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Orthogonal cross joints: do they imply a regional stress rotation?

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Abstract

Orthogonal cross joints extend across intervals between systematic joints in brittle sedimentary strata and abut the systematic joints at about 90° angles. These joints typically form a 'ladder-like' pattern if viewed on a bedding surface. A common interpretation is that orthogonal cross joints define the orientation of the regional stress field during their formation: least compressive stress perpendicular to the joints. It follows that they indicate a rotation of regional principal stresses by 90° after the formation of the systematic joints. Using a three-dimensional boundary element code (Poly3D), we considered a simple geologic case of vertical systematic fractures developing in horizontal strata under a triaxial remote load with: the maximum principal tensile stress being horizontal and perpendicular to the strike of the fractures, the intermediate principal stress being horizontal and parallel to the strike of the fractures, and the least principal tensile stress (i.e. maximum compressive stress) being vertical. The results show that the local maximum principal stress is first perpendicular, and then parallel to, the strike of the systematic fractures as the ratio of fracture spacing to height changes from greater than to less than a *critical value* when the horizontal remote principal stress ratio, the ratio of the intermediate remote principal stress to the maximum remote principal stress under the sign convention of positive for tensile stresses, is greater than a threshold value (~0.2). Thus, the fracturing process changes from infilling of systematic fractures to the formation of orthogonal cross fractures. This provides an alternative mechanism for the formation of orthogonal cross joints that does not require a systematic rotation of the regional stress field by 90°. The critical spacing to height ratio for the local principal stress switch is independent of the least remote principal stress (i.e. overburden). It increases nonlinearly with increasing ratio of the horizontal remote principal stresses, and decreases nonlinearly with increasing Poisson's ratio of the material. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Orthogonal cross joints; Systematic joints; Jointing processes; Layered rocks

1. Introduction

Joints are opening-mode fractures formed as a consequence of the deformation of brittle rock masses in the Earth's crust (Engelder, 1987; Pollard and Aydin, 1988). They are sensitive indicators of the paleo stress field and can be used to infer the orientation of the regional stress field along with its temporal and spatial evolution (e.g. Engelder and Geiser, 1980; Dyer, 1988; Olson and Pollard, 1989). Joints also provide pathways for underground fluid flow. Thus, understanding natural fracture networks in aquifers and hydrocarbon reservoirs as well as engineering sites (National Research Council, 1996) is one of the key steps in modeling ground water flow and predicting hydrocarbon migration and accumulation (Barton and Hsieh, 1989; Gringarten, 1996; Taylor et al., 1999).

Cross joints extend across intervals between systematic joints, without cutting across the systematic joints (e.g. Hodgson, 1961; Hancock, 1985; Dyer, 1988; Gross, 1993; Bai and Gross, 1999). *Orthogonal cross joints*, as one of the commonly-observed categories of cross joints, typically resemble a 'ladder-like' pattern in outcrop (Rawnsley et al., 1992; Rives et al., 1994). They abut the systematic joints at angles near 90° and are limited in length by the intervening distance between the systematic joints (Fig. 1). One of the earliest recorded observations of cross joint geometries was made in the Comb Ridge–Navajo Mountain area of Utah and Arizona by Hodgson (1961). Typical orthogonal cross joint patterns have been reported from outcrops along the Bristol Channel, UK (Rawnsley et al., 1992; Rives et al., 1994; Caputo, 1995; Rawnsley et al., 1998), from the Appalachian Plateau, western New York State (Engelder and Gross, 1993; Zhao and Jacobi, 1997), from the Monterey Formation, Santa Barbara Coastline,

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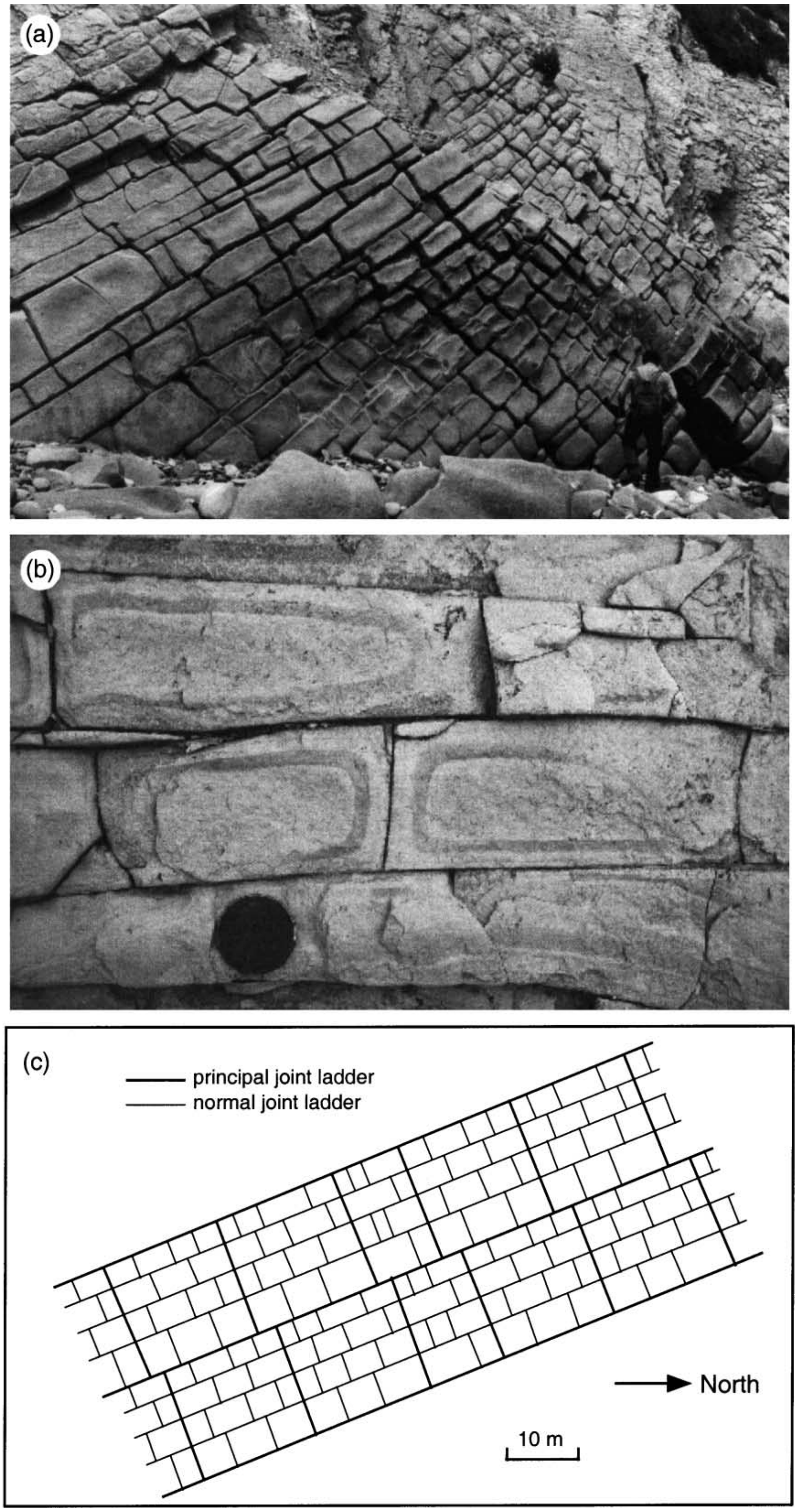


Fig. 1. Field examples of orthogonal cross joints at different scales that developed in between systematic joints. (a). Orthogonal cross joints in a steeply inclined carbonate bed of the Monterey Formation along the Santa Barbara coastline, California. Note geologist for scale. Photograph taken by Wendy Bartlett. (b). Orthogonal cross joints in a porcellanite bed (bedding surface) of the Monterey Formation along the Santa Maria coastline, California. (c). Joint patterns from Nash Point, Bristol Channel, UK (after Rawnsley et al., 1998).

California (Gross, 1993; Finn et al., 1999), and from Arches National Park, Utah (Rives et al., 1994; Cruikshank and Aydin, 1995).

Cross joint paths, which may be curved, are perpendicular to the *local* maximum tensile stress and their main trend is inferred to be approximately perpendicular to the direction of *regional* maximum tensile stress (Dyer, 1988; Engelder and Gross, 1993; Bai and Gross, 1999). Given this interpretation, the presence of orthogonal cross joints implies a regional principal stress *rotation* through 90° after the formation of the systematic joints and before the cross joints formed.

In studying the stress distribution between adjacent equally-spaced fractures in layered rocks, Bai and Pollard (2000) found that there is a *stress transition* for the component of local normal stress in the direction perpendicular to the fractures. Their numerical models suggest that under a remote extension in the direction perpendicular to the fractures, this normal stress changes from tensile to compressive when the fracture spacing to layer thickness ratio changes from greater than to less than a critical value (approximately 1.0). This stress transition implies that, at a certain stage during the formation of a systematic joint set by sequential infilling (Gross, 1993), further infilling is inhibited. On the other hand, because the local stress in the direction perpendicular to the systematic joints becomes compressive, the stress in the direction parallel to these joints may become the maximum tensile (least compressive) stress, and orthogonal cross joints may form in response to this local *switch* in principal stress orientation. In other words, orthogonal cross joints may form in the absence of a regional stress rotation. In this paper, we use the three-dimensional boundary element method to explore the possibility for a local principal stress switch between adjacent systematic joints as a function of the ratio of spacing to layer thickness of the systematic joints, the remotely applied principal stresses and the elastic constants.

2. Previous conceptual models for the formation of orthogonal cross joints

Because opening-mode fractures in isotropic, homogeneous materials propagate in the direction perpendicular to the local maximum tensile stress, or least compressive stress if fluid pressure drives the fracturing (Pollard and Segall, 1987; Pollard and Aydin, 1988), a local principal stress rotation of 90° is needed for orthogonal cross joints to form after the formation of the systematic joints. This stress rotation may be induced by either local or regional mechanisms. Proposed local mechanisms in the geological literature include: rock band warping (Granier and Bles, 1988), stress release under biaxial extension (Simon et al., 1988); regional mechanisms include: visco-elastic strain relaxation (Rives and Petit, 1990; Rives et al., 1994), and regional principal stress rotation (Eyal and Reches, 1983; Hancock et al., 1987; Bahat and Grossmann, 1988; Dunne and North, 1990; Eyal, 1996).

The *rock band warping* mechanism is based on the fact that the systematic joints cut the layer into long and narrow bands. Because of this shape, the bands are very sensitive to bending or warping in the direction parallel to the systematic joints. Any warping of the bands, for example, by inhomogenous settling of the overburden or large-scale folding or faulting, could result in local tensile stresses that would produce orthogonal cross joints (Granier and Bles, 1988; Rives et al., 1994).

The *stress release* mechanism is based on the change of stress in the direction perpendicular to the systematic joints after their formation. Because the surfaces of open (non fluid-filled) systematic joints are traction free, their formation will release the crack-normal tensile stress (Pollard and Segall, 1987). In a rock layer subjected to biaxial extension (e.g. Ghosh, 1988; Simon et al., 1988), it is possible that the stress parallel to the systematic joints becomes the maximum tensile stress (least compressive stress). As a result, orthogonal cross joints may form. Chocolate-tablet boudinage is an example of orthogonal fractures that form as a result of biaxial extension (e.g. Ramsay and Huber, 1983).

Rives and Petit (1990) and Rives et al. (1994) proposed the *visco-elastic strain relaxation* concept based on their experimental observations of fracture formation in a brittle varnish layer bonded to a PVC plate. For a specimen loaded under uniaxial extension, systematic fractures form in the direction perpendicular to the extension and the specimen contracts in the direction perpendicular to the extension. If the extension is released, the specimen recovers to its original shape by expanding in the direction perpendicular to the extension, and this expansion produces fractures orthogonal to the initial fracture set. The key for this mechanism is release of the extension that generates the systematic joints. Geological processes that may induce this release include uplift and erosion, after the rocks have been deformed at depth (Engelder, 1985).

The *stress rotation* mechanism requires the regional principal stresses to change directions by ~90° after the formation of the systematic joints, so cross joints can form perpendicular to the systematic joints. Regional principal stress rotations could be caused by regional fluctuations of tectonic stresses (Hancock et al., 1987; Bahat and Grossmann, 1988; Dunne and North, 1990). Such rotations also could occur during regional uplift and erosion (Engelder, 1985) because of the release of locked-in stress (Friedman, 1972; Rathore et al., 1989).

With the two regional mechanisms cited above, one would expect that orthogonal cross joints would form indiscriminately between all pairs of systematic joints. However, as reported by Engelder and Gross (1993) and Mollema and Antonellini (1999, refer to their Fig. 4), orthogonal cross joints on the same outcrops are most commonly found between systematic joints relatively closely-spaced, whereas between pairs of systematic joints relatively widely-spaced, orthogonal cross joints are less common. This selectivity may in part be attributed to the dependence of cross joint spacing on the spacing of pre-existing systematic joints, with the latter defining an effective mechanical layer thickness (Gross, 1993; Ruf et al., 1998). However, it may also reflect a switch in local principal stress axes that would promote the formation of cross joints once adjacent systematic joints reach a critical spacing. We investigate the latter possibility using numerical models.

3. Numerical modeling and results

3.1. Numerical method and boundary conditions

We use a three-dimensional boundary element code named Poly3D (Thomas, 1993) for the modeling. The code is based on the displacement discontinuity method and the governing equations of linear elasticity theory (Crouch and Starfield, 1983; Becker, 1992). Poly3D uses the solution for an angular dislocation and superimposes several angular dislocations to form a polygonal dislocation surface. Multiple polygonal dislocation surfaces are joined together to make a fracture. Displacement discontinuities along fractures subject to remote loading can be calculated, as well as the stress, strain and displacement fields in their surroundings.

We considered a simple geologic case of vertical systematic fractures developing in horizontal strata under a triaxial remote load with the maximum principal tensile stress, $\sigma_1(\infty)$, being horizontal and perpendicular to the strike of the fractures, the intermediate principal stress, $\sigma_2(\infty)$, being horizontal and parallel to the strike of the fractures, and the least principal stress, $\sigma_3(\infty)$, being vertical. The sign convention used throughout this paper is positive for tensile stresses and negative for compressive stresses. Thus, the least principal stress, $\sigma_3(\infty)$, is the maximum compressive stress, and geologically it is the overburden stress imposed by the overlying strata. The horizontal remote principal stress ratio is defined by the ratio of $\sigma_2(\infty)/\sigma_1(\infty)$. A positive value of this ratio indicates that both $\sigma_1(\infty)$ and $\sigma_2(\infty)$ are positive, and a negative ratio indicates that $\sigma_1(\infty)$ is positive and $\sigma_2(\infty)$ is negative. The model geometry, the meshing for the fractures, and the loading conditions are shown in Fig. 2. Four equally-spaced fractures of the same size and same orientation are placed in a homogeneous, isotropic and elastic whole space. The height of the fractures is T_f and the spacing is S. Limiting the number of fractures to four and using the middle two fractures to represent any two adjacent fractures in a row composed of many equally-spaced members has been shown to introduce maximum errors in fracture aperture and stress distribution of less than 2% (Bai and Pollard, 2000; Bai et al., 2000). The coordinate system is designated with the origin at the middle point of the model (i.e. between the two central fractures), the x-axis is horizontal and is perpendicular to the fractures. The y-axis is horizontal and is parallel to the fractures. The z-axis is parallel to the fractures and pointing upward. The faces of the fractures are specified to be traction free. The mesh on the fractures was refined by reducing the size of the elements until calculated stresses at $x = y = 0$ differed by less than 1%.

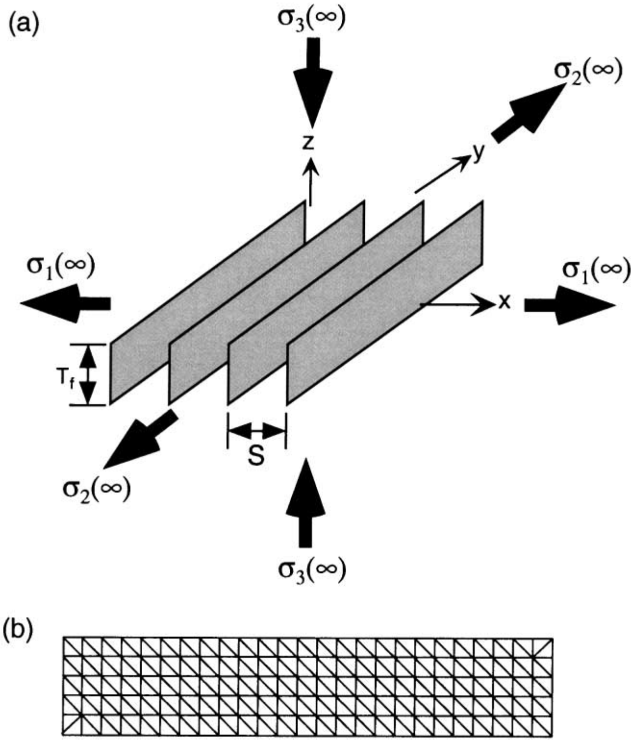


Fig. 2. (a). Model configuration. Four fractures of the same dimension are placed in the whole space of a homogeneous, isotropic and elastic medium. The principal stress switch between the two middle fractures as a function of fracture spacing to height ratio, S/T_f, the horizontal remote principal stress ratio, $\sigma_2(\infty)/\sigma_1(\infty)$, the vertical stress (overburden), $\sigma_3(\infty)$, and the elastic constants, E and ν, was evaluated using this model and Poly3D. (b). The mesh designed for each of the fractures.

3.2. Local principal stress switch and the formation of orthogonal cross fractures

The stresses σ_{xx} and σ_{yy} at $x = y = 0$ were calculated for models with different fracture spacing to height ratios and different horizontal remote principal stress ratios, $\sigma_2(\infty)/\sigma_1(\infty)$, and with the Young's modulus $E = 30$ GPa, and the Poisson's ratio $\nu = 0.25$. The local stresses were then normalized by $\sigma_1(\infty)$. A plot of the normalized stresses versus the fracture spacing to height ratio for the case of $\sigma_2(\infty)/\sigma_1(\infty) = 0.4$ is shown in Fig. 3. We see that there is a critical spacing to height ratio $(S/T_\mathrm{f})^S_\mathrm{cr} \approx 1.7$. When $S/T_\mathrm{f} > (S/T_\mathrm{f})^S_\mathrm{cr}$, $\sigma_{xx} > \sigma_{yy}$. On the other hand, when $S/T_\mathrm{f} < (S/T_\mathrm{f})^S_\mathrm{cr}$, $\sigma_{xx} < \sigma_{yy}$. In other words, the local stress switch occurs as the spacing to height ratio of the systematic fractures changes from greater than to less than the critical

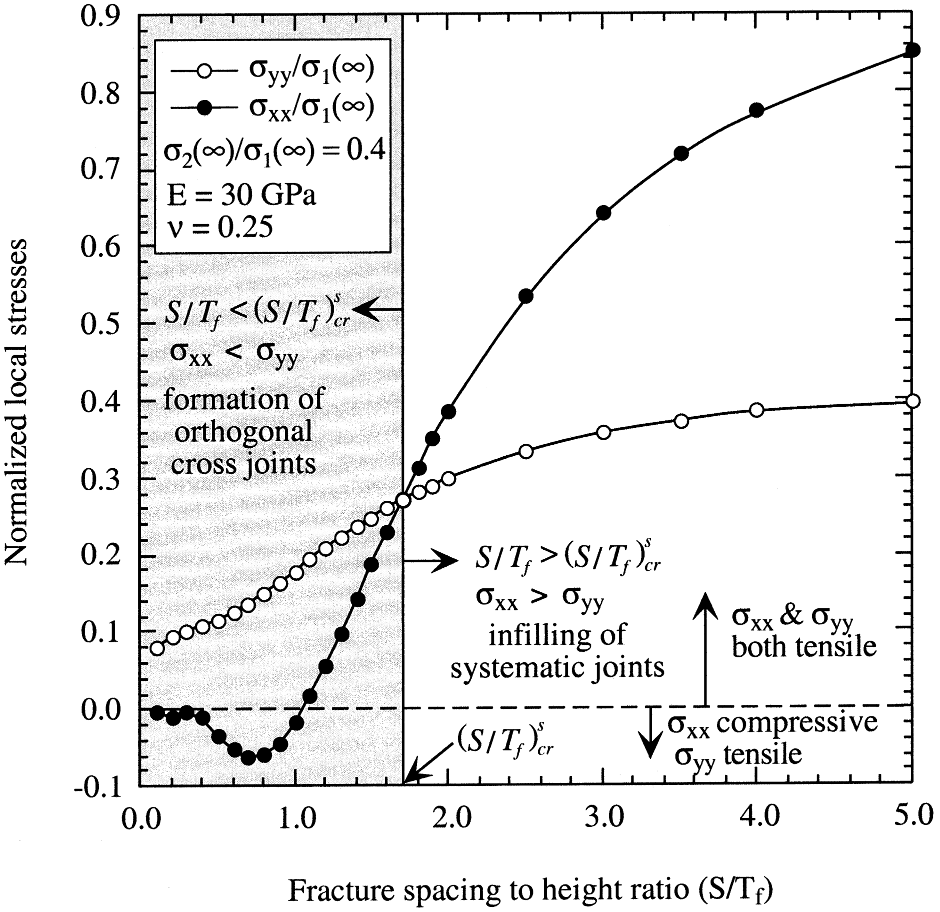


Fig. 3. Results of local stress switch between adjacent equally-spaced fractures. The plots show that when S/T_f is less than about 1.7 (shaded area), σ_{yy} is tensile and $\sigma_{yy} > \sigma_{xx}$, Hence, orthogonal cross fractures will form between the systematic fractures. However, when $S/T_\mathrm{f} > 1.7$, $\sigma_{yy} < \sigma_{xx}$. In this case, infilling of new systematic fractures is going to occur. See Fig. 2 for the coordinate system and the model configuration. The sign convention used here is that tensile stresses are positive and compressive stresses are negative.

ratio. We call this critical ratio the *critical spacing to height ratio for a local principal stress switch*. Trajectories of the local horizontal principal stresses at the $z = 0$ plane and between the two middle fractures are presented for cases with $\sigma_2(\infty)/\sigma_1(\infty) = 0.4$ (i.e. both horizontal stresses tensile) and with different spacing to height ratios (Fig. 4). First, we see that the local principal stresses are approximately aligned with the x- and y-axes. Second, the local principal stresses switch their directions as the spacing to height ratio changes from greater than to less than a critical value. By definition, joints always form in the direction perpendicular to the maximum tensile stress (Pollard and Aydin, 1988). When $S/T_\mathrm{f} = 2.5$ (greater than $(S/T_\mathrm{f})^S_\mathrm{cr}$), the local maximum principal stress is perpendicular to the systematic fractures (Fig. 4a). Thus, potential new fractures will form in the direction parallel to the systematic fractures. When $S/T_\mathrm{f} = 0.9$ (less than $(S/T_\mathrm{f})^S_\mathrm{cr}$), the local maximum principal stress is tensile and approximately parallel to the systematic fractures (Fig. 4b). In this case, fractures are more likely to form in the direction orthogonal to the systematic fractures.

3.3. *Effects of remote loading on the critical fracture spacing to height ratio for the local principal stress switch*

The effects of remote loading were first studied by keeping the two horizontal remote principal stresses constant while changing the vertical remote principal stress (overburden). The results show that the local stresses σ_{xx} and σ_{yy} at $x = y = 0$ are not affected by the vertical remote principal stress. Thus, we conclude that the critical spacing to height ratio for the local principal stress switch is independent of the overburden stress.

We then studied the effects of the horizontal remote principal stresses by keeping the vertical remote principal stress as zero and varying the two horizontal remote principal stresses. For a given model configuration with fixed elastic constants, the normalized local stresses, $\sigma_{xx}/\sigma_1(\infty)$ and $\sigma_{yy}/\sigma_1(\infty)$, depend only on the ratio of the horizontal remote principal stresses, $\sigma_2(\infty)/\sigma_1(\infty)$. The critical spacing to height ratio for local stress switch increases nonlinearly with increasing horizontal remote principal stress ratio (Fig. 5). As the remote principal stress ratio

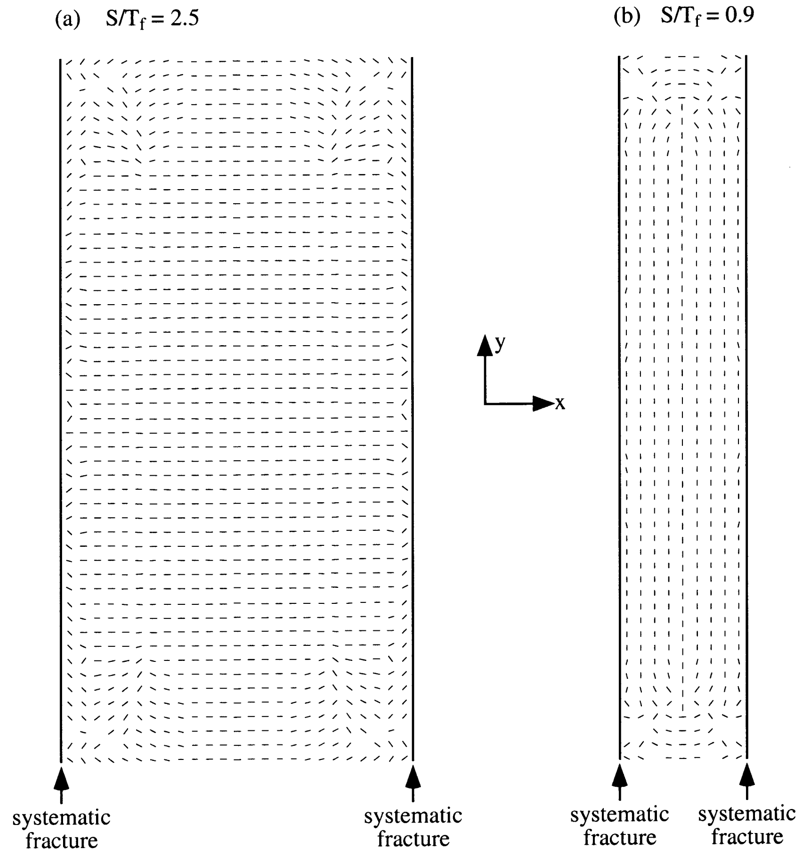


Fig. 4. Trajectories of the maximum horizontal tensile stress on the plane $z = 0$ and between the two middle fractures (refer to Fig. 2). (a). When $S/T_f > (S/T_f)^S_{cr}$, the stress is approximately perpendicular to the systematic fractures, and potential fractures will form in the direction parallel to the systematic fractures. (b). When $S/T_f < (S/T_f)^S_{cr}$, the principal stress is approximately parallel to the systematic fractures in most parts of the plot area. Thus, orthogonal cross fractures will form in the direction perpendicular to the systematic fractures. Data from models with $\sigma_2(\infty)/\sigma_1(\infty) = 0.4$, $E = 30$ GPa, $\nu = 0.25$. For this model configuration, $(S/T_f)^S_{cr} \approx 1.7$. Note the 'abnormal' stress trajectories near the fractures are caused by the boundary element method.

approaches one, the critical spacing to height ratio for the stress switch goes to infinity. Using the regression technique, we obtain the best fit line to this numerical result as:

$$(S/T_f)^S_{cr} = \frac{1.65(\sigma_2(\infty)/\sigma_1(\infty))^{0.33}}{(1 - \sigma_2(\infty)/\sigma_1(\infty))^{0.60}} + f(\sigma_2(\infty)/\sigma_1(\infty)), \quad (1)$$

where $(S/T_f)^S_{cr}$ is the critical spacing to height ratio for the local stress switch, $f(\sigma_2(\infty)/\sigma_1(\infty))$ is a sixth order polynomial:

$$f(x) = -3.618 + 45.90x - 241.9x^2 + 671.3x^3 - 1022.6x^4 + 808.9x^5 - 259.6x^6. \quad (2)$$

Eqs. (1) and (2) are valid for $\nu = 0.25$ and for the remote principal stress ratios in the range from approximately 0.2 to 1. Below this range, the local stresses are either both negative (compressive) or σ_{yy} is always less than σ_{xx} (refer to Fig. 9). Thus, orthogonal cross fractures will not form.

3.4. *Effects of elastic constants on the critical fracture spacing to height ratio for the local principal stress switch*

We studied the elastic constant effects on the critical spacing to height ratio for the local principal stress switch by changing either the Young's modulus or the Poisson's ratio while keeping the other constant. The results show that the critical spacing to height ratio for the stress switch is independent of the Young's modulus. However, it decreases nonlinearly with increasing Poisson's ratio (Fig. 6). The best fit line to the numerical results expressed as a fourth order polynomial is:

$$(S/T_f)^S_{cr} = 2.024 - 1.065\nu - 1.950\nu^2 + 7.396\nu^3 - 13.683\nu^4, \quad (3)$$

where ν is the Poisson's ratio. This equation is valid for $\sigma_2(\infty)/\sigma_1(\infty) = 0.4$.

We plotted the critical spacing to height ratio for the local stress switch as a function of the horizontal principal stress ratio and the Poisson's ratio as an isogram shown in Fig. 7. This plot was constructed based on the numerical results of the critical spacing to height ratios obtained by changing either the Poisson's ratio or the horizontal remote principal stress ratio while keeping the other constant.

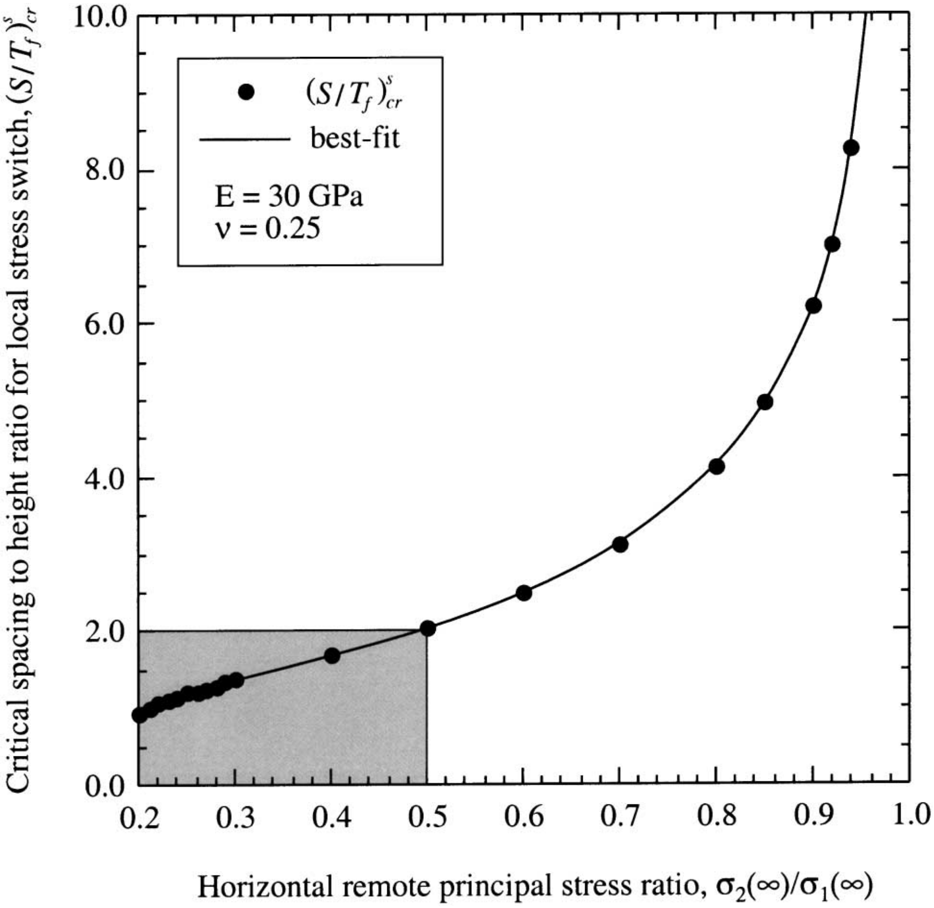


Fig. 5. The critical spacing to height ratio for the local stress switch, $(S/T_f)^S_{cr}$, increases with increasing ratio of the horizontal remote principal stresses, $\sigma_2(\infty)/\sigma_1(\infty)$. Shaded box indicates range of likely horizontal remote stress ratios in light of the observation that $(S/T_f)^S_{cr}$ is less than two for most jointed rock beds.

4. Discussion

In layered sedimentary rocks, systematic joints often are confined by layer boundaries with the joint height roughly equal to the thickness of the fractured layer (e.g. Bahat, 1988; Helgeson and Aydin, 1991; Narr and Suppe, 1991; Gross and Engelder, 1995; Gross et al., 1995). However, the numerical results obtained here are from an isotropic, homogeneous, elastic whole space with parallel equally-spaced systematic fractures because of the limited capability of the numerical code, Poly3D. It has been shown that contrasting elastic moduli of layers only introduce minor quantitative differences, and do not alter the qualitative conclusions regarding the critical spacing to layer thickness ratio for the stress state transition between equally-spaced systematic joints (Bai and Pollard, 2000). We expect a similar outcome for the critical spacing to layer thickness ratio for the local principal stress switch. Therefore, the ratio of spacing to height of the fractures in the model presented here corresponds approximately to the ratio of fracture spacing to the thickness of the fractured layer in a layered system, and we refer to the critical ratio for stress switch as the *critical spacing to layer thickness ratio*.

With this in mind, the numerical results suggest that orthogonal cross joints can form during the infilling process of systematic joints as a result of the local stress switch when the spacing to layer thickness ratio of the systematic joints changes from greater than to less than the critical ratio. In the following discussion, we first evaluate the geologic constraints on the results. Then, we explain the differences between orthogonal cross joint patterns that formed due to regional stress rotation and that formed because of the local stress switch. Finally, we discuss jointing processes in layered rocks with consideration of sequential infilling and formation of orthogonal cross joints.

4.1. Geologic constraints on the numerical results

Geologic constraints can be used to evaluate the ranges over which the numerical results are applicable to the development of natural fractures in rock. First, development of orthogonal cross joints by the proposed mechanism occurs only when the principal stress ratio is positive (Figs. 5–7), which in turn means that both horizontal principal stresses are tensile. Such a state of stress is likely restricted to the uppermost portions of the crust and suggests

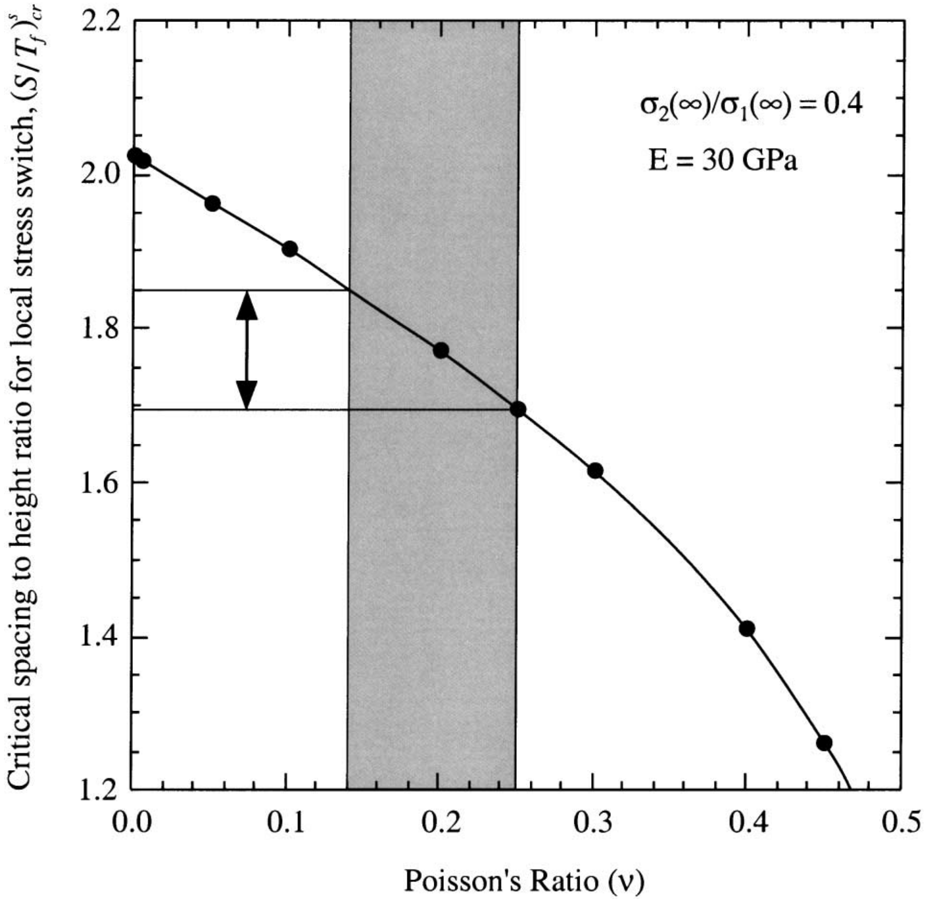


Fig. 6. The critical spacing to height ratio for the local stress switch, $(S/T_\mathrm{f})^S_\mathrm{cr}$, decreases with increasing Poisson's ratio (ν) of the material. Shaded box indicates range of Poisson's ratio for competent lithologies resulting in a limited range (arrow) of critical spacing to height ratios.

that orthogonal cross joints form during uplift and erosion. Second, the fracture pattern consisting of systematic joints and cross joints is generally restricted to the more competent 'jointing' lithologies within a mechanical stratigraphy (e.g. Narr and Suppe, 1991; Gross, 1993). Compilations of rock elastic properties indicate a range in Poisson's ratio of approximately 0.14 to 0.25 for typical 'jointing' lithologies (Blair, 1955, 1956; Hatheway and Kiersch, 1982; Fischer, 1994). This constrains the likely ranges of the critical spacing to height ratio for the local stress switch (Fig. 7). Third, although a wide range in S/T_f has been reported in the literature for layered rocks (e.g. Bai and Pollard, 2000), ratios of $S/T_\mathrm{f} > 2$ are extremely rare for competent lithologies. Thus, the range of possible remote principal stress ratios for orthogonal cross joint development is less than about 0.5 (Figs. 5 and 7). If orthogonal cross joints indeed develop within the same remote stress field as the systematic joints, then our modeling suggests the requirement for a strong anisotropy in principal horizontal stresses (Fig. 7).

4.2. Orthogonal cross joint patterns: regional stress rotation versus local stress switch

If orthogonal cross joints develop because of a switch in the local stress field at a certain stage during the infilling process of systematic joints, these joints would form in the same remote tectonic stress field that promoted propagation of the systematic joints. Furthermore, the cross joints between a pair of systematic joints would have formed after the two systematic joints, but not necessarily after the formation of all systematic joints of that set because infilling of systematic joints could still be occurring elsewhere in the region. In other words, systematic and cross joints formed by this mechanism develop within the same phase of deformation (Fig. 8a).

However, how can a field geologist determine whether or not there was a tectonic stress rotation based on outcrop patterns of orthogonal cross joints? The first step is to evaluate other mesostructures within the outcrop such as tectonic stylolites, fold axes and striated faults. These kinematic and strain indicators may provide constraints on the orientations and timing of regional stress fields (e.g. Eyal and Reches, 1983). We also suggest the following diagnostic test. Consider an outcrop area with widely spaced and narrowly spaced systematic joints. If orthogonal cross joints only occur between pairs of narrowly spaced systematic joints (Engelder and Gross, 1993; Mollema and Antonellini, 1999), it can be concluded that there was no tectonic stress

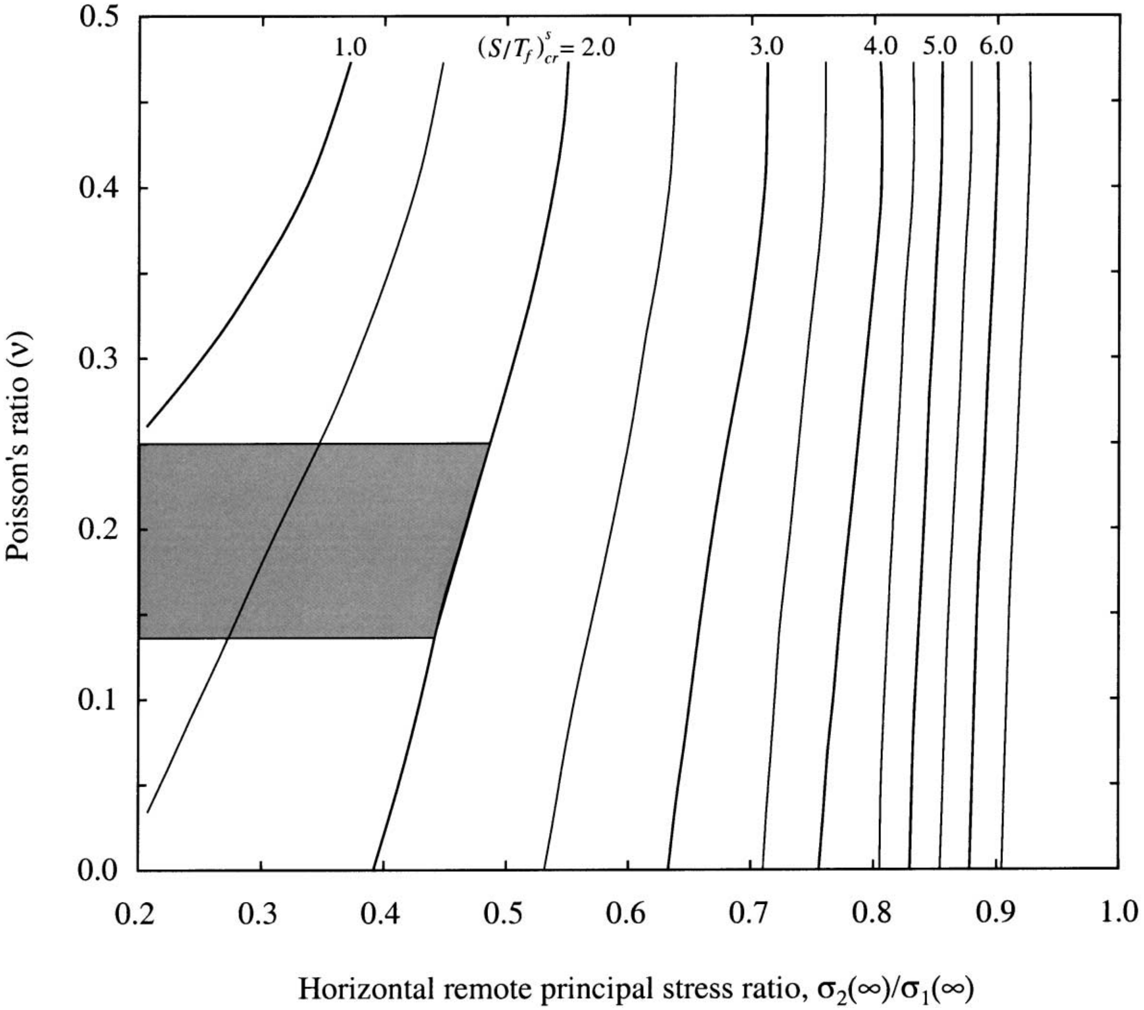


Fig. 7. Plots of the critical spacing to height ratio for the local stress switch, $(S/T_{\mathrm{f}})^{S}_{\mathrm{cr}}$, as a function of the horizontal remote principal stress ratio, $\sigma_2(\infty)/\sigma_1(\infty)$, and the Poisson's ratio, ν. Shaded box indicates range over which the proposed mechanism for orthogonal cross joint formation may occur.

rotation during the joint formation (Fig. 8a). On the other hand, if orthogonal cross joints do not appear selectively between pairs of narrowly spaced systematic joints, a tectonic stress rotation of 90° is a possible interpretation (Fig. 8b).

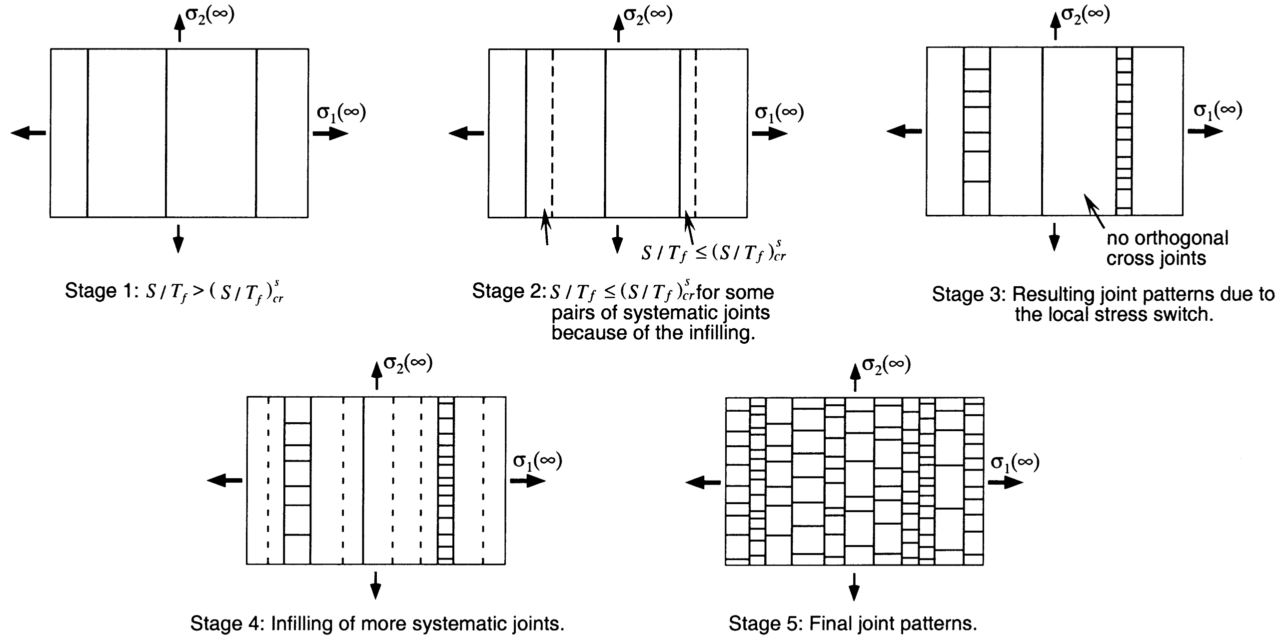
4.3. Jointing processes in layered rocks under an irrotational tectonic stress field

By 'irrotational' we mean that the tectonic stress field does not rotate during the formation of the systematic joints and/or the orthogonal cross joints. In such a stress regime, one process for the formation of the parallel systematic joints has been described as 'sequential infilling' (Gross, 1993). The numerical results of Bai and Pollard (2000) indicate that the process of sequential infilling is restrained by the stress state transition between equally spaced fractures. The critical spacing to layer thickness ratio defined by the stress state transition gives the lower limit for the spacing to layer thickness ratio of the systematic fractures if significant flaws and other mechanisms (i.e. internal fluid pressure) do not exist. The critical spacing to layer thickness ratio for the stress state transition is a function of the elastic constants of the fractured layer and that of the neighboring layers as well as the overburden stress. However, for reasonable ranges of these factors, this critical ratio is approximately 1.0.

The jointing processes is further constrained by the numerical results of the local stress switch. At a certain stage during the infilling process of the systematic joints, orthogonal cross joints may form because of the local stress switch (Figs. 3 and 8a). The switch only occurs when the ratio of the horizontal remote principal stresses is greater than a threshold value (Fig. 5). This threshold depends on the Poisson's ratio of the material (ν). For $\nu = 0.25$, the threshold stress ratio is approximately 0.2.

Considering the two constraints discussed above, we can describe the jointing processes in layered rocks based on the spacing to layer thickness ratio of the systematic joints and the horizontal remote stress ratio. As an example, we show the case with $\nu = 0.25$ in Fig. 9. We see that the formation of orthogonal cross joints can only occur when the ratio of

(a) Local stress switch model



(b) Regional stress rotation model

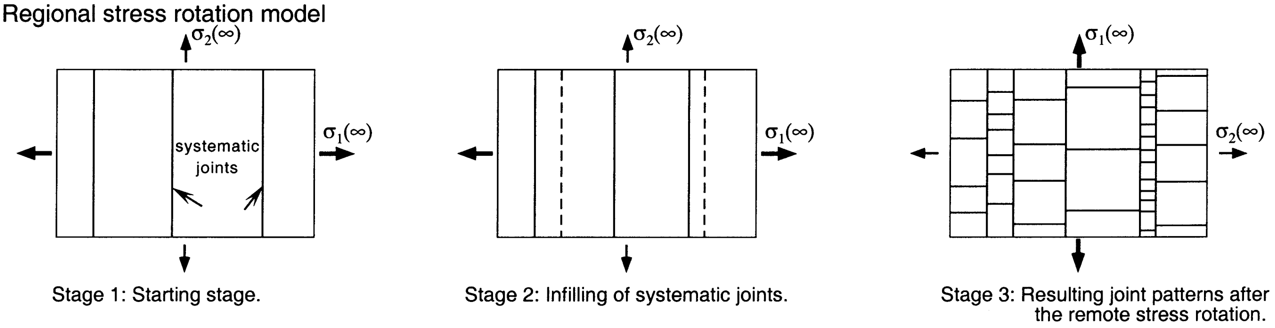


Fig. 8. Schematic representation of two different models that can lead to the development of orthogonal cross joints. (a) Local stress switch model whereby orthogonal cross joints and systematic joints develop within the same stress field. Orthogonal cross joints will be found only between the closely-spaced systematic joints due to the local switch in principal stress axes (Stage 3). However, with further infilling of the systematic joints, orthogonal cross joints may eventually occur between any pair of systematic joints if their spacing to height ratio is less than the critical value, $(S/T_f)^S_{cr}$. (b) Stress rotation model indicating a switch in the orientations of horizontal remote principal stresses through time. Note that in this case orthogonal cross joints will develop between all adjacent systematic joints.

horizontal remote stresses is greater than approximately 0.2 and the spacing to height ratio of the systematic fractures is less than the critical spacing to height ratio for local stress switch described by the curve on the upper right portion of the diagram. When the ratio of horizontal remote stresses is greater than 0.2 and the spacing to height ratio of the systematic fractures is greater than the critical value, further infilling of systematic fractures is more likely to occur. When the ratio of horizontal remote stresses is less than 0.2, further infilling of systematic fractures can only occur when the spacing to height ratio of the systematic fractures is greater than the critical value defined by the stress transition. No jointing occurs when the horizontal remote stress ratio is less than approximately 0.2 and the spacing to height ratio of the systematic fractures is less than the critical value defined by the stress transition. Since the critical spacing ratio for the local stress switch does not depend on the vertical stress (overburden) and the critical ratio for the stress transition only slightly depends on the overburden stress (Bai and Pollard, 2000), we expect the results as described in Fig. 9 only slightly depends on the overburden. This is important in constructing fracture networks at depth.

5. Conclusions

By studying the local stress switch between equally spaced systematic fractures, we have defined a critical spacing to height ratio. In a stress regime with the remote horizontal stress ratio greater than a threshold value, a local stress switch occurs when the spacing to layer thickness

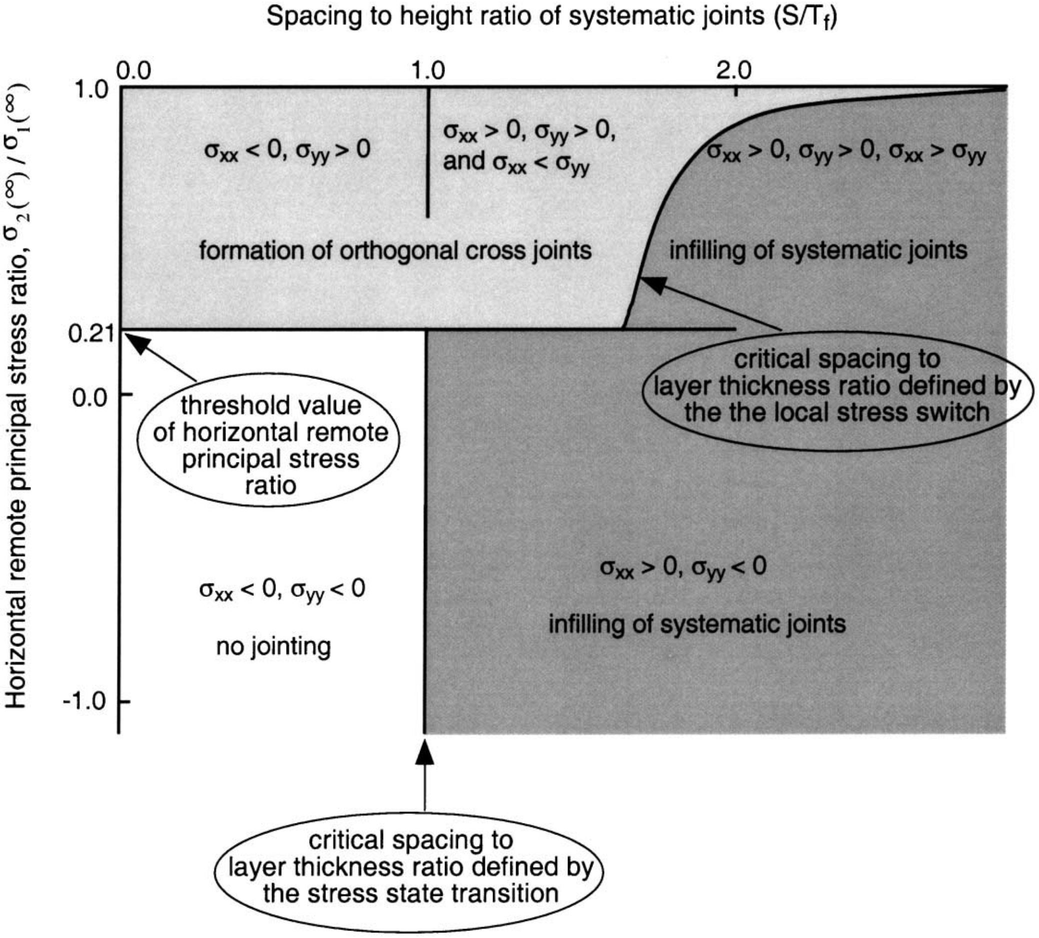


Fig. 9. Jointing processes in layered rocks under irrotational tectonic stresses and their relations to the spacing to layer thickness ratio of the systematic joints, S/T_f, and the horizontal remote principal stress ratio, $\sigma_2(\infty)/\sigma_1(\infty)$. See text for the details.

ratio of the systematic fractures changes from greater than to less than the critical ratio. The critical ratio is independent of the overburden stress and Young's modulus. It increases nonlinearly with increasing ratio of the horizontal remote principal stresses, and decreases with increasing Poisson's ratio of the material. We can conclude that the ladder pattern of systematic joints and orthogonal cross joints does not necessarily imply a 90° rotation of the tectonic stress field. The results put new constraints on jointing processes in layered rocks in an irrotational tectonic stress field.

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