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The transport of U- and Th-series nuclides in a sandy unconfined aquifer

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Abstract—A comprehensive evaluation of the transport of U, Th, Ra, and Rn nuclides of the ²³⁸U- and ²³²Th-decay series in an unconfined sandy aquifer (Long Island, NY) was conducted. Groundwater data are compared with results of a theoretical transport model of weathering of aquifer grains and interaction with surface coatings to establish relationships between the concentrations of the radionuclide activities in the water and flow line distance. The data provide estimates for geochemical parameters including weathering rates and chemical reactivities in both the vadose zone and the aquifer. A theoretical treatment of the transport is presented that considers the reaction between the water and a reactive surface layer. It is found that a model with chemical exchange between the surface layer and the water for all species is not valid, and that the effects of saturation and "irreversible" precipitation of Th is required.

The water table shows a relatively wide range in U activities, the only element in the U-Th series for which vadose zone input is significant in the aquifer. High weathering of U and recoil inputs of ²³⁴U to the water occur in the upper 3 m of the vadose zone, while lower weathering and removal of U from the water occur below. The deeper aquifer has variable 238 U activities that can be accounted for by input from the vadose zone and is not a result of non-conservative behavior. The isotopic composition of U is shown to be directly related to the recoil rate relative to the weathering rate. The wide range of ²³⁸U in the aquifer waters is a reflection of diverse vadose zone inputs, showing that dispersive mixing is not a dominant effect. The higher values of δ^{234} U in the aquifer reflect the recoil/weathering input ratios from within the aquifer where the weathering rate is lower than the vadose zone. Both high U activities and high δ^{234} U values cannot be obtained in the vadose zone or within reasonable flow distances in the aquifer. Radium isotopes are found to be in exchange equilibrium with the surface layer. ²²⁴Ra, ²²⁸Ra, and ²²⁶Ra have comparable activities throughout the aquifer. In the vadose zone, the dominant input of Ra to groundwater is weathering and recoil. As found elsewhere, the ²²²Rn in the water is a large fraction (\sim 5%) of the Rn produced in the aquifer rock. This cannot be due to Ra precipitation onto surface coatings in the aquifer as supported by present weathering with Th in exchange equilibrium with the surface layer. It is found that Th is saturated in the waters under oxidizing conditions so that the weathering input is irreversibly precipitating onto surfaces. However, it is shown that under somewhat reducing conditions, Th activities are much higher and the Th/U ratio in the solution is approximately that of the rock. We propose that under oxidizing conditions the source of Rn is a surface coating enriched in ²³²Th and ²³⁰Th. This Th was precipitated in an earlier phase during rapid dissolution of readily weathered phases that contain $\sim 10\%$ of the U-Th inventory of the rock, with the associated U carried away in solution. Therefore, the previously precipitated ²³⁰Th and ²³²Th produce daughter nuclides in the surface coating which are the dominant contributors of Ra and Rn to the ground water. In particular, Rn is provided by very efficient losses (by diffusion or recoil) from the surface coating. This then does not require recent, large recoil losses from the parent rock or the presence of nanopores in the rock. The first data of both long-lived ²³²Th and short-lived ²³⁴Th and ²²⁸Th in ground water is reported. The Th isotope activities indicate that desorption kinetics are slow and provide the first estimate, based on field data, of the Th desorption rate from an aquifer surface. The mean residence time of Th in the surface coating is ~ 3000 y while in the water it is ~ 1 h. Ra is in partition equilibrium with the aquifer surface layer. However, the strong fixation of Th on surface coatings is very susceptible to changes in oxidation state as is shown by a comparison of two adjacent aquifers. This makes it difficult to define with certainty the retentive characteristics in natural systems. In general, it is shown that the distributions of naturally occurring nuclides can be used to calculate values for transport parameters that are applicable to the transport of anthropogenic nuclides. Copyright © 2001 Elsevier Science Ltd

1. INTRODUCTION

Naturally occurring radionuclides provide analogues for pollutant nuclides. The behavior of U, Th, Ra and Rn isotopes can be studied through their relationships in the ²³⁸U and ²³²Th decay series where the half lives are given in parentheses:

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$$\begin{array}{ccc} \alpha & \beta & \alpha & \alpha & \alpha & \alpha \\ ^{238}\text{U}(4.5\text{Ga}) \xrightarrow{234}\text{Th}(24\text{d}) \xrightarrow{234}\text{U}(0.25\text{Ma}) \xrightarrow{230}\text{Th}(75\text{ka}) \xrightarrow{226}\text{Ra}(1.6\text{ka}) \xrightarrow{222}\text{Rn}(3.8\text{d}) \\ & \alpha & \beta & \alpha \\ ^{232}\text{Th}(14\text{Ga}) \xrightarrow{228}\text{Ra}(5.8\text{a}) \xrightarrow{228}\text{Th}(1.9\text{a}) \xrightarrow{224}\text{Ra}(3.6\text{d}) \end{array}$$

These two decay chains permit comparison of the behavior of two isotopes of the same element which are assumed to have the same chemical properties but very different lifetimes (two isotopes of U, three isotopes of Th, two isotopes of Ra). Although the U-Th series nuclides are expected to be in secular equilibrium in unaltered aquifer host rocks, examples of pronounced disequilibria are found in groundwater. This clearly shows that in open systems such as aquifers, water-rock interactions induce significant elemental and isotopic fractionations (see e.g., Osmond and Cowart, 1992). While the thermodynamic properties of these elements in aqueous solutions under laboratory conditions are well known (see Langmuir, 1997), it is difficult to predict behavior in more complex natural systems. Several studies have considered the aquifer transport of U-Th series nuclides (see Ku et al., 1992; Cowart and Burnett, 1994). Andrews and Kay (1982) and Andrews et al. (1989) calculated the time-scales for nuclide supply and removal. Krishnaswami et al. (1982) calculated supply rates, sorption rate constants, and residence times of short-lived nuclides by deducing recoil inputs of U-Th series nuclides from ²²²Rn activities, but did not consider dissolution, precipitation, and advective transport. Davidson and Dickson (1986) included dispersive flow for U and Ra, but not dissolution and precipitation.

This study uses data from an unconfined sandy aquifer to understand the transport of naturally occurring U, Th, Ra, and Rn nuclides. The approach here differs from earlier works in using a unified transport model by considering:

- more complete sequences of U, Th, Ra and Rn nuclides within the ²³⁸U- and ²³²Th-decay series;
- the vadose zone input into the aquifer through analysis of waters from water table wells;
- 3. the interactions with the aquifer solid through sampling waters of different "ages";
- the distribution of radionuclides between different filtration size fractions;
- the relationship of the observed activities to a water flow model (Tricca et al., 2000) that includes vadose zone input, weathering and recoil within the aquifer and reaction with a surface layer.

A model is used to discuss the data that includes weathering, α -recoil, and a reactive surface coating on the aquifer mineral grains that plays a significant role in storage of Th and Ra radionuclides. If one assumes the sites are in exchange equilibrium with the water, it will be shown that this storage is inadequate to provide sufficient sources of ²²²Rn. From experimental data, Th is close to the saturation limit with thorianite, which is not compatible with an exchange equilibrium model. A modified model is proposed where ²³⁰Th and ²³²Th are quasi irreversibly removed from the water and deposited on surface coating. This Th coating from an earlier stage of weathering can readily provide a surface layer sufficient to support ²²²Rn.

The basic chemical interactions (both equilibrium and non-

equilibrium) depend on the concentrations $({}^{i}c_{j})$ of each species *i* in the different phases (*j*-water, rock, surface coating). However, as the U and Th decay series connect the production of nuclides along the chain by radioactive decay, the activity $(\lambda_{i} {}^{i}c_{j})$ of each species is a governing parameter. We will therefore couch our representation of the transport problem in terms of activities and on occasion refer to the concentrations.

The approach used in this study was to choose a simple aquifer with generally known flow patterns and to follow the abundances of the U-Th series nuclides from the vadose zone input along flow lines. The intent was to determine the evolution of these nuclides along the flow path considering the vadose zone input and subsequent effects of water-rock interaction along the flow paths. This study uses the available hydrologic information available at the site. It does not depend upon the residence times as may be estimated from other tracers but upon the flow rates. The time and distance scales inferred from this study thus result from the rock-water interactions themselves using a typical flow velocity obtained by the hydrologic data. For a more extensive discussion of the theoretical model, the reader is referred to Tricca et al. (2000).

2. SAMPLING AND ANALYTICAL PROCEDURES

This study focuses on the unconfined Upper Glacial Aquifer in Long Island, NY (Figs. 1 and 2) within and around the DOE Brookhaven National Laboratory (BNL). The hydrogeology of the area has been described by de Laguna (1963), de Laguna (1966) and Warren et al. (1968). The aquifer consists of 30 to 70 m of unconsolidated Pleistocene sandy deposits overlain by an unsaturated zone (3-30 m). The aquifer and unsaturated zone consist of quartz, feldspars and traces of micaceous minerals, as well as U/Th bearing zircon and apatite (Faust, 1963; de Laguna, 1963). No carbonate or evaporite minerals were reported. Ground water flows at $1 \times 10^{-4} - 2 \times 10^{-4}$ cm sec⁻¹; the upper value will be used for discussion here. There is an east-west groundwater divide north of the site, and flow across the study area is southward. Available piezometric data provide horizontal flow directions (see Fig. 2 and Geraghty and Miller, 1996), but not vertical flow rates, so unambiguously following the descent of flow lines is not possible. Yet, the ages of waters sampled by deep wells are likely to increase with greater distance from the groundwater divide (see Fig. 2). In this study, well lines have been selected to be along possible flow lines, although incorporation of waters that have evolved along different paths is also considered. The underlying Magothy aquifer is locally separated from the Upper Glacial aquifer by the confining Gardiner's Clay (0-10 m). These Magothy waters are under more reducing conditions and give key complementary data to that found in the Upper Glacial unit.

Ground waters were obtained to determine the vadose zone contribution to the aquifer and effect on the nuclide behavior along flow lines (Fig. 2 and Table 1). In October, 1997, samples were taken along two well lines (W and E) in the Upper Glacial Aquifer. Data for past Ethylene Dibromide contamination (BNL, pers. comm.) indicated that waters from wells $E1(S)_{wt}$ and E2(I) were on the same flowline. A sample was collected in the Magothy (M1) to characterize that aquifer. Note that water from the Magothy was yellow indicating the presence of organic carbon or Fe. No H₂S was detected in the Magothy waters. Other water table samples (W1(S)_{wt}, S1_{wt}, S2_{wt}, S3_{wt}) were collected in March, 1999 to study nuclide behavior in the vadose zone.

Ground water was drawn using a submersible pump run at 0.5 to 20

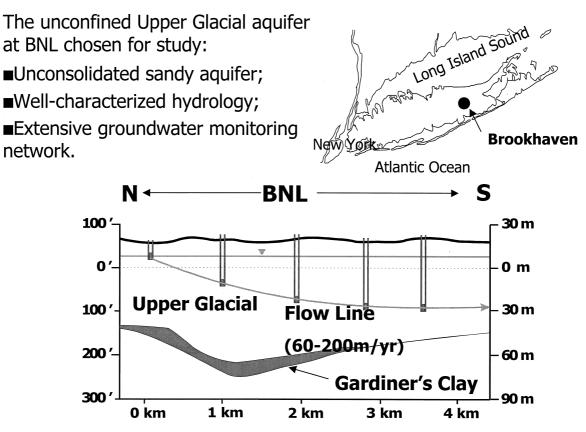


Fig. 1. Schematic diagram showing the location and general layout of the aquifer under investigation. Water table samples are from the base of the vadose zone. Aquifer samples from wells presumed to be along a flow line.

L/min. Temperature, pH, and dissolved O2 concentrations were measured on site along with Rn. Oxygen was measured by O₂ diffusion with a YSI55 oxygen probe. During this measurement, the authors avoided exchange between the sample and the atmosphere. Major anions and cations, Fe, and Mn were measured by SGAB Analyses (Luleå, Sweden). For analysis of ²³⁸U, ²³⁴U, and ²³²Th, separate water samples were collected either unfiltered or filtered on-line through 0.45 μ m cellulose filters. A fraction of the filtered samples were further separated into colloids and 'dissolved' fractions using ultra-filtration systems with a nominal cut-off size of 10 k Daltons. Samples were ultra-filtered with Amicon® polyethersulfone hollow fiber cartridges (see e.g., Porcelli et al., 1997). All filtrations were done, and all samples were acidified, in the field. The filtration system was rinsed between each sample with 1N ultra-pure HCl to collect material remaining in the system. This rinse was kept for analysis. A second HCl rinse was circulated followed by ultrapure water. The $^{238}\text{U},~^{234}\text{U},$ and ^{232}Th of each fraction was concentrated and measured by mass spectrometry (Chen et al., 1986). For analysis of short-lived Ra and Th isotopes, 100 L samples were passed on-line through 0.45 μ m prefilters and through Mn-oxide coated acrylic filters. The Mn-oxide filters were ashed and measured by α and γ counting at Texas A&M, Galveston (Baskaran et al., 1993). Absolute ²²⁶Ra activities were determined from 20 L of 0.45 µm-filtered water, passed through Mn-oxide impregnated fibers that have high Ra adsorption efficiency (Reid et al., 1979).

Total recoveries for the ultra-filtration procedures were evaluated by comparing concentrations in 0.45 μ m-filtered waters with the sum total recovery in <10 k D ultra-filtered water, >10 kD colloids, and HCl rinse. The U recovery for the Amicon ultra-filtration system for sample #W3(D) is only 75%. In addition, a significant amount of U appeared in the acid rinse that was retained during ultra-filtration. The δ^{234} U value (see Table 2 for definition) of the acid rinse shows that the U lost by adsorption belongs to the <10 kD fraction (Table 2). Therefore, the U concentration measured in the filtrate is a minimum value for "dissolved" U.

3. RESULTS

General chemical compositions of the samples are shown in Table 1. The total dissolved solids (TDs) are given in the standard units which include $Si(\sim 10 \text{ mg/L})$ and K (<1 mg/L). The dominant ions are generally Na^+ and either HCO_3^- or Cl^- , with no systematic relationship. The waters are slightly acidic. Major element data for ultra-filtered waters were indistinguishable from those of corresponding 0.45 μ m filtered samples and are not shown. The water table samples are oxidizing, with dissolved oxygen concentrations of 0.25 and 0.47 mM. These samples have the lowest pH values of the Upper Glacial Aquifer of 4.78 to 5.74 (compared to 5.74-6.09 for deeper wells), and also generally have the lowest TDS. Dissolved organic carbon concentrations are between 0.96 to 6.6 mg/L, and are typical for groundwater (see Stumm and Morgan, 1996). Only 0.42 to 0.95 mg/L DOC is associated with > 10 kD colloids. The Total Dissolved Solid (TDS) concentrations are <100 mg/L (except W3(D)). The lack of correlation between TDS and distance from the recharge area or depth requires that the waters sampled on a single well-line have undergone substantially different evolutionary histories due to either changing interaction with the vadose zone, or mixing in the well of waters of different ages, or sampling from different flow lines with different aquifer input conditions. These issues will be evaluated further below. The elements (Ca, Mg, Na, HCO₃, Cl, $SO_4^{=}$ are in millimoles per liter (mM). Fe and Mn are given in micromoles per liter (μ M). The Fe concentrations in the Upper

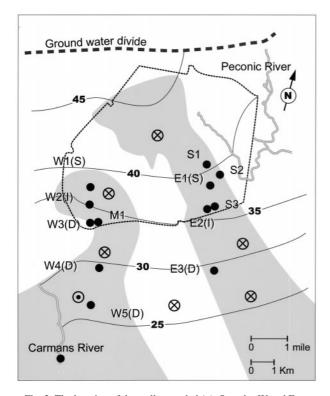


Fig. 2. The location of the wells sampled (•). Samples W and E were taken from the unconfined Upper Glacial Aquifer at either shallow (S), intermediate (I), or deep (D) levels below the water table. Sample M is from the confined Magothy Aquifer. The solid lines show the piezometric levels of the Upper Glacial aquifer. The gray area represents the surface of the Gardiner's Clay Unit discontinuously separating the Upper Glacial and Magothy aquifers. Downward (\otimes) or upward (\odot) vertical flow across the Upper Glacial Aquifer/Magothy boundary is shown. Note the east-west trending groundwater divide located ~0.5 km north of the site.

Glacial are $<0.2 \mu M$ (Table 1) which is in the lower range for ground waters (Hem. 1992). These concentrations are 10^3 larger than the solubility limit of Fe(III), and are too oxidized to form Fe(II); therefore Fe must be associated with colloids. The Mn concentrations are much lower than the solubility limit in the deeper wells ($<0.02-0.16 \mu$ M), with the exception of E3(D), which has a concentration of 7.5 μ M Mn close to the solubility limit that is common in ground waters (Hem, 1992). Therefore Mn is likely to be in the soluble Mn(II) form. For wells $E1(S)_{wt}$ to E2(I), located on the same flow line, the concentration of Cl, a conservative element, is doubled (see Table 1); this cannot be obtained by reasonable weathering rates of the aquifer solid. There is no halite or saline layer in the soil. Therefore, these Cl variations must reflect temporal changes of a surface input induced by anthropogenic inputs such as road salting.

The Magothy waters are chemically distinctive from those of the Upper Glacial Aquifer (Table 1), with lower dissolved oxygen (although no presence of H₂S was observed), higher pH (6.77) and very high Fe and Mn concentrations. The Fe concentration (214 μ M) in the filtered water is $\sim 10^6$ times the solubility limit of Fe(III) (e.g., Fox, 1988, Stumm and Morgan, 1996). The dominant fraction of Fe (70%, see data, Table 1) did not pass through the 10 kD filter and so is likely to be colloidal Fe. The remaining 30%, 54 μ M Fe/L, is still above the solubility limit and is plausibly contained in smaller colloidal particles). It is not due to dissolved organic carbon as the concentrations are the same level as in the overlying aquifer. The Mn concentration (25 μ M) is substantially above the value of 2 μ M commonly found in oxidizing ground waters elsewhere and is above the solubility limit of Mn(II) (Hem, 1992). Note that all the Mn is in the <10 kD fraction, (Table 1), and must be in colloids of size <10 kD or else the solubility is significantly enhanced. The high Mn concentration (e.g., the Mn/Mg ratio is ~10 times that of upper crustal rocks, (Taylor and McLennan, 1985)) could be due to release of Mn oxyhydroxides previously deposited under different redox conditions (Section 5 on Thorium).

Sr was measured in samples from the Upper Glacial and Magothy (Table 2 and Fig. 3). Water table samples have higher ⁸⁷Sr/⁸⁶Sr ratios (0.71191–0.71357) than the deeper samples (0.71144–0.71163) indicating that Sr provided by mineral weathering in the aquifer is less radiogenic than that provided at the water table. The Sr isotopic ratios in the deep wells are indistinguishable from in the Magothy, where ⁸⁷Sr/⁸⁶Sr = 0.71144, suggesting that the deep ground water flows through host rocks of composition similar to that of the Magothy. Sr concentrations range from 12 to 53 μ g/kg in no simple pattern.

For the U-Th series nuclides, we will present the data in terms of specific activity of nuclide *i* in a given phase (*j*) (${}^{i}A_{j}$) (disintegrations per minute per kg). The relative isotopic abundance of 234 U in waters (*w*) will be given by δ^{234} U = (234 U A_w/ 238 UA_w - 1) × 10³ (see Table 2).

The ²³⁸U activities in the aquifer (<0.45 μ m) vary within 2 × 10⁻³ to 18 × 10⁻³ dpm/kg (3–24 ng/kg). This is in the lower part of the extreme range reported for ground waters of 2 × 10⁻³ to 10³ μ g/kg (Osmond and Ivanovich, 1992). They have δ^{234} U = 30 to 1400‰ and so are all enriched in ²³⁴U. Water table U activities vary between 1.1 × 10⁻³ to 1.9 × 10⁻² dpm/kg. This range covers all activities measured in the aquifer. Therefore, U input from the vadose zone to the aquifer can represent an important portion of the U within the

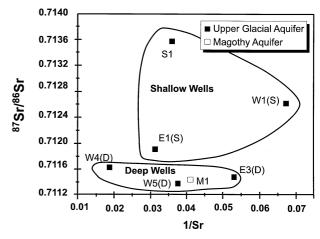


Fig. 3. Sr data of filtered samples from the Upper Glacial and Magothy Aquifers. Samples from deep wells within both aquifers have similar 87 Sr/ 86 Sr ratios, and are distinct from those of water table samples. Note that for a two-component mixing model, the relationship is of the form (87 Sr/ 86 Sr) = A + B/Sr.

		Water					Г	DOC (mg/l)									
Well #	BNL #	level ^b m	depth ^c m	TDS ^d mg/l	Hq	$^{\rm O_2^e}_{\rm mM}$	<0.45 µm	Colloids	$^{<10}_{\rm kD}$	$\mathop{\rm Ca_2}_{\rm mM}$	Mg MM	Na mM	HCO ₃ mM	mM CI	${}^{\rm SO_4}_{\rm MM}$	Fe ^f µM	Mn^{d}
West line																	
$W1(S)_{wt}a^g$	93-04	15	0-5	33	5.42	0.49	1.92		1.02	0.06	0.05	0.21	0.07	0.23		0.05	0.036
$W1(S)_{wt}b^{n}$	93-04	15	0	29	5.40					0.05	0.05	0.19	< 0.16	0.16		0.05	0.13
W2(I)	118-02	15	27	76	5.62	0.31	3.29			0.15	0.11	0.58	0.16	0.68		0.12	0.05
W3(D)	130-03	15	40	119	60.9	0.13	1.34	0.42	1.03	0.27	0.20	0.70	0.44	0.79		<0.007	< 0.02
W4(D)	000-98	24	43	56	5.74	0.31	1.95			0.10	0.08	0.44	0.14	0.59		0.09	0.15
W5(D)	800-40	19	43	62	5.95	0.16	6.61	0.86	5.29	0.12	0.09	0.30	0.43	0.20		0.05	0.16
East line																	
$E1(S)_{wt}$	79-17	4	0-5	91	5.74	0.25	1.56	0.46	0.88	0.32	0.22	0.12	0.33	0.17		0.014	0.13
E2 (I)	100-14	4	30	55	6.03	0.19	1.55	0.95	0.59	0.12	0.10	0.29	0.21	0.37		<0.007	< 0.02
E3(D)	800-25	9	47	42	6.5	0.22	0.96		0.61	0.08	0.05	0.23	0.25	0.20		0.02	7.5
Magothy																	
M1	130-04	16	250	99	6.77	0.06	5.14	0.70	2.52	0.17	0.08	0.25	0.52	0.20		14	25
$M1_{<10 kDfraction}$	stion									0.16	0.09	0.25				54	24.5
Water table																	
$\mathrm{S1}_{\mathrm{wt}}$	58-01	б	0	27	4.82		ĺ	[ĺ	0.03	0.04	0.19	<0.16	0.14	0.09	<0.9	0.35
$S2_{wt}$	100-10	ω	0	28	4.95					0.04	0.10	0.11	<0.16	0.07	0.15	<0.9	0.13
$S3_{wt}$	70-01	L	0	24	4.78					<0.02	0.05	0.20	<0.16	0.16	0.08	<0.9	0.65
^a The units:	^a The units: $m(matere)$: $m\alpha/l = milliareams/litereams M = millim$	illim = Παι	iorame /litar	m = Mm	illimoles/li	- M	- micromolac/litar	itar									

Table 1. General characteristics of groundwater samples.^a

^a The units: m(meters); mg/l = milligrams/liter; mM = millimoles/liter; μ M = micromoles/liter. ^b Water table level given in meters below surface level. ^c Depth of water samples, given in meters below water table. ^d Total Dissolved Load (values include the concentrations of K and Si, not given in the table). ^e O₂ saturation with the atmosphere ≈ 0.4 mM. ^f Data within 5% error measured by Svensk Grundämnesanalys AB, Luleå Tekn. Universitet, 971 87 Luleå, Sweden. ^g Sample collected Fall 97. ^b Sample collected Spring 99.

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		²³⁸ U (10 ⁻³ dpm/kg)			$\delta^{234} U^a$		²³⁴ U Excess	(10^{-5})	²³² Th (10 ⁻⁵ dpm/kg)	÷	598 728
Well	Unfiltered	<0.45 µm	<10 kD	Unfiltered	<0.45 µm	<10 kD	(10 ⁻³ dpm/kg) <0.45 μm	Unfiltered	<0.45 µm	Sr (ppb) <0.45 μm	$< 0.45 \ \mu m$
West line W1(S) a ^b		2 839 + 0 006	0.341 ± 0.002		46 + 12		0.13		2 38 + 0.01	14.87	0.71262 + 4
$W1(S)$, b^c		3.47 ± 0.01							11.6 ± 0.1	19	
W2(I)		3.817 ± 0.007	2.047 ± 0.007		615 ± 18	172 ± 16	2.35		4.03 ± 0.02	40	
W3(D)	10.71 ± 0.04	12.15 ± 0.05	2.570 ± 0.007	166 ± 22	130 ± 11	128 ± 11	1.58	22.6 ± 0.2	1.09 ± 0.02	27	
W4(D)		6.551 ± 0.003	0.822 ± 0.003		1394 ± 23	917 ± 24	9.13		1.42 ± 0.007	53.49	0.71163 ± 5
$W4(D)^d$		6.535 ± 0.002			1390 ± 19		9.08				
W5		2.87 ± 0.01	0.2730 ± 0.0007		673 ± 33		1.93		0.700 ± 0.005	24.39	0.71138 ± 5
East line											
$E1(S)_{wt}$		5.36 ± 0.03			31 ± 17		0.17		1.244 ± 0.007	31.9	0.71191 ± 6
E2(I)		10.04 ± 0.03			40 ± 9		0.40		0.342 ± 0.004	19	
E3(D)		2.122 ± 0.007	0.1326 ± 0.0003		332 ± 21	388 ± 131	0.70		0.464 ± 0.002	18.9	0.71148 ± 7
Magothy											
M1 Water table	35.1 ± 0.5	18.9 ± 0.7	7.53 ± 0.03		197 ± 11	112 ± 15	3.72	6571 ± 20	2148 ± 20	26.61	0.71144 ± 5
S1 _{wt}		18.07 ± 0.04			44 ± 13		0.80			27.8	0.71357 ± 7
$S2_{wt}$		1.083 ± 0.007			698 ± 29		0.76			12	
$S3_{wt}$		3.12 ± 0.007			39 ± 11		0.12			18	
 a δ²³⁴U is th b Sample col c Sample col d Duplicate. 	 a 8²³⁴U is the isotopic shift in performance by Sample collected in Fall 97. ^c Sample collected in Spring 99. ^d Duplicate. 	in permil deviation 7. 99.	^a δ^{234} U is the isotopic shift in permil deviation from secular equilibrium δ^{234} U _w = $(^{234U}$ A _w $-1) \times 10^3$ and 234U A _w and 238U A _w are the activities of 234 U and 238U U in the water. ^b Sample collected in Fall 97. ^c Sample collected in Spring 99.	ium δ^{234} U _w =(²	:34UA _w /238UA _w	$(-1) \times 10^3$ an	d $^{234\mathrm{U}}\mathrm{A_w}$ and $^{238\mathrm{U}}$	$A_{\rm w}$ are the activit	ies of ²³⁴ U and ²³⁸	U in the water.	

Table 2. U and Th activities in the water samples.

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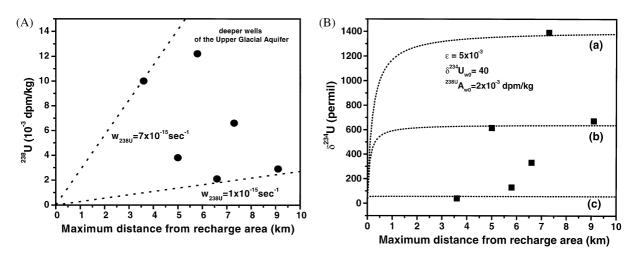


Fig. 4. (A) ²³⁸U activities from deep wells in the Upper Glacial Aquifer. The distance of each well from the groundwater divide to the north (see Fig. 2) is plotted; there is no correlation of activity with this distance. The slopes of the dashed lines define the weathering rates yielding the sample U activities assuming no U input from the vadose zone. (B) $\delta^{234}U_w$ from deep wells in the Upper Glacial Aquifer. The dashed lines show the model evolution of $\delta^{234}U_w$ along flow lines for (a) $w_{238_{U}} = 3 \times 10^{-16} \text{ sec}^{-1}$, (b) $w_{238_{U}} = 7 \times 10^{-16} \text{ sec}^{-1}$, and (c) $w_{238_{U}} = 8 \times 10^{-15} \text{ sec}^{-1}$.

aquifer fluid. The U activity in well $W1(S)_{\rm wt}$ varies by ${\sim}18\%$ between two sampling dates. U activities at the water table (symbol "wt") are not correlated with the thickness of the overlying vadose zone; wells S1_{wt} and S2_{wt}, where the unsaturated zone is ~ 3 m thick, have the highest and lowest 238 U activities, respectively (Table 2). The water table samples usually show moderate U disequilibria (δ^{234} U = 31-46‰). Therefore, the 234 U input by α -recoil into vadose zone water is small compared to that by weathering. An exception is well S2, with $\delta^{234}U = 698\%$. In all wells, the ²³⁸U activity in the dissolved fraction generally follows that of the TDS and shows no correlation with distance from the recharge area (Table 2, Fig. 4a). Note that for the wells $E1(S)_{wt}$ and E2(I), which are likely to be on a flow line (see Sect. 2), the U activity increases from 5 to 10×10^{-3} dpm/kg. However, E3(D), which we thought to be on the same flow line has a lower value than E1(S) or E2(I). Along both sampling lines, the U activity does not increase regularly and the δ^{234} U values vary between 40 and 1394‰ and do not vary regularly with distance (Fig. 4b). Although ultra-filtration experiments indicate that only 12 to 50% of the ²³⁸U in filtered water pass through the 10 kD ultra-filter (Table 2), these values are lower limits (Sec. 3). Note that δ^{234} U values of the ultra-filtered fractions are always smaller than those of the corresponding filtered fraction; thus, the colloids are greatly enriched in ²³⁴U and have not equilibrated with the bulk U in the water. The particulate U load was determined by measuring U in bulk and filtered waters from W3(D); the bulk water contained a slightly lower ²³⁸U activity, due to some variation during pumping, and so particles generated during pumping do not contribute significantly to ²³⁸U in ground water samples.

The ²³⁸U activity in Magothy filtered water $(18.9 \times 10^{-3} \text{ dpm}^{238}\text{U/kg})$ is in the higher range of activities from the Upper Glacial, with $\geq 40\%$ of the ²³⁸U in the <10 kD fraction. The ²³⁸U activity in the bulk water $(35.1 \times 10^{-3} \text{ dpm/kg})$ shows that 46% of the total ²³⁸U is associated with particles. Comparison of the δ^{234} U values in the filtered (δ^{234} U = 197‰) and ultra-filtered (δ^{234} U = 112‰) waters shows that, like in the

Upper Glacial, colloids are enriched in 234 U relative to the dissolved fraction.

²³²Th activities in Upper Glacial filtered fractions lie between 3×10^{-6} to 4×10^{-5} dpm/kg (14–165 pg/kg) (Table 2). Note that for ²³²Th 3×10^{-6} dpm/kg = 3×10^{10} atoms/kg. There are no systematic differences between water table samples and the deeper wells. Data for sample W3(D) shows that 95% of the total ²³²Th is carried by particles. The ²³²Th activities are around the solubility limit (Fig. 5) of thorianite, calculated assuming there are no organic ligands and using the pH-solubility relationship of Langmuir and Herman (1980), which we consider to supercede the values given in Baes and Mesmer (1976). Note that the DOC concentrations measured in

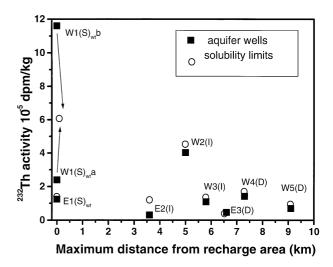


Fig. 5. 232 Th activities in Upper Glacial Aquifer samples versus the distance from the ground water divide (see Fig. 2). The 232 Th activities appear to correspond to solubility limits estimated for the corresponding pH (open circles). Solubility limits were calculated for each sampling site and were indicated for each sample. In most cases the measurement agree with the solubility limit except for sample W1(S)_{wt}b that is largely above the solubility limit.

Table 3. Short-lived nuclides in 0.45-µm filtered groundwater.

Well #	²³⁴ Th 10 ⁻³ dpm/kg	²²⁶ Ra 10 ⁻³ dpm/kg	²²² Rn dpm/kg	²²⁸ Ra 10 ⁻³ dpm/kg	²²⁸ Th 10 ⁻³ dpm/kg	²²⁴ Ra 10 ⁻³ dpm/kg	²²⁶ Ra/ ²²⁸ Ra Activity ratio	²²⁴ Ra/ ²²⁸ Ra Activity ratio
West line								
West fille W1(S) _{wt}	12	26 ± 6	109	37 ± 7	1.8	58 ± 12	0.7	1.6
W2(I)	15	59 ± 12	65	193 ± 38		145 ± 29	0.3	0.8
W3(D)	30	33 ± 6	336	42 ± 8	6.7	35 ± 7	0.7	0.8
W4(D)	26	63 ± 12	108	150 ± 30		159 ± 31	0.4	1.0
W5(D)	15	26 ± 6	84	45 ± 9	7.1	92 ± 18	0.6	2.1
East line								
E1(S) _{wt}	74	32 ± 6	174	35 ± 7		36 ± 7	0.9	1.0
E1 _{<10kD}	44	<1.5		<5		<3		
E2(I)	22	22 ± 4	162	56 ± 9		40 ± 8	0.4	0.7
E3(D)	22	22 ± 4	522	24 ± 4	3.4	26 ± 5	0.9	1.1
Magothy								
M1	34	27 ± 6	80	45 ± 9	5.2	23 ± 4	0.6	0.5

All errors (2σ) for nuclide activities are errors equal to 20%, unless otherwise noted.

our samples might allow solubility limits that are orders of magnitude higher than the measured activities. However, there is no clear correlation between the DOC and 232 Th A_w . In addition, although both nuclides are provided by weathering, the 232 Th/ 238 U activity ratio in the water is 10^{-2} that of the rock. This shows that ²³²Th precipitates and that the organic matter is not a ligand controlling the Th solubility. We will not consider Th-DOC complexes in this case. The activity of one sample $(W1(S)_{wt})$ is clearly below saturation, while $W1(S)_{wt}b$ collected at the same location but at a different time is above the predicted solubility limit ($\sim 2-4 \times 10^{-5}$ dpm/kg) (Table 2). This discrepancy may be due to the presence of colloids in the second sample. The Th/U ratio of the colloids in sample W3(D) is $\sim 2 \times 10^{-3}$ times that of the average continental crust, assuming all the 232 Th and 40% of the U (Sect. 3) are colloid-bound.

The 234 Th activities in the Upper Glacial $\sim 15 \times 10^{-3}$ to 74×10^{-3} dpm/kg. Due to the short half-life of ²³⁴Th, ²³⁴Th input from the water table does not influence the composition of deeper waters. The ${}^{234}\text{Th}A_{\mu}/{}^{238}\text{U}A_{\mu}$ ratios (${}^{i}A_{\mu}$ is the activity of nuclide *i* in the filtered water) range from 2 to 10 and show that in addition to decay of ²³⁸U in the water, ²³⁴Th is provided by recoil or weathering from the aquifer rock. The ratio 232 Th $A_{u}/$ ${}^{A^{a}Th}A_{w}$ in filtered waters varies by a factor of 80, reflecting the different controls on the activities of these isotopes. The data for E1(S)_{wt} (Table 3) shows that 60% of ²³⁴Th is in ultrafiltered water. The ²²⁸Th activities in the Upper Glacial of 1.8×10^{-3} to 7.1×10^{-3} dpm/kg (Table 3) are 100 times higher than that of ²³²Th. Therefore, ²²⁸Ra decay is more important to the supply of ²²⁸Th than weathering of Th from the rock. However, ${}^{228}\text{Th}A_w/{}^{228}\text{Ra}A_w \sim 0.1$ (Table 3) and so both nuclides are not in secular equilibrium.

The ²³²Th activity in filtered water from the reduced Magothy is 21×10^{-3} dpm/kg (88 ng/kg); this is 10^3 times that of the shallow aquifer (Table 2), and therefore greatly exceeds the Th solubility limit calculated above. The DOC concentration (~5 mg/L) is comparable to that of the Upper Glacial and, as indicated previously, is not likely to account for this high activity. This water also has high Mn and Fe concentrations and ²³²Th may then be bound to Fe-rich colloids. In contrast, the ²³⁴Th (34×10^{-3} dpm/kg) and ²²⁸Th (5.2×10^{-3} dpm/kg) activities are comparable to those in the Upper Glacial waters. The particulate ²³²Th activity, deduced from bulk and filtered data, is $\sim 44 \times 10^{-3}$ dpm/kg. A similar calculation for U yields (²³²Th/²³⁸U) ~ 2.7 for the particles, similar to upper crustal values.

These data on Th do not indicate any controlling complexation with organic matter and the activities in filtered samples generally coincide with the solubility limits of ThO2. Therefore we consider the ²³²Th activity in the groundwater is limited by saturation, so that once this limit is reached, ²³²Th added by weathering is precipitated onto the surface of the aquifer solid. Changes in solubility can occur only with pH changes. In particular, due to weathering of the aquifer rock, the pH of ground waters will increase along a flow line, so that the solubility of Th will decrease, and progressive precipitation will occur. Note that our calculation of the saturation limit is based upon the assumption that ²³²Th in the filtered water is largely dissolved. There is no data for ²³²Th on colloids in the ground water to confirm this; however, for ²³⁴Th (see below), one water table well (E1(S)_{we}) appeared to have $60 \pm 15\%$ ²³⁴Th in solution (Table 3) and so $\leq 40\%$ of ²³²Th is expected to be associated with colloids. At the water table, the ²³²Th activity of one of the samples is below the calculated solubility limit. Yet, the ²³²Th/²³⁸U in the water table sample is lower than in the aquifer rocks; suggesting that if U and Th are weathered at similar rates, Th is removed more effectively than U in the vadose zone (Langmuir, 1997, Ivanovich and Harmon, 1989).

 226 Ra activities in the Upper Glacial fall in a narrow range $(22-63 \times 10^{-3} dpm/kg)$, with 226 Ra/ 228 Ra and 224 Ra/ 228 Ra ratios of 0.7 to 2.1 (Table 3). Most samples have $(22-33) \times 10^{-3}$ dpm 226 Ra/kg. Samples W2(I) and W4(D) are exceptions (59 and 63 $\times 10^{-3}$ dpm 226 Ra/kg), so that Ra activities alternate along the western well line (Table 3). No Ra was detected in ultra-filtered water; so Ra is carried mainly on colloids as expected for a surface-reactive element (Langmuir, 1997). However, in the "Discussion" we use the Ra activities without distinguishing between the dissolved and colloidal Ra, and the data show that adsorbed Ra is rapidly exchanging with Ra in the water. The Magothy has comparable Ra activities.

The Rn activities must be in steady state and equal to the supply rate into solution. The ²²²Rn activities in the Upper Glacial generally vary over a narrow range of 65 to 174 dpm/kg

Symbol	Parameter	Units
${}^{i}A_{w}$	Activity of <i>i</i> in the water	dpm/kg-water
${}^{i}A_{sc}^{W}$	Activity of <i>i</i> in the surface coating	dpm/kg-coating
$^{i}A_{r}$	Activity of <i>i</i> in the rock $(^{238U}A_r \sim ^{232Th}A_r)$	440 dpm/kg-rock
${}^{i}A'_{w}$	Activity of <i>i</i> in the water in the vadose zone	dpm/kg-water
^{i}D	Distribution coefficient of <i>i</i> between surface coating and water	
ε_i	Fraction of nuclides <i>i</i> produced in the rock and ejected or leached into the water after alpha-recoil	$(0.6-6) \times 10^{-5}$
f_i	Fraction of <i>i</i> produced in the surface coating by decay of parent p and ejected into the water	
F_i	Total fraction of i in surface coating released by desorption and ejection during production (see Appendices)	
$i\hat{k}_{-1}$	"Velocity" of <i>i</i> desorbing from surface coating	$cm sec^{-1}$
$i\hat{k}_1$	"Velocity" of <i>i</i> adsorbing onto surface coating	$cm sec^{-1}$
L_i	α -recoil length of nuclide <i>i</i>	2×10^{-8} m
	Decay rate of nuclide <i>i</i>	sec ⁻¹
$\lambda_i \\ S^i \hat{k}_1$	Removal rate of <i>i</i> from solution (per atom per unit volume)	
$S^i \hat{k}_{-1}$	Desorption rate of <i>i</i> from surface	
n	Porosity	0.3
q	= n/(1 - n)	
r	Average grain radius	$(10-100) \times 10^{-6} \text{ m}$
R _{eman}	Emanation fraction of Rn	2
ρ_r	Rock density	2.7 g/cm^{3}
$ ho_w$	Water density	1 g/cm^3
$ ho_{sc}$	Surface coating density	2.7 g/cm^{3}
S	Moisture content	0-1
S	Surface coating area per volume of rock	cm^{-1}
Sξ	Volume of surface coating per volume of rock	
ν	Advective water velocity in the aquifer (water flux/m)	$(1-2) \times 10^{-4} \text{ cm} \cdot \text{sec}^{-1}$
Wi	Weathering rate	sec ⁻¹
ξ	Surface coating thickness	μ m
χ_i	Atoms i in surface coating/atoms of i in water	
\bar{x}_i	Characteristic length scale for transport of <i>i</i>	m

Table 4. Model parameters: Typical values for the upper Glacial aquifer in Long Island, New York.

(Table 3) with higher values in W3(D) (336 dpm/kg) and E3(D) (522 dpm/kg). Such high Rn contents have been observed previously (e.g., Krishnaswami et al., 1982). The ²²²Rn activity in the Magothy (~80 dpm/kg) is comparable to the values of the Upper Glacial. No correlation exists between ²²²Rn and ²³⁸U or ²²⁶Ra. The ²²²Rn activities are 10³ times those of parent ²²⁶Ra in the water, and cannot be produced by ²²⁶Ra from the water. With a ²²⁶Ra activity in the aquifer solid of 440 dpm/kg of rock and ~ 2800 dpm/kg of water (Copenhaver et al., 1993), the emanation fractions (the fraction of Rn produced by ²²⁶Ra in the rock that is released into solution) are generally (3–6) × 10⁻², with two high values of 0.12 and 0.19.

4. TRANSPORT IN THE VADOSE ZONE AND UPPER GLACIAL AQUIFER

The groundwater activities differ by several orders of magnitude, with $^{222}R^{a}A > ^{236}R^{a}A > ^{234}T^{h}A > ^{234}UA \sim ^{238}UA$, due to differences in groundwater input rates, as well as in removal rates. A transport model will be used to interpret the measured disequilibria. After a brief introduction of the model, each element will be discussed with respect to vadose zone input and aquifer transport so that the behavior of U, Th, Ra and Rn in groundwater can be characterized.

4.1. Radionuclide Transport Model

The one-dimensional model used here (Tricca et al., 2000) includes advective transport and physico-chemical reactions between three phases: water of advective velocity ν , mineral

grains, and a reactive surface coating of specific surface area S and thickness ξ . All parameters used in the model are listed in Table 4 and the model is presented schematically in Figure 6. Note that each nuclide (*i*) in the chain is produced by its respective parent (*p*). This model can be applied to the average transport rate in the vadose zone by assuming that the pores are filled with water by a fraction equal to the moisture content *s*. The symbols used for variables in the vadose zone are the same as those in the saturated zone but with a prime. The moisture content *s* is unity in the saturated zone below the water table and has a value between 0 and 1 in the vadose zone.

Input of each species into the water occurs through:

- weathering characterized by a nuclide-specific weathering rate constant for the bulk rock (*w_i*);
- recoil ejection or preferential weathering as suggested in Hussain and Lal (1986) of a fraction ε_i of atoms produced in the rock by α -decay of parent p. Note that in case of no preferential weathering, ε_i is directly related to the radius r of the emitting mineral and to the recoil length L; for homogeneous spherical grains $\varepsilon_i \cong 3L/r$ when $L \ll r$ (Kigoshi, 1971). Here, $L \sim 200$ Å (Fleischer, 1982).
- production in the surface coating: *f_i* is the fraction of atoms *i* produced in the surface coating that are ejected into the water after production by *α* or *β* decay;
- desorption from the surface coating (of volume $S\xi$ per volume of rock) characterized by ${}^{i}\hat{k}_{-1}$ (cm sec⁻¹), the velocity of atoms *i* that cross the surface-water boundary to go into solution; ${}^{\text{Th}}\hat{k}_{-1}/\xi$ is equivalent to a first order kinetic constant (see Eqn. 9);

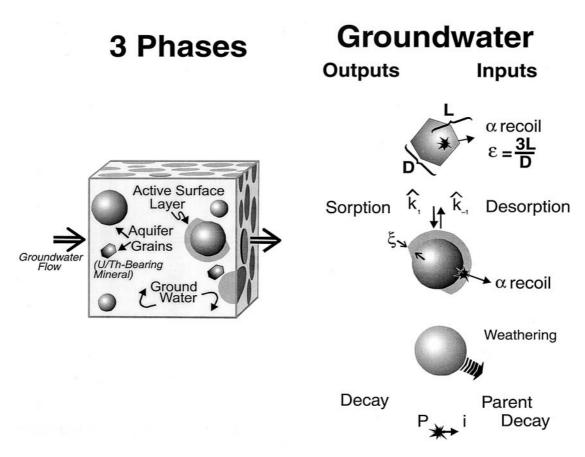


Fig. 6. Model of the aquifer and the reactions taking place between the different phases.

• decay of the parent nuclide *p* within the aquifer.

Removal from the water occurs through:

- decay of the nuclide within the groundwater;
- adsorption onto the surface coating characterized by ${}^{i}\hat{k}_{-1}$ (cm sec⁻¹), the average velocity with which atoms *i* in solution impact and sorb onto the surface layer.

We first present in Eqn. 1 (Tricca et al., 2000), the transport equation in terms of concentrations $({}^{i}c_{\beta})$ and then convert to activities $({}^{i}A_{\beta})$. This equation represents the transport in terms of mol/kg of species *i* in phase $\beta ({}^{i}c_{\beta})$. The velocity of the water is the macroscopic water flux in the medium divided by the porosity and the density.

$$\frac{1}{\nu} \frac{\partial^{i} c_{w}}{\partial t} + \frac{\partial^{i} c_{w}}{\partial x} = \frac{\rho_{r}}{\rho_{w} \cdot q \cdot \nu} \\
\cdot \left(w_{i} \cdot {}^{i} c_{r} + \varepsilon_{i} \cdot \lambda_{p} \cdot {}^{p} c_{r} + {}^{i} \hat{k}_{-1} \cdot \frac{\rho_{sc} \cdot S}{\rho_{r}} \\
\cdot {}^{i} c_{sc} + f_{i} \cdot \frac{\rho_{sc} \cdot S \cdot \xi}{\rho_{r}} \cdot \lambda_{p} \cdot {}^{p} c_{sc} \right) \\
+ \frac{\lambda_{p}}{\nu} \cdot {}^{p} c_{w} - \left(\frac{\lambda_{i}}{\nu} + {}^{i} \hat{k}_{1} \cdot \frac{S}{q \cdot \nu} \right) \cdot {}^{i} c_{w} \quad (1)$$

As all nuclides within a decay series are linked by radioactive

decay reactions, the activity of a nuclide depends on that of its precursors. To convert the concentrations to activities $({}^{i}A_{\beta})$ we multiply both sides of Eqn. 1 by λ_i and replace $\lambda_i {}^{i}c_{\beta}$ with $({}^{i}A_{\beta})$ and $\lambda_p {}^{p}c_k$ with ${}^{p}A_k$. The following transport equation for nuclide activity must be integrated for each nuclide progressively through the decay series, starting with 238 U and 232 Th.

$$\frac{1}{\nu} \frac{\partial^{i} A_{w}}{\partial t} + \frac{\partial^{i} A_{w}}{\partial w} = \frac{\rho_{r}}{\rho_{w} q s v}$$

$$\cdot \left(w_{i}^{i} A_{r} + \varepsilon_{i} \lambda_{i}^{p} A_{r} + {}^{i} \hat{k}_{-1} \frac{\rho_{sc}}{\rho_{r}} {}^{i} A_{sc} + f_{i} \frac{\rho_{sc} S \xi}{\rho_{r}} \lambda_{i}^{p} A_{sc} \right)$$

$$+ \frac{\lambda_{i}}{\nu} {}^{p} A_{w} - \left(\frac{\lambda_{i}}{\nu} + {}^{i} \hat{k}_{1} \frac{S}{q \nu} \right) {}^{i} A_{w} \quad (1a)$$

Here ${}^{i}A_{k}$ is the activity of nuclide *i* in phase *k* (water *w*, rock *r*, or surface coating *sc*), and ρ_{k} are the densities of the phases. The rock being weathered provides the source of U-Th series nuclides. We assume that the activities in the rock $({}^{i}A_{r})$ are constant over the time range of interest. The void ratio is q = n/(1-n), where *n* is the porosity. At the boundary between two regions with different properties, the flux of all species (including water flow) must be conserved.

The solution to equation (1a) depends on the boundary condition at some surface and a constitutive equation describing the relationship and state of the surface coating. The variation with time of the total number of species i in the surface coating in activity units is:

$$\rho_{sc} \frac{d(A_{sc}S\xi)}{dt} = S\rho_w^i \hat{k}_1^i A_w + \rho_{sc} S\xi(1-f_i) \lambda_i^p A_{sc} - \rho_{sc} S(i\hat{k}_{-1} + \lambda_i \xi)^i A_{sc} \quad (1b)$$

All parameters are defined in Table 4. The parameter f_i is the fraction of the production of *i* by decay which is lost from the surface layer by recoil. f_i is dependent on the thickness (ξ) of the surface layer with $f_i \sim 1/2$ when $\xi << L$ (the recoil distance for α decay). For production by β decay, f_i is then 1 if the daughter is not adsorbed and so completely released into solution, or 0 if it is strongly adsorbed.

There are two distinctive types of behavior of the surface layer that require attention. The simplest case is where the surface layer can exchange species with the fluid and where, for stable species, it is in local chemical equilibrium. In the steady state case, $\frac{d}{dt} ({}^{i}A_{sc}S\xi) = 0$ and a relationship between ${}^{i}A_{w}$ and ${}^{i}A_{sc}$ is then explicitly given. This approach is similar to the usual treatment given in box model calculations. The functional form for ${}^{i}A_{sc}$ in terms of ${}^{i}A_{w}$ from equation (1b) can then be substituted into equation (1a) and the steady state solutions determined. A different case is when a species is saturated in the solution so that further additions above the saturation value by weathering or by production are precipitated out on the surface layer. The total number of atoms (or total activity) of this species in the surface layer ($\rho_{sc} {}^{i}A_{sc}S\xi$) must then increase and the surface layer is not in steady state

$$\frac{d}{dt}\left(\rho_{sc}^{i}A_{sc}S\xi\right)>0.$$

To expand on the behavior at saturation of a species, we consider a simple box model of the aquifer with no flow and where a surface phase is growing from a solution. Then the rate constant for removal of a species from solution is \hat{k}_1 S/q in terms of the parameters used here. The time scale corresponding to this chemical removal is much shorter than the transit time for water motion to cover a few cm. If a system was initially saturated with a concentration of ${}^{i}c_{w}^{sat}$ in the water and ${}^{i}c_{sc}^{sat}$ in the surface phase, it can be shown that the addition of more species *i* to the water at say a constant rate $w_i^{i}c_r$ will be rapidly removed and have an excess concentration Δc_w above saturation of $\rho_r w_i^{\ i} c_r / \hat{k}_1 S \rho_w$. In terms of the rate constants used here, this is very small compared to ${}^{i}c_{w}^{sat}$. It also follows that equation (1b) becomes, for species with no precursor (e.g., ²³²Th, ²³⁸U): $\rho_{sc} \frac{d}{dt} ({}^{i}A_{sc}S\xi) = \rho_r w_i {}^{i}A_r = \rho_w S \hat{k}_1 \Delta^i A_w$, where $\Delta^i A_w \cong \lambda_i \Delta^i c_w$. This leads to a regular growth in the content of *i* in the surface coating. It also fundamentally alters the fluid transport equation. For example, if either of the parent nuclides ²³⁸U and ²³²Th are saturated, then the transport equation reduces to their saturation values ${}^{^{238}\text{U}}c_w$ or ${}^{^{232}\text{Th}}c_w$ and are not related to the weathering rate. The resulting equations for the

daughter species are, in general, then different from those obtained for the exchangeable mechanism given earlier.

4.2. Transport for an Exchangeable Surface Coating

We will first develop the model of an exchangeable surface coating site and derive the general conclusions from this. It will be shown that this appears to be a reasonable description but that it is not consistent with the observation that ²³²Th appears to be saturated. It completely fails to explain the Rn data. We will then develop a model that includes the saturation behavior of ²³²Th and show that a model of no exchangeable sites for Th isotopes and exchangeable sites for the other nuclides appears to present a self-consistent picture.

When production from the decay of a parent within the surface coating is negligible, then at steady state we obtain from equation (1b):

$$\hat{k}_1 \rho_w{}^i A_w = \rho_{sc} (\hat{k}_{-1} + \lambda_i \xi)^i A_{sc}$$
(2a)

The ratio of the numbers of atoms in the surface coating $[\rho_{sc}{}^{i}A_{sc}S\xi(1-n)]$ to those in the water $(\rho_{w}{}^{i}A_{w}sn)$ is χ_{i} . Using equation (2a), for a water-saturated zone (s = 1) we obtain:

$$\chi_i \equiv \frac{{}^i \hat{k}_1}{{}^i \hat{k}_{-1} + \lambda_i \xi} \left(\frac{S\xi}{q} \right) \equiv {}^i D \, \frac{S\xi}{q} \tag{2b}$$

Here ${}^{i}D \equiv {}^{i}k\!/({}^{i}k_{-1} + \lambda_{i}\xi)$ is the effective distribution coefficient between surface coating and the water. It may also be shown, using equation (1a) that using χ_{i} , there is a characteristic distance for each species $\bar{x}_{i} \equiv vs/\lambda_{i}(1 + \chi_{i})$.

As described in Tricca et al. (2000), the general solution to equation (1a) is of the form ${}^{i}A_{w} = {}^{i}A_{w0}e^{-x/\bar{x}} + {}^{i}A_{w\infty}(1-e^{-x/\bar{x}_{i}}) +$ ${}^{i}J(x)$ (see Appendix A in this paper). The function ${}^{i}J(x)$, with ${}^{i}J(0) = 0$, represents the component of the activity (the 'transitional activity') of nuclide *i* that evolves with distance, produced by the 'transitional' decay of the precursors. The constant value of ${}^{i}A_{w\infty}$ is reached when $\lim({}^{i}J)_{x\to\infty} = 0$ and $x \gg$ \bar{x}_i . The activities increase along a flow line from ${}^iA_{w0}$ towards a limiting value ${}^{i}A_{w\infty}$, resulting from a balance between the rate into, and the removal rate out of, the groundwater. The term ${}^{i}A_{w\infty}$ is simply the value of ${}^{i}A_{w}$ corresponding to the limiting case where the total rate of decay of *i* nuclides in the water $(\lambda_i \rho_w^i c_w n)$ plus the surface coating $(\lambda_i \rho_{sc} S\xi (1-n)^i c_{sc})$ is equal to the production rate of *i* in the rock (both recoil and weathering) plus the production rate in the surface coating. For the case of exchangeable sites this uses equation (2b) for the ratio of the number of atoms in the surface coating to that in the water. Also note that for all reactive species that are treated here that $\chi_i >> 1$ so that $\chi_i + 1$ is extremely close to χ_i . Most of the expressions in Appendix A can be written down by inspection from the above considerations. The term ${}^{i}A_{w0}$ is the boundary condition at x = 0. If we take the upper boundary of the vadose zone to be at x = 0, then ${}^{i}A_{w0} = 0$ for all species where rainwater and dust are the input (except for ²¹⁰Pb, which is not treated here). For the underlying aquifer, if we take x =0 to be the boundary at the water table, then ${}^{i}A_{w0}$ is the value at the base of the water table.

At steady state for ²³⁸U and ²³²Th, the primary nuclides with no parents, the solutions to equations (1a) and (1b) are of the form:

Table 5. Estimated* \bar{x}_i and χ_i values for the unconfined aquifer.

	$\bar{\tau}_i = 1/\lambda_i$		
Nuclide	sec	$\chi_{ m i}$	$\bar{\mathbf{x}}_i(\mathbf{m})$
²³⁸ U	2×10^{17}	0	4.07×10^{11}
²³⁴ Th	3×10^{6}	700	1×10^{-2}
²³⁴ U	1.1×10^{13}	0	2×10^{7}
²³⁰ Th	3.4×10^{12}	2×10^{6}	3
²²⁶ Ra	$7.3 imes 10^{10}$	700	3×10^{2}
²²² Rn	4.7×10^{5}	0	9×10^{-1}
²³² Th	6.3×10^{17}	2×10^{6}	6×10^{5}
²²⁸ Ra	2.6×10^{8}	100	5
²²⁸ Th	8.7×10^{7}	1000	9×10^{-3}
²²⁴ Ra	4.5×10^{5}	0.2	0.9
²²⁰ Rn	$8.0 imes 10^1$	0	1.59×10^{-4}

* From model of exchangeable sites on the surface layer.

$${}^{i}A_{w} = {}^{i}A_{0}e^{-x/\bar{x}_{i}} + \frac{\rho_{r}(1-n)}{n\rho_{w}} \frac{w_{i}}{\lambda_{i}(1+\chi_{i})} {}^{i}A_{r}(1-e^{-x/\bar{x}_{i}})$$
(2c)

As the mean lives $1/\lambda_i$ for 238 U and 232 Th are very long, it follows that even for χ_i as large as 10^6 that $x/\bar{x}_i \ll 1$ for the scale of any aquifer (see Table 5). As a result, the solution of equation (2c) reduces to the simple form:

$${}^{i}A_{w} = {}^{i}A_{w0} + \left[\frac{\rho_{r}(1-n)}{n\rho_{w}}\frac{w_{i}}{vs}{}^{i}A_{r}\right]x$$
(2d)

where we have used the definition of \bar{x}_i in equation (2b). At steady state for ²³⁸U and ²³²Th, the distribution coefficient (or the retardation factor) plays no role as the surfaces are saturated.

The simplest case of a decay product is for ²³⁴Th. The behavior of this short-lived nuclide can be seen from inspection of equation (1a). In the first brackets the ratio of the first term to the second term is $(w^{234}\text{Th} A_r/\epsilon^{234}\text{Th}\lambda^{234}\text{Th}^{238}U_{r})$. As the rock is taken to be in secular equilibrium, the ratio of the activities is unity. For $\epsilon_{234}\text{Th} \sim 5 \times 10^{-3}$ and $w_{238_{\rm U}} \sim 2 \times 10^{-16} \text{ sec}^{-1}$, the ratio of the terms is 4×10^{-6} . Thus, weathering of ²³⁴Th is negligible relative to recoil. The last term within the brackets is for the decay of ²³⁸U in the surface coating. As U is soluble under the conditions studied here, this term is zero. The term between the brackets represents the decay of ²³⁸U in the water $(\lambda_{234_{\rm Th}} e^{234}\text{U}A_w/\nu)$. In equation (1a), the ratio of the weathering term to the ²³⁸U decay in the aquifer water (s = 1) is $\frac{\rho_r}{\rho_w q} w_{234_{\rm Th}} e^{234_{\rm Th}} A_r e^{5 \times 10^{-3}}$. Thus, ²³⁸U decay in the water is negligible. The other terms vanish because of Eqn. 1b or 2a.

For steady state, the equation for ²³⁴Th reduces to:

$${}^{234\text{Th}}A_{w} \simeq {}^{234\text{Th}}A_{w\infty} = \left[\frac{\rho_{r}}{\rho_{w}q} {}^{234\text{Th}}A_{r}\right] \frac{\varepsilon_{234\text{Th}}}{(1+\chi_{234\text{Th}})}$$
(3)

Using the data on ²³⁴Th in Table 3 in Eqn. 3, we find that typically $\varepsilon_{234_{Th}}/\chi_{234_{Th}} \sim 5 \times 10^{-6}$ in the aquifer water. For $\varepsilon_{234_{Th}} = 5 \times 10^{-3}$, $\chi_{234_{Th}} = 1 \times 10^3$, while for a large value of $\varepsilon_{234_{Th}} = 5 \times 10^{-2}$ (Krishnaswami et al., 1982), $\chi_{234_{Th}} =$

 1×10^4 . It follows that ²³⁴Th is strongly adsorbed. For a homogeneous aquifer (${}^{i}A_r$ constant), then ${}^{{}^{234}\text{Th}}A_w$ should be constant. As can be seen in Table 3 the total range in ${}^{{}^{230}\text{Th}}A_w$ is constant to within a factor of two in accord with this inference.

We now estimate the ratio of the activities of ²³⁴Th to ²³⁸U in the water. Using Eqn. 3 for ²³⁴Th and Eqn. (2d) for ²³⁸U, we obtain: ²³⁴Th A_w /²³⁸U $A_w = \nu \varepsilon_{234_{Th}}/(1 + \chi_{234_{Th}})xw_{238_U}$. From the values above and using $x \sim 1$ km, we find ²³⁴Th A_w /²³⁸U $A_w \approx 5 \times 10^4/(1 + \chi_{234_{Th}})$. It follows that even for a high value of $\chi \sim 10^4$, the activity of ²³⁴Th in the water is substantially greater than that of ²³⁸U. This is what is typically observed for all aquifer samples except the Magothy (see later discussion). Only in the case where the concentration of ²³⁸U in the water is exceptionally high or the removal of Th is much more efficient will ²³⁴Th A_w /²³⁸U $A_w \sim 1$ be achieved.

[In treating ²³⁴Th, we have assumed a steady state and an exchange reaction (i.e., $\hat{k}_{-1} > 0$). However, if we consider a condition where $\hat{k}_{-1} = 0$ for no transport for ²³⁴Th from the surface coating to the water (i.e., irreversible precipitation), then a steady state for ²³⁴Th is still possible due to the short lifetime of ²³⁴Th. The value of $\chi_{234_{Th}}$ would be the same as calculated above and $\chi_{234_{Th}} = \frac{234_{Th}\hat{k}_1}{\lambda_{234_{Th}}}S$ corresponding to $\xi\lambda_{234_{Th}}$ > $^{234}Th\hat{k}_{-1}$. As we consider ²³⁸U to be soluble, the above solutions for both ²³⁸U and ²³⁴Th are more generally applicable (see Sections 4.3 and 4.4)].

For ²³⁴U, which is produced by ²³⁴Th, there is a surface production term but as U is not kept in the surface layer, all terms in Eqn. (1b) vanish. The steady state problem then reduces to Eqn. (1a) for ²³⁴U. All terms involving ²³⁸U \hat{k}_{-1} and ²³⁸U \hat{k}_1 vanish. As $\lambda_{234_{\rm U}}$ is very small, $\frac{\lambda_{234_{\rm U}}}{\nu}$ ²³⁴UA_w is negligible. As there is almost no ²³⁴Th in the water (see above), $\frac{\lambda_{234_{\rm Th}}}{\nu}$ ²³⁴ThA_w is negligible. The key term is $f_{234_{\rm U}} \delta_{\rho_r} S\xi \lambda_{234_{\rm Th}} c_{sc}$. For this β decay, $f_{234_{\rm U}} \varepsilon_{134_{\rm Th}}$ ²³⁴ThA_r yielding $\frac{\partial}{\partial x}$ ²³⁴UA_w = $\frac{\rho_r}{\rho_w S\nu}$ ($w_{234_{\rm U}}$ ²³⁴UA_r + $\lambda_{234_{\rm U}} \varepsilon_{234_{\rm Th}}$ ²³⁴ThA_r). The solution for ²³⁴U is then:

$${}^{234}\mathbf{U}_{A_{w}} = \frac{\rho_{r}(1-n)}{\rho_{w}n} \frac{\left(w_{234\mathbf{U}} + \varepsilon_{234\mathrm{Th}} \lambda_{234\mathbf{U}}\right)}{sv} {}^{238\mathbf{U}}_{A_{r}} \cdot x + {}^{238\mathbf{U}}_{W_{0}} \qquad (4)$$

We note that as ²³⁸U and ²³⁴U are considered soluble, and that ²³⁴Th has a short life time, that Eqn. (2d) for ²³⁸U, Eqn. 3 for ²³⁴Th, and Eqn. 4 for ²³⁴U may be used for these nuclides as a starting point in case of saturation-precipitation of other species further down the decay series.

The approach as outlined above is applied sequentially to each of the nuclides in the decay series in determining the steady state concentrations for the exchange model.

4.3. Uranium in the Vadose Zone

We will now use the above equations to discuss the uranium isotopes. As the rainfall contributes essentially none of the nuclides under consideration (input of ²¹⁰Pb is important and not considered here), the input term ${}^{i}A_{w_{0}} = 0$ in the atmospheric

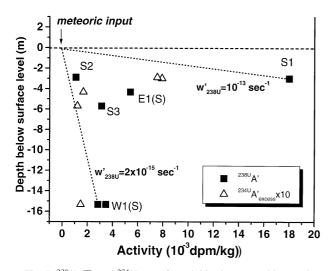


Fig. 7. ²³⁸U (\blacksquare) and ²³⁴U_{excess} (Δ) activities in water table samples representing vadose zone input into the shallow aquifer. The slopes of the dashed lines give the minimum $w'_{238_{\rm U}}$ values yielding measured ²³⁸U activities. Both ²³⁴U_{excess} and ²³⁸U activities in the vadose zone decrease with increasing depth. Note that the ²³⁴U_{excess} activities at 3 m depth are identical, while the ²³⁸U activities are very different.

interface with the top of the vadose zone. As discussed in Sect. 3, the vadose zone input can account for the ²³⁸U in the deeper aquifer. Note that samples from water table wells are affected to a small degree by processes occurring within the aquifer; but generally contain waters that have had limited aquifer interaction. Therefore, while the input from the surrounding aquifer rock often dominates over vadose zone input for short-lived nuclides, the vadose zone contributions of the long-lived nuclides ²³⁸U, ²³⁴U, ²³²Th, and ²²⁶Ra remain important. In vadose zone waters (all parameters in the vadose zone will be noted with a prime), the activity ^{²³⁸U}A'_w is from weathering of aquifer minerals and follows the relationship:

$${}^{^{238}\text{U}}\!A'_w = \left[\frac{\rho_r}{\rho_w q}\right] \left[\frac{w'_{^{238}\text{U}}}{sv'}\right] x \tag{5}$$

The parameters in the second term (the moisture content *s*, the water velocity ν' , and $w'_{238_{U}}$) are highly variable, and estimated average values will be used. Within the vadose zone, where recharge by rainwater is $R \sim 50$ cm/yr, $\nu' = R/n'$. With n = 0.3 and s = 0.5, then the average $\nu' = 1 \times 10^{-5}$ cm/s. The distance *x* is the distance along the macroscopic flow line. The value for *x* that we will use (which is the distance between two sampling points on an averaged smooth flow line) therefore may well underestimate the true distance over which a stream line of water has flowed. It will be used as a first approximation in our discussion.

As seen in Figure 7, the ²³⁸U activities are not linearly correlated with depth and cannot be explained by ²³⁸U input by a single weathering rate constant. If a rate constant w'_{238_U} is calculated for each water table sample, then a wide range of 2×10^{-15} sec⁻¹ to 1×10^{-13} sec⁻¹ is obtained; these correspond to a bulk removal of (0.3-2) mg/cm² per yr from the vadose zone. The highest weathering rate corresponds to complete removal of the upper 3 m of the vadose zone within 2×10^5 years. The enhanced mobilization of U in the vadose zone is consistent with widely observed high chemical weathering rates in the subsurface (e.g., Langmuir, 1997), which reflect the effects of increased mechanical weathering, small grain sizes, high levels of pCO₂, and high concentrations of organic acids (see e.g., Domenico and Schwarz, 1990). These values for the sandy deposits here are somewhat lower than the average for continental chemical weathering of $\sim 3 \text{ mg/cm}^2$ per yr (Garrels and McKenzie, 1971), which, however, is highly influenced by the weathering of carbonates. In general, the comparison indicates that the values obtained here for $w'_{238 \text{ u}}$ are reasonable.

Using the solution for ${}^{^{234}\text{U}}A'_w$ from Eqn. 4, we obtain:

$$\sum_{w'}^{2^{234}} (M'_{w'})^{2^{38}} (M'_{w}) = \frac{(w'_{2^{34}} U + \varepsilon'_{2^{38}} Th \lambda_{2^{34}} U)}{w'_{2^{38}} U}$$
(6)

Here $w'_{234_{\rm U}}/w'_{238_{\rm U}} \sim 1$. Using the definition of δ^{234} U, the isotopic shift (in permil deviation from secular equilibrium) in the vadose zone is:

$$\delta^{234} U'_{w} = \left[\frac{\lambda_{^{234}U} \varepsilon'_{^{234}Th}}{w'_{^{238}U}} \right] \times 10^{3}$$
(7)

Note that δ^{234} U is simply related to the ratio of recoil to weathering and is independent of depth.

From the data in Table 2 samples $S1_{wt}$ and $E1(S)_{wt}$, with $\delta^{234}U \sim 30$ to 40‰ and the weathering rates calculated above, have ε' values of 6×10^{-4} to 3×10^{-2} (requiring grain sizes down to 2 μ m). Well $S1_{wt}$ also has the highest ^{238}U activity, and so has the highest value for w'_{238_U} . Overall, the U data require both high weathering and high recoil rates in the shallow portion of the vadose zone. For most of the samples listed in Table 2 (see Figs. 4 and 7), there is a limited range in $\delta^{234}U'$, so that $\varepsilon'_{234_{Th}}/w'_{238_{U}}$ ratios are relatively constant (Eqn. 7), while the range of ^{238}U activities requires a wide range of $w'_{238_{U}}$ (Eqn. 5).

The co-variation of ${}^{238}UA'_{w}$ and $\delta^{234}U'_{w}$ at the water table is obtained by combining Eqn. 6 and 7 to eliminate $w'_{238,.}$:

$$\delta^{234} \mathbf{U}'_{w} = \left[\frac{\rho_{r}^{2^{38} \mathrm{U}} A_{r} \, \lambda_{2^{34} \mathrm{U}} \, \varepsilon_{2^{34} \mathrm{Th}}^{*} 10^{3}}{\rho_{w} q s v'} \right] \frac{x}{2^{38} \mathrm{U} A'_{w}} \tag{8}$$

This is plotted in Figure 8a for $\varepsilon'_{234_{Th}} = 5 \times 10^{-2}$ and x = 15 m (the maximum unsaturated zone thickness) thus establishing the possible range of vadose zone compositions. For a fixed vadose zone thickness there is thus a hyperbolic relationship between $\delta^{234}U'_w$ and ${}^{^{238}U}A'_w$. For smaller x or $\varepsilon'_{234_{Th}}$, the curve shifts to downward. High values of either ${}^{^{238}U}A'_w$ or $\delta^{^{234}}U'_w$ can be obtained, but not both together. Water table data for ${}^{^{238}U}A'_w$, while one samples have low $\delta^{234}U'_w$ and a range of ${}^{^{238}U}A'_w$, while one sample has high $\delta^{^{234}U'_w}$ but low ${}^{^{238}U}A'_w$. Samples S1_{wt} and S2_{wt} fall on the calculated curve for $\varepsilon'_{234_{Th}} = 5 \times 10^{-2}$, while deeper samples in the vadose zone require $\varepsilon'_{234_{Th}} = 5 \times 10^{-3}$.

4.3.1. U transport in the unconfined aquifer

For nuclides with very long half-lives, the solution of Eqn. 1 is a curve uniformly increasing with flow distance. As noted in Sect. 3, U activities do not show such a pattern along each well

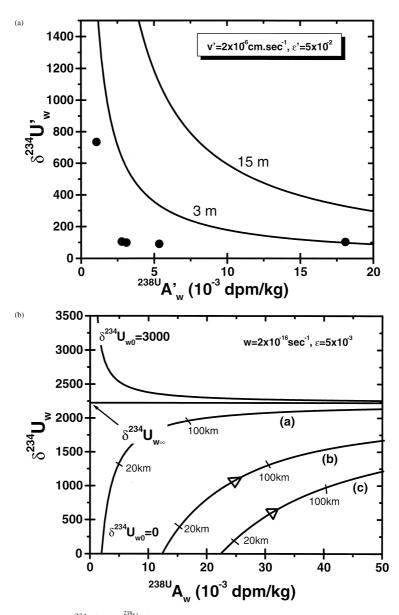


Fig. 8.a. The evolution of $\delta^{234}U'_w$ and ${}^{238}UA'_w$ at the water table. The curves show the model prediction for $\varepsilon'_{234_{Th}} \sim 5 \times 10^{-2}$ and for different vadose zone thicknesses (3 m and 15 m, that is the maximum thickness of the non-saturated area). Since the thickness of the vadose zone is variable, all U compositions generated in the vadose zone are located in the area below the 15m curve. The water table samples (•) are all within that region. 8b. The evolution of $\delta^{234}U_w$ and ${}^{238}U_A_w$ along flow lines for different $\delta^{234}U_{w0}$ and ${}^{238}U_A_{w0}$. As U is added, and the influence of the water table input diminishes, $\delta^{234}U_w$ asymptotically approaches $\delta^{234}U_{w\infty}$. In the Upper Glacial Aquifer $\delta^{234}U_{w0}$ values are likely to be $<\delta^{234}U_{w\infty}$, then $\delta^{234}U_w$ is expected to increase with U addition. Curves a, b and c are for ${}^{238}U_{Aw0} = 2$, 12 and 22 × 10⁻³ dpm/kg, respectively. The flow distances are marked on each trajectory; note that these depend upon the value for w_{238u} .

line, and so cannot be readily explained by evolution along a single flowline with uniform conditions, but rather are considered to be on different flow lines with different vadose zone inputs to the aquifer inputs. The data can be used to limit the range of conditions yielding the observed U isotope activities. The solution to Eqn. (1a) for ²³⁸U, that is provided to the groundwater by input at the water table and weathering within an homogenous aquifer, is:

$${}^{238}U\!A_{w} = \left(\frac{\rho_{r}}{\rho_{w}qv}\right) w_{238U} + {}^{238U}\!A_{w0}$$
(9)

Here x is the distance along a flow line, which is not actually known. The maximum x value for each sample is the distance between the well and the ground water divide to the north of the site. Figure 4a shows measured U activities versus distance from the water divide. A maximum estimate of $w_{238_{\rm U}} \sim (1-7) \times 10^{-15} \, {\rm sec}^{-1}$ is obtained from the $^{238}{\rm U}$ activities in the aquifer, assuming no vadose zone input, $^{238}{\rm U} A_{w0} = 0$. For comparison, weathering rate constants of 1 to $6 \times 10^{-14} \, {\rm sec}^{-1}$ can be obtained from the Sr data, using as the host rock Sr concentration that of the average upper crust of 350 mg/kg

(Taylor and McLennan, 1985). Since the 238 U in the aquifer can be provided entirely by the unsaturated zone, $w_{238_{\rm U}}$ may be much smaller.

²³⁴U is provided both by weathering (along with ²³⁸U) and decay of ²³⁴Th going into solution. Combining the solutions of Eqn. 1a for ²³⁴U and ²³⁸U (and assuming $w_{234_{\rm U}} = w_{238_{\rm U}}$) we have:

$$\delta^{234} \mathbf{U} = \begin{bmatrix} \frac{\rho_r}{\rho_w q} \left(\lambda_{234_{\mathrm{U}}} \varepsilon_{234_{\mathrm{Th}}}\right) \frac{2^{238_{\mathrm{U}}} A_r}{2^{38_{\mathrm{U}}} A_{w0}} \frac{x}{v} + \delta^{234} \mathbf{U}_0 \times 10^{-3} \\ \hline \left(\frac{\rho_r w_{238_{\mathrm{U}}}}{\rho_w q} \cdot \frac{2^{38_{\mathrm{U}}} A_r}{2^{38_{\mathrm{U}}} A_{w0}}\right) \frac{x}{v} + 1 \end{bmatrix} \times 10^{-3}$$

At long distances, an asymptotic isotope composition is approached:

$$\delta^{234} \mathbf{U}_{w^{\infty}} = \left(\frac{\lambda_{234_{\mathrm{U}}} \boldsymbol{\varepsilon}_{234_{\mathrm{Th}}}}{\boldsymbol{w}_{238_{\mathrm{U}}}}\right) \cdot 10^3 \tag{11}$$

Eqn. 10 represents mixing of U supplied at the water table (with $\delta^{234}U_{w0}$ from the vadose zone) with a progressively added, aquifer-derived component with composition $\delta^{234}U_{w\infty} \propto \varepsilon_{234_{Th}}/w_{238_U}$. As both ε and w are proportional to the surface areas of the grains, changes in $\varepsilon_{234_{Th}}/w_{238_U}$ cannot be obtained by changing the mineral surface area, but rather must be due to changes in weathering reaction rates caused by changes in water or mineral chemistry (Sect. 4.2). A relationship between $\delta^{234}U$ and $^{238}U_A_w$ independent of flow distance is obtained by eliminating *x* combining Eqn. 8 and 10 and using the vadose zone input (primes):

$$\delta^{234} \mathbf{U}_{w} = \left(\delta^{234} \mathbf{U}_{w0}' - \delta^{234} \mathbf{U}_{w\infty}\right) \frac{{}^{238} \mathbf{U} A_{w0}'}{{}^{238} \mathbf{U} A_{w}} + \delta^{234} \mathbf{U}_{w\infty} \quad (12)$$

Samples along different distances on a flowline will fall on a trajectory defined by values for $\varepsilon_{234_{Th}}/w_{238_{U}}$, $\delta^{234}U_{w0}$, and $^{15}_{20}A_{w0}$ (see Fig. 9b). If there were no vadose zone input $({}^{^{238}\text{U}}A_{w0} = 0), \, \delta^{^{234}}\text{U} = \delta^{^{234}}\text{U}_{w\infty}$ (a constant) and ${}^{^{238}\text{U}}A_{w}$ would increase proportionately to distance, so that a horizontal trajectory is followed. Otherwise, $\delta^{234}U_w$ will asymptotically approach $\delta^{234}U_{w\infty}$ (Eqn. 12); whether this approach will be ascending or descending will depend upon the sign of $(\delta^{234}U_{w0} - \delta^{234}U_{w\infty})$ which depends on w_{238u}/w'_{238u} . As the weathering rate in the aquifer is expected to be much less than in the valoes zone, $(\delta^{234}U_{w0} - \delta^{234}U_{w\infty}) < 0$ and $\delta^{234}U_{w}$ increases from $\delta^{234}U'_{w0}$ upward toward $\delta^{234}U_{w\infty}$ (see Fig. 8b). Note that for the parameters used, the maximum measured value for ${}^{^{238}\text{U}}A_w$ is 20×10^{-3} dpm/kg and the distance required to reach $\delta^{234} U_{\scriptscriptstyle W}$ of 2000‰ is 100 km. Consequently, only the lower part of Figure 8b is pertinent for the present discussion.

The measured ground water values of $\delta^{234}U_w$ and ${}^{^{238}U}A_w$ are compared with the model in Figure 9a. Most of the Upper Glacial Aquifer well samples are in the area defined for the vadose zone input (Eqn. 5), and so the bulk of the U data in the aquifer can be explained by variability of the vadose zone with no significant input from the aquifer. However, sample W4(D) lies outside the region indicated, and so requires U input from the aquifer. No deep aquifer sample has high values for both

 $\delta^{234}U_w$ and ${}^{^{238}U}A_w$. Note that the data do not fall on a single curve, and so cannot be explained by a single set of values for $\varepsilon_{234_{\text{Th}}}/w_{238_{\text{U}}}, \delta^{234}\text{U}_{w0}, \text{ and } \delta^{234}A_{w0}$. If the vadose zone input is significantly modified by aquifer input, then U compositions of ground water can be described by Eqn. 12, and by curves shown in Figure 9a. Such trajectories are shown (curves b and c in Fig. 9a) for $\delta^{234}U_{w0} = 0$ and different ${}^{238}U_{w0}$ values. The deeper aquifer data which also show high $\delta^{234} U_{\scriptscriptstyle W}$ values are close to curves for $w_{238_{\rm U}} \sim 2 \times 10^{-16} \text{ sec}^{-1}$, $\varepsilon_{234_{\rm Th}} = 5 \times 10^{-3}$, $v = 2 \times 10^{-4} \text{ cm sec}^{-1}$, $^{238_{\rm U}}A_{w0} = 2 \times 10^{-3}$ dpm/kg and $\delta^{234}U_{w0} = 0$. It follows that vadose zone inputs with low U concentration even with no excess ²³⁴U will show substantial increase in $\delta^{234}U_w$ due to substantial recoil of ^{234}U . When $\varepsilon_{234_{\text{Tb}}}/w_{238_{\text{H}}}$ goes up, $\delta^{234}U_w$ goes up markedly. The (10) when $\varepsilon_{234_{Th}}/w_{238_U}$ goes up, ε_{w} given $\delta^{234}U_w$ depends mainly on the distance to achieve a given $\delta^{234}U_w$ depends mainly on the weathering rate. In the case of curve c where vadose zone U input is much higher, then much longer trajectories (x) are required to reach the same $\delta^{234}U_w$ (see Fig. 9b). However, the ²³⁸U activity of sample W4(D) (increased by a factor of 3 with respect to the assumed initial input) and its high $\delta^{234}U_w$ value increased by more than 1000‰, requires a flow distance ~ 20 km (Eqn. 9 and 10). This is 2 times further than the distance to the ground water divide (9 km, Fig. 2). For a flow velocity that is 2 times lower, which is within the range given in Table 4, the flow distance is correspondingly reduced to 10 km. Alternatively, a higher $\delta^{234}U_{w0}$ value would readily allow achieving the W4 values; however, similar $w_{238_{\text{U}}}$ and $\varepsilon_{234_{\text{Tb}}}$ values are still required. Note that while a high weathering input within the aquifer could yield high $^{^{238}\text{U}}A_w$ values, it could not generate samples with high $\delta^{234}U_w$ values. The Magothy water sample (M1) lies slightly above the vadose zone envelope and may result from, either a high $^{^{238}\text{U}}A_{w0}$ from the vadose zone and subsequent recoil within the aquifer with low weathering rates, or a low $^{^{238}\text{U}}A_{w0}$ and U input from the aquifer at higher weathering rates.

4.3.2. Redistribution of uranium in the vadose zone

A noteworthy feature of the U isotope distribution in the vadose zone is that both the highest ²³⁸U, and highest ²³⁴U excess, activities ${}^{234}UA'_{excess}$ were obtained for the two shallow wells, $S1_{wt}$ and $S2_{wt}$ (Figs. 7, 8a, and Table 2). The ${}^{238}U$ and excess ²³⁴U activities appear to systematically decrease with increasing vadose zone thickness. An exception to this pattern is the data for one shallow well, with a relatively low ²³⁸U activity but an excess ²³⁴U activity comparable to the other well of same depth. That the shallowest wells have the highest $H^{*}UA'_{excess}$ values may be a general feature within the vadose zone, and would suggest that high U activities are generated at shallow depths throughout the study area, with progressive removal of U from the water during downward migration. From this point of view, samples collected at locations with different vadose zone depths then represent waters that are at different stages in this evolutionary history. The highest value for $w'_{238_{\text{H}}} = 1 \times 10^{-13} \text{ sec}^{-1}$ calculated for the top 3m, may be applicable in the upper portion of the vadose zone throughout the site. Such enhanced mobilization of U from the upper vadose zone agrees with the largely observed depletions of Fe, Al, and other elements in the A horizon due to complexation by organic humic acids (Domenico and Schwarz, 1990). The un-

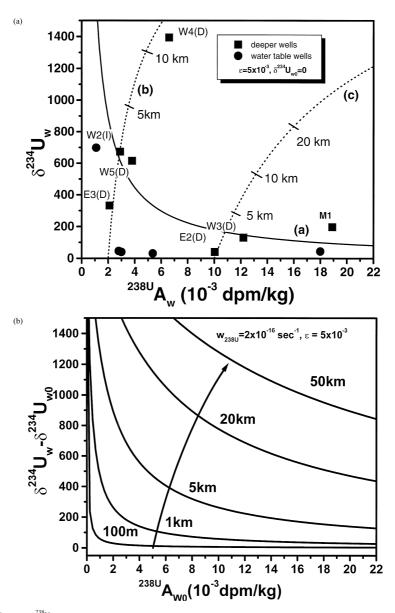


Fig. 9.a. $\delta^{234}U_w$ vs. $^{^{238}U}A_w$ in water table (•) and deeper (•) Upper Glacial Aquifer samples. Curve a represents the envelope of the vadose zone input (calculated for the maximum thickness of the vadose zone) and shows the maximum values expected in the vadose zone; since the vadose zone thickness varies, all water table values fall below this line. Most deep well $^{^{238}U}A_w$ values have the same range as water table values, and so may result mainly from vadose zone input. Dashed lines represent two evolutions along flow lines with flow distances marked. Curve b starts from $^{^{238}U}A_{w0} = 2 \times 10^{-3}$ dpmU/kg a $\delta^{234}U_{w0} = 0$ and has a low $w_{238_U} (2 \times 10^{-16} \text{ sec}^{-1})$ and increasing $\delta^{234}U$ due to recoil input. Curve c has similar w_{238_U} and starts from $^{^{238}U}A_{w0} = 12 \times 10^{-3}$ dpmU/kg. Note that curve b passes close to W4, which U isotopic composition obtained from this initial input requires a flow distance ~20 km. This distance can be halved if the flow velocity is twice lower than assumed here. The sample M1 from the Magothy can be explained by a high U initial activity from the vadose zone input and subsequent recoil from within the confined aquifer at low weathering rates. 9b. Distances for trajectories like curve c (Fig. 9a) but a range of values for $^{^{238}U}A_{w0}$. Note high values for both $\delta^{^{234}U_w}$ and $^{^{238}U}A_w$ can only be generated over very long distances.

derlying soil layers are a sink for these less soluble constituents, forming a zone of precipitation characterized by maxima in Fe and other elements along soil profiles (Langmuir, 1997; Land, 1998; Greenman, 1999). Incorporation of U into these layers may possibly be a mechanism for U removal at depths > 3 m.

The U removal can be examined by comparing calculated U-depth profiles and the decrease in U activity below the maximum at 3 m for sample $S1_{wt}$ (Fig. 7). The model described

above for radionuclide transport in ground water can be used to model removal of U including reversible adsorption onto host mineral surface coatings (see Sect. 4.1 and Figs. 6, 8a), with a $\bar{x}'_{238_{\rm U}} \equiv {\rm v's}/(\lambda_{238_{\rm U}}(1 + \chi'_{238_{\rm U}}))$ (Eqn. 2b). The deep vadose zone value ~ 3×10^{-3} dpm ²³⁸U/kg (Fig. 7), is reached in $\bar{x}'_{238_{\rm U}} \sim 6$ m. Then from Eqn. 2a $\chi'_{238\rm U} > 2 \times 10^9$, which implies that 6×10^6 dpm ²³⁸U/kg of water) is associated with the surface coating in the deeper zone, and that the bulk solid has a U

concentration >1000 ppm. This is unreasonable; so, the model assumption of reversible equilibrium between the water and the surface coating is not valid. Alternatively, U removal may occur onto a surface layer that is not in steady-state equilibrium with the water. If U is removed from the water by irreversible adsorption, then ²³⁸U desorption may be neglected in Eqn. 1a, so:

$${}^{238U}\!A'_{w} = \frac{\rho_{r}}{\rho_{w}} \frac{\bar{x}_{238U}}{sqv'} {}^{238U}\!A_{r} \left(1 - q^{\frac{x}{\bar{x}_{238U}}}\right)$$
(13)

where

$$\frac{1}{\bar{x}_{238U}^{\prime}} = \left({}^{U}\hat{k}_{1}^{\prime} \frac{S}{sqv'} \right)$$
(14)

Using $\bar{x}'_{238_{\rm U}} \sim 6$ m (deduced from Fig. 7), v' ~ 5 × 10⁻⁶ cm/s, and a void ratio $q \sim 0.5$, then ${}^{\rm U}\hat{k}'_1 \sim 4 \times 10^{-9}$ /S. This can be related directly to the removal rates by

$$\left[\frac{d^{^{238}}A_w}{dt}\right]_{sorb} = -\left[{}^{U}\hat{k}'_1 \frac{S}{sq}\right]{}^{^{238}}A_w = -\left[\frac{\nu'}{\bar{x}'_{^{238}}U}\right]{}^{^{238}}A_w \quad (15)$$

where (v'/\bar{x}'_{238_U}) is the constant controlling the rate of U removal; the mean time-scale calculated for removal is then ~0.7 yrs. A corresponding weathering rate for the deep vadose zone can be obtained from Eqn. 2 of $w'_{238_U} \sim 1 \times 10^{-14} \text{ sec}^{-1}$, which is equivalent to an average weathering time of 3×10^6 years. This w'_{238_U} value is lower than the weathering rate of $1 \times 10^{-13} \text{ sec}^{-1}$ required to obtain the high ²³⁸U activity of well S1_{wt}. Therefore, the vadose zone appears to be divided into an upper region (0–3 m) characterized by a high weathering rate, and the lower vadose zone (below 3m) characterized by a lower weathering rate. Note that the time-scale for removal is longer than might be expected for chemical reaction rates. A more rapid removal would imply a smaller value for \bar{x}'_{238_U} . If \bar{x}'_{238_U} is lower by as much as a factor of 10, a uniform weathering rate would be applicable throughout the vadose zone.

The U isotopic compositions in the valoes zone vary over a limited range (Fig. 7) with the exception of one sample S2_{wt}, discussed further below, so that $\varepsilon'_{234_{\text{Th}}}/w'_{238_{\text{U}}}$ is relatively constant. Similar δ^{234} U values for the upper and lower regions of the valoes zone (where $w'_{238_{\text{U}}}$ was found to decrease with depth by an order of magnitude) requires that $\varepsilon'_{234_{\text{Th}}}$ decreases with $w'_{238_{\text{U}}}$. Only an increase in the size of U-bearing grains will decrease the effective value of $w'_{238_{\text{U}}}$ and keep $\varepsilon'_{234_{\text{Th}}}/w'_{238_{\text{U}}}$ constant. As discussed above, the higher $w'_{238_{\text{U}}}$ in the upper valoes zone is likely due to differences in water chemistry, and so may have led to an overall decrease in grain size (and thus an increase in $\varepsilon'_{234_{\text{Th}}}$).

An exception to the trends discussed above is the shallow well sample S2_{wt}, which has a substantially lower ²³⁸U activity than the other shallow well sample (S1_{wt}), screened at the same depth. In contrast, these wells have the same ²³⁴UA'_{excess} values (and so different $\delta^{234}U'_w$ values). This suggests that while the ²³⁴U input rates by α -recoil ejection are identical at these sites, the weathering rates are different. The chemical compositions of the two samples (Table 1) also may indicate interactions with different solids.

4.4. The Behavior of Th in the Unconfined Aquifer

The first insight into the behavior of Th is seen in the measured ratios of the activities of 232 Th to 238 U. From inspection of the data in Table 2 and the discussion in Section 3, we can see that $^{^{232}\text{Th}}A_w/^{^{280}}A_w \sim (10^{-2} \text{ to } 10^{-3}) (^{^{^{232}\text{Th}}}A_w/^{^{^{238}}}A_w)$. From the exchangeable surface layer model, the ratio $(^{^{232}\text{Th}}A_w/^{^{238}}A_w)/((^{^{^{232}\text{Th}}}A_w/^{^{^{238}}}A_w)) = w_{232_{\text{Th}}}/w_{238_{\text{U}}}$. This indicates that the weathering rate for Th is 10^{-2} to 10^{-3} that of U. This means that the intrinsic weathering rates of minerals supplying U is far greater than for those minerals supplying Th. However, this requires that U-providing minerals have extremely low Th contents. It is difficult to reconcile this with the Th/U ratios in radioactive minerals. The fact that the Th concentration of 232 Th appears to be fixed by the solubility limit is also in conflict with an exchange model for Th.

We follow the behavior of the isotopes for the case of an exchangeable surface coating. At steady state the solution for ²³²Th is then given by equation (2d). Note that as $x/\bar{x}_{232_{Th}} \ll$ 1, there is no adsorptive term. The ²³⁸U decay series produces ²³⁰Th. There is no ²³⁰Th data yet available. However, some inferences on the ²³⁰Th isotopes can be made using the data on ²³⁴Th and the exchange model for ²³⁴Th (see Eqn. 3) is:

$${}^{^{234}\text{Th}}\!A_{_{W^{\infty}}} = \left[\frac{\rho_r}{\rho_w q} {}^{^{234}\text{Th}}\!A_r\right] \frac{\varepsilon_{^{234}\text{Th}}}{(1+\chi_{^{234}\text{Th}})}$$
(16)

If we assume $\hat{k}_{-1} >> \lambda_{234_{Th}}\xi$, (see Eqn. 2b) then $\chi_{230_{Th}} \approx \chi_{234_{Th}} = 1 \times 10^3$ to 1×10^4 and we obtain $\bar{x}_{230_{Th}} = \chi_{230}\lambda_{230}/\nu = 1.3$ to 13 km. The relative importance of recoil and weathering inputs can be seen by comparing the terms in the first brackets of Eqn. 16. With $w_{230_{Th}} = w_{238_U} \leq 7 \times 10^{-15} \text{sec}^{-1}$ (calculated from Table 5), $w_{230_{Th}}/\lambda_{230_{Th}} \leq 2 \times 10^{-2}$; it follows that for $\varepsilon_{230_{Th}} \sim 5 \times 10^{-3}$, either weathering or recoil could be the dominant source of 230 Th.

Using the asymptotic solution the maximum activity for ²³⁰Th (when $x >> \bar{x}_{230_{Tb}}$), we have

$${}^{^{230}\text{Th}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{w_{^{230}\text{Th}} / \lambda_{^{230}\text{Th}} + \varepsilon_{^{230}\text{Th}}}{\chi_{^{230}\text{Th}}} \right) {}^{^{230}\text{Th}}\!A_r \qquad (17)$$

This maximum ²³⁰Th activity can be related to ²³⁴Th by combining Eqn. 16 and 17:

$$\frac{{}^{230\text{Th}}\!A_{w^{\infty}}}{{}^{234\text{Th}}\!A_{w^{\infty}}} = 1 + \frac{w^{230}\text{Th}/\lambda_{^{230}\text{Th}}}{\varepsilon_{^{230}\text{Th}}}$$
(18)

Therefore, an estimate of the maximum ²³⁰Th activity can be obtained from the measured ²³⁴Th activity from Eqn. 16. For $w_{230_{\text{Th}}} < 7 \times 10^{-15} \text{ sec}^{-1}$ and $\varepsilon_{230_{\text{Th}}} = 5 \times 10^{-3}$, ${}^{230_{\text{Th}}}A_{w\infty}/{}^{234_{\text{Th}}}A_{w\infty} \leq 6$. If we consider a weathering rate of $2 \times 10^{-16} \text{sec}^{-1}$ (corresponding to the U data) then ${}^{230_{\text{Th}}}A_{w\infty}/{}^{234_{\text{Th}}}A_{w\infty} \approx 1$.

A value for the desorption rate of Th from the surface coating can be obtained knowing the distribution of ²²⁸Th produced by ²²⁸Ac ($\tau_{1/2} = 6.2$ h) from ²²⁸Ra. As there is no recoil input of ²²⁸Th (produced by β decay), the dominant source for ²²⁸Th is decay of ²²⁸Ra in the water and desorption of ²²⁸Th from the surface coating. Therefore, once the limiting activities are reached (when x >> $\bar{x}_{228_{Ra}} \approx 5$ m and x >> $\bar{x}_{228_{Ra}} \approx 1$ cm), the total activity of ²²⁸Th in the water and

surface coating equals that of ²²⁸Ra in these two phases. The activity of ²²⁸Th in solution (correlated with the recoil supply of ²²⁸Ra) is compared to that of ²³⁴Th (supplied by recoil) by combining the equations for the water activities of each (see Appendix A):

$$\frac{^{228}\text{Th}A_{w^{\infty}}}{^{234}\text{Th}A_{w^{\infty}}} = \begin{bmatrix} ^{\text{Th}}\hat{k}_{-1} + \lambda_{228}\text{Th}\xi \\ \frac{}{\text{Th}}\hat{k}_{-1} + \lambda_{234}\text{Th}\xi \end{bmatrix} F_{228}\text{Th} \begin{bmatrix} 1 + \frac{F_{228}Ra}{\varepsilon_{234}} \frac{^{232}\text{Th}}N_{sc} \\ \frac{}{\varepsilon_{234}} \frac{F_{228}Ra}{\varepsilon_{234}} \frac{F_{228}Ra}{\varepsilon_{234}} \end{bmatrix} F_{228} + \frac{F_{228}Ra}{\varepsilon_{234}} \frac{F_{22}}{\varepsilon_{234}} \frac{F_{22}}{\varepsilon_$$

 ${}^{^{232}\text{Th}}N_{\text{sc}}$ and ${}^{^{238}\text{U}}N_r$ represent the number of atoms of ${}^{^{232}\text{Th}}$ in the surface coating and ²³⁸U in the rock respectively. The first bracket varies between 3×10^{-2} for very slow desorption rates and 1 in case of very rapid desorption. The second bracket represents the ²²⁸Ra and ²³²Th on the surface coating supplying ²²⁸Th (direct production from the rock is negligible) relative to the ²³⁸U atoms in minerals supplying ²³⁴Th. We will assume that ${}^{\text{Th}}\hat{k}_{-1} >> \lambda_{228_{\text{Th}}}\xi$ in which case $F_{\frac{228_{\text{Th}}}{21.5}} \sim 1$ (see Appendix A). If the desorption rate is fast $({}^{\text{Th}}\hat{k}_{-1}^{\text{in}} >> \lambda_{234_{\text{Th}}}\xi)$, then ${}^{234}\text{Th}A_{w^{\infty}} \ge {}^{228}\text{Th}A_{w^{\infty}}$. However, measurements give ${}^{228}\text{Th}A_{w^{/234}\text{Th}}A_{w} = 0.1$ (Table 3) and indicate (for ${}^{232\text{Th}}N_{sc} \approx 0$) that $\xi/^{\text{Th}}\hat{k}_{-1} = 3 \times 10^7$ sec (Eqn. 19). Note that while ²³⁴Th is supplied directly into solution by recoil (and then adsorbs), ²²⁸Th is largely supplied into the surface coating, and the activity in solution is determined by desorption. Note that larger $\xi^{/\text{Th}}\hat{k}_{-1}$ values would be obtained if a significant amount of ²³²Th existed on the surface coating. The significance of $\xi^{\text{Th}}\hat{k}_{-1}$ is clearer by noting that (Tricca et al., 2000):

$$\left[\frac{d^{228\mathrm{Th}}N_{sc}}{dt}\right]_{desorb} = \frac{{}^{228\mathrm{Th}}N_{sc}({}^{\mathrm{Th}}\hat{k}_{1}S)}{\xi S} = {}^{228\mathrm{Th}}N_{sc}\left[\frac{{}^{\mathrm{Th}}\hat{k}_{-1}}{\xi}\right] \quad (20)$$

Then $\xi^{\text{Th}}\hat{k}_{-1} \sim 1$ yr is the mean lifetime of ²²⁸Th in the surface coating. This calculation therefore provides the first estimate, based upon field measurements, of the desorption rate parameter of Th from aquifer host rock surfaces. Then, $\chi_{234_{Th}}$ should be ~ 1/10 of χ_i due to its short lifetime where $i = {}^{228}$ Th, 230 Th or ²³²Th, and $\chi_{228_{\text{Th}}} = \chi_{230_{\text{Th}}} = \chi_{232_{\text{Th}}} = {^{\text{Th}}\hat{k}_1}/{^{\text{Th}}\hat{k}_{-1}}$ (from Eqn. 2b). The sorption rate of Th can be obtained from (Tricca et al., 2000):

$$\chi_{^{234}\text{Th}} = \frac{{}^{^{234}\text{Th}}DS\xi}{q} = \frac{{}^{\text{Th}}\hat{k}_{1}S/q}{{}^{\text{Th}}\hat{k}_{-1}/\xi + \lambda_{^{234}\text{Th}}}$$
(21)

If $\chi_{234_{\text{Th}}} = 10^3$, and assuming that ${}^{\text{Th}}\hat{k}_{-1}/\xi << \lambda_{234_{\text{Th}}}$ (desorption of Th is slower than decay of 234 Th) then ${}^{\text{Th}}\hat{k}_1S/q =$ $3 \times 10^{-4} \text{sec}^{-1}$. Noting that adsorption follows the relationship:

$$\left[\frac{d^{^{228}\mathrm{Th}}N_{sc}}{dt}\right]_{sorb} = {}^{^{228}\mathrm{Th}}N_{w}\left[\frac{S^{\mathrm{Th}}\hat{k}_{1}}{q}\right] = -\left[\frac{d^{^{228}\mathrm{Th}}N_{w}}{dt}\right] \qquad (22)$$

a mean residence time of Th in the groundwater ~ 1 h is obtained (see also Krishnaswami et al., 1982). In this treatment of exchangeable sites on the surface for Th, we see that all estimates of $\chi_{\rm Th}$ are very large so that the surface layer always has almost all of the Th isotopes in the system surface layer plus water. As will be shown in the subsequent section, the amount of Th inferred to be on the surfaces in this model is insufficient to support the Rn.

4.5. Behavior of Ra Isotopes

The ²²⁶Ra input in the vadose zone occurs by recoil and weathering of the rock; assuming exchange between the solid and the water, then for $x < < \bar{x}'_{226_{P_a}}$ (see Sect. 4.1)

$${}^{^{226}\text{Ra}}A'_{w} = \frac{\rho_{r}}{\rho_{w}q} \left(\frac{\lambda_{^{226}\text{Ra}}\varepsilon'_{^{226}\text{Ra}} + w'_{^{226}\text{Ra}}}{v'} \right)^{^{230}\text{Th}}A_{r} \cdot x$$
(23)

Considering the term in the brackets, assuming $w'_{238_{\rm U}} = w'_{226_{\rm Ra}}$ and using $w'_{238_{\rm U}} = 1 \times 10^{-13} \, {\rm sec}^{-1}$ calculated for the top 3 m of the vadose zone (Sect. 4.3), then $w'_{226_{Ra}}/\lambda_{226_{Ra}} \sim 1 \times 10^{-2}$. If $\varepsilon'_{226_{R_a}} \sim \varepsilon'_{234_{T_b}} \sim 2 \times 10^{-2}$, then the weathering and recoil input are comparable. At 3 m below the surface level, a ²²⁶Ra activity of 32×10^{-3} dpm/kg is reached. For a thicker layer with a high weathering rate, ²²⁶R^aA_w would increase proportionately. Consequently, the measured water table activities of $^{226}\text{Ra}~(\sim 40\times 10^{-3}~\text{dpm/kg})$ can be reached over vadose zone thicknesses of ≥ 3 m. Since $\bar{x}'_{226_{Ra}} = 300$ m (this section and Table 5), the vadose zone input may be a dominant source of ²²⁶Ra in the upper 300 m of the aquifer. The Ra isotope activities measured in two samples from the water table (Table 3) are comparable to those further down in the aquifer.

In the aquifer, the ²²⁶Ra activities will be governed by the In the aquifer, the Ka activities will be governed by the aquifer input for $x >> \bar{x}'_{226_{Ra}}$. Assuming $w_{238_{U}} \sim w_{226_{Ra}}$ then $w_{226_{Ra}}/\lambda_{226_{Ra}} << \varepsilon_{226_{Ra}}$ and recoil is the dominant input $^{226_{Ra}}A_{w\infty} = \frac{\rho_r}{\rho_w q} \left(\frac{\varepsilon_{226_{Ra}}}{1+\chi^{226_{Ra}}}\right)^{230_{Th}}A_r$ (see Appendix A). With $\varepsilon_{226_{Ra}} \sim 5 \times 10^{-3}$, $^{230_{Th}}A_r \sim 440$ dpm/kg and no adsorption $(\chi_{226_{Ra}} = 0)$, $^{226_{Ra}}A_{w\infty} \sim 13$ dpm/kg of water. In contrast, the observed values are $\sim 30 \times 10^{-3}$ dpm/kg. observed values are $\sim 30 \times 10^{-3}$ dpm/kg. Therefore the supply of ²²⁶Ra by α -recoil is far more than is necessary to account for the ²²⁶Ra observed in the water. Each of the Ra isotopes has quasi-constant activities in the deeper wells of the aquifer indicating that they have reached the limiting value ${}^{iRa}A_{w\infty}$ (Appendix A). Here, as Ra is dominated by recoil input,

$${}^{{}_{46}\!Ra}\!A_{{}_{W^{\infty}}} = \frac{\rho_{r}}{\rho_{w}q} \, \frac{\left(\varepsilon_{{}^{226}\!Ra} + 0.5\varepsilon_{{}^{230}\!Th}\right)}{\chi_{{}^{226}\!Ra}} \, {}^{{}_{230}\!Th}\!A_{r}$$
(24)

$${}^{^{228}\text{Ra}}\!A_{_{W^{\infty}}} = \left(\frac{\rho_r}{\rho_w q}\right) \, \frac{\varepsilon_{^{228}\text{Ra}} \, {}^{^{232}\text{Th}}\!A_r + 0.5^{^{232}\text{Th}}\!A_{sc}}{\chi_{^{228}\text{Ra}}} \tag{25}$$

$$\frac{\rho_r}{\rho_w q} \frac{\left(\varepsilon_{224_{\text{Ra}}} + 0.25\varepsilon_{228_{\text{Ra}}}\right)^{230_{\text{Th}}} A_r + 0.25 \times 0.74^{232_{\text{Th}}} A_{sc}}{\chi_{224_{\text{Ra}}}} \quad (26)$$

From Eqn. 24, using $\varepsilon_i \sim 5 \times 10^{-3}$ and ²²⁶Ra activities between 30 to 60×10^{-3} dpm/kg, values of $\bar{x}_{226_{Ra}} \approx 300$ to 700 and $\bar{x}_{226_{Ra}} = 100$ to 300 m (Eqn. 2b) are obtained. Similar calculations done for ²²⁸Ra and ²²⁴Ra assuming that the production by ²³²Th decay from the surface coating is negligible $^{(^{22}\text{Th}}A_{sc} \sim 0)$ lead to similar χ_{Ra} values (500–1000) as well as $\bar{x}_{228_{Ra}} \sim 7 \text{ m}$ and $\bar{x}_{224_{Ra}} \sim 0.3 \text{ m}$ (Table 5). This agrees with the well-known adsorptive behavior of Ra (e.g., Langmuir, 1997; Osmond and Ivanovich, 1992). Using the data presented in Table 3, the ²²⁶Ra activity on the surface coating is calculated to be ~10 dpm/kg of rock for $\chi_{Ra} = 10^3$. This surface coating activity is much less than the 440 dpm/kg ²²⁶Ra inventory of the rock but is a substantial portion ($\sim 2\%$) of it.

Using Eqn. 24 and 25 (and if ²³²ThA_{sc} \approx 0), then ²²⁶RaA/²²⁸RaA \sim 1.5 $\chi_{228_{Ra}}/\chi^{226}_{Ra}$. From Eqn. 2a, $\chi^{228}_{Ra}/\chi^{226}_{Ra} \ge 1$ (also $\chi^{224}_{Ra}/\chi^{226}_{Ra} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu} \ge 1.5$. The measured ratios of ²²⁶Ra $A_{\nu}/^{228}_{Ra} A_{\nu}$ are relatively close to this value. Note that if the Ra isotopes reacted irreversibly with the surface coating, then the activity ratios in the water would be inversely proportional to the decay constants of the nuclides and the Ra activity ratios in the water would deviate by orders of magnitude from unity. As this is not the case, the assumption of reversible exchange for Ra appears to be generally valid.

All Ra isotopes have a precursor Th isotope; therefore, the parent-daughter decay pairs provide information on the relative behavior of Ra and Th. The model predicts 236 Ra $A/{}^{234}$ Th $A \sim 1.5 \chi_{234}$ Th $/\chi_{226}$ Ra (see Appendix A); the measured ratios vary from 1 to 3, and indicate that χ_{234} Th $\sim \chi_{226}$ Ra. Therefore, Ra and Th have similar sorption characteristics by this model. The observed relatively small variations in 226 Ra $A_w/{}^{234}$ Th A_w can only be explained by a change in χ_{226} Ra $/\chi_{234}$ Th Since $\chi_i \propto {}^i$ DS ξ and i D $\sim {}^i \hat{k}_1 / {}^i \hat{k}_{-1}$, different values of χ_{226} Ra $/\chi_{234}$ Th can only occur through a change in ${}^i \hat{k}_1$ or ${}^i \hat{k}_{-1}$.

We see that the model of exchangeable sites on the surface coating appears to describe the Ra data. However, we note that the estimates of χ_{Ra} are quite critical to the question of whether there is sufficient input from the rock by recoil and weathering.

4.6. Supply of ²²²Rn

As noted above, the exchangeable site model appears to give a reasonable semiquantitative description of the transport through the Ra isotopes, if we ignore the fact that ²³²Th appears to be at saturation equilibrium. The radon isotopes are particularly important as this element is (when not outgassed) completely in solution. As a result, the Rn inventory is a much stricter constraint on the problem since it does not depend on a precise estimate of χ_{Rn} . Inspection of Table 3 shows that the ratio of activities of 222 Rn/ 226 Ra in the waters range from 10³ to 10⁴. The ratio 222 Rn/ 226 Rn/ 226 represents the ratios of the recoil and weathering rates. As ²²⁶Ra and ²²²Rn are in the same chain, z is of order unity. It can be seen that the high observed ratio can only be explained by values of $\chi_{\rm Ra}$ which are even greater than estimated above. If we use the Rn data to calculate R_{eman}, the ratio of Rn in the water phase in a volume of aquifer to the production of Rn by the rock in that same volume, we have:

$$R_{eman} = \frac{\rho_{w} n^{222 \text{RA}}}{\rho_{r} (1-n)^{222 \text{Rn}}} \approx \frac{1 \times 0.3 \times 150}{2.5 \times 0.7 \times 440} = 5 \times 10^{-2}$$
(27)

Thus, about 5% of the total ²²²Rn produced by the rock must be in the water phase. This would require a very high net recoil fraction of $\varepsilon \sim 5 \times 10^{-2}$. If recoil for other nuclides were this large (e.g., ²³⁴U), then there would be greater δ^{234} U effects observed here.

It has been previously recognized that high ²²²Rn activities in the water as measured here, require emanation fractions that are too high to be explained by direct recoil from typical grain sizes. This point has been earlier emphasized for instance by Krishnaswami et al. (1982) and is again confirmed here. In the

present study we have included all possible contributions by both weathering and recoil as well as a reactive, exchangeable surface layer that greatly enriches the Ra content in that layer. As shown here and noted by Tricca et al. (2000), this does not provide the amount necessary to explain the high Rn levels that have been presented in the literature. Among the proposals are: the special siting of Ra within the U mineral grains to provide ready Rn losses (Krishnaswami and Seidemann, 1988; Semkov, 1990; Semkov and Parekh, 1990; Morawska and Phillips, 1992), and the existence of nanopores within a mineral and preferential diffusion of ²²²Rn produced by ²²⁶Ra (Rama & Moore, 1984; 1990). However, laboratory experiments comparing the losses of both Ar and Rn in silicate minerals failed to find any evidence for preferential diffusion (Krishnaswami and Seidemann, 1988). The authors of that work proposed that "the relatively high loss of 222Rn in the samples has to be attributed to preferential enrichment of ²²⁶Ra on grain surfaces or to the existence of accessory U-Th minerals in or around grain boundaries from which substantial quantities of ²²²Rn escape." The present report has sought to provide a consistent set of relationships for all of the U-Th decay series nuclei with consideration of all contributions, including an exchangeable surface layer. However, the 222Rn behavior cannot be explained with an ε_i value that is compatible with other data but requires values over a factor of ten greater. However, such an increased ε value is not compatible with observed U data as it would require a similar $w_{238_{11}}$ increase. This would imply that both high δ^{234} U values and high ${}^{^{238}U}A_w$ values should be easily observable in ground water samples at short distances (Fig. 8a), as the distances calculated in Sect. 4.3 decrease by a factor of 10. Therefore, while high ε_i and w_i values are valid in the vadose zone, it is not possible to apply them to all the decay series nuclides in the aquifer. The explanation of high Rn activities appears to be specific to Rn and its immediate production and other explanations have to be considered.

5. MODEL OF SATURATION-PRECIPITATION OF THORIUM

We now take into consideration the observations that:

- 1. ²³²Th appears to be controlled by its saturation limit;
- the weathering rate of ²³²Th is very much lower than that of U for the exchangeable surface model;
- the Ra isotopes appear to be in exchange "equilibrium" with the surface; and
- the Rn can not be provided for by recoil, weathering and calculated Ra concentrations in the exchangeable surface layer.

Our approach is now to consider all the Th isotopes to be controlled by precipitation for additions to the water above the saturation limit for ²³²Th; and that the surface layer, with greatly enhanced ²³²Th and ²³⁰Th, is the source of the Ra and the Rn. This results in a "mixed model" of the surface layer, dependent on the different chemical species.

To account for the Rn would require that the surface layer contain ~ 10% of the ²³⁰Th in the rock. The evolution of ²³²Th for saturation is now at steady state for the water, $^{^{232}\text{Th}}A_w = ^{^{232}\text{Th}}A_{wsat}$. Thus, the ratio of $^{^{232}\text{Th}}A_w / ^{^{238}\text{U}}A_w \neq w_{232_{\text{Th}}} / w_{238_{\text{U}}}$. Using the discussion in Section 4 of behavior at saturation, we

have for the surface layer: $\rho_{sc}\frac{d}{dt} \left({}^{232}\text{Th}A_{sc}S\xi \right) = \rho_{r}W_{232}\text{Th}A_{r}$, so that the surface layer content increases with time and:

$$\rho_{sc}({}^{^{232}\text{Th}}A_{sc}S\xi) = \rho_r w_{^{232}\text{Th}}A_r \cdot t$$
(28)

As we require that at the present time, to provide the Rn, $\rho_{\rm sc}(^{^{232}{\rm Th}}A_{sc}S\xi) \approx 10^{-1} \rho r^{^{^{232}{\rm Th}}}A_r$, this yields:

$$w_{232\rm Th} t = 10^{-1} \tag{29}$$

For a time scale of $\sim 10^4$ years this gives $w_{232_{\rm Th}} \sim 3 \times 10^{-13}$ sec^{-1} , which is comparable with the weathering rates found for U in some of the vadose zone samples. To obtain a weathering rate of a factor of ten lower would require a time scale of $\sim 10^5$ years which we consider too long. While the rates are compatible with a Th source in the vadose zone, the amount of Th required to coat the aquifer grains can not be provided by the vadose zone as its volume is several orders of magnitude too small. If weathering of the aquifer were to provide this amount of Th, with weathering rates typically from 2×10^{-15} to 2×10^{-16} sec⁻¹, would require time scales of 10 to 100 million years. It follows that to provide the Th inventory requires a high initial weathering rate, comparable with that of the vadose zone, and that this weathering is pervasive throughout the rock. We therefore propose that the aquifer rock originally contained two populations of U and Th minerals, one of which (called a) contains 10% of the U-Th inventory of the rock and undergoes rapid weathering (w_{ia}) , and the other (b)which undergoes much slower weathering (w_{ib}) . In an initial stage, the weathering input is governed by mineral "a" and the highly reactive nuclides from mineral "a" will be deposited on the surface coating.

5.1. Thorium Transport in the First Weathering Phase

From some initial state of the system the activities of all the nuclides must be found by integrating Eqn. 1a and 1b, taking into account the Th solubility limit and the weathering behavior of the U-Th phases "*a*." When phases "*a*" have been essentially exhausted by this initial weathering regime at some time *t*, then after that the ²³²Th on the surface phase will be governed by Eqn. 28) and ²³⁰Th by Eqn. 31.

At time *t*, for the short-lived 234 Th using Eqn. 1b and assuming that the rate constant for 234 Th is the same as for 232 Th, we have:

$${}^{^{234}\text{Th}}A^{a}_{w} \approx \frac{\rho_{r} \left[\varepsilon^{a}_{^{234}\text{Th}} \lambda_{^{234}\text{Th}} + w^{a}_{^{234}\text{Th}} \right]}{\rho_{w} S^{\text{Th}} \hat{k}_{1}} {}^{^{238}\text{U}}A^{a}_{r}$$
(30)

For the surface coating (from Eqn. 1c)

$$\lambda_{^{234}\text{Th}} \rho_{sc}^{^{234}\text{Th}} A^{a}_{sc} \xi S = \rho_{r} (w^{a}_{^{234}\text{Th}} + \varepsilon^{a}_{^{234}\text{Th}} \lambda^{^{234}\text{Th}})^{^{234}\text{Th}} A^{a}_{r} \qquad (31)$$

It can be seen that ${}^{234}\text{Th}A_w$ is now related to the rate of supply by the removal rate and the connection between ${}^{234}\text{Th}A_{sc}$ and ${}^{234}\text{Th}A_w$ is changed. We further note that the rate constant for ${}^{234}\text{Th}$ removal from the water is ${}^{\text{Th}}\hat{k}_1S/q \sim 3 \times 10^{-4} \text{ sec}^{-1}$ (as found for the equilibrium model). This corresponds to a mean lifetime of ~ 1 h for Th in the water.

For ²³⁰Th (using Eqn. 1a and 1b and neglecting the ²³⁰Th

decay) the equation for the surface layer, using Eqn. 28 and 29, is given by:

$$(\rho_{sc}^{230\text{Th}}A_{sc}^{a}S\xi) = \rho_{r}(w_{230\text{Th}}^{a} + \lambda_{230\text{Th}}\varepsilon_{230\text{Th}}^{a})^{238\text{U}}A_{r}^{a} \cdot t$$
(32)

so that

$${}^{230\text{Th}}A^{a}_{sc}/{}^{232\text{Th}}A^{a}_{sc} = \frac{\left(w^{a}_{230\text{Th}} + \lambda_{230\text{Th}} + \lambda_{230\text{Th}}^{a}\right)}{w^{a}_{232\text{Th}}} \frac{{}^{238\text{U}}A^{a}_{r}}{{}^{232\text{Th}}A^{a}_{r}}$$
(33)

Under the condition of rapid weathering we expect $\lambda_{230_{\text{Th}}} \varepsilon_{230_{\text{Th}}}^{a} / w_{232_{\text{Th}}}^{a}$ to be much less than one. If the weathering rates of U and Th are equal $(w_{230_{\text{Th}}}^{a} \approx w_{232_{\text{Th}}}^{a})$, then the ratio of $^{230}\text{Th}/^{232}\text{Th}$ in the surface coating is the ratio in the parent rock. As the Ra isotopes are all daughters of the Th isotopes, the hypothesized surface layer would provide Ra from both series in the proper proportions. The subsequent evolution during phase *b* will then depend on the subsequent transport and production when the more resistant phases "b" remain.

5.2. The Present Regime with Inherited Surface Coating

After about 300 to 1000 yr (the flushing time for this aquifer), the transport equation reduces to a steady state $(\partial/\partial t = 0)$ in the new regime. The governing equation is now symbolically the same as Eqn. 1b.

$$\frac{\partial^{i}A_{w}^{*}}{\partial x} = \frac{\rho_{r}}{\rho_{w}qsv}$$

$$\cdot \left(w_{ib}^{i}A_{rb} + \varepsilon_{ib}\lambda_{i}^{p}A_{rb} + {}^{i}\hat{k}_{-1}\frac{\rho_{sc}S}{\rho_{r}}{}^{i}A_{sc}^{*} + f_{i}\frac{\rho_{sc}S\xi}{\rho_{r}}\lambda_{i}^{p}A_{sc}^{*}\right)$$

$$+ \frac{\lambda_{i}}{v}{}^{p}A_{w}^{*}\left(\frac{\lambda_{i}}{v} + {}^{i}\hat{k}_{1}\frac{S}{qv}\right){}^{i}A_{w}^{*} \quad (34)$$

but where ${}^{i}A_{sc}^{*}$ and ${}^{i}A_{w}^{*}$ represent the activities of nuclide *i* in the surface coating and in the water and ${}^{i}A_{r}^{b}$ the rock during the second stage of weathering. A rigorous treatment would require that the activities in a different precipitated phase (i.e., different surface coatings such as ThO₂) must be described by different terms. However, this distinction is not pursued for simplicity.

The surface coating activities in Eqn. 34 depend on the integrals of Eqn. 1a and 1b during the earlier stage of weathering given above. Since the amounts of nuclides deposited during the earlier stage are presumed to be orders of magnitude higher than the ones precipitated during the later stage of lower weathering rates, ${}^{i}A_{sc}^{*}$ for long-lived reactive species that are at saturation will be dominated by the amount of nuclide *i* deposited during the earlier stage of weathering. We will assume that all other nuclides have grown back to secular equilibrium with their respective sources.

For all species that are at saturation, and because $w_{ib} < w_{ia}$, any changes in the activities (of ²³²Th and ²³⁰Th in the surface coating) due to further additions in the more recent regime will not be significant. As a result ${}^{^{232}\text{Th}}A_{sc}^*$ and ${}^{^{230}\text{Th}}A_{sc}^*$ in the present regime are the values resulting at time *t* at the end of the first regime. For exchangeable or dissolved nuclides, their activities from the earlier phase will be replaced by interaction with the water in the later phase. However, any production by the "initial" ${}^{^{230}\text{Th}}2{}^{^{230}\text{Th}}$ content in the surface

coating will have to be taken into account. Thus, in the second regime the equations governing the radium isotopes must include the ²³⁰Th and ²³²Th incorporated earlier in the surface coating. The values in the surface coating are given by Eqn. 28, 29 and 32.

The equation for ²²⁸Th is found to be:

$${}^{^{228}\mathrm{Th}}A^{b}_{w} \approx \frac{\rho_{r}\lambda_{^{228}\mathrm{Th}}}{\rho_{w}{}^{\mathrm{Th}}\hat{k}_{1}S} \left[\varepsilon_{^{228}\mathrm{Th}} + \frac{f_{^{228}\mathrm{Th}}}{10}\right]{}^{^{232}\mathrm{Th}}A^{b}_{r} \qquad (35)$$

for the case that 1/10 of the ²³²Th inventory is on the surface coating. Using Th $\hat{k}_1 S/q = 3 \times 10^{-4}$ and ²³²Th $A_r \sim 440$ dpm/kg, we obtain ²²⁸Th $A_w \sim 1.1 \times 10^{-1} \left(\varepsilon_{^{228}\text{Th}} + \frac{f_{^{228}\text{Th}}}{10} \right)$. For $\varepsilon \sim 5 \times 10^{-3}$ and $f_{^{228}\text{Th}} \sim 10^{-1}$, ²³²Th $A_w \approx 1.7 \times 10^{-3}$ dpm/kg, which is at the low range of the observed values. If $f_{^{228}\text{Th}} \sim 0.5$ (requiring a very thin surface coating) then this would cover the high range of observed values. Overall there is good agreement and self-consistency for the Th isotopic abundances. Note that ²³²Th A_w is typically about 1/10 of ²²⁸Ra, in accord with the idea that Th is precipitated out while Ra is in exchangeable sites.

5.3. Ra and Rn from Surface Th

We now treat the radium isotopes ²²⁸Ra, ²²⁶Ra, and ²²⁴Ra, considering them to be in exchangeable sites that are predominantly in the surface phase and that their parents were deposited during the first stage of weathering. Using Eqn. 34 at steady state in the recent regime, we obtain:

$${}^{228\text{Ra}}A_{w} = \frac{\rho_{sc}}{\rho_{w}{}^{\text{Ra}}\hat{k}_{1}} \left[{}^{\text{Ra}}\hat{k}_{-1} + f_{228\text{Ra}} \xi \lambda_{228\text{Ra}} \right] {}^{232\text{Th}}A_{sc}^{*} \qquad (36a)$$

$${}^{226\text{Ra}}A_{w} = \frac{\rho_{sc}}{\rho_{w}^{\text{Ra}}\hat{k}_{1}} \left[{}^{\text{Ra}}\hat{k}_{-1} + f_{226\text{Ra}} \xi \lambda_{226\text{Ra}} \right] {}^{230\text{Th}}A_{sc}^{*} \qquad (36b)$$

$$^{224\text{Ra}}A_{w} = \frac{\rho_{sc}}{\rho_{w}^{\text{Ra}}\hat{k}_{1}} \left[{}^{\text{Ra}}\hat{k}_{-1} + f_{224\text{Ra}} \,\xi\lambda_{224\text{Ra}} \right] {}^{232\text{Th}}A_{sc}^{*} \qquad (36c)$$

We showed (Eqn. 33) that $^{230}\text{Th}A_{sc}^{a} / ^{232}\text{Th}A_{sc}^{a} \approx ^{230}\text{Th}A_{r}^{a} / ^{232}\text{Th}A_{r}^{a}$. As discussed in Section 4.5, if $w_{\rm U} \approx w_{\rm Th}$, the ratios of the activities of these isotopes are again all about unity. The greatest est enhancement should be for 224 Ra which has the shortest lifetime (by over a factor of ~ 580) so that the term $f_{224_{\rm Ra}}\xi\lambda_{224_{\rm Ra}}$ makes some contribution. This may be seen in the results in Table 3. Note that the expressions in Eqn. 36a,b,c are of the form $^{\rm Ra}A_w^* = \frac{\rho_{sc}S\xi}{\rho_wq\chi_{\rm Ra}^*} {}^{P}A_{sc}^*$, which is identical to the form given for an exchangeable surface coating in Section 4.5 (Eqn. 24, 25, 26): $^{\rm Ra}A_w^*/^{\rm Ra}A_w = \frac{\rho_{sc}S\xi^{P}A_{sc}^*\chi_{\rm Ra}}{\rho_r\chi_{\rm Ra}^*\epsilon_{\rm Ra}^*PA_r}$. Since the $\chi_{\rm Ra}$ are calculated from the same observed $^{\rm Ra}A_w$ values in both models the left hand side is equal to one. However, for the case of Th being saturated (precipitated without exchange) we have taken $\rho_{sc}S\xi^PA_{sc}^*\approx 0.1^PA_r$ in order to supply the Rn. It follows that $\chi_{\rm Ra}^*/\chi_{\rm Ra} \approx \frac{10^{-1}}{\varepsilon_{\rm Ra}} \sim 20$. A similar argument can be made for Th. Hence the $\chi_{\rm Ra}^*$ values that are obtained for the Th saturation-precipitation model yield values over an order of magnitude greater than obtained earlier (i.e., $\chi_{\rm Ra}^* \sim 10^4$). This large increase in χ_{Ra} yields a change in the characteristic distances \bar{x}_{Ra} , decreasing them by a factor of ten.

Using Eqn. 36a,b,c, the equations for Rn are then simply:

$${}^{220\text{Rn}}A_{w}^{*} = \frac{\rho_{r}}{\rho_{w}q} w_{{}^{232}\text{Th}}^{a} A_{r}^{a} \cdot t$$
(37a)

$${}^{^{222}\text{Rn}}A_{w}^{*} = \frac{\rho_{r}}{\rho_{w}q} \left(w_{^{230}\text{Th}}^{a} + \lambda_{^{230}\text{Th}} \varepsilon_{^{230}\text{Th}}^{a} \right)^{^{238}\text{U}}A_{r}^{a} \cdot t$$
(37b)

The ratios of these activities are around unity. As we have taken $w^a_{232_{\text{Th}}} \cdot t^{232_{\text{Th}}} A_r^a \approx 10^{-1} \cdot ^{232_{\text{Th}}} A_r$ (see Eqn. 28), this will thus satisfy the ²²²Rn observations.

5.4. Summary of Results for the Th Saturation Model

We here summarize the results obtained in this section. It was shown that if sufficient ²³²Th and ²³⁰Th are precipitated in the surface coating in non-exchangeable sites, then it is possible to explain the Ra and Rn data. However, this requires that the deposited ²³²Th and ²³⁰Th be the result of an early weatheringdeposition regime before the present. If this earlier regime was of duration $t \sim 10^4$ years, then the rate of weathering is that found for the vadose zone. This weathering would have to be pervasive through the aquifer rock and cannot be provided by the present vadose zone. In the present regime, assumed to occur \sim 1000 yr after the end of the initial stage of rapid weathering, the system is then described by the current input of weathering and recoil of the rock, but with the added source of ²³²Th and ²³⁰Th in the surface layer. The behavior of all nuclides except for Th isotopes is one of exchange with the surface layer, in agreement with the Ra observations. The implications of this model are that there should be a "removable" surface layer in the aquifer rock that contains $\sim 10\%$ of the total inventory of ²³²Th and ²³⁰Th, ²²⁶Ra, ²²⁸Ra, and ²²⁴Ra of the bulk rock system. We infer that the weathering rate for Th is about equal to that of U. The great difference between ${}^{^{238}\text{U}}A_w$ and ${}^{^{232}\text{Th}}A_w$ is due to saturation-precipitation of Th. It is also found that the characteristic distances of \bar{x}_i for Ra and Th are much smaller than for the equilibrium exchange model and reduce the equations for these nuclides to a local model, approximating a "box model."

5.5. The Magothy Aquifer

As mentioned in Sect. 4, the waters within the more reducing Magothy are distinctive from those of the Upper Glacial in having Fe, Mn and Th concentrations that are orders of magnitude higher, but with Sr and U concentrations and isotopic compositions that are similar (Tables 1 and 2). These results, as well as those of the low O2 concentrations (down by a factor of 5), show the Magothy aquifer to be more reducing and to contain a high amount of colloids. The enormous increases in Fe, Mn and Th concentrations is not directly attributable to the decrease in O2 but is most likely due to the activity of microorganisms causing the reduction with a large effect on Fe, Mn and Th (Arnold et al., 1988). The water from the Magothy is recharged from the same area as the water from the Upper Glacial Aquifer and the flow velocities are comparable. The U isotopic composition and U activities in the Magothy are similar to those of the unconfined aquifer. Therefore, the $^{238}\mathrm{U}$ and ²³⁴U activities in water from the Magothy can be explained similarly to those from the Upper Glacial Aquifer (Sect. 4.3). The δ^{234} U values are constrained by the ratio $\epsilon_{234_{TM}}/w_{^{238}U}$ and the U activities are dependent on the weathering rate. For instance, using a $\epsilon_{234_{Th}}\sim 5\times 10^{-3}$ and $w_{238_{U}}\sim 10^{-14}~\text{sec}^{-1},$ measured U activities in the Magothy are obtained within a few km of flow. However, the ²³²Th activities are extremely high and $^{^{238}\text{U}}\!A_{\scriptscriptstyle W} \sim {}^{^{232}\text{Th}}\!A_{\scriptscriptstyle W}$ (Table 2) which indicates that U and Th input into the Magothy waters occurs by rock weathering without any fractionation or Th removal. The Ra activities (similar to those of the Upper Glacial Aquifer samples) are also shown to reflect reversible Ra adsorption onto the surface coating. The ²²²Rn activity is at the same very high level as in the Upper Glacial Aquifer (80 dpm/kg). So ²²²Rn data in the Magothy also require storage of Th on the surface coating during an earlier stage of weathering. The U-Th series in the aquifer economy in the Magothy is thus in consonance with the two-stage model taking into account the change in solubility of Th under somewhat reducing conditions. At present, the Th is transported on the colloids in the Magothy. This high release of Th observed in the Magothy is indicative of the fact that rather modest changes in the chemistry of an aquifer can lead to extension and rapid release of Th from what were insoluble precipitates in another circumstance. The Magothy certainly requires extensive study to understand the chemical processes involved.

6. CONCLUSIONS

The study represents a comprehensive evaluation of U- and Th-series radionuclide transport in an unconfined aquifer, and provides a model for interpreting an extensive nuclide data set measured within an aquifer. The theoretical model considerations used here are based upon a one-dimensional advective transport model considering weathering, recoil and interaction with a surface layer (Tricca et al., 2000). Values for input rates were calculated for weathering, α -recoil into the waters (both in the vadose zone and in the aquifer) and removal by adsorption/ precipitation. Rate constants for adsorption and desorption were determined from the field data. Samples from water table wells have been used to evaluate input to the aquifer from the vadose zone. The activities (per kg of water) of the long-lived nuclides (238U, 234U, and 232Th) in water table samples are comparable to those from deeper in the aquifer, demonstrating that vadose zone weathering is a dominant supply of these nuclides to the aquifer. There is a relatively wide range in measured U activities and δ^{234} U values within different water table samples. However, the vadose zone does not yield water samples with both high U activities and high isotopic shifts. The range in measured U activities from water table wells suggests that the upper 3 m of the vadose zone is characterized by high fractional weathering rate ($w_{238_{\rm U}}$ up to 10^{-13} sec⁻¹) and recoil radionuclide inputs ($\varepsilon_{\rm i}$ up to 5×10^{-2}) to the water. Lower weathering and scavenging of U from the water occur at locations where the vadose zone is thicker. In the aquifer, the U activities follow two trends: a wide range of U activities with low δ^{234} U values, and low U activities with higher δ^{234} U values. The variable U activities in the aquifer are thus a reflection of diverse vadose zone inputs that are preserved within the aquifer along different flow lines, rather than the result of complex non-conservative U behavior within the

aquifer. Some higher δ^{234} U values for waters with low U activities in the deep aquifer show that further contribution by recoil occurs within the aquifer associated with low rates of aquifer weathering. For limited flow distances, generating high δ^{234} U requires low U input activities from the aquifer. Values of $w = 2 \times 10^{-16} \text{ sec}^{-1}$ and $\varepsilon_{234_{\text{Th}}} \sim 5 \times 10^{-3}$ have been calculated to explain the measured activities in the aquifer. Observations of δ^{234} U and 238 U indicate that both higher $w_{238_{\text{U}}}$ and $\varepsilon_{234_{\text{Th}}}$ values are not found in the aquifer. Hence, high U activities along with high δ^{234} U values cannot be attained in the ground water for the transit distances of the aquifer.

The present study provides the first groundwater data set of both long-lived ²³²Th and short-lived ²³⁴Th and ²²⁸Th. The ²³²Th activities in the groundwater appear to be controlled by the solubility of thorianite. If we assume that Th in the water is in exchange equilibrium with a surface coating with a partition coefficient, it is found that Th is strongly adsorbed. Using the ²³⁴Th data, the amount of Th in the surface coating relative to that in the water (χ) is calculated to be 1×10^3 . The mean residence time in the water is 1 h for Th isotopes and 1 yr in the surface coating. For Ra in the vadose zone, the main input to the water is weathering. Calculated values of $\bar{x}_{226_{Ra}} \sim 300$ m, $\bar{x}_{228_{Ra}}\sim7$ m, and $\bar{x}_{224_{Ra}}\sim0.3$ m indicate that while the vadose zone input of ²²⁶Ra is significant in the upper portions of the aquifer, the Ra activities within the deeper aquifer are the result of local processes in the neighborhood of each sampling site. Data for ²²⁴Ra, ²²⁶Ra and ²²⁸Ra activities show that Ra adsorbs reversibly on the surface and the calculated amount of Ra on the surface coating relative to that in the water is ≈ 300 to 700

The measured activities of ²²²Rn in the water require input of a large (\sim 5%) fraction of the Rn produced in the solid phases as also shown by previous workers (Krishnaswami et al., 1982; Fleischer, 1982). This cannot be accounted for with the previous model, even considering contributions from a strongly adsorbing surface layer in equilibrium with the water. Ra siting on the surface of the grain has been already suggested in previous work (Krishnaswami and Seidemann, 1988), where high ²²²Rn loss "has to be attributed to preferential enrichment of ²²⁶Ra on grain surfaces or to the existence of U-Th accessory minerals in or around grain boundaries." However it is impossible to reach the required amount of ²²⁶Ra by precipitation of Ra within a system in steady-state. Considering that Th appears to be at saturation limit, we propose that the source of ²²²Rn is due to precipitation of Th at saturation and not in exchangeable sites in the surface coating. We suggest that ²³²Th and ²³⁰Th were deposited on the grain surfaces during an earlier stage of high weathering rates in the formation of the aquifer. Note that the process of dissolution, and precipitation onto the grains, of Th spatially separates ²³⁴U and ²³⁸U from the Ra and Th isotopes in the surface coating of the solid aquifer. This surface coating corresponds to $\sim 10\%$ of the total Th content of the rock and thereby provides the source of all daughters which are readily available to the water. Then, the effective value of Th_{sc}/Th_{water} is shown to be $\sim 5 \times 10^7$ which is a factor of 10^3 greater than in the case of exchangeable sites discussed above. It is also shown that Ra is still in exchange equilibrium with the surface coating while Th is adsorbed quasi irreversibly. The Th activities in the water are then controlled by the saturation condition. The Rn is lost by recoil or diffusion from the thin surface coating. This model predicts that ²³⁰Th and ²³²Th and

their daughter products should be removable from the surface coating by appropriate leaching techniques. From the data of Krishnaswami and Seidemann (1988), it appears that the surface layer must be formed even in granitic rocks. Therefore weathering during first exposure to the hydrosphere is considered to be responsible for forming the ThO₂ surface layer. We believe this model is broadly applicable in most geological circumstances. We cannot identify the specific rapidly weathered primary U-Th phases that are hypothesized to provide the ²³⁰Th and ²³²Th surface coatings.

We have found that Th is precipitated on surface coatings for time scales of $\sim 10^4$ years in the Upper Glacial Aquifer. However, in the underlying Magothy formation, the Th is essentially released. The water chemistry of these two aquifers is almost indistinguishable except for the drop (by a factor of ~ 5) of the O₂ content and a concurrent increase of Fe and Mn by many powers of ten. This decrease in O₂ is plausibly the result of bacterial action. It follows that under "nearly identical" conditions, species that are fixed on surface substrates can, with a minor change in chemistry (particularly O₂), release these precipitated species. This shows that the stability of retentive sites for storage is very sensitive to changes and these sites can readily become non-retentive, particularly if in situ reduction processes may take place. We have not established a causal mechanism for this.

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APPENDIX A

²³⁸U-SERIES:

• ${}^{238U}A_{w\infty} = \left(\frac{\rho_r}{\rho_w \cdot q}\right) \frac{\psi_{238U}}{\lambda_{238U}} {}^{238U}A_r$ • ${}^{234Th}A_{w\infty} = \left(\frac{\rho_r}{\rho_w q}\right) \frac{\left(\psi_{234Th} + \varepsilon_{234Th} \lambda_{234Th}\right)}{\lambda_{234Th} (1 + \chi_{234Th})} {}^{238U}A_r$ • ${}^{234U}A_{w\infty} = \left(\frac{\rho_r}{\rho_w q}\right) \frac{\left(\psi_{234Th} + \psi_{234Th} \frac{\lambda_{234U}}{\lambda_{234Th}} + \varepsilon_{234Th} \lambda_{234U}\right)}{\lambda_{234U}} {}^{238U}A_r$

•
$${}^{230\text{Th}}A_{w\infty} = \frac{\rho_r}{\rho_w q} \left(\frac{(w_{230\text{Th}} + \varepsilon_{230\text{Th}}\lambda_{230\text{Th}})}{\lambda_{230\text{Th}} (1 + \chi_{230\text{Th}})} \right)^{238\text{U}}A$$

•
$$^{226\text{Ra}}A_{w^{\infty}} = \frac{1}{1 + \chi_{^{226}\text{Ra}}} \left(\frac{\rho_r}{\rho_w q} \left(\frac{W_{^{226}\text{Ra}}}{\lambda_{^{226}\text{Ra}}} + \varepsilon_{^{226}\text{Ra}} \right)^{^{230}\text{Th}}A_r + F_{^{226}\text{Ra}} \chi_{^{230}\text{Th}}A_{w^{\infty}} \right)$$

•
$$^{230}\text{Rn}A_{w\infty} = \frac{\rho_r}{\rho_w q} \left(\varepsilon_{222\text{Rn}} + f_{222\text{Rn}} \left(\frac{w_{226\text{Rn}}}{\lambda_{226\text{Rn}}} + \varepsilon_{226\text{Ra}} + (1 - f_{226\text{Ra}}) \left(\frac{w_{230\text{Th}}}{\lambda_{230\text{Th}}} + \varepsilon_{230\text{Th}} \right) \right) \right)^{238\text{U}}A_r$$

²³⁰Th-Series:

$$\begin{array}{ll} \bullet & ^{232\mathrm{Th}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{W^{232}\mathrm{Th}}{\lambda_{232\mathrm{Th}} (1 + \chi^{232}\mathrm{Th})} \right)^{^{232\mathrm{Th}}}\!A_r \\ \bullet & ^{238\mathrm{Rn}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{W^{228}\mathrm{Ra}}{\lambda_{228}} \!+ \! \varepsilon_{228} \! \varepsilon_{\mathrm{Ra}} \!+ \! F_{228} \! \varepsilon_{\mathrm{Ra}} \! \frac{W^{232}\mathrm{Th}}{\lambda_{232} \! \mathrm{Th}} \right)^{^{232\mathrm{Th}}}\!A_r \\ \bullet & ^{238\mathrm{Rn}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{W^{228}\mathrm{Th}}{\lambda_{228} \! \mathrm{Th}} \!+ \! F_{228} \! \varepsilon_{\mathrm{Ra}} \! \left(\frac{W^{228}\mathrm{Ra}}{\lambda_{228} \! \mathrm{Ra}} \!+ \! \varepsilon_{228} \! \varepsilon_{\mathrm{Ra}} \! + (1 - f_{228} \! \varepsilon_{\mathrm{Ra}}) \frac{W^{232}\mathrm{Th}}{\lambda_{232} \! \mathrm{Th}} \right) \right)^{^{232\mathrm{Th}}}\!A_r \\ \bullet & ^{224\mathrm{Ra}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{W^{224}\mathrm{Ra}}{\lambda_{228} \! \mathrm{Th}} \!+ \! F_{228} \! \varepsilon_{\mathrm{Ra}} \! \left(\frac{W^{228}\mathrm{Ra}}{\lambda_{228} \! \mathrm{Ra}} \!+ \! \varepsilon_{228} \! \varepsilon_{\mathrm{Ra}} \! + (1 - f_{228} \! \varepsilon_{\mathrm{Ra}}) \frac{W^{232}\mathrm{Th}}{\lambda_{232} \! \mathrm{Th}} \right) \right)^{^{232\mathrm{Th}}}\!A_r \\ & ^{224\mathrm{Ra}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{W^{224}\mathrm{Ra}}{\lambda_{224} \! \mathrm{Ra}} \!+ \! \varepsilon_{224} \! \varepsilon_{\mathrm{Ra}} \right)^{^{228}\mathrm{Th}}\!A_r \\ & ^{224\mathrm{Ra}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{W^{224}\mathrm{Ra}}{\lambda_{224} \! \mathrm{Ra}} \!+ \! \varepsilon_{224} \! \varepsilon_{\mathrm{Ra}} \right)^{^{228}\mathrm{Th}}\!A_r \\ & ^{224\mathrm{Ra}}\!A_{w^{\infty}} = \frac{\rho_r}{\rho_w q} \left(\frac{M^{222}\mathrm{Ra}}{\lambda_{224} \! \mathrm{Ra}} \!+ \! \varepsilon_{224} \! \varepsilon_{\mathrm{Ra}} \right)^{^{228}\mathrm{Th}}\!A_r \\ & + \frac{F_{224\mathrm{Ra}} \chi^{228\mathrm{Th}}}{1 + \chi^{224\mathrm{Ra}}} \, ^{228\mathrm{Th}}\!A_w \\ & ^{228\mathrm{Th}}\!A_{w^{\infty}} + \frac{F_{224\mathrm{Ra}} \left(1 - f_{228\mathrm{Th}} \right) \left(\frac{\lambda_{228} \! \mathrm{Th}}{1 + \chi^{224\mathrm{Ra}}} \right)^{^{228\mathrm{Th}}}\!A_{w^{\infty}} \\ & + \frac{F_{224\mathrm{Ra}} \left(1 - f_{228\mathrm{Th}} \right) \left(\frac{\lambda_{228} \mathrm{Th}}\!A_{w^{\infty}} \!+ \frac{F_{224\mathrm{Ra}} \left(1 - f_{228\mathrm{Th}} \! \mathrm{K} \right) \left(1 - f_{228\mathrm{Ra}} \right) \left(\frac{\lambda_{228} \! \mathrm{Ra}}{\mathrm{Ra}} \! \mathrm{K} \right)^{^{228\mathrm{Ra}}}\!A_{w^{\infty}} \\ & + \frac{F_{224\mathrm{Ra}} \left(1 - f_{228\mathrm{Th}} \right) \left(\frac{\lambda_{228} \mathrm{Th}}{\mathrm{Th}} \! \mathrm{K}_{-1} + \lambda_{228\mathrm{Th}}} \mathrm{K} \right) \left(1 - f_{228\mathrm{Ra}} \right) \left(\frac{\lambda_{228} \mathrm{Ra}}{\mathrm{Ra}} \! \mathrm{K} \right)^{^{222}\mathrm{Ra}} \\ & + \frac{F_{224\mathrm{Ra}} \left(1 - f_{228\mathrm{Th}} \right) \left(\frac{\lambda_{228} \mathrm{Th}}{\mathrm{Th}} \! \mathrm{K}_{-1} + \lambda_{228} \mathrm{Th}} \mathrm{K} \right) \left(\frac{\lambda_{228} \mathrm{Ra}}{\mathrm{Ra}} \! \mathrm{K} \right) \left(\frac{\lambda_{228} \mathrm{Ra}}{\mathrm{Ra}} \! \mathrm{K} \right)^{^{222}\mathrm{Ra}} \\ & + \frac{F_{224\mathrm{Ra}} \left(1 - f_{228} \mathrm{Th} \! \mathrm{K} \right) \left(\frac{\lambda_{228} \mathrm{Th}} \mathrm$$

 $F_{i} = \left((1 - f_{i}) \left(\frac{{}^{\text{Th}} \hat{k}_{-1}}{{}^{\text{Th}} \hat{k}_{-1} + \lambda_{i} \xi} \right) + f_{i} \right)$