2) to continuous form, we obtain a normalizing equation for $\varphi(t)$,

$$\int_0^\infty \varphi(t)dt = 1 \tag{5}$$

The infinity in the upper limit of the integral (5) is the result of transformation of (2) to continuous form, as K contains an infinite number of elementary time intervals. The appearance of zero in the lower limit of (5) and (4) is because $t \gg \Delta t$. The typical view of the function $\varphi(t)$ is schematically shown in Figure 1.

In the estimates of the need in proved reserves for some developing oil-producing region, usually the

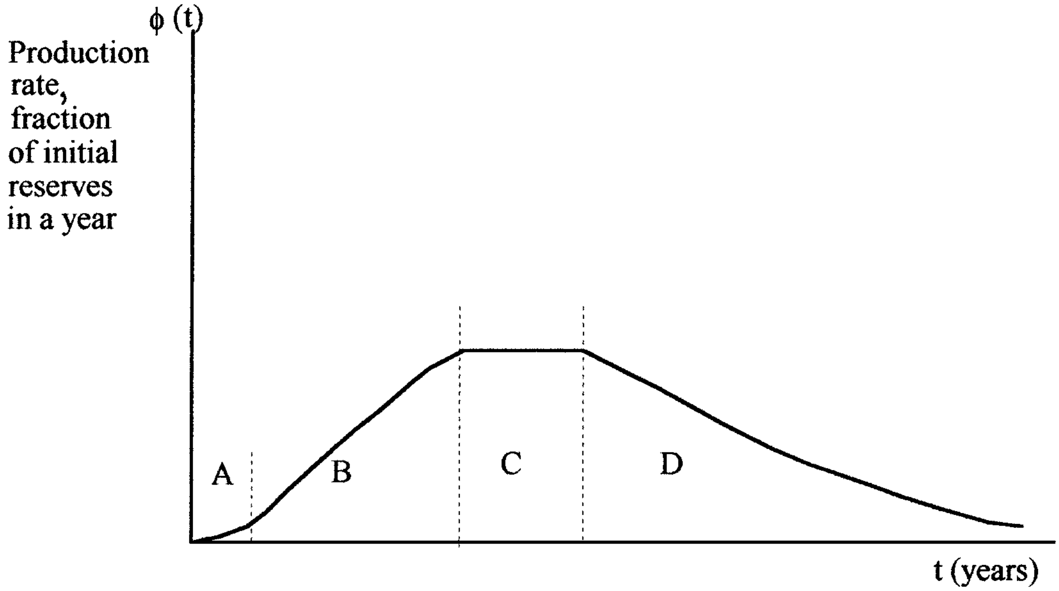

Figure 1. Typical form of production curve from reservoir. A, initial period; B, period of growing production rate; C, stable period; D, period of decreasing production.

values of total reserves, V^T, and the total production rate, Q^T, are used. They may be presented as

$$V^T(t) = V^o(t) + V(t),$$
$$Q^T(t) = Q^o(t) + Q(t) \qquad (6)$$

where $V^o(t)$ and $Q^o(t)$ are, considered as known, correspondingly, the reserves and annual production rate at time t, of the "old" oil reservoirs, already producing at $t = 0$. The values $V(t)$ and $Q(t)$ are the reserves and production rate of the new reservoirs, developed at $t > 0$. The value $Q(t)$ is defined by (4), and

$$V(t) = \int_0^t v(\tau)d\tau \qquad (7)$$

In considering the total reserves and production of some region, we will name the value $Q(t)$ as "production deficit" or "production shortage," needed to ensure the total level of production $Q^T(t)$ at the time t. The production $Q(t)$ must be obtained from new reserves, to be put into development from $t = 0$ to the year t.

CALCULATION OF THE FUNCTION $\varphi(t)$

The function $\varphi(t)$ describes the rate of production from some typical oil reservoir in fractions of initial reserves per year. The function $\varphi(t)$ is related to some selected unit of reserves and its dimension is 1/year. The function $\varphi(t)$ enters the main equation of the model (4) and must be determined before Equation (4) is solved.

To select a proper form of the function $\varphi(t)$ for the following calculations, we assume, that all the reservoirs of the considered formation are, in some sense, "similar," that is, they are developed by the wells with the same quantity of reserves per well and that the flow rate of oil from the well is declining in time with the same rate of decline for all the wells. We assume also that the rate of drilling (number of the wells drilled per year) is the same function of time for the every unit of the reserves, for example, 10 million bbl.

We characterize "quality of reserves" by the following system of geologic and technologic parameters, which permit the calculation of the function $\varphi(t)$.

- q_0 (million bbl/year)—initial production rate of the oil from a single well;
- t_0 the characteristic time of the well production decline in years (see description next);
- ω_0 (million bbl) initial reserves per well [of the three parameters, q_0, t_0, and ω_0, only two are independent; they are related by (9)];
- T (years) characteristic time of drilling, that is, the full time of drilling of a some given fraction of the number of wells in the unit of reserves, depending on the accepted expression for the drilling rate (see example).

The listed parameters (more correctly, their averaged by reservoir values) determine the rate of the development of a given oil reservoir. To explicitly obtain the expression for the function $\varphi(t)$, we make some assumptions related to the technology of drilling and production. These assumptions, and the form of the function $\varphi(t)$ obtained, must be considered as examples, appropriate for further calculations. For the situations when these assumption are unacceptable, they may be changed in the frames of the same general modeling technique.

We assume (which is acceptable, in many instances) that oil production from a well may be described by the equation

$$q = q_0 \exp\left(-\frac{t}{t_0}\right); \qquad t_0 > 0, \quad t > 0 \tag{8}$$

where q is production at the time t (years) from the beginning of well production. Equation (8) describes the exponential decline of a single well production and may be considered as the definition of the constant t_0 (years) The reciprocal to t_0 value, $1/t_0$, may be termed the coefficient of the production decline.

From (8) it follows that

$$\omega_0 = \int_0^\infty q(t)dt = q_0 t_0 \tag{9}$$

Equation (9) defines the relation between parameters ω, t_0, and q_0. Note that this relation may change if we use for well production an equation other than (8).

Our purpose now is to calculate (using the given system of parameters) the function $\varphi(t)$ Equation (3), and approximately also (4), may be applied to the unit of reserves, using the reserves around a single well as a elementary cell. This procedure permits us to calculate the function $\varphi(t)$ for a unit of reserves. In this calculation, we may use Equations (1) or (4) for a unit of reserves and assume that the function $\varphi(\tau)$ describes the production rate of one well.

In all calculations, the volume of reserves in every unit is considered as a given constant (i.e., 10 million bbl). The unit of reserves is considered as consisting of a number of elementary cells drained each by one well.

The rate of putting the reserves into development, $v_o(t)$, in such model is proportional to the number of wells drilled in a year, m [if we multiply m to ω_0, we obtain exactly the value of $v_o(t) = v(t)$ for the unit of reserves].

Let us assume, for further examples, that the total number of wells, producing in a unit in a year *t*, $M(t)$, and number of wells put into production in a year, $m(t)$, may be expressed by equation

$$\begin{aligned} M(t) &= M_0\left[1 - \exp\left(-\frac{t}{T}\right)\right], \\ m(t) &= \frac{dM(t)}{dt} = \frac{M_0}{T}\exp\left(-\frac{t}{T}\right) \end{aligned} \tag{10}$$

where M_0 is the total number of wells in a unit.

Equation (10) approximately describes the situation, when the number of wells in the unit, drilled in the first year, is maximal and in the following years the number of drilled wells per year declines exponentially. The volume of reserves of the unit, put into production in a year, $v_o(t)$, is expressed by

$$v_o(t) = m(t)\omega_0 = \frac{M_0\omega_0}{T}\exp\left(-\frac{t}{T}\right) \tag{11}$$

Using (8), (10), and (11), we obtain from (4) the expression for $\varphi(t)$:

$$\begin{aligned} \varphi(t) &= \frac{Q_o(t)}{M_0\omega_0} = \frac{1}{M_0\omega_0}\int_0^t v(t)q(t-\tau)d\tau \\ &= \frac{T+t_0}{t_0^2}\left\{\exp\left(-\frac{t}{t_0}\right) - \exp\left[-t\left(\frac{1}{t_0} + \frac{1}{T}\right)\right]\right\}; \\ &\qquad t_0 > 0, T > 0 \end{aligned} \tag{12}$$

where $Q_o(t)$ is the yearly production from the unit, which, being divided to initial reserves of the unit, $M_0\omega_0$, gives the value of $\varphi(t)$. The Expression (12) which meets the condition

$$\int_0^\infty \varphi(t)dt = 1$$

may be easily checked. This expression also is convenient for modeling purposes, for example, for solving (4) using analytical methods.

The constants T and t_0, having the dimension of time (years), entering into expression (12), may be defined from the field data, or estimated from geologic properties of the reservoir. The value of T in Equation (12) may be interpreted as the time necessary for drilling of $\exp(-1) = 0.3679$ (or about 37%) of total number of producing wells in the unit of reserves. The value t_0 is reciprocal to the coefficient of production decline for a single well.

Two typical curves $\varphi(t)$, calculated using (12), with different values of T and t_0, are shown in Figure 2. In the same Figure 2, the curve $\varphi(t)$ used in our previous work (Ryzhik and Feygin, 1972) also is shown.

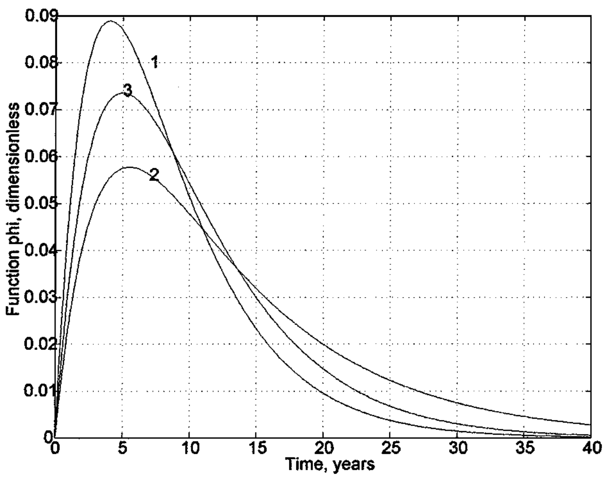

Figure 2. Three typical curves $\varphi(t)$. (1) Equation (12) with $t_0 = 5$ years, $T = 10$ years; (2) same with $t_0 = 10$ years and $T = 5$ years; (3) Equation (13), $T_1 = 5$ years.

It is described by equation

$$\varphi(t) = \frac{t}{T_1^2} \exp\left(-\frac{t}{T_1}\right) \tag{13}$$

where T_1 is a parameter, equal to the time, when a peak of production rate is reached for the unit element.

For the curve described by (12) the peak of production is reached in the year t_m, equal to

$$t_m = T \ln\left(1 + \frac{t_0}{T}\right) \tag{14}$$

PROBLEM STATEMENT. APPLICATION OF THE LAPLACE TRANSFORM

Our main purpose is to calculate the function $v(t)$, the rate of putting the reserves into production, if the production rate (production deficit), $Q(t)$, is prescribed, also assuming the function $\varphi(t)$ to be given.

Equation (4) considered as equation for $v(t)$ is linear; the integral in its right-hand side has a convolution form. This permits us to apply the Laplace transform method for its solution.

If we apply the Laplace transform to (4), we obtain

$$\bar{Q}(p) = \bar{v}(p)\bar{\varphi}(p) \tag{15}$$

where, by bars over functions, the corresponding Laplace transforms are designated, for example,

$$\begin{aligned} \bar{Q}(p) &= \int_0^\infty Q(t)\exp(-pt)\,dt, \\ \bar{v}(p) &= \int_0^\infty v(t)\exp(-pt)\,dt, \\ \bar{\varphi}(p) &= \int_0^\infty \varphi(t)\exp(-pt)\,dt \end{aligned} \tag{16}$$

where p is the parameter of the Laplace transform.

If the function $\varphi(t)$ is known, for a given function $v(t)$ [or $\bar{v}\ (p)$], the solution of (15) in Laplace transformations is

$$\bar{v}(p) = \frac{\bar{Q}(p)}{\bar{\varphi}(p)} \tag{17}$$

To obtain the unknown function $v(t)$, first we calculate functions $\bar{\varphi}(p)$ and $\bar{Q}(p)$. Then, from (17), we obtain the function $\bar{v}(p)$. To calculate the final expression for $v(t)$, we must make the inversion of Laplace transform of the function $\bar{v}(p)$.

If the functions $Q(t)$ and $\varphi(t)$ are expressed via elementary functions, their images (Laplace transform) may be obtained easily, in the most situations. However, the inversion of the Laplace transform can be made analytically only in some special situations (these situations are collected in many handbooks, e.g., Bateman and Erdelyi, 1954).

There exist also some numerical methods for inverse Laplace transform. Application of these methods may make the field of model applications more wide. Because the main purpose of this paper is the illustration of the model, we shall not describe and use these methods here.

EXAMPLE CALCULATIONS

The Laplace transform of the expression (12) is rather simple and permits one to obtain analytically inverse Laplace transform of $\bar{v}(p)$ in some practically important examples, when, for example, $Q(t)$ is expressed by polynomials or by a combination of the exponential functions. We consider these examples in more detail here. As is shown in the next section, these examples may be generalized for application in a rather wide number of examples of practical use.

The Laplace transform of the function $\varphi(t)$, expressed by (12), is

$$\bar{\varphi}(p) = \frac{a(a+b)}{b}\left(\frac{1}{p+a} - \frac{1}{p+a+b}\right) = \frac{a(a+b)}{(p+a)(p+a+b)} \quad (18)$$

where

$$a = \frac{1}{t_0}, \qquad b = \frac{1}{T}$$

We shall now give some examples of the relation between the demand in annual growth of reserves, $v(t)$ and "production deficit," $Q(t)$. First, it is necessary to note that if $Q(t)$ is assumed to be constant (from the moment $t = 0$), or proportional to t (assumed the linear growth of the production from new reserves), there exist no solution of Equation (4). The reason is that to achieve a constant or linearly growing production, we need some value of initial reserves put into production simultaneously at $t = 0$. In accordance with (12), at the initial moment ($t = 0$), the production rate from the element of reserves is zero and also the first-time derivative of production rate is zero.

As an example of the function $Q(t)$, for which there exist the analytical solution of (4), we may consider the function

$$Q_1(t) = Q_0[1 - \exp(-ct)]^2 \quad (19)$$

where Q_0 and c are constants. The function $Q_1(t)$ at small t grows as t^2, but at the values of $t \gg 1/c$, it is close to the constant Q_0.

It is easy to calculate, using the tables of Laplace transform, that the function $v(t)$ for the choice of $Q(t) = Q_1(t)$, if $\varphi(t)$ may be described by (12), has the form

$$v(t) = Q_0\left[1 - \frac{2(a-c)(a+b-c)}{a(a+b)}\exp(-ct) + \frac{(a-2c)(a+b-2c)}{a(a+b)}\exp(-2ct)\right] \quad (20)$$

It is important to note that although the initial value of $Q(0) = 0$, the initial value of $v_0 = v(0)$ is not zero, but

$$v_0 = \frac{2Q_0c^2}{a(a+b)} \quad (21)$$

To ensure the production rate growth as t^2, we must put into production initially the reserves equal to v_0.

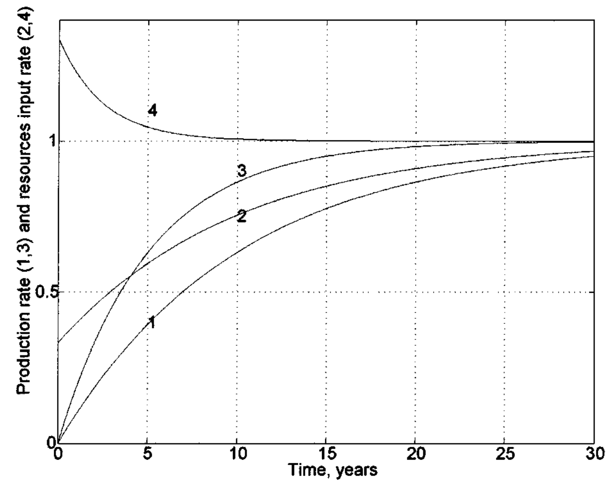

Figure 3. Curves $v(t)$ and $Q(t)$, for example (19) and (20), $a = 0.2$, $b = 0.1$. (1) $Q(t)$ for $c = 0.1$; (2) $v(t)$ for $c = 0.1$; (3) $Q(t)$ for $c = 0.2$; (4) $v(t)$ for $c = 0.2$. Values *a, b,* and *c* are in 1/year.

As an example, in Figure 3, two pairs of curves are shown, $Q(t)$ and $v(t)$, for the values of constants $a = 0.2$, $b = 0.1$, $v_0 = 1$, and for two values of $c = 0.1$ and $c = 0.2$; values *a, b,* and *c* are expressed in 1/year.

From the curves presented in Figure 3, we see that after a relatively long period of time, both the annual production and the annual input of reserves are almost equal constants.

The other example is an approximation to the linear growth of production, with the seriously mentioned condition that the production deficit $Q(t)$ and its time derivative are equal to zero at $t = 0$. This approximation may be described by the equation

$$Q_2(t) = Q^* t[1 - \exp(-ct)] \quad (22)$$

where Q^* and c are constants.

The solution for $v_2(t)$ for this example may be obtained easily using the Laplace transform method, but is rather cumbersome. Thus, we present here only two pairs of graphs in Figure 4, corresponding to the same function $\varphi(t)$ as in previous example and two values of c, 0.1 and 0.2 1/year. Q^* is equal to 1.

The characteristic feature of this example, seen in Figure 4, is that the curves $v(t)$, corresponding to a given $Q(t)$, after some initial time interval, are parallel to $Q(t)$ with a shift of about $1/c$ years. This feature is specific for every situation of growing annual "production deficit."

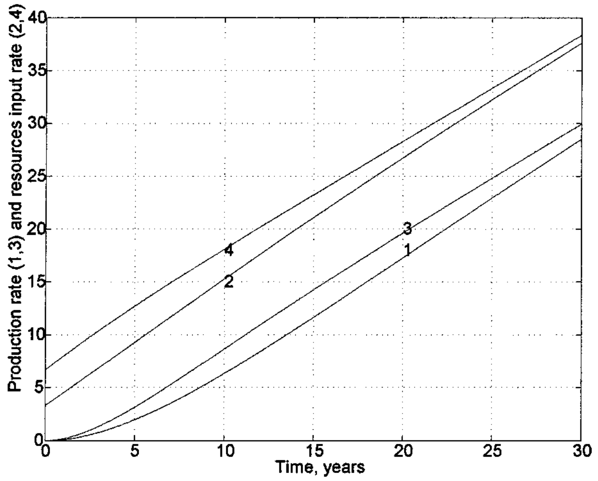

Figure 4. Curves $Q(t)$ and $v(t)$, for example (22) ($a = 0.2$, $b = 0.1$. (1) $Q(t)$ for $c = 0.1$; (2) $v(t)$ for $c = 0.1$; (3) $Q(t)$ for $c = 0.2$; (4) $v(t)$ for $c = 0.2$.

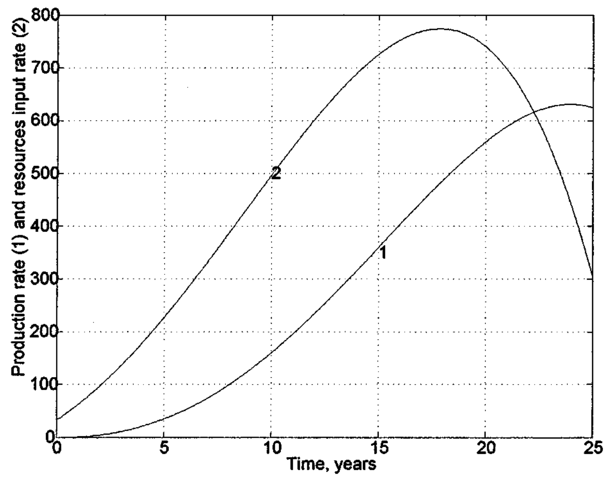

Figure 5. Graphs $Q(t)$ (curve 1) and $v(t)$ (curve 2) for polynomial approximation.

APPROXIMATE CALCULATION OF THE REQUIREMENTS IN NEW RESERVES

If the information for calculating parameters of the function $\varphi(t)$, namely, T and t_0, is available, the simple expressions may be suggested for approximate practical calculations of the requirement in new reserves input, $v(t)$, ensuring the given production rate, $Q(t)$ (production deficit).

In most of the practical situations, the projected "deficit" curve $Q(t)$ may be well approximated by a polynomial

$$Q(t) = At^2 + Bt^3 + Ct^4 \tag{23}$$

where A, B, and C are rather easily calculated approximation constants if a curve $Q(t)$ is given in any form. We begin the expression (23) with the second power of t, not including a constant and a linear by t component, because of condition $Q(0) = Q'(0) = 0$, as discussed, and we restrict ourselves by the three first terms for the sake of simplicity (it is not a necessary condition).

Using expression (12) for $\varphi(t)$ performing the inverse Laplace transform, we obtain

$$v(t) = Q(t) + \frac{1}{ad}\{2A + [2A(a+d) + 6B]t + [3B(a+d) + 12C]t^2 + 4C(a+d)t^3\} \tag{24}$$

where $Q(t)$ is expressed by (23), $d = a + b$. (Note, that $a = 1/t_0$, and $b = 1/T$).

As an example, in Figure 5 the graphs $Q(t)$ and $v(t)$, in some arbitrary units, calculated using (23) and (24), are shown for the following values of parameters

$$A = 1, \quad B = 0.1, \quad C = -0.004,$$
$$a = 0.2, \quad d = 0.3$$

The comparison of Figures 5, 3, and 4 shows, that the rule of the parallelism between the curves $v(t)$ and $Q(t)$ holds only for the situation of growing production rate.

The function $v(t)$ expresses the annual input of the reserves into production. To obtain the total requirements in new reserves, $V(t)$, to the year t, we must integrate $v(t)$ by time from zero to t. This gives

$$V(t) = \int_0^t v(\tau)d\tau = \Omega(t) + \frac{1}{ad}\{A + [A(a+d) + 3B]t^2 + [B(a+d) + 4C]t^3 + C(a+d)t^4\} \tag{25}$$

where

$$\Omega(t) = \int_0^t Q(\tau)d\tau = \frac{At^3}{3} + \frac{Bt^4}{4} + \frac{Ct^5}{5}$$

is the total expected production of new reserves from $t = 0$ to t.

The expressions, given in this section, permit us to calculate the requirements in new oil reserves necessary to ensure a given production rate. The most important restriction of these calculations is that we assume all the new reserves to be of similar "quality," that is, their development may be described by a single function $\varphi(t)$, characterizing the production rate from the unit of reserves. If, in the considered region, there are several reservoirs with different properties, the problem of distributing the "production deficit" between these reservoirs arises. This problem may be solved by the optimization approach using economic considerations. When the production distribution is made, the requirements in reserves may be separateley calculated for every reservoir.

SUMMARY AND CONCLUSIONS

In this paper the mathematical model of the reserves planning is proposed. The approximate continuous equation is constructed based on the discrete scheme of material balance equations. The model permits us to calculate the requirements in new reserves, if the potential production rate and the parameters of the reservoir are given. Several examples are presented for such calculations using the Laplace transform method and a simple approximate general algorithm is proposed. The proposed model also may be used as part of the more general optimization models, including the economic considerations for heterogeneous reservoirs.

The model discussed in this paper permits us to calculate the necessary annual increase of proved reserves; this helps to solve a number of practical problems of oil production and exploration.

It is shown that the annual reserves demand in the situation of growing production rate may be estimated roughly as equal to the "production deficit" at some year in the future. The time lag between annual production and reserves demand depends on the given (economically acceptable) production rate of the new oil reservoirs. The greater the production rate, the less must be the time lag between production and increase of reserves. In the examples given in the paper, the time lag ranges from 5 to 10 years.

It is evident that the delay in placing reserves into production has a negative effect on the value of the future production rate. Nevertheless, if the reserves deficit in initial period may be compensated by subsequent placing of greater reserves into production, this negative effect may be prevented.

The annual and total demand of proved reserves calculated by the proposed scheme permits the assessment of the feasibility of exploration and of preparation for development of the necessary number of oil pools, and the necessary expenses for the required period of time.

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