Probabilistic interpretation of indirect risk criteria and evaluation of rock burst hazard during exploitation of anthracite seams

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PROBABILITY INTERPRETATION OF INDIRECT RISK CRITERIA AND ESTIMATE OF ROCK-BURST HAZARD IN MINING ANTHRACITE SEAMS

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A method of risk calculation is proposed for determining the geodynamic hazard by indirect criteria. A new variant of nomogram is obtained for estimating the rock-burst hazard with respect to culm yield in drilling the blastholes in the anthracite seams being mined, which makes it possible to reject casual outbursts during production measurements and reveal hazardous situations that cannot be recorded by means of the existing procedures.

Geodynamic hazard, one-parameter criterion, frequency and interval estimates, culm yield, nomogram

STATEMENT OF THE PROBLEM

According to the Federal Law No. 116-FL "On production safety of hazardous industrial objects" [1] and the directives of State Mining Technical Inspection of Russia [2], the declarations of production safety should be developed at the mining enterprises; they are to contain the estimates of emergency risk and the analysis for efficiency of measures taken to prevent any emergencies. The most severe emergencies in the mines are those caused by geodynamic hazard manifestations, i.e., sudden outbursts and rock bursts [2, 3] leading not only to the great material losses, but also to the human victims.

By the Russian traditions, it is not accepted to consider industrial injuries and human death in terms of money, therefore, the estimate of risk for such cases should consist in probability calculation per one person a year and comparison with the admissible degree of hazard. In the international practice, the probability $10^{-5} - 10^{-6}$ per one person a year is assumed the most admissible. However, the lower estimates can be found, in particular, there is a suggestion to introduce the norm of order of 10^{-4} for Russia, which is connected with almost ten-fold distinction in level of the industrial injuries between Russia and the western countries.

Proceeding from the definition adopted [4], the risk from the geodynamic phenomena can be estimated by the formula

$$R = \frac{K_1 N_1 + K_2 N_2 + K_3 N_3 + \cdots}{S}, \tag{1}$$

where K_1, K_2, and K_3 is the number of injured employees due to the rock burst in the development workings, sudden outburst in the development workings, and rock burst in stopings, respectively; N_1, N_2, and N_3 is the number of corresponding dynamic phenomena; S is the number of employees at the enterprise (mine, pit, etc.). In many cases, it is worthwhile to estimate more differentiated, i.e., according to the type (force) of dynamic phenomena, mine-field sections, and separate professions (drift miners, faceworkers, powdermen, etc.). In these cases, the values of K_i, N_i, and S_i should characterize the phenomena under study, sections, or categories of employees. For some dynamic phenomena in hydraulic dumps and tailing ponds, not only miners, but also local residents should be reckoned among the endangering ones.

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We emphasize that all the values entering into expression (1) and other similar formulas are random. In most cases, distribution of the values of K_i type can be obtained from the emergency statistics. For lack of such information, distributions of K_i can be estimated on the basis of expert judgment or preliminary calculations. To take account of the real situation at a specific mining enterprise, the average statistical values should be corrected in the following way:

$$K_i' = K_i k_1 k_2 , \tag{2}$$

where k_1 is the efficiency coefficient of short-term prediction; k_2 is the coefficient characterizing industrial efficiency (observance of safety regulations, professional skills, etc.). Based only on the opinion of the specialists knowing this production, k_1 and k_2 can be estimated.

If the question were about the material damages, we could use mathematical expectation of risk

$$M(R) \approx \frac{\sum M(K_i)M(N_i)}{M(S)} . \tag{3}$$

However, when the question is about the human lives, we judge by those years, in which the number of emergencies and victims was the utmost. Therefore, we use the higher estimates: a) determination of one-sided confidence intervals which will not be exceeded for each separate year with the prescribed probability (0.9, 0.95, and 0.99); b) determination of the average maximal risks for a certain fixed period of time or all the time of mine operation. To do this, we employ extreme distributions.

Estimate b) seems to be more preferable, since it conforms better to the essence of the problem and accepted methods of risk estimate in the emergency situations. However, for lack of information about K and N distributions, the confidence intervals are calculated with more accuracy than the moments of extreme distributions. In many cases, the volume information on these indices is so insufficient that it is required to restrict ourselves to calculation of the average risk (mathematical expectation or median). Note that for large objects (branch or basin), we can use mathematical expectation of risk without essential errors.

Evaluation of N_i is a complicated problem. Approximate values can be obtained just the same way as for K_i on the basis of emergency statistics for the previous years. More accurate estimates are obtained from calculations for various conditions that take place in the object in question, for example

$$N_i = \sum_{k,j} l_{k,j} \, n_{k,j} , \tag{4}$$

where $l_{k,j}$ is the length of k-type workings under j-th geological conditions, $n_{k,j}$ is the probability of rock burst or outburst per unit of length of such type working under present conditions. Note that for large periods of time, $l_{k,j}$ are also random variables.

We can apply four basic groups of methods (and their combinations) to determine N_i: 1) direct measurements and calculations of the factors hazardous to human lives and health; 2) indirect measurements and calculations; 3) use of statistic data obtained in similar objects; and 4) the method of expert estimates. The fundamental difference between the methods of the second group and the methods of the first group lies in the fact that the measured or calculated values do not characterize the hazardous factors directly, but they are just indirectly connected with them.

The indirect criteria based on the correlations between the values of some parameters and the hazard of dynamic events are widely used at present to estimate the hazard associated with dynamic phenomena. In-situ observations, laboratory experiments or theoretical statements showed the existence of these relations are the basis of such criteria. Systematic measurements of these parameters in various situations made it possible to differentiate the obtained values according to the degree of hazard. The criteria of this type can be divided into one-dimensional and two-dimensional (which use the values of two parameters). In this paper, one-dimensional criteria are examined.

PROBABILITY INTERPRETATION OF INDIRECT RISK CRITERIA

1. Frequency Estimate of Probability. Figure 1 illustrates an example of one-dimensional criterion; as a matter of fact, regions *I–IV* express different probabilities of hazard. The problem consists in giving the numerical probability estimates to this criterion, proceeding from the measurements that were made during establishment of these boundaries. The experiment includes two possible results: *I* — the event (rock burst, outburst, etc.) occurred (or was to take place, but was stopped by some means) and 0 — the event did not take place, or *I* — hazardous situation and 0 — safe situation. Further we shall speak about the events of ● and ○ type, respectively. Using the introduced designations, the results of the experiments can be presented in the following form (Fig. 2). The cases *a* and *b* differ by the character of boundary region. In the first case, the information about the boundary region (between the last point of ● type and the first point of ○ type) is absent, in the second case, the probability of both events is nonzero.

Proceeding from the experiments, we assume it established that the probability of event $P(x)$ grows monotonically with an increase in value of x. Note that such dependence is not the only possible; in addition to increase or decrease of hazard with an increase in x, the criteria of other types can also exist (Fig. 2*c*, *d*). In the case of monotonic dependence, it is natural to assume that if at $x = x_1$ the event ● took place, it indicates that similar events are possible at all $x > x_1$ and gives no information about domain $x < x_1$. Analogously, the absence of ○ event for $x = x_2$ indicates the possibility of the absence of events at $x \le x_2$ and gives no information about domain $x > x_2$. Then, for any value of x, the probability of ● event can be found from the formula

$$P(x) = \frac{k}{k+m} = \frac{k}{n}, \tag{5}$$

where k is the number of experiments of ● type at $z \le x$; m is the number of experiments of ○ type at $z \ge x$; $n = k + m$ is the total number of experiments which give the information on the degree of hazard with this value of the parameter. Calculations of P and n for the situations a and b are shown in Fig. 3; it is evident, that in the case of a, it is impossible to estimate the probability in the boundary region $(n = 0)$.

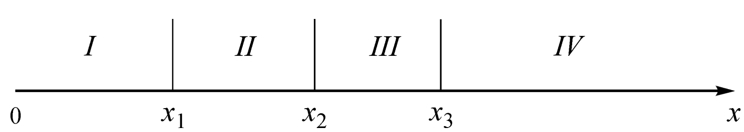

Fig. 1. Example of one-dimensional indirect criterion of hazard

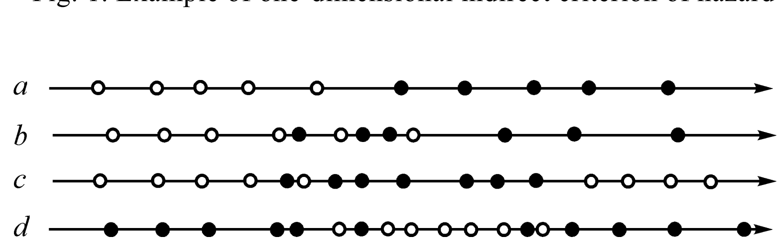

Fig. 2. Examples of experimental data location in determining the hazard criterion

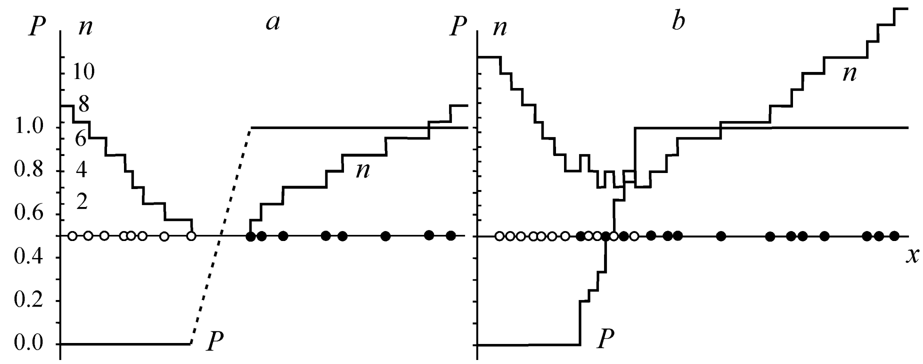

Fig. 3. Examples of dependences P and n on x for situations a and b

2. Probability Estimate with Regard for Measurement Error. The values of x are impossible to determine absolutely accurately due to the following reasons: first, the measurement errors exist; second, the value of x varies at different measurements (from experiment to experiment). At first, let us consider that x has normal distribution. Estimating the probabilities by formula (5), assume that k and m can take nonintegral values. None of probability distributions corresponds to this assumption (except the normal one which approximates binomial distribution at great values of n). However, the accepted conditions make it possible to extend the characteristics of binomial distribution (mathematical expectation, median, dispersion, and confidence intervals) to the nonintegral values of the parameters. The examples of calculations of P and n with regard for this assumption are presented in Fig. 4. It is easy to observe that the distinction between the cases a and b reduces; some information on the probability of the ● event in the boundary region for the case of a appears, which is based on a very small number of experiments $(n < 1)$. The form of curves $P(x)$ and $n(x)$ depends on the dispersion of x; as the dispersion increases, the curves become more smooth, and the dimension of the transition zone increases with $P \approx 0.5$.

It is possible to distinguish five basic distribution variants of the values of x.

1. The variant examined above for the additive deviations: x is distributed by approximately symmetric law (normal or close to it), and the standard deviation σ_x does not depend on the mean value of x_{av}.

2. The multiplicative deviations: x is distributed by approximately symmetric law and the standard deviation is in proportion to the mean value of $\sigma_x \approx k_v x_{av}$.

3. Complex multiplicative deviations: x is distributed by approximately symmetric law; the standard deviation is in proportion to the function of the mean value of $\sigma_x \approx kf(x_{av})$. In this case, when dispersion is low, it is efficient to change the variable leading to variants 1 or 2. For binomial distribution, the transition to variant 1 is realized by means of "arcsine transformation" [5].

4. Essentially asymmetric distribution of deviations.

Among asymmetric distributions, the geological characteristics and mechanical properties of rocks often have logarithmically normal and Weibull (Rozin – Rammler) distributions. They describe similar statistical models, and their distribution functions are close at $0.1 < F(x) < 0.9$; therefore, it makes sense to consider only one of them for small number of measurements. Later on logarithmically normal distribution with the constant value of the logarithmic standard deviation β will be used. If $\beta \le 0.15 - 0.2$, then the asymmetry coefficient does not exceed $0.5 - 0.6$ and the distribution is close to the normal one, $\sigma_x \approx \beta x_{av}$ (variant 2).

5. The variations of x change considerably from experiment to experiment, but the dependence on x_{av} is either absent, or it is impossible to describe by a single law. The situation, when the dependence σ_x on x falls apart into two sections and the transition from one law to another is associated with the growth of hazard is of great interest; here, the variation parameter is also the sign of hazard, therefore, it is helpful to use the values of x and σ_x to improve the efficiency of prediction.

Figures 4*c* and *d* show the examples of calculations of *P* and *n* for multiplicative deviations and logarithmically normal distribution of *x*. It is obvious, that the cases of normal distribution with $\sigma_x / \sigma = 0.2$ and logarithmically normal distribution with $\beta = 0.2$ slightly differ from each other. The feature of multiplicative deviations consists in displacement of point $P = 0.5$ to the side of smaller values of *x*; the neglect of multiplicative character of the error leads to hazard underestimate, respectively.

3. Interval Estimates of Probability. The estimates of probability of *P* obtained above are not reliable. For example, at $P = 0$, there will not be a single positive result in 3 experiments, we shall probably fail to obtain them at $P = 1/20$ and $P = 1/10$; at $P = 1/3$ the probability to have such result in three experiments is high enough $(8/27 \approx 0.3)$. Let us plot one-sided confidence intervals with significance levels $\varepsilon = 0.5$, 0.25, 0.1, 0.05, and 0.01 to estimate the reliability of *P*. The curves presented in Fig. 5 are to be understood in the following way: the probability that the interval $(0, P_\varepsilon)$ covers the ideal value of *P* is $1 - \varepsilon$. For example, at $x = 0$, the probability of finding *P* in the interval (0, 0.25) is 0.9, but in the interval (0, 0.32) it is 0.95. It is easy to note that the intervals indicated are wide enough, and the probabilities of falling the ideal value of *P* outside their limits are high enough (0.1 and 0.05, respectively). Thus, even at $x = 0$ (nominal value of the criteria quantity), we cannot be absolutely convinced that the probability of ● event is less than 0.25 or even 0.32.

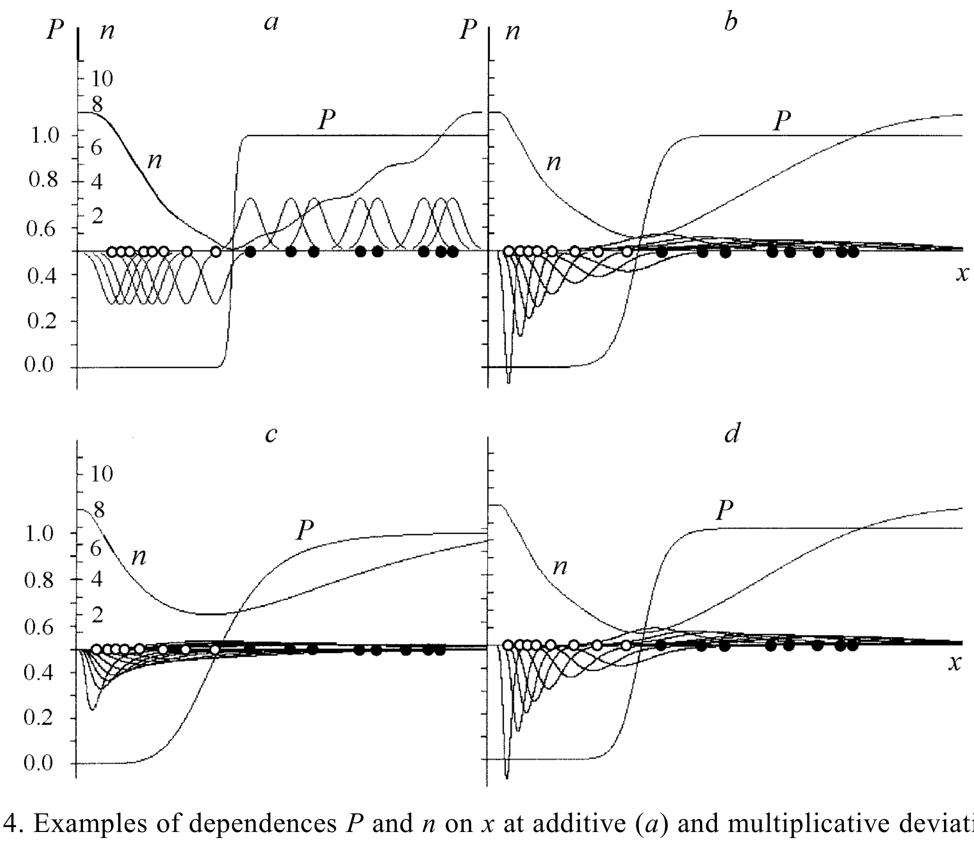

Fig. 4. Examples of dependences *P* and *n* on *x* at additive (*a*) and multiplicative deviations of *x*: *b* — $\sigma_x = 0.2x$; *c* and *d* are logarithmically normal distributions $\beta = 0.5$ and $\beta = 0.2$, respectively

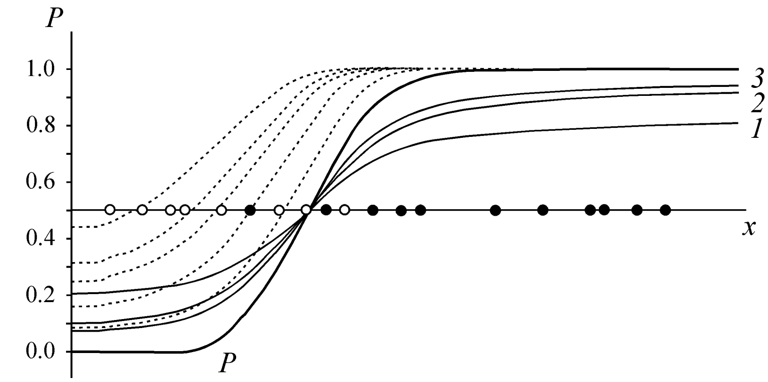

Fig. 5. Comparison of confidence intervals (dotted lines) and estimates (7) – (9) (curves *1*, *2* and *3*, respectively) in 19 experiments (8 events of ○ type and 11 events of ● type)

With increase in number of experiments (Fig. 6), the bound of confidence intervals approach considerably the curve $P(x)$, but do not merge with it. Even if $x = 0$, a certain probability of nonzero values of *P* remains. From Fig. 6 it is difficult to determine exact interval bounds, therefore, the corresponding values are given in Table 1.

According to the data from Table 1, even in 500 experiments with reliability of 95% we can only indicate that $P(0) < 0.01\%$ and consequently, $P = k / n$ is not the mean value of ● event probability; in any case, it is situated not in the center of the interval $(0, P_\varepsilon)$ covering the possible ideal values of *P*. It is necessary to have some other estimates of *P* closer to the center of the interval $(0, P_\varepsilon)$. However, this approach prevents us from obtaining other point estimates of *P* except the unreliable ones $P = k / n$. To obtain them, we shall understand the probability not as the frequency of event occurrence, but as the confidence degree. This point of view is cardinal during estimations of risk [6].

4. The Bayes and Minimax Estimates of Probability. The distinction between two approaches to the concept "probability" can be compared with experimentalist's position before and after performing the experiments. Prior to conducting the experiments, the problem is understood as an estimate of *nonrandom* value of x according to random results of experiments $x_1, x_2, ..., x_n$. After carrying out the experiments, the values of x_1, x_2 ,..., x_n are known and *nonrandom*, respectively; at the same time, it is possible to ascribe the values of $x_{av} = (x_1 + x_2 + ... + x_n) / n$, $(x_1 + x_n) / 2$, $x_{av} + \sigma_x$, $x_{av} - \sigma_x$, and so on to the measured value of x with different degree of confidence. If we understand the degree of confidence to these values as the probability, then x is interpreted as *the random* value.

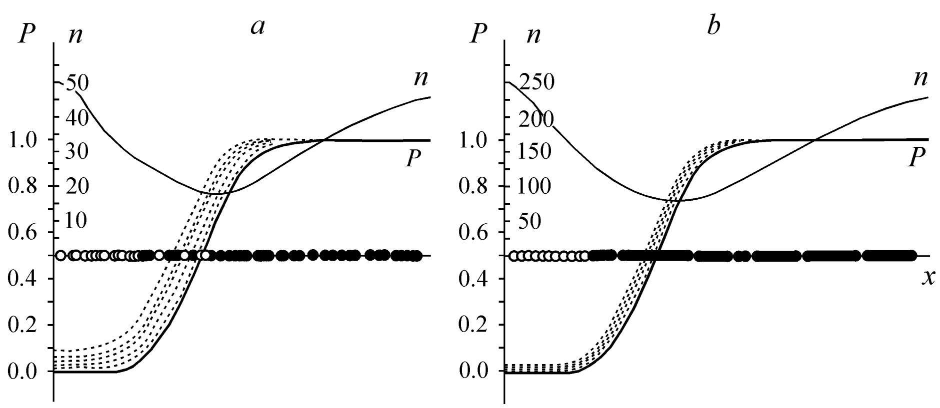

Fig. 6. Confidence intervals in 100 (50 events of ○ type and 50 events of ● type) and 500 (250 events of ○ and ● types each) experiments (the deviations are distributed according to logarithmically normal law, $\beta = 0.2$)

TABLE 1. Upper Limits of Confidence Intervals P_ε (%) at $x = 0$

ε	20 experiments (10 events of ○ and ● types each)	50 experiments (25 events of ○ and ● types each)	200 experiments (100 events of ○ and ● types each)	500 experiments (250 events of ○ and ● types each)
0.50	6.70	2.70	0.69	0.28
0.25	13.0	5.40	1.40	0.55
0.10	20.7	8.80	2.30	0.92
0.05	26.0	11.3	3.00	1.20
0.01	37.0	16.9	4.50	1.80

However, the point estimates of the measured value are not the only ones according to the confidence degree to them. They depend on the estimation of losses, when the value of desired parameter θ (the most widespread forms of the loss function are quadratic $(\theta_{\text{true}} - \theta_{\text{estimation}})^2$ and absolute $|\theta_{\text{true}} - \theta_{\text{estimation}}|$) is chosen incorrectly, and the choice of the decision rule in comparing the risks of inaccurate estimates of θ.

The basic approaches we use are the minimax and Bayes approaches. According to the first approach, such a decision rule is chosen, for which the maximal value of risk is equal to its least possible maximum. The essence of the minimax approach is to take into account the worst result (when choosing θ) and minimize the maximum of potential risk. The essence of the second approach consists in considering the preliminary information about the probability of possible values of θ and consequently, the choice of the rule is made in favor of that one which ensures the minimal risk. The decision rules are called admissible ones, if there is no other rule which ensures the smaller values of the risk function at any values of θ. According to the basic theorems of decision theory, the solution should be the Bayes solution to become admissible; if the Bayes decision function is single, then it is admissible; if the minimax solution is single, then it is admissible; if the solution is admissible and the risk function for this solution is constant at all possible values of θ, then it is the minimax solution. These theorems do not relieve us of subjectivism which is connected with the Bayes approaches. *A priori* distribution of $p(\theta)$ is assigned by the researcher and depends on the initial concepts (or results of the previous experiments); in many cases, the range of possible values of the desired parameter is also established by the researcher. For lack of information, we usually take $p(\theta) = \text{const}$ at any values of θ, which makes *a priori* distribution the only one, but the problem how to justify this assumption remains.

The desired parameter is the probability *P* of ● events at the prescribed *k* and *n*. *A priori* knowledge about *P* is absent (all the quantitative information is used when estimating *P*), therefore, *a priori* probabilities of the number of cases of α ● and β ○ events are equal. The main difficulty is how to determine the value of *a priori* information.

A priori density of *P* distribution has the form of beta-distribution with the parameters α and β. Therefore,

$$p(P) = P^{\alpha-1}(1-P)^{\beta-1}\,, \tag{6}$$

a posteriori density has the same form with the parameters $\alpha + k$ and $\beta + n - k$, respectively. At the quadratic loss function and the values of *a priori* probabilities $\alpha = \beta = 0.5\sqrt{n}$, the risk does not depend on θ, therefore, the minimax estimate of probability is

$$P_M = \frac{k + 0.5\sqrt{n}}{n + \sqrt{n}}\,. \tag{7}$$

The second estimate to be used is mathematical expectation of the confidence degree (or, to be more exact, the likelihood function) to the values of *P*; the distribution of the confidence degree *p* has the form of beta-distribution with the parameters $k+1$ and $n-k+1$; at $\alpha=\beta=1$, this is the Bayes estimate that makes up

$$P_B=\frac{k+1}{n+2}. \tag{8}$$

The third estimate is the median of confidence degree distribution to the values of *P*

$$P_{med}\approx\frac{k+0.693}{n+2\cdot 0.693}. \tag{9}$$

Before we compare (7) – (9), we pay attention to the following circumstance. When in process of direct measurements the value of some variable *x* is found, then whatever decision rule was chosen, the estimates obtained (for example, $(x_1+x_2+...+x_n)/n$, $(x_1+x_n)/2$, $x_{n/2}$) bring us to the close results. Only certain *a priori* information can change them considerably, but it is absent here. In the case of estimate of probability *P*, when there are no results of ● type $(k=0)$ in *n* experiments, the picture is different: all our estimates substantially differ from $P=k/n=0$. Moreover, to some extent they contradict to the common sense — for lack of ● events observed in the experiments, they indicate nonzero event probability (and quite sufficient, for example, $P=0.11-0.15$ at $n=5$ and $P=0.06-0.12$ at $n=10$). On the other hand, these estimates make it clear that it is possible to prove the existence of any fact experimentally, but one cannot completely disprove it (without additional logical arguments).

When choosing one of the estimates (7) – (9), formula (9) is the most incorrect, since it mixes up the quadratic and absolute estimates of deviations. Minimax estimate (7) is the most acceptable for the problems of safety evaluation, but it has one considerable disadvantage. Too much importance is attached to the values of α and β characterizing only the absence of *a priori* information; but the method of determining assumes the growth of their values with increase in number of experiments $\alpha=\beta=0.5\sqrt{n}$. Therefore, we dwell on estimate (8), since it corresponds to a simple idea, i.e., it is required to add two hypothetical experiments with the opposite results to the experimental data in order to obtain the ideal value of *P*.

Compare the estimates of *P* with the confidence intervals by formulas (7) – (9). One can note (Fig. 5) that formulas (8) and (9) correlate with the confidence intervals better than (7) at any number of experiments. If $k=0$, then the upper interval bound with the confidence probability $\varepsilon=0.5$ (i.e., approximately the center of the region, where the ideal value of *P* can lie) is close to estimates (8) and (9). When the values of k/n are high, such coincidence is absent due to characteristics of the confidence intervals of binomial distribution, but as *n* increases, the range of values of k/n, where curves (8) and (9) are close to the bound of confidence interval, also increases. This comparison is an additional argument in favor of formula (8).

The point estimates (as well as the interval ones) indicate the high risk of ● event at any values of *x* including $x=0$. However, all the estimates obtained up to now did not consider one very important circumstance: when *x* decreases independently of the number of experiments performed, the probability of ● decreases, and vice versa. In fact, according to the calculations for the case shown in Fig. 7, $P(x_1)=P(x_2)$, but $P(x_2)$ differs from $P(x_3)$ only by existence of the point with measurement of ○ type between them. At low dispersion of x, in accordance with formula (8) the probability ratio is $P(x_2)/P(x_3)=(3/4):(2/3)\approx 1.125$. The fact that the probability of ● event (if criterion x has physical foundations) should decrease with reduction in *x* is obvious, i.e., $P(x_3)>P(x_2)>P(x_1)$.

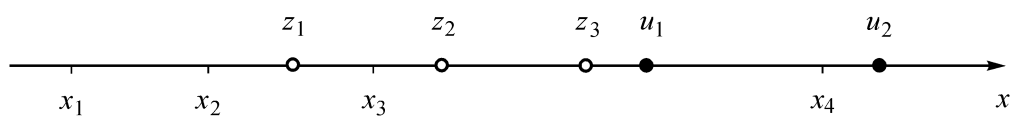

Fig. 7. Example for location of the values of parameter x in base experiments and control measurements

5. Estimates of Probability with Regard for *A Priori* Dependence of *P* on *x*. Let us correct definition (5) of probability $P(x)$ in order to solve the stated problem. Then k was defined as a number of experiments of • type at $x \le z$, and m — as ○ type when $x \ge z$. However, if we know that hazard increases monotonically with growth in x, the experiments give different information about the hazard level at the given parameter value, i.e., when calculating the probability $P(x)$, different significance should be attached to different base experiments. Introduce the function characterizing an increase in degree of safety (probability of event ○) when x decreases. Designate it as $L(z, x)$, where z is the parameter of the base experiment, x is its value measured with the use of criterion. Mathematically, the function L represents the “distance” and should increase monotonically as the difference $z - x$ or the ratio z / x increases. One of the most important problems of the probabilistic approach is to establish the form of this function, since it is possible to propose a number of such functions: $z - x$, $(z - x)^2$, z / x, $(z / x)^2$, $\ln(z / x)$, etc.

Before we discuss the form of L, note that both x and z are the random variables, and it is impossible to establish their exact position. Therefore, we introduce the function $G(z, x)$ characterizing the distance between two random points with the prescribed distribution functions

$$G(x, z) = A(Dx, Dz) \iint\limits_{z>x>0} L(x, z) f_{\bar{x}}(x) f_{\bar{z}}(z) dx dz, \tag{10}$$

where $\bar{x}$ and $\bar{z}$ are the sample mean values of x and z; $f_{\bar{x}}(x)$ and $f_{\bar{z}}(z)$ are the distribution densities of x and z; $A(Dx, Dz)$ is the normalized factor depending on the dispersions of x and z.

We introduce similar functions $G^*(u, x)$ and $L^*(u, x)$ for the points located in the influence zone of • type events (for example, x_4 in Fig. 7). Since the influence zones of points of ○ and • types spread in different directions, then $L^*(u, x) = L(u, x)$ and similarly, $G^*(u, x) = G(u, x)$.

Note the properties of $G(z, x)$:

1. $G(z, x)$ is monotonically increasing (from z at $x = \text{const}$) or decreasing function (from x at $z = \text{const}$).

2. $G(z, z) = 0.5$, and according to this

$$A = \frac{0.5}{\iint\limits_{z>x>0} L(x, z) f_{\bar{z}}(x) f_{\bar{z}}(z) dx dz}. \tag{11}$$

The essence of the proposed normalization method is that the probabilities of obtaining the greater or smaller values of x relative to the ideal value are equal, i.e., x is the distribution median of $f(x)$.

The median is the scale parameter in both types of distribution (normal and logarithmically normal) and coincides with mathematical expectation of x or $\ln x$.

3. The form of $L(z, x)$ depends on the type of distribution $f(x)$ [1)]. This dependence is incapable of determining $L(z, x)$, but makes it possible to restrict in each specific case the class of functions, from which the choice is made.

[1)] It is obvious that the distribution densities $f(x)$ and $f(z)$ may have different parameters, but the functions themselves relate to the same class.

Consider the most widespread case of relation between x and z: $|z-x| >> \max(\sigma_x, \sigma_z)$, where σ_x and σ_z are the root-mean-square deviations of x and z.

1. Additive deviations: x and z have normal or close to normal distribution and $\sigma_x = \sigma_z$ at any $\bar{x}$ and $\bar{z}$. In this case, it is natural to choose linear function $L(z,x) = z - x$ or quadratic function $L(z,x) = (z-x)^2$.

For the quadratic function,

$$G(\bar{z},\bar{x}) = 0.5\frac{\iint\limits_{z>x>0}(z-x)^2 f_{\bar{x}}(x) f_{\bar{z}}(z)\,dxdz}{\iint\limits_{z>x>0}(z-x)^2 f_{\bar{z}}(x) f_{\bar{z}}(z)\,dxdz} \approx 0.5\left(\frac{\bar{z}-\bar{x}}{\sigma}\right)^2; \tag{12}$$

for the linear function,

$$G(\bar{z},\bar{x}) = 0.5\frac{\iint\limits_{z>x>0}(z-x)\, f_{\bar{x}}(x) f_{\bar{z}}(z)\,dxdz}{\iint\limits_{z>x>0}(z-x)\, f_{\bar{z}}(x) f_{\bar{z}}(z)\,dxdz} \approx 0.5\sqrt{\pi}\left(\frac{\bar{z}-\bar{x}}{\sigma}\right). \tag{13}$$

2. Multiplicative deviations: x and z have normal or close to normal distribution; σ_x and σ_z are proportional to $\bar{x}$ and $\bar{z}$, and the variation coefficient $k = \sigma / x = \text{const}$, respectively. At first, consider the same functions $L(z,x)$, for example, the quadratic one, then

$$G(z,x) \approx 0.5\left(\frac{\bar{z}-\bar{x}}{k\bar{z}}\right)^2 = \frac{1}{2k^2}\left(1-\frac{\bar{x}}{\bar{z}}\right)^2.$$

In accordance with this, at $x << z$ the function G has constant value $0.5/k^2$, which contradicts to the initial prerequisites when forming $G(z,x)$. In this case, it is most reasonable to choose $L(z,x) = z/x$ or $L(z,x) = (z/x)^2$.

For $L(z,x) = z/x$,

$$G(\bar{z},\bar{x}) = 0.5\frac{\iint\limits_{z>x>0}\frac{z}{x} f_{\bar{x}}(x) f_{\bar{z}}(z)\,dxdz}{\iint\limits_{z>x>0}\frac{z}{x} f_{\bar{z}}(x) f_{\bar{z}}(z)\,dxdz} \approx \frac{\bar{z}/\bar{x}}{1+\frac{2}{\sqrt{\pi}}k}; \tag{14}$$

for $L(z,x) = (z/x)^2$,

$$G(\bar{z},\bar{x}) = 0.5\frac{\iint\limits_{z>x>0}\left(\frac{z}{x}\right)^2 f_{\bar{x}}(x) f_{\bar{z}}(z)\,dxdz}{\iint\limits_{z>x>0}\left(\frac{z}{x}\right)^2 f_{\bar{z}}(x) f_{\bar{z}}(z)\,dxdz} \approx \frac{(\bar{z}/\bar{x})^2}{1+\frac{4}{\sqrt{\pi}}k}. \tag{15}$$

If $z >> x$ and $k << 1$, then for the linear function, $G \approx x/z$, and for the quadratic function, $G \approx (x/z)^2$, respectively. On the basis of the examples analyzed we can conclude that when the choice of L is correct, the function G has the same form as the function L (up to the constant factor).

3. Logarithmically normal distribution with the constant value of logarithmic standard deviation $\beta < 1$. By analogy with the case of additive deviations, it is natural to choose the functions $L(z,x) = \ln(z/x)$ or $L(z,x) = \ln^2(z/x)$, then

for the first function,

$$G(\bar{z},\bar{x})\approx 0.5\frac{\ln\dfrac{\bar{z}}{\bar{x}}}{\beta}\ ; \tag{16}$$

for the second function,

$$G(\bar{z},\bar{x})\approx 0.5\frac{\ln^2\dfrac{\bar{z}}{\bar{x}}}{\beta^2}\ . \tag{17}$$

The above-mentioned condition of coincidence for the functions *G* and *L* is fulfilled, however, a considerable disadvantage of logarithmic functions consists in weak dependence of probability on *x*. Therefore, we consider the same functions $L(z,x)=z/x$ or $L(z,x)=(z/x)^2$, as in the case of multiplicative normal deviations, and check the form of *G* in the cases in question.

For $L(z,x)=z/x$,

$$G(\bar{z},\bar{x})=0.5\frac{\iint\limits_{z>x>0}\frac{z}{x}f_{\bar{x}}(x)f_{\bar{z}}(z)dxdz}{\iint\limits_{z>x>0}\frac{z}{x}f_{\bar{z}}(x)f_{\bar{z}}(z)dxdz}\approx\frac{\dfrac{\bar{z}}{\bar{x}}}{1+\dfrac{4}{\sqrt{\pi}}\beta}\ ; \tag{18}$$

for $L(z,x)=(z/x)^2$,

$$G(\bar{z},\bar{x})=0.5\frac{\iint\limits_{z>x>0}\left(\frac{z}{x}\right)^2 f_{\bar{x}}(x)f_{\bar{z}}(z)dxdz}{\iint\limits_{z>x>0}\left(\frac{z}{x}\right)^2 f_{\bar{z}}(x)f_{\bar{z}}(z)dxdz}\approx\frac{\dfrac{\bar{z}}{\bar{x}}}{1+\dfrac{8(1-\beta^2)}{\sqrt{\pi}}\beta}\ . \tag{19}$$

Thus, the functions *G* and *L* coincide up to the constant factors, therefore, we can use $L(z,x)=z/x$ and $L(z,x)=(z/x)^2$ at logarithmically normal distribution of *x*. We are to note a diverse role of the variation parameters in the cases of $L=(z-x)^n$ on the one hand, and $L=(z/x)^n$ on the other hand. In the first situation, the standard deviation σ takes the part of normalized factor: an increase of σ by a factor of 2 leads to a decrease in *G* by a factor of 2^n. In the second situation, even a considerable increase in variation parameters *k* and β leads only to a small decrease in value *G*, but changes the relation between the functions *G* and *L*.

These reasonings make it possible to restrict the classes of functions, but do not permit us to establish their parameters (first of all, the exponent).

There are two ways to determine the exact form of the functions. First, for some criteria, their connection with the real hazard source is known, at least, approximately. The type of this connection can show the form of the functions *L* and L^*. Second, the method of expert estimates makes it possible to calculate the probability of rock-burst hazard according to culm yield.

Figure 8 presents the examples for calculations of probabilities of ● type events for different variants of deviations and functions *L*. It is easy to observe that taking account of *a priori* dependence of *P* on *x* shows considerably smaller probabilities of ● type events at small values of *x*. Even 20 experiments (10 events of ○ type and 10 events of ● type) is enough to demonstrate that at the nominal value of the measured quantity $(x=0)$, the probability of ● event is about 10^{-4} or 10^{-5}.

1. Initial Data. The method of local estimate of rock-burst hazard according to culm yield in mining the anthracite seams was chosen as an example of using the proposed procedure [7, 8]. The initial data for construction of this criterion were 26 experiments carried out by the researchers of the All-Russian Scientific-Research Institute of Surveying in the "Yuzhnaya", "Maiskaya", and "Zapadnaya-Kapital'naya" mines of "Rostovugol" Enterprise (Table 2); 9 measurements fell on the rockburst-hazardous conditions (categories I and II) and 17 measurements fell on the conditions which do not require actions to prevent rock bursts (categories III and IV).

The investigations are based on the following assumptions: first, the rock-burst hazard monotonically increases with the increase in culm yield; and, second, the more is the blasthole depth, the greater is the boundary value corresponding to the rockburst-hazardous situation. These assumptions rely on the numerous measurements made in the mines in the USSR and Germany, as well as on the laboratory and mine tests.

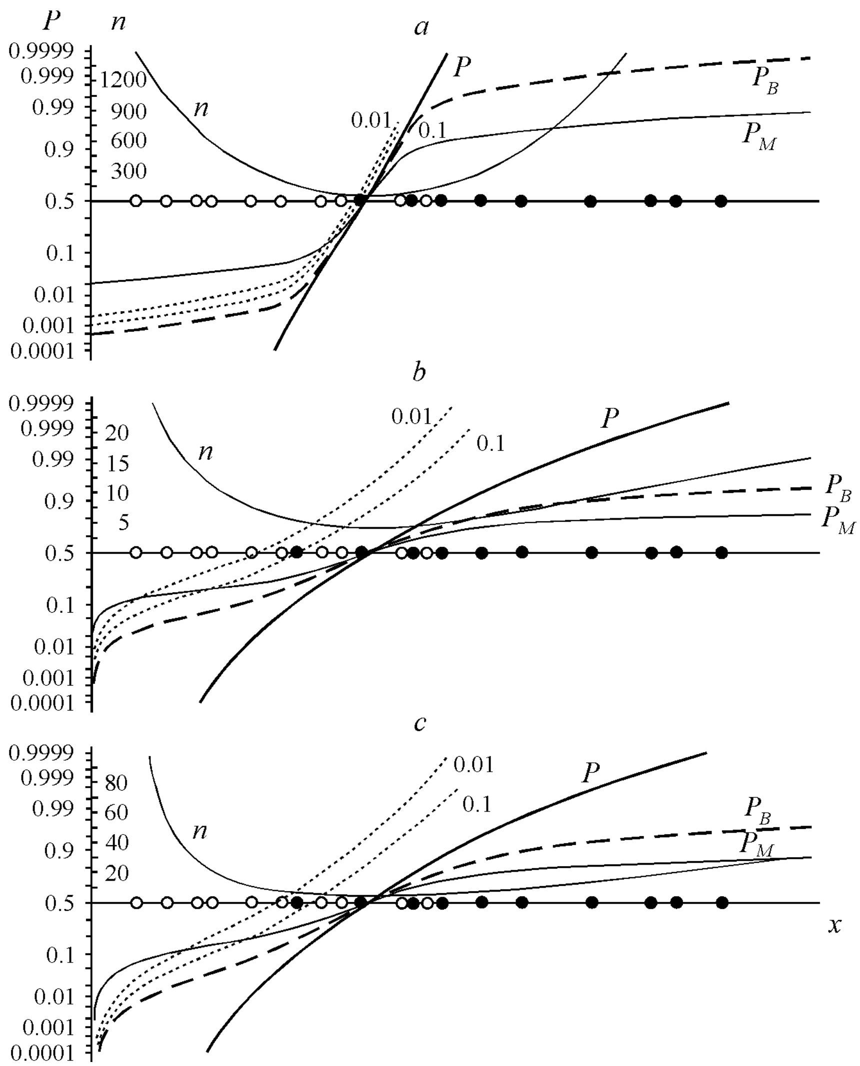

Fig. 8. Examples of probability calculation with regard for *a priori* dependence of P on x: a — additive deviations $L=(z-x)^2$; b — logarithmically normal deviations $L=z/x$; c — logarithmically normal deviations $L=(z/x)^2$

2. Causes of Culm Yield Variation. Assume that the rock-burst hazard criterion falls into a series of one-dimensional criteria (for each meter of the blasthole). However, such separation does not correspond to the essence of criterion, since we do not know to which meter of the blasthole walls relates the failed coal formed during drilling of the next blasthole section. It was shown in [8] that in most cases, the depth of die penetration begins increasing sharply (or reaches the maximum) by 1 – 2 m closer to the blasthole mouth than the culm yield. Large fractions formed at the boundary of the fracture zone may probably enter the blasthole after coal failure when drilling the next meters (Fig. 9). The data of acoustic emission also indicate this fact [8, 9]. The same cause can apparently determine frequently encountering decrease in culm yield and actual blasthole diameter in the last meters of drilling (jamming).

TABLE 2. The Measurement Results of Culm Yield during Blasthole Drilling in Anthracite Seams [8]

No.	Hazard	Thickness of seam m, m	Culm yield p (l/m) from each meter of the blasthole									
			Depth l, m									
			1	2	3	4	5	6	7	8	9	10
1	●	1.5	1	2	2.5	3	5.5	6.5	10.5	27.5	40	60
2	○	1.05	1.5	2.5	9	18.5	26	28.5	35	35.5	36	36
3	○	1.0	1.5	3.3	3.5	14	27.5	29.5	31.5	–	–	–
4	○	1.3	1	3.5	7	12.5	13	13.5	13	12	12.5	10
5	○	1.0	0.6	1.2	1	3.2	4	5	7	–	–	–
6	○	1.3	1	2	3.5	7	–	–	–	–	–	–
7	○	1.3	2	5	10	12.5	–	–	–	–	–	–
8	○	1.25	1.5	5	5	7.5	15	–	–	–	–	–
9	○	1.1	0.6	1.2	1.8	3.2	4	5	6	–	–	–
10	●	0.95	3	12	18	–	–	–	–	–	–	–
11	○	0.95	3.5	7.5	8.8	–	–	–	–	–	–	–
12	●	1.0	3.5	6	24	–	–	–	–	–	–	–
13	●	1.6	2	3	4.5	5	6.5	7	10.5	20	40	55.5
14	●	1.15	2.5	3	5.5	10	12	20.5	58	60	20.5	15
15	●	1.0	10	11.5	12	20	42.5	30	21	12	–	–
16	●	1.2	1	2.5	2.5	18.5	20	52	–	–	–	–
17	○	1.0	3	5.5	9	–	–	–	–	–	–	–
18	○	1.4	1	1.8	2.7	3.7	5	6	8.5	–	–	–
19	●	1.0	3	22	33	–	–	–	–	–	–	–
20	○	1.4	1	2	4	6.5	–	–	–	–	–	–
21	○	1.4	1	2	5.5	7	–	–	–	–	–	–
22	○	1.4	1.5	2.3	12.5	16.5	–	–	–	–	–	–
23	○	1.4	1	2	2.5	3.5	4.5	5.4	6	7.5	8	9.5
24	●	1.3	0.5	1	1.5	3	15	48.5	26	–	–	–
25	○	1.25	1	1.5	14	20.5	20.5	19.5	10.5	–	–	–
26	○	1.6	2	2.5	6	6.5	8	10	12.5	–	–	–

N o t e : hazard and thickness of seams are determined by B. N. Yavorskii

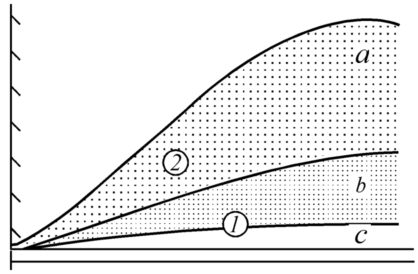

Fig. 9. Scheme of crushing: *a*, *b*, *c* are the regions of coarse, medium, and fine crushing, respectively. Culm from section *2* enters the blasthole only after failure of section *1*

Besides the failure incompleteness near the boundary of the crushed zone, the physical essence of this phenomenon consists in the fact that coarse crushing makes the granulometric composition more homogeneous (in the logarithmic scale) than fine crushing [10]. Crushed materials with homogeneous granulometric composition are characterized by higher loosening factors and rates of dilatancy at small shears $(\gamma \sim 0.02-0.1)$ as compared with materials with inhomogeneous granulometric composition [11]. Therefore, small movements of the pieces relative to each other lead to the fast increase in volume of the failed coal in region *a* (Fig. 9) and appearance of compressive stresses preventing the culm from entering the blasthole.

Failure and jamming depend on the parameters of fracturing pattern in the coal seam, rate of drilling, and a number of random unpredictable circumstances. Consequently, it is expedient to examine the measurement results by a single blasthole not as a data set but as an integration and choose the maximum value with allowance for the blasthole depth. We should bear in mind that due to nonuniformity of the stress state of rock mass, as well as irregularity of the failure and jamming processes, the errors in both directions are possible.

Firstly, a large section of the blasthole can be subjected to jamming, and one very large (random) number (Fig. 10, curve *1*) can be among the volumes of culm entered the blasthole even at the low stress concentration. When drilling is not accompanied by sharp vibrations of the forward force, shocks, etc., we should repeat the check drilling two or three times more, and if the hazardous values are note achieved, the first result is to be rejected.

Secondly, the failure can be uniform enough, and the jamming can manifest itself weaker than usual. In this case, the relation $p(l/m)$ will also have a smooth character (Fig. 10, curve *2*). In hazardous situation, a number of measurements may approach to the boundary of category II, but none of them lies above it, i.e., formally, the section can be considered safe. In such cases, it is useful to repeat measurements under existing rules, however, unlike the first situation, it is difficult to describe the formal criteria of selection for the second situation even in general form without probabilistic approach. The question about isolating the method for such situations will be discussed below.

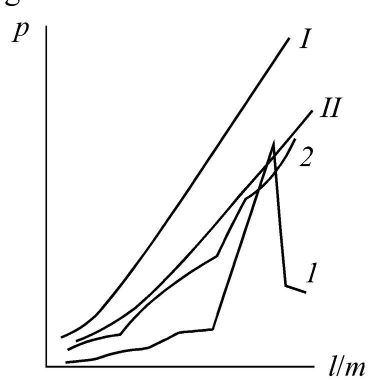

Fig. 10. Influence exerted by irregularity of jamming on the estimate of rock-burst hazard

Thirdly, even having excluded extreme cases, the influence exerted by variations of the stress state of rock mass, failure, and the step of jamming will affect the results of rock-burst hazard estimate. For example, not only dispersion of the jamming step, but also its mean value affect the results of estimate. The smaller is the mean value, the closer to the blasthole mouth the great values of p will be obtained, and consequently, the conclusion about the necessity for taking measures to prevent rock bursts is more probable. On the contrary, the larger is the average step of jamming, the greater is the distance from the blasthole mouth, where the great values of p manifest themselves, and the more failed coal is not extracted from the blasthole; as a result, the conclusion about the absence of direct hazard is more probable.

We can partially take into account another source of errors when processing the data, i.e., if the thickness of the coal seam is small, the segment 1 m long covers the sections with substantially different stress state. This leads to distortion of the curve $p(l/m)$ and underestimate of $P_{\max}$. For example, if two great and close values are situated next to each other, then the true maximum lies between them; another example — in the sections of rapid increase in culm amount, the growth in $p(l/m)$ actually begins before the first great value. In our opinion, in such cases, we should not reduce the distances between the reference points (this will lead to the growth in error, complexity of measurements, and even stronger influence of the jamming step on their results), but calculate the intermediate values of change in culm yield along the blasthole dp/dl by means of quadratic approximation

$$p_{1\frac{1}{2}} = \frac{3p_1 + 6p_2 - p_3}{8},$$
$$p_{i+\frac{1}{2}} = \frac{-p_{i-1} + 9p_i + 9p_{i+1} - p_{i+2}}{16}, \qquad 1 < i < k-1, \tag{20}$$
$$p_{k-\frac{1}{2}} = \frac{3p_k + 6p_{k-1} - p_{k-2}}{8},$$

where k is the blasthole length, m.

The example of using formulas (20) is given in Table 3.

If we introduce this method of calculations into safety regulations [7], then we should strictly specify the seam thickness, for which it is valid. The exact boundary values of thickness are difficult to determine (judging by the steepness of curves in the nomograms, they should be approximately 0.5 – 1 m for bituminous coal and 1 – 1.5 m for anthracite[2]).

TABLE 3

	Initial data													
Distance from the blasthole mouth, m	1		2		3		4		5		6		7
Culm yield, l/m	1		3		12		6		24		24		12
	Supplementary data													
Distance from the blasthole mouth, m	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
Culm yield, l/m	1	1	3	8	12	8.5	6	14.5	24	26.5	24	19.5	12

[2] Note that the calculated intermediate values can change a casual outburst into the regular change of the curve $p(l/m)$ and bring to hazard overestimate.

3. Choice of the Maximum Value along the Blasthole. First of all, it is required to choose the maximum values in the series obtained during drilling of one blasthole in order to apply the proposed procedure to the data of Table 2. We can suggest a set of alternative approaches to perform this operation and the criteria to compare them, but it is difficult to choose the best one. In this article, the choice is made with the help of the base line. It demarcates the hazardous and non-hazardous conditions in the best way and describes approximately the trend of the curve $p(l/m)$ at the boundary of these conditions. The form of the curve was chosen on the basis of 26 experiments (Table 2) and the type of nomogram used for the bituminous coals of lower metamorphism degree [7]. As a result, the equation

$$y = a\left(\frac{l}{m} + b\right)^n, \qquad (21)$$

was chosen, where a, b, and n are the desired coefficients.

The following criteria were used to compare the sets of their values: k_0 is the number of • type experiments lying below the base line (k_0 should be equal to zero according to safety regulations); k_1 is the number of ○ type experiments lying above the base line (the minimum value of k_1 for curve *2* was 2 from experiments Nos. 22 and 25; curves with $k_1 > 2$ were excluded from further consideration); n_0 is the number of experiments of • type, where two and more points lie above the base line; n_1 is the total number of points lying above the base curve in the experiments of • type; s_0 is the total index of relative deviations of $P_{\max}$ upward for the experiments of • type (s_0 was calculated from the formula $s_0 = \sum_{\bullet} \ln \max_l p(l)/y$); s_1 is the total index of relative deviations of $P_{\max}$ (upward — for the experiments of • type and downward — for the experiments of ○ type; s_1 was calculated from the formula $s_1 = \sum_{\bullet} \ln \max_l p(l)/y + \sum_{\circ} \ln \max_l y/p(l))$. Two variants have been chosen with the help of the criteria enumerated. As is seen from Fig. 11, the base lines are located very close to each other, therefore, the choice of any of them should not considerably change the probabilistic estimates. However, the further calculations were performed with both selected curves: $y = (l/m + 1.7)^{1.85}$ and $y = 1.15(l/m + 1.7)^{1.75}$ in order to confirm this statement.

4. Estimates of Probabilities for Arising the Hazardous Situations. As the experiments carried out in the “Yuzhnaya”, “Maiskaya”, “Zapadnaya – Kapital’naya”, “Komsomol’skaya”, and “Yanovskaya” mines have shown [8, 12], the scattering in the values of *P* has the multiplicative character with the coefficient of variation approximately 15 %. In this case, the deviations in the direction of overstating *p* exceed the deviations towards the lesser side, i.e., the values of *p* have asymmetric distribution. Therefore, it was assumed that the variation of *p* has logarithmically normal distribution with the logarithmic standard deviation $\beta = 0.15$. The form of function $L(z, x)$ was determined by means of expert estimates. Based on the opinion of experts in rock bursts, it was assumed $L = (z/x)^2$.

The values of probabilities P, P_B, and P_M for both forms of the base lines calculated from formulas (19), (5), (7), and (8) are very close. As an example, the calculation results for the base line $y = (l/m + 1.7)^{1.85}$ are presented in Fig. 12 by analogy with Fig. 8. However, similarity of the curves $P(P_{\max}/y)$ obtained does not guarantee the same proximity of the curves $p(l/m)$ at the specified values of *P*, since in Fig. 12 the axis of ordinates has a scale which conceals slight distinctions.

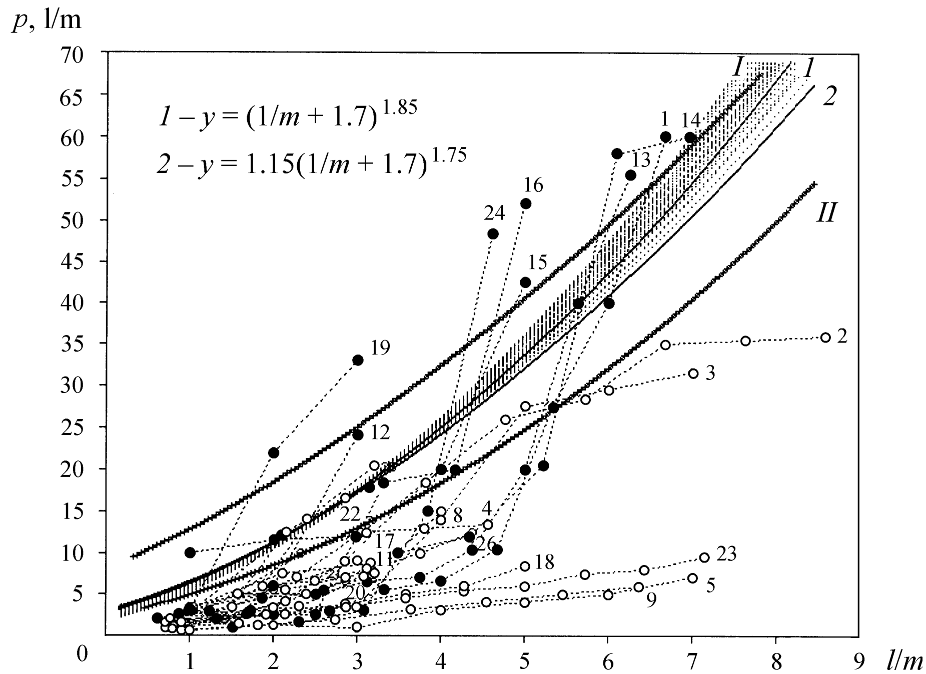

Fig. 11. Comparison between the variants of base lines (crosshatched region): *I* and *II* — the boundaries of I and II rockburst-hazardous categories, respectively

The curves $p(l/m)$ for both types of the base line are shown in Fig. 13. First of all, the proximity of the boundary of II rockburst-hazard category and the curve corresponding to 15 % probability attracts our attention, especially at $y=(l/m+1.7)^{1.85}$. The boundary of category I approaches 75 % line closer of all; however, the trend of two curves has more considerable distinctions than for category II. The distinction between the values obtained for p at fixed probabilities P for two different base lines in the linear scale is markedly great than between the curves $P(P_{\max}/y)$, but is not substantial either. Taking account of the simpler equation (less by one parameter) and higher proximity to the boundary of category II, the final choice was made in the base line $y=(l/m+1.7)^{1.85}$.

Let us discuss the probability values obtained. It is difficult to estimate to which probability of hazard the boundary of categories I and II should correspond; the probability should be apparently high and the estimate of 75 % does not raise doubts. The probability corresponding to the boundary of categories II and III is more definite. Since no measures are taken to prevent rock bursts for category III, this boundary should correspond to a very low probability. Nevertheless, it practically coincided with the line $P\approx 0.15$, i.e., approximately 1 case of 7, which is quite a lot. If it were actually so, the criterion would “work” only due to the fact that in a certain place a rock burst occurs in short terms. However, this case is more complicated. As pointed out above, we defined the probability as the confidence degree in order to obtain point estimates of probability that are different from the frequency estimate. Therefore, the probability value (e.g., $P\approx 0.15$) is formed from two components. The first component is the actual variation of culm yield at equal degree of hazard, the second one is the ignorance measure of this variation. Due to introduction of the function $L(z,x)$ characterizing confidence in monotonous reduction of probability with decrease in culm yield, the second component is considerably reduced as compared with the estimate by the classical Bayes formula. According to (8), for 17 experiments of ○ type, the probability at any low culm yield would be not less than $1/19\approx 0.052$.

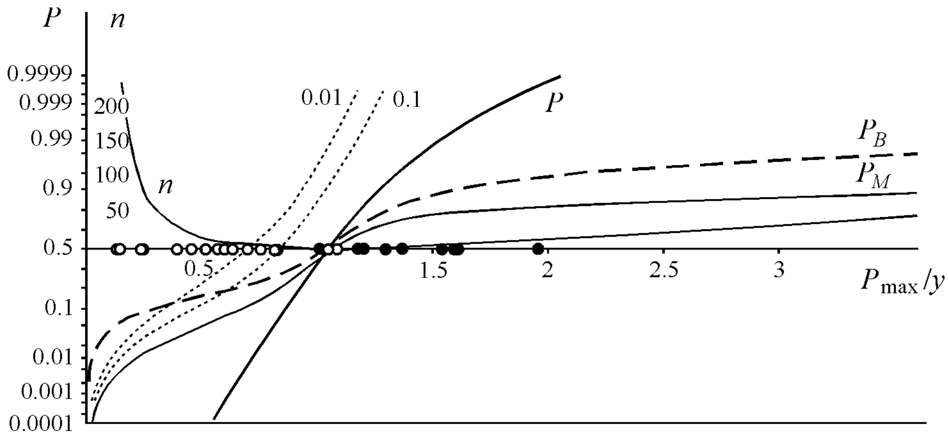

Fig. 12. Dependences of hazard probability on $P_{\max} / y(l/m)$ at the base line $y = (l/m + 1.7)^{1.85}$

A great number of base experiments will obviously lead to reduction in the second component at the limit (at the infinite number of experiments) to its tendency to zero. However, it is impossible to determine exactly the "weight" of the second component, since the true probabilities are unknown. The approximate estimate can be obtained by means of artificial increase in experiment number, i.e., using random numbers, we can obtain a certain number of results which have logarithmically normal distribution relative to the initial results with the coefficient of variation 15 %. The calculation results for the increased number of experiments are given in Table 4.

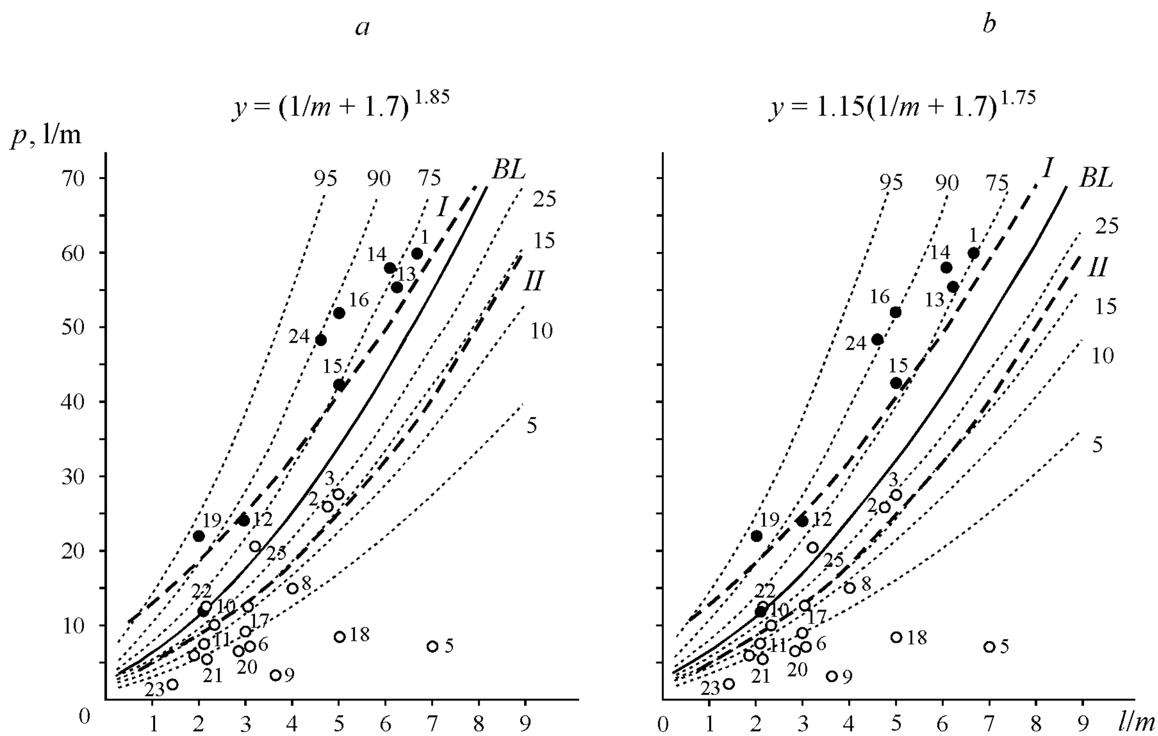

Fig. 13. Type of the curves $p(l/m)$ at different forms of the base line BL. Probabilities of hazard are indicated in percents near the curves (the Bayes estimate): a — $P_B \approx 43\%$; b — $P_B \approx 40\%$

TABLE 4

Actual probabilities	Probabilities with increased number of experiments,							
	Number of experiments							
	52	130	260	390	520	760	1040	1300
0.05	0.028	0.11	0.06	0.04	0.03	0.02	0.015	0.012
0.10	0.061	0.30	0.22	0.16	0.13	0.12	0.10	0.10
0.15	0.101	0.67	0.54	0.50	0.45	0.39	0.39	0.37
0.25	0.198	0.169	0.152	0.157	0.147	0.137	0.138	0.133
Base line ≈ 0.427	0.419	0.427	0.401	0.429	0.408	0.407	0.402	0.398
0.75	0.810	0.832	0.843	0.863	0.860	0.857	0.861	0.858
0.90	0.941	0.974	0.980	0.984	0.985	0.986	0.988	0.987
0.95	0.972	0.990	0.994	0.996	0.997	0.998	0.998	0.9985

N o t e : random numbers were used when generating the results of non-existent experiments, therefore, we should not pay attention to slight deviations from the basic tendency.

As is seen from Table 4, beyond the limits of narrow band near the base line our ignorance constitutes the principal part of probability (the probability distinction in the base line from 0.5 is explained by unequal number of experiments of ● and ○ types, as well as asymmetric distribution of culm yield). The boundary of category II (0.15) corresponds approximately to 3 – 4 %, the line of 0.05 probability — approximately to 0.1 % of the true probability. Therefore, we should understand the examples shown in Fig. 13 as the upper limit (the lower limit lies above the base line) of probability of arising the rock-burst hazardous situation taking into account a small number of the base experiments, which is regularly associated with the true probability.

5. Recommendations and Examples. The probability estimates, which are necessary to calculate the degree of risk, should not be identified with the estimates of hazard level used for making prompt decisions in specific situations. The number of separated hazard levels should correspond to the number of measures for rock burst prevention. Since the sets of measures for rock-burst hazard categories I and II do not practically differ [7], then it makes no sense to separate two levels of hazard. On the other hand, it makes sense to distinguish extremely hazardous situations (conditionally, category 0); the people stay in the risk area should be excluded.

Proceeding from the stated above, we can propose the following variants of rock-burst hazard estimate by the culm yield for anthracite seams. The situation is considered hazardous if, at least:

a) one measurement lies above the base line;

b) two measurements lie above the line of 0.15 (boundaries of category II) or one measurement was obtained in the last meter of the blasthole;

c) three measurements or two, including the measurement obtained in the last meter of the blasthole, lie above the line of 0.1.

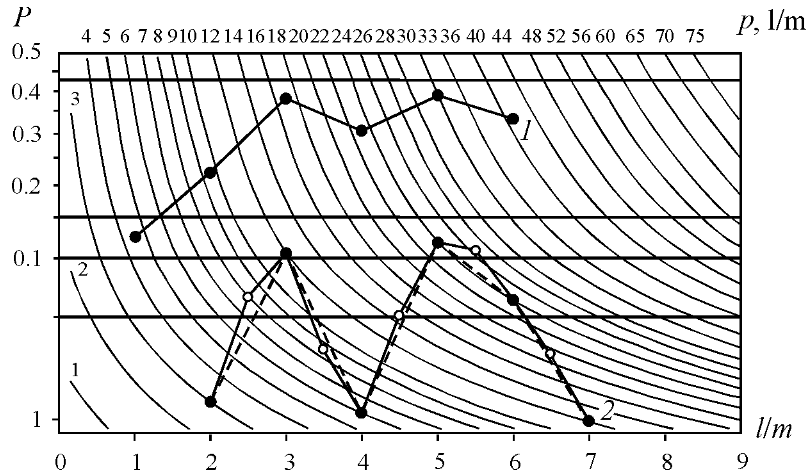

Fig. 14. Nomogram in "probability – reduced length of blasthole" coordinates

If one measurement lies above the line of 0.15, the others lie below the line of 0.05, and drilling is not accompanied by shocks and jamming of the drilling instrument, then three more check blastholes are drilled in the same section of the face at the distances not less than 1.5 m from each other. If the data indicating I or II categories of rock-burst hazard ($a-c$) are obtained in none of the cases, then the results of the first measurements should be rejected.

To use the method proposed, a special nomogram is developed (Fig. 14), where the values of probability are plotted on the axis of ordinates. In the nomogram, the curves corresponding to different volumes of culm yield (l/m) are plotted. In obtaining the intermediate values the points are to be located in the segments of vertical lines between the curves corresponding to the basic values.

Let us consider the examples of nomogram use. In the first example, the culm yield monotonically increases along the blasthole: $p(1)=4.5$, $p(2)=9.5$, $p(3)=17$, $p(4)=23$, $p(5)=33$, and $p(6)=41$ l/m. This situation (Fig. 14, curve *1*) is hazardous: 5 points lie above the line of 0.15. In the second example (Table 3), the amount of culm changes along the blasthole in a complicated way. The high thickness of seam obliges us to use the initial data only (Fig. 14, dotted line *2*), and the situation is estimated as safe, since only two points lie above the line of 0.1. If the seam thickness is low, the intermediate values are calculated additionally (Fig. 14, solid line *2*); the situation is estimated as hazardous, since three points lie above the line of 0.1. The difference can be explained by the fact that in the second case, the measurements are relatively rare. Thus, the risk of missing the hazardous situation is higher and the confidence degree to the conclusion about situation safety is lower, respectively.

CONCLUSIONS

The new approach is proposed for calculating the risk in estimating the hazard by indirect criteria.

On the basis of this approach, a new variant of nomogram is plotted for rock-burst hazard estimate according to culm yield in process of blasthole drilling in the anthracite seams being mined. This variant makes it possible to reveal hazardous situations which are impossible to record by the existing procedures.

The proposed probabilistic method can be also used for the choice of intervals between the control determinations of rock-burst hazard and periodicity of their performing in stoping and development workings.

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