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Permanent components of the crust, geoid and ocean depth tides

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Abstract

The tidal deformation caused by the luni-solar potential includes not only a periodic part, but also a time-independent part, called the permanent tide. How to deal with the tidal correction in gravimetric observations, especially the treatment of the permanent tide, has been discussed for a long time, since some practical and physical problems exist anyhow. A resolution adopted by IAG (1983) was that the permanent tidal attraction of the Moon and the Sun should be eliminated, but the permanent tidal deformation of the Earth be maintained. This is called zero gravity, and the geoid associated with it is the zero geoid. As to the crust deformation, Poutanen et al. (Poutanen, M., Vermeer, M., Mäkinen, J., 1996. The permanent tide in GPS positioning. Journal of Geodesy 70, 499–504.) suggested that co-ordinates should be reduced to the zero crust, i.e. the crust that includes the effect of the permanent tide. This research shows that horizontal components of the permanent earth tides, which are not considered in recent studies, are also important in GPS positioning and geoid determination. Since the tide-generating potential can be expanded into harmonics and divided into two parts (geodetic coefficients and the group of harmonic waves), the permanent earth tides can be easily obtained by multiplying the amplitude of the zero-frequency wavelength by the corresponding geoid geodetic coefficient. Formulas for both elastic and fluid cases are presented. Numerical results for the elastic case show that he vertical permanent crust (zero crust), geoid and ocean depth tides reach −12.0, −5.8 and 6.1 cm at the poles, and 5.9, 2.9 and −3.0 cm at the equator, respectively. The horizontal permanent crust, geoid and ocean depth tide components reach as much as 2.5, 8.7 and 6.3 cm, respectively. According to the solution of IAG (1983), the permanent vertical components are kept in GPS positioning and geoid computation. Thus, it is natural to include the horizontal components correspondingly.

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1. Introduction

In the last decades many scientists were involved in the Earth tide research, especially gravity, tilt and strain tides. Melchior (1978) developed a complete fundamental work in his famous "The tides of the planet Earth". Wahr (1981) proposed an advanced tidal model for a rotating ellipsoidal Earth, where the nearly diurnal response of the core relevant to the ellipsoidal figure of the Earth was taken into account. Dehant (1987) further considered the effects of mantle inelasticity on the tidal response. The tidal models for core resonance and the viscoelastic response were described in the frequency domain.

Scherneck (1991) discussed the tidal displacement model by compiling the results in the literature in a comprehensive and conveniently formulated displacement model for application in VLBI analysis and the discussion is also valid for GPS data processing. He mainly considered the solid Earth and ocean loading tides, with incorporating effects from anelasticity, ellipsoidal figure, and fluid core resonance.

A very important contribution has to be mentioned here is the IERS Conventions (McCarthy, 1996), where one can find a relevant discussion on tidal applications in conventional celestial and terrestrial reference systems. They are widely accepted as standards for the processing of satellite and space-geodetic observations for coordinate determination. They contain guidelines for treatment of solid Earth, ocean, and geopotential tides in the analysis of satellite geodetic observations. They also give the procedure for tidal corrections and the treatment of the permanent component of station coordinates.

All the above-mentioned studies have been carried out for the surface Earth tides, i.e. for the tides on the solid Earth surface. Recently Sun and Sjöberg (1998) studied and discussed the internal tidal responses, load Love numbers and Green's functions, of the Earth to a surface mass load. Their work can be easily extended to an internal tidal study of the Earth by the luni-solar potential.

The tidal deformation caused by the luni-solar potential includes not only a periodic part, but also a permanent (time-independent) part, usually called the permanent tide. In earlier days, the gravimetric observations were reduced by subtracting the whole tidal effect, including the permanent part. However, this correction leads to a gravity value which differs from the long-term temporal mean value. Many scientists have studied the permanent tide problem; among them are Honkasalo (1959), Heikkinen (1979), Groten (1980, 1982), Yurkina et al. (1983), Ekman (1989a,b, 1996), Rapp (1989), Moritz (1992) and Poutanen et al. (1996).

Honkasalo (1959) first pointed out that the tidal corrections imply a constant low tide at the pole and a permanent high tide at the equator due to the constant potential terms. It means that we are not using the real mean sea level as geoid but a geoid which must be defined so that the Moon and the Sun are considered as being at an infinite distance. So, Honkasalo (1959) suggested a correction to gravity, i.e. to restore the permanent part, which leads to the concept of mean gravity. Similarly, when treating the potential in the same way, one obtains the mean geoid. This is known as the Honkasalo correction.

The use of the Honkasalo correction, however, causes a problem when the height of the mean geoid above the reference ellipsoid is computed, since the difference of the mean geoid computed by applying the permanent term correction and applying the Stokes's formula amounts to as much as 72 cm at the pole. The reason for the discrepancy is that the mean gravity includes the permanent tidal attraction of the Sun and the Moon. Both masses are outside the geoid and thus violate the assumptions of Stokes's formula (Heikkinen, 1979). On the other hand, a more theoretical objection one could make is the violation of physical reality if the permanent tide is eliminated, since the mean shape of the Earth is changed as well. To maintain the consistency of other geodetic parameters, one should change also the moment of inertia, rotational velocity and centrifugal force of the Earth, all of these against physical reality (Ekman 1989b).

To avoid the problems of both using and not using the Honkasalo correction, Ekman (1979) and Groten (1980) proposed a new concept that the permanent tidal attraction of the Moon and the Sun are eliminated, but the permanent tidal deformation of the Earth is maintained. This is called zero gravity and the geoid associated with it is the zero geoid. This concept was accepted in a resolution of IAG in 1983.

As to the crust deformation, as pointed out by Poutanen et al. (1996), in GPS literature there are practically less discussion on the handling of the solid Earth tide. In the manuals of software packages the handling of the tide is completely unmentioned. Poutanen et al. (1996) suggest that coordinates be reduced to the zero crust (i.e. the crust that includes the effect of the permanent tide). Note that the mean and zero crusts are equal. The relation between the non-tidal geoid (crust), the zero geoid (crust) and the mean geoid (crust) can be found in Fig. 3 of Poutanen et al. (1996).

It should be emphasized that the horizontal permanent tides are also important in GPS positioning and computation of the gravimetric deflection of the vertical. Unfortunately, they have never been considered up to now, in contrast to the vertical component that has been studied by many scientists. In the following we discuss the permanent tide problem for three earth tides: the crust (displacement) tide, ocean depth tide and geoid tide.

It should be pointed out that the ocean (depth) tide, like the Earth tide, also contains a permanent tide, theoretically speaking. In practice, however, scientists (e.g. Schwiderski, 1980a,b,c) often describe the ocean tide by decomposing them into tidal components (M_2, S_2, O_1, ...), which do not include very long wavelengths. Therefore, we cannot practically obtain loading effects of the permanent ocean tide, although it exists, theoretically. On the other hand, we will not discuss it in this article, because the effects are small and negligible.

2. Permanent tide-generating potential

We start with the tidal potentials caused by the Moon and the Sun. Let $W_m(r,\phi,\lambda)$ be the gravitational potential raised at point $P(r,\phi,\lambda)$ by the Moon at a distance $d_{\rm m}$ from the centre of the Moon, where the subscript m indicates the Moon, r is the radial distance, ϕ the latitude and λ the longitude. Then we have (Melchior, 1978)

$$W_{\rm m}(r,\phi,\lambda)=\sum_{n=2}^{\infty}W_{{\rm m}n}(r,\phi,\lambda)=GM_{\rm m}\sum_{n=2}^{\infty}\frac{r^n}{d_{\rm m}^{n+1}}P_n(\cos z_{\rm m}), \tag{1}$$

where G is the gravitational constant, $M_{\rm m}$ the mass of the Moon, $P_n(\cos z_{\rm m})$ the Legendre's polynomial of degree n, and $z_{\rm m}$ the geocentric zenithal distance of the Moon at the considered point P. Similarly, we have the gravitational potential $W_{\rm s}({\rm r},\phi,\lambda)$ of the Sun at the same point $P(r,\phi,\lambda)$

$$W_s(r,\phi,\lambda)=\sum_{n=2}^{\infty}W_{sn}(r,\phi,\lambda)=GM_s\sum_{n=2}^{\infty}\frac{r^n}{d_s^{n+1}}P_n(\cos z_s),\tag{2}$$

where d_s is the distance between the centres of the Earth and the Sun, the subscript s indicates the Sun, M_s the mass of the Sun, and z_s the geocentric zenithal distance of the Sun at the considered point P. The main contribution to the tide-generating potential comes from the Moon, since it is nearer to the Earth than the Sun. However, since the mass of the Sun is much larger than that of the Moon, the contribution of the Sun can give approximately half of the Moon's potential contribution. Combining the luni-solar potentials, we obtain the total tide-generating potential

$$W(r,\phi,\lambda)=W_m(r,\phi,\lambda)+W_s(r,\phi,\lambda).\tag{3}$$

Since the factors $\frac{r^n}{d_m^{n+1}}$ in (1) and $\frac{r^n}{d_s^{n+1}}$ in (2) decay very fast as n increases, the potentials $W_m(r,\phi,\lambda)$ and $W_s(r,\phi,\lambda)$ are usually truncated at $n=3$ and $n=2$, respectively. Using the fundamental formula of the position triangle of spherical astronomy, the potential of degree 2 in (1) becomes Laplace's tidal formula, which contains three terms. The first term (sectorial function) expresses the tides with the semi-diurnal period. Their amplitude has a maximum at the equator when the declination of the perturbing body is zero. Amplitudes are zero at the poles. The second term (tesseral function) divides the sphere into areas, which change sign with the declination of the perturbing body. The corresponding tide period is diurnal and the amplitude is maximum at latitude 45° north and 45° south when the declination of the perturbing body is maximum. The amplitude is always zero at the equator and at the poles. The third term (zonal function) depends only on the latitude. The nodal lines are at $\pm 35°16'$. Since it is a squared sine function of the declination of the perturbing body, its fundamental period will be 14 days for the Moon and 6 months for the Sun. However, the most important thing to the present study is the constant part of the zonal function. The effect of this permanent tide is a slight increase of the Earth's flattening. The amplitudes of the equipotential surface and the corresponding crust changes can be obtained through a further development of the zonal function.

For theoretical and numerical convenience, the tidal potential $W(r,\phi,\lambda)$ can be decomposed into two parts: a term related with position $N^{ij}(r,\phi)$– a function of the radial distance and the latitude, and a term in related with time $M_{ij}(\lambda,\tau)$– a function of the longitude and the hour angle, so that

$$W(r,\phi,\lambda)=\sum_{i=2}^{3}\sum_{j=0}^{3}N^{ij}(r,\phi)M_{ij}(\lambda,\tau).\tag{4}$$

Doodson (1922), Tamura (1987) and Xi (1987) expanded the tide-generating potential into harmonics. To have a generalized standard for comparing amplitudes of the harmonic waves, they set up normalized coefficients $N^{ij}(r,\phi)$– the geodetic coefficients. The superscript ij of N^{ij} (or the subscript ij of M_{ij}) expresses the second ($i=2$) and third ($i=3$) harmonic degrees of the luni-solar potential, and the permanent ($j=0$), diurnal ($j=1$), semi-diurnal ($j=2$) and one third ($j=3$) period tides. In our case we are interested only in the permanent tide. From the structure of the potential and the harmonic expansions of the potential, we know that only the case $ij=20$ contain permanent potential. So that the corresponding geodetic coefficient N^{20} of the permanent tide will be used in this study, and it has the following format

$$N^{20}(r,\phi)=\frac{1}{2}D_{m2}\left(\frac{r}{a}\right)^2\left(1-3\sin^2\phi\right), \tag{5}$$

where D_{m2} is the Doodson's constant for the Moon, defined as

$$D_{m2}=\frac{3}{4}GM_m\frac{a^2}{c_m^3}, \tag{6}$$

where a is the mean Earth radius, c_m the mean distance between the centres of the Moon and the Earth. Similarly the Doodson's constant D_{s2} for the Sun is

$$D_{s2}=\frac{3}{4}GM_s\frac{a^2}{c_s^3}=\left(\frac{c_m}{c_s}\right)^3\frac{M_s}{M_m}D_{m2}, \tag{7}$$

where c_s is the mean distance from the centre of the Sun to the centre of the Earth. Numerically, the Doodson's constants D_{m2} and D_{s2} can be determined according to the definitions (6) and (7) and the Sun and Moon's parameters (Xi, 1982): $D_{m2}=26248$ cm^2/s^2, $D_{s2}=12054$ cm^2/s^2.

At the same time, $M_{ij}(\lambda,\tau)$ in (4) has the following form

$$M_{ij}(\lambda,\tau)=\sum_k K_k\cos\left(\alpha_k+\beta_{ij}\right), \tag{8}$$

where K_k is the amplitude coefficient of a harmonic wave, α_k the phase of the wave corresponding to K_k. K_k and α_k are determined by a harmonic development of the tidal potential (Doodson 1922; Tamura, 1987; Xi, 1987). The upper limit of summation of k depends on an accuracy requirement, usually to several hundreds. β_{ij} is the phase correction. For the case of $ij=20$, $\beta_{20}=0$. The harmonic expansions of the potential by Doodson (1922), Tamura (1987) and Xi (1987) indicate that the amplitude coefficient of the permanent tide is $K_{M0}=0.50458$ for the Moon and $K_{M0}=0.23411$ for the Sun. Therefore, according to (4), the permanent potential $W_p(r,\phi)$ caused by the Moon and the Sun can be expressed as

$$\begin{aligned}W_p(r,\phi)&=N^{20}(r,\phi)\cdot(K_{M0}+K_{S0})\\&=0.73869N^{20}(r,\phi)\\&=9694.57\left(\frac{r}{a}\right)^2\left(1-3\sin^2\phi\right).\end{aligned} \tag{9}$$

In the following, we will discuss the permanent deformations caused by the permanent potential.

3. Three kinds of permanent tides: crust, geoid and ocean depth

The Earth deforms with a mass redistribution when the tide-generating potential is applied to it. The mass redistribution naturally causes an (indirect) potential change, which is proportional to the tide-generating potential with a factor k_n, i.e. $k_n W(r,\phi,\lambda)$, where k_n is one tidal Love number in the triplet (h_n, l_n, k_n). The Love number triplet (h_n, l_n, k_n) describes the displacement (h_n for vertical and l_n for horizontal) and the gravitational potential change (k_n) caused by the luni-solar attraction. The Love numbers are dimensionless. For a SNREI (spherically symmetric, non-rotating, elastic and isotropic) earth model, $h_n = h_n(r)$. Otherwise, the Love numbers depend on not only n, but also m, and $h_{nm} = h_{nm}(\mathbf{r}) \neq h_n(r)$, i.e. frequency- and/or space-dependent. They can be obtained from real geodetic observations or from a theoretical calculation based on an Earth model, such as 1066A (Gilbert and Dziewonski, 1975) or PREM (Dziewonski and Anderson, 1981). Both theories and observations show that the Earth appears almost elastic in a short time scale, such as from a few seconds to a few months. Almost all of the Earth tides are within this time scale. Therefore, in practice, an elastic tidal model describes the Earth tides very well.

However, in case of the permanent tide, the Earth is likely considered to be approximately in a state of hydrostatic equilibrium. The elastic Love numbers should be replaced by the fluid limit Love numbers, which are $h = 1.94$, $l = 0$, $k = 0.94$ (Munk and MacDonald, 1960; McCarthy, 1996). Unfortunately, the real Love numbers for the permanent tide are not known, because the real Earth is neither elastic nor liquid. Actually, they are not only not known, but also fundamentally not knowable (Groten, 1982). There is not any empirical way to separate the permanent deformation from the original shape of the Earth. There is no any practical earth model to calculate them. There is no any unique way to observe the permanent deformation so far, maybe forever. One thing is clear, that is, the real Love numbers for the permanent tide should lie between the elastic and the fluid limit ones. The pure elastic and the pure liquid can be considered as two extreme cases of the real Earth. Due to the limited life of the Earth, one cannot prove that the Earth is fully isostatic adjusted to an equilibrium status. In the spatial aspect, the geoid undulations indicate that the Earth is far too well adjusted, for any harmonic degrees. In the time aspect, as mentioned above, the Earth appears almost elastic in a short time scale; while it is difficult to observe or prove that the Earth is a liquid equilibrium in a long but limited time scale. Therefore, it is difficult to determine the Love numbers of the permanent deformation for the real Earth. On the other hand, since the permanent deformation is a constant term and to be removed in order to obtain the temporally varying part of the tide-induced site displacement, choosing any Love number values for the permanent tide will not affect the observable tidal quantities. From this point of view, one is free to choose any set of the Love numbers. However, in any case, it is very important and essential to use the same kind of Love numbers to keep all the corrections or resolutions in the same scale. In fact, McCarthy (1996) recommended using the elastic Love numbers to remove the permanent deformation, and then restore it in the next step. Poutanen et al. (1996) also recommended the elastic Love numbers for considering the permanent tide in GPS positioning. This paper is trying to extend the study of the permanent deformation to geoid tide and ocean depth tide, including the crust tide. Therefore, to keep a coincidence with McCarthy (1996) and Poutanen et al. (1996), we also use the elastic Love numbers for a practical numerical calculation. Note that choosing Love numbers does not affect the theoretical discussion. For a numerical result, one can just replace the Love numbers by desired ones.

When the Earth undergoes a tidal deformation, any point on the Earth is displaced. The vertical displacement causes another (indirect) potential $W_h(r,\phi,\lambda)$, which is also proportional to the tide-generating potential, with a ratio factor h_n (Love number), i.e.-$h_n W(r,\phi,\lambda)$. Correspondingly, the potential changes related to the permanent potential $W_p(r,\phi,\lambda)$ can be expressed by

$k_2 W_p(r, \phi, \lambda)$ and $-h_2 W_p(r, \phi, \lambda)$, respectively. Then the total permanent potential change at point P is the sum of the above mentioned three terms (one original, two extra potentials), i.e.

$$W_t(r, \phi, \lambda) = (1 + k_2 - h_2) W_p(r, \phi, \lambda). \tag{10}$$

Since the Moon and the Sun move around the Earth, the distances d_m and d_s and the angular distances z_m and z_s vary correspondingly, so that the Earth deforms in the form of Earth tides under the luni-solar attraction. However, the permanent potential is time-independent. It causes the Earth's flattening forever. From the total tidal permanent potential $W_t(r,\phi,\lambda)$, we can easily obtain the permanent equipotential surface change by dividing by the gravity. It is valid for any point only if the point is fixed on the crust of the Earth. When the observing point is located on the oceanic surface, the crust displacement related h_2 term disappears. In this case the tidal deformation just coincides with the geoid, called permanent geoid tide. Below we define and discuss these Earth tides.

3.1. Permanent crustal deformation

Permanent crust Earth tide, also called permanent displacement tide, is defined as the permanent displacement of the point P fixed on the crustal surface of the Earth caused by the permanent luni-solar potential. Let the deformation vector be $\boldsymbol{\xi}$. According to the elasticity theory, the crustal tide deformations can be easily written as

$$\begin{aligned} \boldsymbol{\xi}(r, \phi, \lambda) &= \xi_r(r, \phi, \lambda)\mathbf{e}_r + \xi_\phi(r, \phi, \lambda)\mathbf{e}_\phi + \xi_\lambda(r, \phi, \lambda)\mathbf{e}_\lambda \\ &= \frac{1}{g}\left[h_2\mathbf{e}_r + l_2\left(\mathbf{e}_\phi \frac{\partial}{\partial\phi} + \mathbf{e}_\lambda \frac{\partial}{\cos\phi\partial\lambda}\right)\right] W_p(r, \phi, \lambda), \end{aligned} \tag{11}$$

where $(\mathbf{e}_r, \mathbf{e}_\phi, \mathbf{e}_\lambda)$ are unit vectors in the spherical co-ordinates, g is the gravity. Then the vertical, north and east components of the crust tide are

$$\xi_r(r, \phi, \lambda) = \frac{1}{g} h_2 W_p(r, \phi, \lambda) \tag{12}$$

$$\xi_\phi(r, \phi, \lambda) = \frac{1}{g} l_2 \frac{\partial W_p(r, \phi, \lambda)}{\partial\phi} \tag{13}$$

$$\xi_\lambda(r, \phi, \lambda) = \frac{1}{g} l_2 \frac{\partial W_p(r, \phi, \lambda)}{\cos\phi\partial\lambda}. \tag{14}$$

Note that the formulas (11)–(14) are valid for any point in the solid Earth, while the deformation of a liquid part should be treated differently.

3.2. Permanent ocean depth tide

As we will show below, with neglecting ocean dynamic effects, the ocean depth tide is nothing but the equipotential surface point tide, which describes a point movement at an equipotential surface under the attraction of the Moon and the Sun. The movement of the point happens in vertical and horizontal directions. The vertical component is also called equipotential surface change; however, to our knowledge, the horizontal component of the point movement at equipotential surface has not been studied before. Generally, we define the movement of the point as equipotential surface point tide.

At first, let us see what is the equipotential surface change. On any given equipotential surface, the geo-potential $V(r,\phi,\lambda)$ is a constant C and its equation can be expressed as

$$V(r, \phi, \lambda) = C. \tag{15}$$

When the tidal potential is applied to the elastic Earth, tidal deformations must happen. First we assume that point $P(r,\phi,\lambda)$ is transported radial to $P(r+\zeta,\phi,\lambda)$, where the geo-potential becomes

$$V(r + \zeta, \phi, \lambda) = V(r, \phi, \lambda) + \zeta \frac{\partial V(r, \phi, \lambda)}{\partial r}. \tag{16}$$

At point $P(r+\zeta,\phi,\lambda)$, the total potential is the sum of geo-potential $V(r+\zeta,\phi,\lambda)$, and the total tide-generating potential $W_t(r,\phi,\lambda)$. Then the potential equation of the deformed equipotential surface (15) becomes

$$V(r, \phi, \lambda) + \zeta \frac{\partial V(r, \phi, \lambda)}{\partial r} + W_t(r, \phi, \lambda) = C, \tag{17}$$

or

$$\zeta \frac{\partial V(r, \phi, \lambda)}{\partial r} + W_t(r, \phi, \lambda) = 0. \tag{18}$$

With considering the definition of the acceleration of gravity

$$g = -\frac{\partial V(r, \phi, \lambda)}{\partial r}, \tag{19}$$

we obtain the equipotential-surface tide as (Bruns's formula)

$$\zeta(r, \phi, \lambda) = \frac{W_t(r, \phi, \lambda)}{g}. \tag{20}$$

Eq. (20) implies that the equipotential surface tide corresponds to the total tide-generating potential. Note that since the total potential W_t defined in (10) consists of the potential caused by the earth deformation and the luni-solar potential, Eq. (20) actually represents the difference between the ocean surface and bottom displacement.

Now, let us define the equipotential surface point tide so that we can obtain the permanent surface point tide or the permanent ocean depth tide. Actually, a point at the equipotential surface changes not only in the vertical direction, but also in the horizontal direction. The horizontal components along latitude and longitude directions can be easily obtained by horizontal components of the luni-solar potential dividing by the gravity on the Earth's surface. Here, we define a new concept — equipotential surface point movement, noted by the vector $\boldsymbol{\zeta}$, to express the movement of a point on the equipotential surface. The equipotential surface point movement consists of the equipotential surface change $\zeta_r(r,\phi,\lambda)$ and the movement in horizontal directions $\zeta_\phi(r,\phi,\lambda)$ and $\zeta_\lambda(r,\phi,\lambda)$. Since we are interested only in the permanent part, we only consider the permanent ocean depth tide corresponding to the permanent potential $W_p(r,\phi,\lambda)$. It follows that $\boldsymbol{\zeta}$ can be mathematically expressed as

$$\begin{aligned}\boldsymbol{\zeta}(r,\phi,\lambda) &= \zeta_r(r,\phi,\lambda)\mathbf{e}_r + \zeta_\phi(r,\phi,\lambda)\mathbf{e}_\phi + \zeta_\lambda(r,\phi,\lambda)\mathbf{e}_\lambda \\ &= \frac{1}{g}\left[(1+k_2-h_2)\mathbf{e}_r + (1+k_2-l_2)\left(\mathbf{e}_\phi\frac{\partial}{\partial\phi} + \mathbf{e}_\lambda\frac{\partial}{\cos\phi\partial\lambda}\right)\right]W_p(r,\phi,\lambda) \\ &= \frac{1}{g}(1+k_2)\left(\mathbf{e}_r + \mathbf{e}_\phi\frac{\partial}{\partial\phi} + \mathbf{e}_\lambda\frac{\partial}{\cos\phi\partial\lambda}\right)W_p(r,\phi,\lambda) - \boldsymbol{\xi}(r,\phi,\lambda),\end{aligned} \tag{21}$$

where three components are

$$\zeta_r(r,\phi,\lambda) = \frac{1}{g}(1+k_2-h_2)W_p(r,\phi,\lambda) \tag{22}$$

$$\zeta_\phi(r,\phi,\lambda) = \frac{1}{g}(1+k_2-l_2)\frac{\partial W_p(r,\phi,\lambda)}{\partial\phi} \tag{23}$$

$$\zeta_\lambda(r,\phi,\lambda) = \frac{1}{g}(1+k_2-l_2)\frac{\partial W_p(r,\phi,\lambda)}{\cos\phi\partial\lambda}. \tag{24}$$

Note that the whole deformation caused by the total tide generating potential $W(r,\phi,\lambda)$ has a similar expression as (21) by replacing $W_p(r,\phi,\lambda)$ by $W(r,\phi,\lambda)$.

3.3. Permanent geoid tides

The geoid is a special equipotential surface, which is a level surface coinciding (with neglecting some small effects, such as ocean dynamic topography) with the free surface of the oceans together with its continuation through the continents. This level surface, of course, changes dynamically by the Earth tide, like other equipotential surfaces. The tidal deformation of the ocean level surface is defined as geoid tide, while the permanent part is defined as the permanent geoid tide $\boldsymbol{\zeta}^o$.

Note that the concept of the conventional definition of the geoid is not changed here, and it is still the static surface of the ocean, not the dynamically changed one. The geoid tide is considered as a perturbing part of the static geoid. When an observation is performed on the ocean surface, e.g. a GPS observation on a ship, the tidal effects on the observation are caused by the geoid tide. Here again, any dynamic effects are neglected, such as the ocean loading, the ocean dynamic topography, and so on. Like the equipotential surface point tide, the geoid tide also changes in the vertical and horizontal directions. This means that the permanent geoid tide vector $\boldsymbol{\zeta}^o$ consists of ζ_r^o, ζ_ϕ^o and ζ_λ^o, along the vectors $\mathbf{e}_r$, $\mathbf{e}_\phi$ and $\mathbf{e}_\lambda$, respectively. Since the geoid tide has nothing to do with the vertical crustal displacement, the terms containing h_2 in (21) disappear. Hence, the permanent geoid tide $\boldsymbol{\zeta}^o$ can be obtained from $\boldsymbol{\zeta}$ by replacing $(1+k_2\text{-}h_2)$ by $(1+k_2)$, i.e.

$$\begin{aligned}\boldsymbol{\zeta}^o(r,\phi,\lambda) &= \zeta_r^o(r,\phi,\lambda)\mathbf{e}_r + \zeta_\phi^o(r,\phi,\lambda)\mathbf{e}_\phi + \zeta_\lambda^o(r,\phi,\lambda)\mathbf{e}_\lambda \\ &= \frac{1}{g}(1+k_2)\left(\mathbf{e}_r + \mathbf{e}_\phi\frac{\partial}{\partial\phi} + \mathbf{e}_\lambda\frac{\partial}{\cos\phi\partial\lambda}\right)W_p(r,\phi,\lambda).\end{aligned} \tag{25}$$

The three components are

$$\zeta_r^o(r,\phi,\lambda) = \frac{1}{g}(1+k_2)W_p(r,\phi,\lambda) \tag{26}$$

$$\zeta_\phi^o(r,\phi,\lambda) = \frac{1}{g}(1+k_2)\frac{\partial W_p(r,\phi,\lambda)}{\partial\phi} \tag{27}$$

$$\zeta_\lambda^o(r,\phi,\lambda) = \frac{1}{g}(1+k_2)\frac{\partial W_p(r,\phi,\lambda)}{\cos\phi\partial\lambda}. \tag{28}$$

The first component changes the geoidal height, while the others change the deflection of the vertical. The discussion in this section is for the oceanic geoid, but it is also valid for the continental geoid, although it cannot be directly observed.

3.4. Relations of the three tides

Above we have discussed the definitions of the three types of Earth tides. Here we prove that they are not independent. From Eqs. (11), (21) and (25), we can easily obtain the following simple relation

$$\boldsymbol{\zeta}^o(r,\phi,\lambda) - \boldsymbol{\zeta}(r,\phi,\lambda) = \boldsymbol{\xi}(r,\phi,\lambda), \tag{29}$$

i.e. the three Earth tides are closely related. In practice, we may only consider any two of them, and the third one can be obtained from the relation (29).

On the other hand, we can prove that the equipotential surface tide also expresses the ocean depth change (tide). It is known that the geoid tide $\boldsymbol{\zeta}^o$ makes the ocean surface change to a new level. At the same time, the bottom of the sea changes in the form of a crust tide, i.e. $\boldsymbol{\xi}$. Then the ocean depth change is the difference of the ocean surface change (geoid tide) and the bottom change (crust tide), i.e. $\boldsymbol{\zeta}^o(r,\phi,\lambda)-\boldsymbol{\xi}(r,\phi,\lambda)$. Notice that the difference is nothing but the equipotential surface point tide $\boldsymbol{\zeta}$, given in (29). Therefore, it proves that the ocean depth tide $\boldsymbol{\zeta}(r,\phi,\lambda)$ is just the equipotential surface point tide.

4. Calculations of the permanent tides

In this section, we discuss practical formulas for calculating the three permanent tides. Like calculating the gravity, tilt and strain Earth tides, geodetic coefficients and phase corrections in this method are essential in spectral form. Using the above harmonic expansion of the tide-generating potential, we can easily obtain the spectral formulas for the crust tides and the geoid tides

$$\xi_t'(r,\phi,\lambda)=\sum_{i=2}^{3}\sum_{j=0}^{3}\sum_{k}\xi_t^{ij}(r,\phi)K_k(ij)\cos\left(\alpha_k(ij)+\beta_t^{ij}\right) \tag{30}$$

$$\zeta_t'^{0}(r,\phi,\lambda)=\sum_{i=2}^{3}\sum_{j=0}^{3}\sum_{k}\zeta_t^{ij}(r,\phi)K_k(ij)\cos\left(\alpha_k(ij)+\beta_t^{ij}\right), \tag{31}$$

where the subscript t stands for r, ϕ and λ. The equipotential surface tide (or ocean depth tide) can be derived from the two tides, according to the relation (29). Since $\cos(\alpha_k(20)+\beta^{20})=1$ for the permanent tides, we know that the permanent tides are longitude-independent. Then the index of the permanent tides is reduced from (r,ϕ,λ) to (r,ϕ). Therefore, from Eqs. (30) and (31), we can write the permanent crust and geoid tides as

$$\xi_r(r,\phi)=\frac{h_2}{2g}D_{m2}\left(\frac{r}{a}\right)^2\left(1-3\sin^2\phi\right)(K_{M0}+K_{S0}) \tag{32}$$

$$\xi_\phi(r,\phi)=-\frac{3l_2}{2g}D_{m2}\left(\frac{r}{a}\right)^2\sin2\phi(K_{M0}+K_{S0}) \tag{33}$$

$$\xi_\lambda(r,\phi)=0 \tag{34}$$

and

$$\zeta_r^0(r,\phi)=\frac{1+k_2}{2g}D_{m2}\left(\frac{r}{a}\right)^2\left(1-3\sin^2\phi\right)(K_{M0}+K_{S0}) \tag{35}$$

$$\zeta_\phi^0(r,\phi)=-\frac{3(1+k_2)}{2g}D_{m2}\left(\frac{r}{a}\right)^2\sin2\phi(K_{M0}+K_{S0}) \tag{36}$$

$$\zeta_\lambda^0(r,\phi)=0, \tag{37}$$

where K_{M0} and K_{S0} are the amplitude coefficient of the permanent tides for the Moon and the Sun, respectively. The coefficients of the factor $(K_{M0}+K_{S0})$ are the crust geodetic coefficients and the geoid geodetic coefficients, respectively. As mentioned above, the geodetic constants are functions of position (latitude). Once a computation point is decided, the geodetic constants are determined.

According to the solution of IAG (1983), it is the zero geoid, not the mean geoid, which should be used. To fit this purpose, the geodetic coefficient [the coefficients in (35) and (36) must be modified, by omitting the 1 in the factor $(1+k_2)$, so that (35)–(37) become

$$\zeta_r^0(r,\phi) = \frac{k_2}{2g} D_{m2}\left(\frac{r}{a}\right)^2 \left(1 - 3\sin^2\phi\right)(K_{M0} + K_{S0}) \tag{38}$$

$$\zeta_\phi^0(r,\phi) = -\frac{3k_2}{2g} D_{m2}\left(\frac{r}{a}\right)^2 \sin 2\phi (K_{M0} + K_{S0}) \tag{39}$$

$$\zeta_\lambda^0(r,\phi) = 0, \tag{40}$$

In a numerical calculation, how to choose Love numbers is still a not well-solved problem, due to the difficulties discussed above. In the following calculation, as the first approximation, we use the elastic Love numbers, the extreme case of the Earth. This is to keep the coincidence with the IERS Conventions (McCarthy, 1996), so that the results can be compared each other.

The Love numbers are taken the following values calculated from the PREM Earth model (McCarthy, 1996): $h_2 = 0.6026$, $l_2 = 0.0831$ and $k_2 = 0.29525$. Then the zero crust can be obtained from (32) (the values of K_{M0} and K_{S0} are given above)

$$\xi_r(r,\phi) = 5.95\left(\frac{r}{a}\right)^2 \left(1 - 3\sin^2\phi\right) \tag{41}$$

$$\xi_\phi(r,\phi) = -2.46\left(\frac{r}{a}\right)^2 \sin 2\phi \tag{42}$$

$$\xi_\lambda(r,\phi) = 0, \tag{43}$$

where $g = 982$ cm/s^2 is used. Comparing the above results with the IERS Conventions shows that they are basically identical. A little difference is that a height factor $(r/a)^2$ is included in (41)–(43), but not in Eqs. (17a) and (17b) of the IERS Conventions. In the maximum case when we take $r = 6380$ km, the factor $(r/a)^2$ can cause a difference of 0.28%.

Similarly, we can obtain the permanent geoid tide by multiplying the amplitude of the zero-frequency wavelength $(K_{M0} + K_{S0})$ by the corresponding geoid geodetic coefficient.

The three components of the permanent (zero) geoid are

$$\zeta_r^0(r,\phi) = 2.92\left(\frac{r}{a}\right)^2 \left(1 - 3\sin^2\phi\right) \tag{44}$$

$$\zeta_\phi^0(r,\phi) = -8.74\left(\frac{r}{a}\right)^2 \sin 2\phi \tag{45}$$

$$\zeta_\lambda^0(r,\phi) = 0. \tag{46}$$

Note that the deflection component of the vertical can be obtained by $-\zeta_\phi^0(r,\phi)/a$, where a is the radius of the Earth. To see how much the geoid moves in the horizontal direction, in practice, we consider the component $\zeta_\phi^0(r,\phi)$ instead of the deflection. The equipotential surface point (ocean depth) changes can be derived from $\xi_p(r,\phi)$ and $\zeta_p(r,\phi)$, i.e.

$$\zeta_r(r,\phi) = \zeta_r^0(r,\phi) - \xi_r(r,\phi) = -3.03\left(\frac{r}{a}\right)^2\left(1-3\sin^2\phi\right) \tag{47}$$

$$\zeta_\phi(r,\phi) = \zeta_r^0(r,\phi) - \xi_r(r,\phi) = -6.28\left(\frac{r}{a}\right)^2\sin 2\phi \tag{48}$$

$$\zeta_\lambda(r,\phi) = 0. \tag{49}$$

The units of the results in Eqs. (41)–(43) and (44)–(49) are centimetres. Note that the Eqs. (32)–(49) are main results of this study. Most of them are new, since the permanent tides of the geoid tide and the ocean depth tide have less been studied before, especially for the horizontal components. While the IERS Conventions present only the permanent deformation in the crustal tide.

For another extreme case, one can use the fluid limit Love numbers ($h=1.94$, $l=0$, $k=0.94$). The formulas for the fluid case are presented here, so that we can use them when the fluid limit case is considered necessary, e.g. when the IERS Conventions are updated. Following the exact procedure above, we obtain the three permanent tides for the fluid limit case as below (note that the superscript f indicates the fluid case).

Zero crust:

$$\xi_r^f(r,\phi) = 19.16\left(\frac{r}{a}\right)^2\left(1-3\sin^2\phi\right) \tag{50}$$

$$\xi_\phi^f(r,\phi) = 0 \tag{51}$$

$$\xi_\lambda^f(r,\phi) = 0, \tag{52}$$

Permanent geoid:

$$\xi_r^{0f}(r,\phi) = 9.30\left(\frac{r}{a}\right)^2\left(1-3\sin^2\phi\right) \tag{53}$$

$$\zeta_\phi^{0f}(r,\phi) = -27.83\left(\frac{r}{a}\right)^2\sin 2\phi \tag{54}$$

$$\zeta_\lambda^{0f}(r,\phi) = 0. \tag{55}$$

Equipotential surface point (ocean depth) changes:

$$\zeta_r^f(r,\phi) = -9.86\left(\frac{r}{a}\right)^2(1-3\sin^2\phi) \tag{56}$$

$$\zeta_\phi^f(r,\phi) = -27.83\left(\frac{r}{a}\right)^2\sin 2\phi \tag{57}$$

$$\zeta_\lambda^f(r,\phi) = 0. \tag{58}$$

The above formulas show that the horizontal component of the crust vanishes since $l=0$. The non-zero components of the three permanent tides are almost three times larger than that of the elastic case. The true permanent deformation of the Earth should lie somewhere in between the two cases, maybe closer to the fluid limit.

The numerical distributions of the three permanent tides along latitude for the elastic case are calculated according to the above formulas (41)–(43) and (44)–(49), and plotted in Figs. 1–3.

Fig. 1 gives the distribution along latitude, including the vertical and horizontal components, of the permanent (zero) crust tide caused by the permanent potential. The horizontal component along longitude of the permanent crust tide is zero. The unit is cm. The figure shows that the vertical permanent crust change (zero crust) reaches −12 cm at the poles and 6 cm at the equator. The horizontal permanent crust displacement also reaches as much as 2.5 cm. Based on the solution of IAG (1983) for permanent potential and gravity tide, Poutanen et al. (1996) suggested that a crust deformation should imply that computed co-ordinates of bench marks or other fixed points on the crust be reduced to the zero crust, i.e. the crust that includes the effect of the

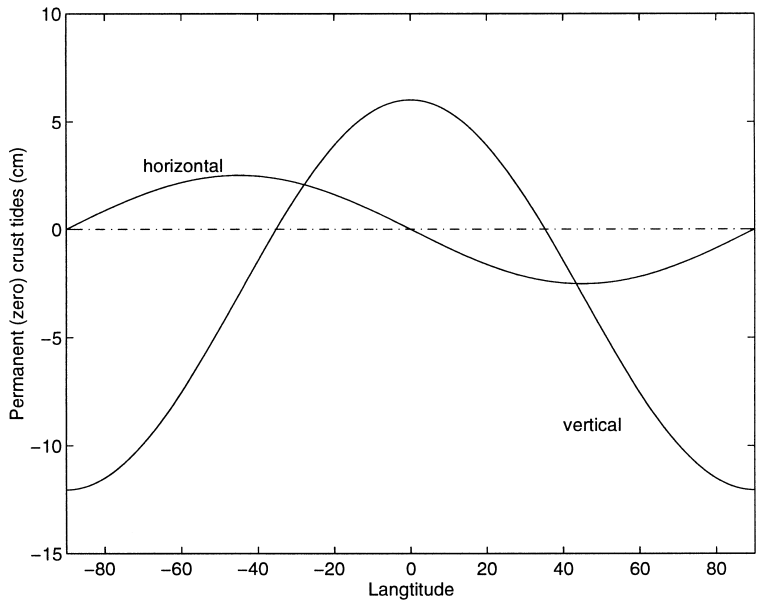

Fig. 1. Distributions of the permanent (zero) crust tides caused by the permanent part of the luni-solar potential along latitude. The horizontal component along longitude of the permanent crust tide is zero. The unit is cm.

permanent tide. It means that we should keep the vertical permanent (zero) crust tide in any results of crust measurement, e.g. by GPS, leveling and so on. However, the crust deformation, including the permanent deformation, contains not only a vertical component, but also a horizontal one. Such a displacement vector should be treated properly with all its significant components. In practice, the 2.5 cm horizontal permanent crust displacement is big enough to be considered. Therefore, we suggest that the horizontal component be correspondingly considered, as the vertical one.

Fig. 2 shows the distribution along latitude of the permanent geoid tide (vertical and horizontal components) caused by the permanent part of the luni-solar potential. The horizontal geoid tide (deflection) along longitude is zero. The unit is cm. The figure shows that the vertical component reaches −5.8 cm at the poles and 3.0 cm at the equator. The horizontal permanent geoid deflection (horizontal displacement) reaches as much as 8.7 cm at ±45°. Similarly, not only the vertical component of the permanent geoid tide should be considered in GPS positioning (at see) and geoid computation, but also the horizontal components should be correspondingly considered.

It should be mentioned also that the geoid tide has special applications in geodesy and marine geodesy. As GPS is widely used as a powerful geodetic tool, tidal corrections are certainly important in GPS data processing. Especially, when precise GPS observations are performed simultaneously on both land and ocean (on a ship), tidal corrections must be carefully considered. Such a combined observation scheme was recently proposed by Jansson (1994), with the purpose to determine a point on the bottom of the sea. The point can be considered as a position to set an (e.g. seismic) instrument or to construct a pillar of a bridge, or simply a reference point for future use. The basic idea to measure the position of the point is to combine the GPS and Sonar observations. As precise marine geodetic applications require, say, 1 cm accuracy of positioning in the sea, the above mentioned permanent geoid tide must be considered.

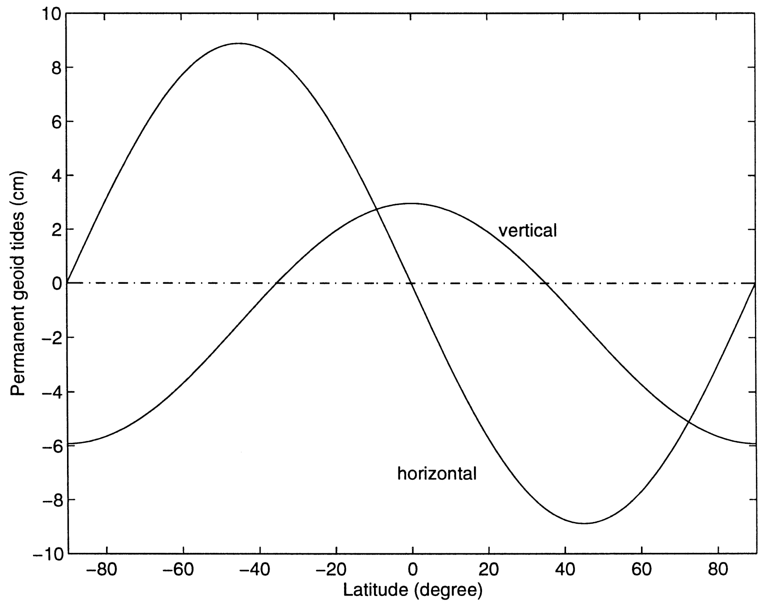

Fig. 2. Distributions of the permanent geoid tide caused by the permanent part of the luni-solar potential along latitude. The horizontal geoid tide (deflection) along longitude is zero. The unit is cm.

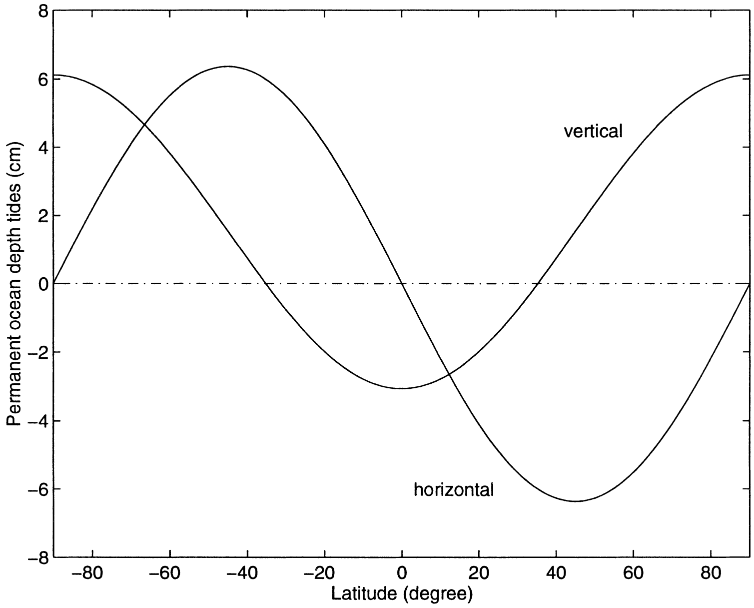

Fig. 3. Distributions of the permanent ocean depth tide caused by the permanent part of the luni-solar potential along latitude. The horizontal component along longitude is zero. The unit is cm.

The distribution along latitude of the permanent ocean depth tide caused by the permanent potential can be found in Fig. 3. Again, the horizontal component along longitude is zero. The figure shows that the vertical component changes 6.1 cm at the poles and −3.0 cm at the equator, while the horizontal component reaches 6.3 cm at ±45°. The results indicate that the permanent ocean depth tides are big enough to be considered in any high accuracy applications.

5. Conclusions

The results of our study on the three permanent tides show that the vertical permanent crust (zero crust), geoid and ocean depth tides reach −12.0, −5.8 and 6.1 cm at the poles, and 5.9, 2.9 and −3.0 cm at the equator, respectively. The horizontal permanent crust, geoid and ocean depth tides reach as much as 2.5, 8.7 and 6.3 cm, respectively. According to the solution of IAG (1983), IERS Conventions (McCarthy, 1996) and the suggestion by Poutanen et al. (1996), vertical components of the permanent tides should be included in GPS positioning and geoid computation. We suggest here that the horizontal components should be correspondingly considered. More precisely, the horizontal GPS co-ordinates should retain the permanent tide component, and the corresponding horizontal geoid components are of particular interest in GPS and precise marine positioning.

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