

# Kinetic energy of rain and its functional relationship with intensity

Christian Salles<sup>a,\*</sup>, Jean Poesen<sup>b</sup>, Daniel Sempere-Torres<sup>c</sup>

<sup>a</sup>Laboratoire Hydrosiences Montpellier (UMR 5569), Université Montpellier II, Case Courrier MSE, 34095 Montpellier Cedex 5, France

<sup>b</sup>Laboratory for Experimental Geomorphology, K.U. Leuven, Redingenstraat 16, B-3000 Leuven, Belgium

<sup>c</sup>Departament d'Enginyeria Hidràulica, Marítima i Ambiental, Universitat Politècnica de Catalunya, Xarxa d'Hidrologia Mediterrània, Jordi Girona, 1-3, D-1; E-08034 Barcelona, Spain

Received 10 November 2000; revised 24 August 2001; accepted 23 October 2001

## Abstract

The rain kinetic energy (KE) is a widely used indicator of the potential ability of rain to detach soil. However, rain kinetic energy is not a commonly measured meteorological parameter. Therefore, empirical laws linking the rain kinetic energy to the more easily available rain intensity ( $I$ ) have been proposed based on drop-size and drop-velocity measurements. The various mathematical expressions used to relate rain kinetic energy and rain intensity available from the literature are reported in this study. We focus our discussion on the two expressions of the kinetic energy used: the rain kinetic energy expended per volume of rain or volume-specific kinetic energy ( $KE_{\text{mm}}$ ,  $\text{J m}^{-2} \text{mm}^{-1}$ ) and the rain kinetic energy rate or time-specific kinetic energy ( $KE_{\text{time}}$ ,  $\text{J m}^{-2} \text{h}^{-1}$ ). We use statistical and micro-physical considerations to demonstrate that  $KE_{\text{time}}$  is the most appropriate expression to establish an empirical law between rain kinetic energy and rain intensity. Finally, considering the existing drop-size distribution models from literature, we show that the most suitable mathematical function to link KE and  $I$  is a power law. The constants of the power law are related to rain type, geographical location and measurement technique. © 2002 Elsevier Science B.V. All rights reserved.

**Keywords:** Rain erosivity; Kinetic energy; Drop-size distribution

## 1. Introduction

Empirical and process-based soil erosion models often use rain kinetic energy (KE) as the rain erosivity index: e.g. in splash erosion modelling (e.g. Poesen, 1985) and in modelling sheet and rill erosion, such as in SLEMSA (Elwell, 1978), in EUROSEM (Morgan et al., 1998a,b) or in RUSLE (Renard et al., 1997).

Basically, the rain kinetic energy results from the kinetic energy of each individual raindrop that strikes the soil. The information provided by drop-size distribution

(DSD) measurements combined with fall velocity measurements or empirical laws linking terminal fall velocity ( $V_t$ ) and drop diameter ( $D$ ), allow one to calculate the rain kinetic energy. DSD data have been obtained using various techniques (e.g. flour pellet, filter paper, oil immersion, electro-mechanical or optical devices, meteorological radar). Such measurements usually do not provide continuous data in space and time. An exception is the study from Doelling et al. (1998) who reports 7 years of DSD measurements in northern Germany. Hence, the introduction of more specific devices that allow continuous and direct rain kinetic energy measurements (e.g. Madden et al., 1998; Jayawardena and Rezaur, 2000a) will hopefully enlarge the availability of rain kinetic energy datasets.

\* Corresponding author. Fax: +33-4-67-14-47-74.

E-mail address: christian.salles@msem.univ-montp2.fr (C. Salles).

Nevertheless, rain kinetic energy is still widely calculated from DSD measurements combined with empirical  $V_t(D)$  laws (e.g. Laws, 1941; Gunn and Kinzer, 1949; Beard, 1976). Due to the sporadic availability of DSD measurements, data obtained from measurement campaigns were analysed in order to establish empirical relationships between KE and rain intensity ( $I$ ). Assuming that the DSD samples used to establish the KE– $I$  relationship were representative, KE can be calculated directly for any rainfall event from  $I$  using the KE– $I$  relationship. Actually, rain intensity data, which are widely available, are obtained in a straightforward manner in comparison to KE.

The objective of this study is to demonstrate how the rain kinetic energy should be expressed when one wants to relate KE to  $I$  and then to find the most suitable mathematical expression linking both parameters. In Section 2, the two existing expressions of KE are discussed. A (non-exhaustive) review of the literature yields the different formulations used to relate KE and  $I$  in Section 3. Next, we discuss the statistical and micro-physical basis which needs to be considered when linking KE and  $I$ .

## 2. Two expressions for specific rain kinetic energy

As reported by Kinnell (1981) and Rosewell (1986), the specific kinetic energy of rain can be expressed in two ways: i.e. volume-specific and time-specific kinetic energy. Kinetic energy of rain is usually expressed as the amount of rain kinetic energy expended per unit volume of rain (volume-specific kinetic energy,  $KE_{\text{mm}}$ ; e.g. Wischmeier and Smith, 1958; Hudson, 1965; Kinnell, 1973; Carter et al., 1974; Zanchi and Torri, 1980; Coutinho and Tomás, 1995; Cerdà, 1997; Jayawardena and Rezaur, 2000b).  $KE_{\text{mm}}$  has units of energy per unit area and per unit rain depth ( $\text{J m}^{-2} \text{mm}^{-1}$ ) and is derived from the drop flux by

$$KE_{\text{mm}} = 10^{-3} \frac{\rho}{2} \frac{\sum_i X(D_i) D_i^3 V_t^2(D_i)}{\sum_i X(D_i) D_i^3}, \quad (1)$$

where  $\rho$  ( $\text{kg m}^{-3}$ ) is the water density in standard conditions, the drop flux density  $X(D_i)$  (drops  $\text{m}^{-2} \text{s}^{-1}$ ) is the number of drops with diameter

$D_i$  (cm) arriving per unit time and per unit area and  $V_t(D)$  ( $\text{m s}^{-1}$ ) is the terminal fall velocity of a raindrop with diameter  $D_i$ .  $KE_{\text{mm}}$  expresses the ratio between kinetic energy and the volume of rainwater involved or as pointed out by Sempere-Torres (1994)  $KE_{\text{mm}}$  expresses the average squared velocity of the raindrop population arriving at a surface weighed by the raindrop volume.

With a similar definition as the rain intensity (i.e. the volume of water falling on a unit horizontal surface during a unit time), rain kinetic energy can also be expressed as the rain kinetic energy expended per unit area and per unit time (i.e. the rate of kinetic energy or time-specific kinetic energy,  $KE_{\text{time}}$ ).  $KE_{\text{time}}$ , with units of energy per unit area and per unit time ( $\text{J m}^{-2} \text{h}^{-1}$ ), is derived from:

$$KE_{\text{time}} = 3.6 \times 10^{-3} \frac{\rho \pi}{12} \sum_i X(D_i) D_i^3 V_t^2(D_i). \quad (2)$$

$KE_{\text{time}}$  reflects according to Kinnell (1981) the rate of expenditure of rainfall kinetic energy, or according to Smith and De Veaux (1992) the rainfall power ( $\text{W m}^{-2}$ ), or according to Madden et al. (1998) and Steiner and Smith (2000) the rainfall kinetic energy flux density ( $\text{J m}^{-2} \text{h}^{-1}$ ).

The two expressions of kinetic energy,  $KE_{\text{mm}}$  and  $KE_{\text{time}}$ , are related to each other through the rain intensity by

$$KE_{\text{time}} = c I KE_{\text{mm}}, \quad (3)$$

where  $c$  is a constant which adjusts for any difference that exists in the units of time employed.

The rain intensity  $I$  ( $\text{mm h}^{-1}$ ) is derived from the drop flux density by the equation:

$$I = 3.6 \frac{\pi}{6} \sum_i X(D_i) D_i^3. \quad (4)$$

Thus with  $KE_{\text{mm}}$  expressed in  $\text{J m}^{-2} \text{mm}^{-1}$ ,  $c$  is equal to 1 if  $KE_{\text{time}}$  is expressed in  $\text{J m}^{-2} \text{h}^{-1}$ , or is equal to 1/3600 if  $KE_{\text{time}}$  is expressed in  $\text{J m}^{-2} \text{s}^{-1}$ .

$KE_{\text{mm}}$  is the most widely used expression for specific rain kinetic energy because of two historical reasons: (i) kinetic energy was derived from DSDs mainly with non-automatic techniques such as the filter paper or the flour pellet method. To prevent overlapping of drops during rain sampling, the paper (or flour) was manually exposed during very short periods (usually less than 1 s). The exposure time

Table 1  
Reported relationships between time-specific kinetic energy ( $KE_{\text{time}}$ ) and intensities ( $I$ ) of rains. The expression of  $KE_{\text{min}}$  not reported in this table can easily be obtained by the relation (see Eq. (3))  $KE_{\text{min}} (\text{J m}^{-2} \text{mm}^{-1}) = KE_{\text{time}} (\text{J m}^{-2} \text{h}^{-1})/I$  (DSD, drop-size distribution; n.a., not available)

Reference	$KE_{\text{time}} (\text{J m}^{-2} \text{h}^{-1})-I (\text{mm h}^{-1})$ relation	Location	Range of $I$ ( $\text{mm h}^{-1}$ )
Bollinne et al., 1984	$12.32I + 0.56I^2$	Belgium	0.27–38.6
Brandt, 1990	$I (8.95 + 8.44 \log_{10} I)$	USA, DSD from Marshall and Palmer (1948)	n.a.
Brown and Foster, 1987	$29I (1 - 0.72e^{-0.05I})$	USA	0–250
Carter et al., 1974	$11.32I + 0.5546I^2 - 0.5009 \times 10^{-2}I^3 + 0.126 \times 10^{-4}I^4$	South Central USA	1–250
Cerro et al., 1998	$38.4I (1 - 0.538e^{-0.029I})$	Barcelona, Spain	n.a.
Coutinho and Tomás, 1995	$35.9I (1 - 0.559e^{-0.034I})$	Portugal	0–120
Hudson, 1965	$29.86I (I - 4.29)$	Zimbabwe	n.a.
Jayawardena and Rezaur, 2000b	$36.8I (1 - 0.691e^{-0.038I})$	Honk Kong	0–150
Kinnell, 1981	$I (17.124 + 5.229 \log_{10} I)$	Miami, Florida	1.89–309
Kinnell, 1981	$30.132I (I - 5.484)$		
	$29.31I (1 - 0.281e^{-0.018I})$		
	$I (9.705 + 9.258 \log_{10} I)$		
	$29.863I (I - 4.287)$		
	$29.22I (1 - 0.894 e^{-0.0477I})$		
	$I (27.3 + 21.68e^{-0.048I} - 41.26e^{-0.072I})$		
McGregor and Mutchler, 1976	$I (9.81 + 10.6 \log_{10} I)$	Rhodesia (from Hudson (1961))	18.5–228.6
Onaga et al., 1988	$21.1069I^{1.136}$	Mississippi, USA	n.a.
Park et al., 1980	$29I (1 - 0.72e^{-0.05I})$	Okinaawa, Japan	n.a.
Renard et al., 1992	$29I (1 - 0.596e^{-0.0404I})$	USA	n.a.
Rosewell, 1986	$26.35I (1 - 0.669e^{-0.0349I})$	USA	n.a.
Rosewell, 1986	$24.48 (I - 1.253)$	Gunnedah, Australia	1–145.9
Rosewell, 1986	$24.80 (I - 1.292)$	Brisbane, Australia	1–161.2
Sempere-Torres et al., 1992	$34I - 190$	Melbourne, Australia	n.a.
Smith and De Veaux, 1992	$13I^{1.21}$	Cowra, Australia	n.a.
Smith and De Veaux, 1992	$11I^{1.23}$	Cévennes, France	20–100
Smith and De Veaux, 1992	$18I^{1.24}$	Oregon, USA	n.a.
Smith and De Veaux, 1992	$11I^{1.17}$	Alaska, USA	n.a.
Smith and De Veaux, 1992	$10I^{1.18}$	Arizona, USA	n.a.
Smith and De Veaux, 1992	$11I^{1.14}$	New Jersey, USA	n.a.
Steiner and Smith, 2000	$11I^{1.25}$	North Carolina, USA	n.a.
Tracy et al., 1984	$210Ie^{-0.0766I^{0.15}} - 211.8$ if $I < 76 \text{ mm h}^{-1}$	Florida, USA	n.a.
Uijjenhoet and Stricker, 1999a	$7.20I^{0.32}$	Northern Mississippi, USA	n.a.
	$8.53I^{1.29}$	Arizona	n.a.
	$8.46I^{1.17}$	Based on Marshall and Palmer parameterisation	n.a.
	$8.89I^{1.28}$		
	$10.8I^{1.06}$		
	$7.74I^{1.35}$		
	$23.4I - 18$		
Usón and Ramos, 2001	$I (11.87 + 8.73 \log_{10} I)$	NE Spain	< 20
Wischnmeier and Smith, 1958	$I (9.81 + 11.25 \log_{10} I)$	Washington, USA; DSD from Laws and Parsons (1943)	n.a.
Zanchi and Torri, 1980		Italy	n.a.

was not accurately known, which did not allow one to accurately determine  $KE_{time}$ . (ii) Rain intensity has been measured since long by rain gauges and represents the volume of rain recorded during a given period, thus the total event kinetic energy could be easily deduced from the product of rain depth and  $KE_{mm}$ . Additionally, Wischmeier and Smith (1958) contributed significantly to the use of the  $KE_{mm}$  expression by proposing a  $KE_{mm}-I$  relation and by introducing this relation in the  $R$ -factor of the universal soil loss equation (Wischmeier and Smith, 1978).

### 3. An overview of rain kinetic energy–rain intensity relationships

Various types of mathematical formulations derived from measured rain intensity and calculated kinetic energy data have been proposed to describe  $KE_{mm}-I$  relationships. Most of the mathematical functions are inspired by the empirical relationship established by Wischmeier and Smith (1958) which is based on the  $V_t(D)$  data of Laws (1941) and Gunn and Kinzer (1949) and the DSD data of Laws and Parsons (1943)

$$KE_{mm} = a + b \log_{10} I, \quad (5a)$$

where  $a$  and  $b$  are constants derived through the regression.

At this point, it should be mentioned that Wischmeier and Smith did not provide any indication on the physical basis of this linear-log equation. The linear-log formulation has been and is still widely used to relate  $KE_{mm}$  to  $I$  (see Table 1). Discussion in Section 4 demonstrates that such formulation is in contradiction with the observed DSD and  $V_t(D)$ . Nevertheless, Morgan (1995) reported several studies that considered the DSD of rainfall described by Marshall and Palmer (1948) as representative for a wide range of environments. Therefore, he recommended the formula for calculating the kinetic energy given by Brandt (1989). Assuming that the DSD equals the DSD presented by Marshall and Palmer, Brandt (1989) computed the rain intensity ( $I$ ) and  $KE_{mm}$  and then adjusted an empirical relation between the two calculated parameters:

$$KE_{mm} = 8.95 + 8.44 \log_{10} I. \quad (5b)$$

The RUSLE (Renard et al., 1997) revised the kinetic energy formulation of Wischmeier and Smith and proposed the conditional relations:

$$KE_{mm} = 11.9 + 8.73 \log_{10} I \quad \text{if } I \leq 76 \text{ mm h}^{-1}, \quad (6a)$$

$$KE_{mm} = 28.3 \quad \text{if } I > 76 \text{ mm h}^{-1}. \quad (6b)$$

Eq. (6b) combined with Eq. (3) implies that  $KE_{time}$  increases linearly with  $I$  for  $I$  values larger than  $76 \text{ mm h}^{-1}$ .

Apart, from the widely used Eq. (5a), several other mathematical expressions have been proposed in the literature. Kinnell (1981) reported that the essentially linear  $KE_{time}-I$  relationship observed by Hudson (1961) over Zimbabwe was found to be valid for rainfall data collected in Miami (Kinnell, 1973). As stated by Kinnell (1981), relating  $KE_{time}$  linearly to  $I$  implies the following relationship between  $KE_{mm}$  and  $I$ :

$$KE_{mm} = c^{-1}(d - eI^{-1}), \quad (7)$$

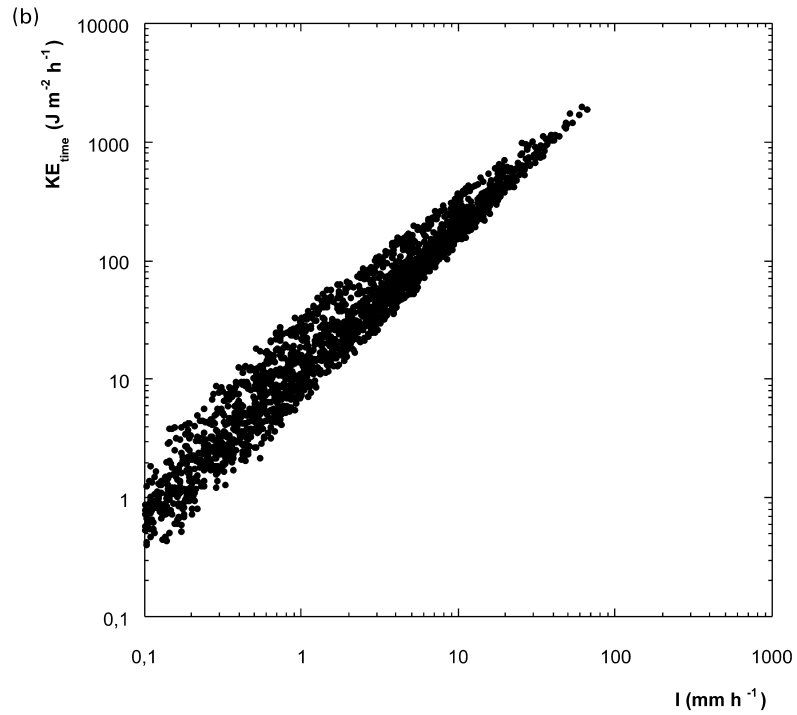
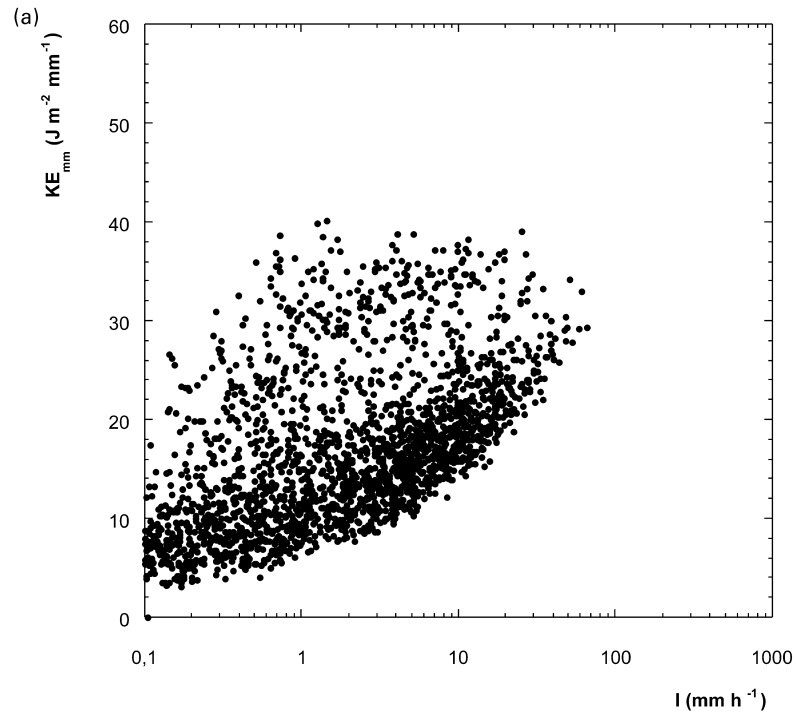
where  $d$  and  $e$  are positive constants.

Sempere-Torres et al. (1992) found a linear relation between  $KE_{time}$  and  $I$ , for rain intensity values larger than  $20 \text{ mm h}^{-1}$ . Rosewell (1986) collected drop-size data over Australia with a disdrometer of Joss and Waldvogel (1967). Hudson (1965) measured drop-size with the flour pellet method for Zimbabwe. Despite the fact that climatic conditions and the measuring techniques used in these studies were different, they both related  $KE_{mm}$  to the inverse of  $I$  as shown in Eq. (7).

Carter et al. (1974), from DSD measurements in south central United States, developed an equation of polynomial form:

$$KE_{mm} = a' + b'I + c'I^2 + d'I^3. \quad (8)$$

Polynomial equations were also tested on kinetic energy derived from DSD measured in Belgium, with degree from one to three, by Renard (1983). She found that a polynomial with degree one gave the best regression and therefore kept a linear relation. As reported by Govers (1991) such a linear relationship presents some limitations. As an example,  $KE_{mm}$  derived from the equation proposed by Bollinne et al. (1984) (see Table 1) with a rain intensity equal to  $40 \text{ mm h}^{-1}$  will predict an unrealistically high kinetic



energy. This value exceeds the kinetic energy of rain containing only drops with diameter equal to 5 mm.

McGregor and Mutchler (1976) proposed an alternative expression by introducing the exponential function:

$$KE_{\text{mm}} = a_0 + b_0 e^{-b_1 I} + c_0 e^{-c_1 I}. \quad (9)$$

Kinnell (1981) re-examined the relationships that relate  $KE_{\text{mm}}$  to  $1/I$  and to  $\log_{10} I$ , which are producing negative values for  $KE_{\text{mm}}$  corresponding to low intensity values, and concluded that the following empirical  $KE_{\text{mm}}-I$  relationship is more appropriate:

$$KE_{\text{mm}} = z(1 - p e^{-hI}). \quad (10)$$

Brown and Foster (1987) recommended in their analysis to use  $z = 29 \text{ J m}^{-2} \text{ mm}^{-1}$ ,  $p = 0.72$  and  $h = 0.05 \text{ h mm}^{-1}$  and stated that this equation is a superior analytical form by having a finite positive value at zero intensity and approaching an asymptote at high intensities as a continuous function. It is also important to note that for small values of  $I$ , Eq. (10) reduces to a linear form. Kinnell (1987) reported that the  $z$  parameter in Eq. (10) was shown to vary little from a value of about  $29 \text{ J m}^{-2} \text{ mm}^{-1}$ , for many geographic locations (i.e. USA, Zimbabwe and Eastern Australia) and that  $p$  and  $h$  may be more site specific. The constancy of the  $z$  parameter is contradicted by the results of (i) Coutinho and Tomás (1995) at the Vale Formoso Erosion Station—Portugal (i.e.  $z = 35.9 \text{ J m}^{-2} \text{ mm}^{-1}$ ), (ii) Cerro et al. (1998) in Barcelona (i.e.  $z = 38.4 \text{ J m}^{-2} \text{ mm}^{-1}$ ) and (iii) Jayawardena and Rezaur (2000b) in Hong Kong (i.e.  $z = 36.8 \text{ J m}^{-2} \text{ mm}^{-1}$ ).

Tracy et al. (1984) proposed a  $KE_{\text{mm}}-I$  equation based on data collected in southern Arizona (USA) limited to rain intensities smaller than  $76 \text{ mm h}^{-1}$ :

$$KE_{\text{mm}} = 210 e^{-0.0766I^{0.175}} - 211.8. \quad (11)$$

Above this threshold rain intensity, they consider  $KE_{\text{mm}}$  to become constant, equalling  $33.5 \text{ J m}^{-2} \text{ mm}^{-1}$ . The observation that  $KE_{\text{mm}}$  of rain is substantially constant at intensities exceeding a

threshold value is also reported by Rosewell (1986), Kinnell (1987) and Renard et al. (1997).

Most of the reported relationships consider the volume-specific kinetic energy. As many equations with time-specific kinetic energy can be derived by considering the  $KE_{\text{time}}-KE_{\text{mm}}$  relationship (Eq. (3)).

From this non-exhaustive review of  $KE_{\text{mm}}-I$  relationships, we want to emphasise the tendency towards an increasingly sophisticated formulation and consequently a large uncertainty in the appropriateness of the mathematical expressions used. The widespread scatter in the  $KE_{\text{mm}}-I$  plots used to obtain the empirical equations is a typical feature (e.g. Bollinne et al., 1984; Rosewell, 1986; Kinnell, 1987; McIsaac, 1990; Coutinho and Tomás, 1995). These scatterplots have two characteristic features. (i) At low  $I$  values, a large number of  $KE_{\text{mm}}$  data having a large range of values are found. (ii) As a consequence, the number of  $KE_{\text{mm}}$  data corresponding to large  $I$  values is very limited and the fit of the selected equation is largely controlled by the values of these few data points. Hence, extrapolation towards the large  $I$  values suffers from a very large uncertainty and can therefore result in unreliable KE predictions.

The second expression of specific kinetic energy,  $KE_{\text{time}}$ , has been related to  $I$  by linear or power relations. Kinnell (1973) and Rosewell (1986) proposed a linear relationship, Sempere-Torres et al. (1992) indicated a linear relation for intensities larger than  $20 \text{ mm h}^{-1}$ . Mihara (1951), Smith and De Veaux (1992), Uijlenhoet and Stricker (1999b) and Steiner and Smith (2000) used a power law relation between  $KE_{\text{time}}$  and  $I$ :

$$KE_{\text{time}} = aI^b. \quad (12)$$

The scatterplots given in the cited  $KE_{\text{time}}-I$  studies present less heteroscedasticity than the corresponding  $KE_{\text{mm}}-I$  scatterplots. As an illustration, we have plotted the  $KE_{\text{mm}}$  and  $KE_{\text{time}}$  values versus  $I$  from the dataset collected by Sempere-Torres et al. (1992) using a Joss and Waldvogel disdrometer (Fig. 1(a) and (b)). These figures are an excellent

Fig. 1. (a) Scatterplot of volume-specific kinetic energy ( $KE_{\text{mm}}$ ) versus intensity ( $I$ ) of rain both deduced from DSD data collected by Sempere-Torres et al. (1992) in the Cévennes (south of France) with a Joss and Waldvogel disdrometer ( $n = 2432$ ). (b) Scatterplot of the time specific kinetic energy ( $KE_{\text{time}}$ ) versus intensity ( $I$ ) of rain both deduced DSD data collected by Sempere-Torres et al. (1992) in the Cévennes (south of France) with a Joss and Waldvogel disdrometer ( $n = 2432$ ).

example of the characteristic features of  $KE_{\text{mm}}$  and  $KE_{\text{time}}-I$  scatterplots. A comparison of both figures yields the conclusion that  $KE_{\text{time}}$  is more appropriate to be linked to  $I$  than  $KE_{\text{mm}}$ . In Fig. 1(a), there are very few data points in the range of large  $I$  values and the wide dispersion of the data points corresponding to small  $I$  values makes the identification of a suitable mathematical function to be fitted very difficult.  $KE_{\text{mm}}$  becomes unstable when  $I$  approaches zero because: (i) when  $I$  becomes small,  $KE_{\text{mm}}$  which depends on the ratio  $KE_{\text{time}}/I$ , is very sensitive to  $I$  fluctuations or  $I$  uncertainties which enhance the scatter. Hence, sampling effects are increasing with the decrease of rain intensity; (ii)  $KE_{\text{mm}}$  of rain is highly sensitive to the DSD of the rain (e.g. droplets during fog versus average drop-size during convective rain events).

The various listed  $KE-I$  relationships and their associated constants are reported in Table 1. In Fig. 2(a), all the listed  $KE_{\text{mm}}-I$  relationships are drawn whereas Fig. 2(b) depicts the corresponding  $KE_{\text{time}}-I$  relations. The potential zone in which  $KE-I$  relations plot is limited by two physical boundary curves defined by monodisperse DSDs. Steiner and Smith (2000) determined the equation for the lower boundary. This boundary is obtained by assuming that all raindrops have sizes equal to 0.02 cm (i.e. corresponding to a cloud/raindrop-boundary drop-size with  $V_t(0.02) = 0.72 \text{ m s}^{-1}$ ) and the equation becomes (Steiner and Smith, 2000):

$$KE_{\text{mm}} = 0.26 (\text{J m}^{-2} \text{ mm}^{-1}). \quad (13a)$$

In a similar way, the upper boundary curve is calculated assuming all raindrops (i) have sizes equal to 0.8 cm (i.e. corresponding to the maximum drop-size observed in natural rain (Pruppacher and Klett, 1998)) and (ii) have  $V_t$  equal to  $9.65 \text{ m s}^{-1}$  (i.e. maximum  $V_t$  according to the empirical relation proposed by Atlas et al. (1973)) and the equation becomes:

$$KE_{\text{mm}} = 46.6 (\text{J m}^{-2} \text{ mm}^{-1}). \quad (13b)$$

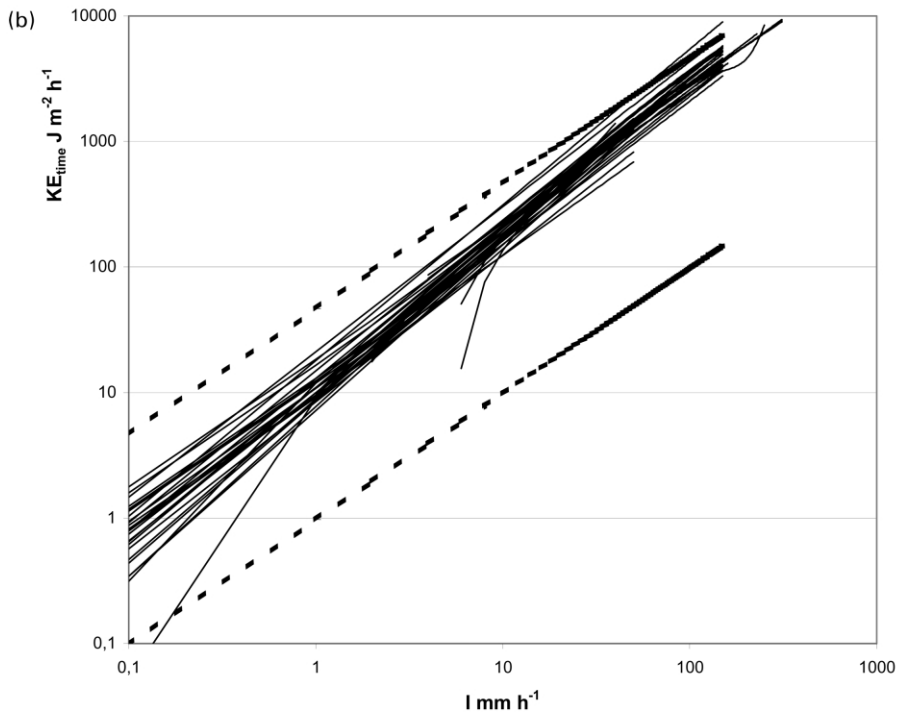
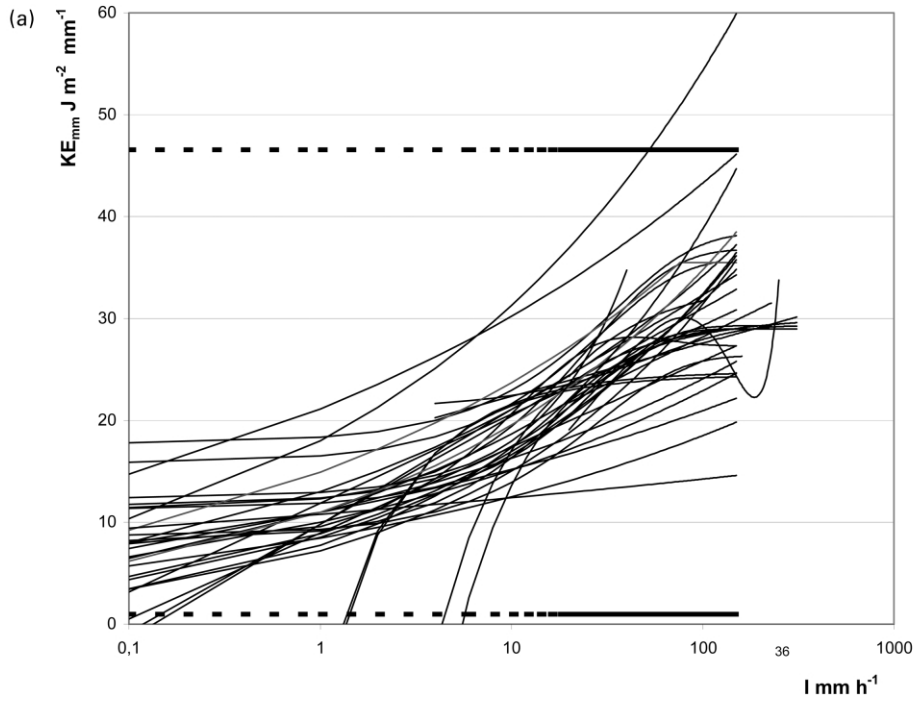
Obviously, most of the different empirical relation-

ships shown in both Fig. 2(a) and (b) plot inside these boundaries. Exceptions occur for low and very high  $I$  values. The  $KE-I$  relations plotting partly outside the physical boundaries are essentially the result of a misuse by extrapolation or of inappropriate functions linking  $KE$  and  $I$  for the low or very high  $I$  values. As reported by Mualem and Assouline (1986), from a physical point of view,  $KE_{\text{mm}}$  does not tend towards zero when  $I$  approaches zero (Eq. (13a)) because the raindrop size does not tend towards zero but has a finite size. Kinnell (1981) has already identified equations that produce negative values for  $KE_{\text{mm}}$  at low  $I$  values.

#### 4. Selecting an appropriate rain intensity–kinetic energy relationship

The two expressions of specific rain kinetic energy,  $KE_{\text{time}}$  and  $KE_{\text{mm}}$  are both valid ones and can both be related to  $I$ . Nevertheless, fitting  $KE_{\text{mm}}$  versus  $I$  in order to identify an empirical relationship does not strictly satisfy statistical rules. From a statistical point of view, relating  $KE_{\text{mm}}$  to  $I$  produces erroneous results. It is a typical example of spurious self-correlation as described by Kenney (1982).  $KE_{\text{mm}}$  is the kinetic energy of the rain divided by the rain volume for a given period. Statistically linking  $KE_{\text{mm}}$  to  $I$  is equivalent to linking the ratio  $KE_{\text{time}}/I$  to  $I$ . Such an operation artificially modifies the correlation coefficient between both parameters. Usually, the value of the spurious self-correlation coefficient is much higher than the one between the original variables. Kenney (1982) states that spurious self-correlation can also act in a negative manner. A correlation coefficient is being reduced by a ‘spurious ratio correlation’ (i.e.  $y/x$  correlated to  $x$ ) when the original data are well correlated and the coefficients of variation of the parameters ( $x$  and  $y$ ) are of the same order of magnitude. Nevertheless, whatever the effect on the correlation coefficient, overestimation or underestimation, linking  $KE_{\text{mm}}$  to  $I$  suffers from an incorrect

Fig. 2. (a) Volume specific kinetic energy ( $KE_{\text{mm}}$ ) versus rain intensity ( $I$ ) relations ( $n = 29$ ) calculated from equations listed in Table 1. The two dashed lines correspond to the physical boundaries defined by monodisperse DSD. (b) Time specific kinetic energy ( $KE_{\text{time}}$ ) versus rain intensity ( $I$ ) relations ( $n = 29$ ) calculated from equations listed in Table 1. The two dashed lines correspond to the physical boundaries defined by monodisperse DSD.





use of statistics. It does not mean that the  $KE_{mm}$  expression is wrong but that if one wants to identify an empirical relationship between KE and  $I$ , then the  $KE_{time}$  expression is more appropriate and should be used for fitting with  $I$ . From this  $KE_{time}$  relation a  $KE_{mm}$ – $I$  relationship can easily be deduced using Eq. (3).

As shown in Section 3, there is no agreement on the mathematical formulation of the  $KE_{mm}$ – $I$  relationships. This is due to the combined results of (i) the large variety in methods used to measure DSD and their influence on the derived empirical model, (ii) important variability of DSD following geographic location and rain type, and (iii) the statistical artefact in the  $KE_{mm}$ – $I$  relationship due to the inclusion of  $I$  in the  $KE_{mm}$  expression.

The mathematical expressions derived from  $KE_{mm}$ – $I$  relationships, to express  $KE_{time}$  become very complicated. The physical justification of such formulation, if any, is not obvious. As an example, Eqs. (3) and (5a) allow one to express  $KE_{time}$  versus  $I$  using the following expression:

$$KE_{time} = cI(a + b \log_{10} I). \tag{14}$$

Despite the fact that numerical values given by Eq. (14) are not so far from more simple and intuitive relationships, such as the power law relationship, it is difficult to support a physical basis for this equation. Eq. (14) cannot be deduced from the definitions of  $KE_{time}$  and  $I$  (Eqs. (2) and (4)). In order to identify a more rational formulation, we consider models used to characterise DSD and then derive the relationship between  $KE_{time}$  and  $I$  from a micro-physical point of view.

DSDs have received a large attention in several other research fields, such as meteorology, propagation of electromagnetic waves through the atmosphere and rain measurement by radar. The DSDs are usually expressed using a distribution function  $N(D,I)$ , i.e. the number of drops per unit of air volume and per size range  $D$  to  $D + \Delta D$  (usually expressed in  $cm^{-4}$ ) for a given rain intensity  $I$ . Sempere-Torres et al. (1994, 1998) have shown that the various mathematical expressions describing the DSD in earlier studies, i.e. the exponential function proposed by Marshall and Palmer (1948), the Weibull distribution of liquid water content over drop-size by Best (1950) which translates itself to a generalised gamma raindrop

size distribution (Ulbrich, 1983), and the lognormal function by Feingold and Levin (1986), can be written in terms of a general scaling formulation, which in the particular case of using the rain intensity  $I$  as reference variable reads:

$$N(D,I) = I^\alpha g(DI^{-\beta}), \tag{15}$$

where  $\alpha$  and  $\beta$  are constants,  $I$  is the reference variable (any other rainfall bulk variable may be chosen) and  $g$  is a function independent of  $I$  but dependent on the choice of  $I$  as reference variable, called general distribution function.

This general formulation can now be applied to find out an analytical relation between  $KE_{time}$  and  $I$ . Using the terminal fall velocity  $V_t(D)$  to transform the rainfall DSD recorded at ground level (i.e.  $X(D)$  in Eq. (1)) into the DSD in a volume of falling rain ( $N(D,I)\Delta D$ ), one can deduce that:

$$X(D) = 10^6 N(D,I)\Delta D V_t(D). \tag{16}$$

From Eqs. (2) and (16), the expression for  $KE_{time}$  becomes:

$$KE_{time} = 3.6 \times 10^3 \frac{\rho\pi}{12} \int_{D_{min}}^{D_{max}} N(D,I) D^3 V_t^3(D) dD, \tag{17}$$

and replacing  $N(D,I)$  by Eq. (15), this leads to:

$$KE_{time} = 3.6 \times 10^3 \frac{\rho\pi}{12} I^\alpha \int_{D_{min}}^{D_{max}} g(DI^{-\beta}) D^3 V_t^3(D) dD. \tag{18}$$

Now, using a power law for  $V_t(D)$  (see Eq. (A5) in the Appendix A) and introducing the variable  $x = DI^{-\beta}$ , Eq. (18) reads

$$\begin{aligned} KE_{time} &= 3.6 \times 10^3 \frac{\rho\pi}{12} I^\alpha \int_{D_{min}}^{D_{max}} g(DI^{-\beta}) D^3 (a_0 D^{b_0})^3 dD \\ &= 3.6 \\ &\quad \times 10^3 \frac{\rho\pi}{12} a_0^3 I^{\alpha+\beta(4+3b_0)} \int_{x_{min}}^{x_{max}} g(x) x^{3(b_0+1)} dx, \end{aligned} \tag{19}$$

with  $D$  in cm and  $I$  in  $mm\ h^{-1}$ .

Note that the integral in Eq. (19) has a value which depends only on the  $g$  function and is independent of the value of  $I$ . Therefore, Eq. (19) implies a power

relation between  $KE_{\text{time}}$  and  $I$  according to

$$KE_{\text{time}} = AI^{\alpha+\beta(4+3b_0)}, \tag{20}$$

where  $A$  is a constant that reads

$$A = 3.6 \times 10^3 \frac{\rho\pi}{12} a_0^3 \int_{x_{\min}}^{x_{\max}} g(x)x^{3(b_0+1)} dx. \tag{21}$$

Moreover, if the Atlas and Ulbrich (1977) relationship  $V_t(D) = 17.67D^{0.67}$  is used, these equations become

$$KE_{\text{time}} = AI^{\alpha+6.01\beta}, \tag{22}$$

$$A = 5.18 \times 10^9 \int_{x_{\min}}^{x_{\max}} g(x)x^{5.01} dx, \tag{23}$$

with  $KE_{\text{time}}$  expressed in  $J m^{-2} h^{-1}$ , and  $I$  in  $mm h^{-1}$ .

As a consequence, all the DSD models proposed in the literature, which are particular cases of the general scaling formulation of Sempere-Torres et al. (1994), lead to the conclusion that a power law is the most suitable function to relate  $KE_{\text{time}}$  and  $I$ . Furthermore, from Eq. (3) it follows that  $KE_{\text{mm}}$  should also be related to  $I$  by a power law:

$$KE_{\text{mm}} = c^{-1}AI^{\alpha+6.01\beta-1}. \tag{24}$$

However, a further simplification of these equations can still be obtained. If the self-consistency constraints (see Eqs. (A6) and (A7) in the Appendix A) are now applied, Eq. (22) will finally read

$$KE_{\text{time}} = AI^{1+1.34\beta}. \tag{25}$$

Note that the exponent of the  $KE_{\text{time}}-I$  relation depends on just one single parameter,  $\beta$ . On the other hand, the coefficient  $A$  will be the integral of the general function  $g$ . In a variety of climates, this function is well fitted by an exponential function with just one single free parameter,  $\mu$ , due to the constraint (A7) (see for instance Sempere-Torres et al. (1998) or Salles et al. (1999))

$$g(x) = \frac{3.0 \times 10^{-8}}{\Gamma(4.67)} \mu^{4.67} \exp(-\mu x). \tag{26}$$

Thus,  $A$  can be written as

$$\begin{aligned} A &= 5.18 \times 10^9 \frac{3.0 \times 10^{-8}}{\Gamma(4.67)} \mu^{4.67} \frac{\Gamma(6.01)}{\mu^{6.01}} \\ &= 1288.17\mu^{-1.34}, \end{aligned} \tag{27}$$

where both equations have been obtained integrating from 0 to  $\infty$  instead of using  $D_{\min}$  and  $D_{\max}$ . This useful mathematical simplification is frequently used in DSD studies instead of calculating the truncated integral. The differences can be neglected in most common types of rain, where  $D_{\min}$  and  $D_{\max}$  are sufficiently smaller and larger than the median volume drop diameter (Ulbrich, 1985).

Finally we can write  $KE_{\text{time}}$  (in  $J m^{-2} h^{-1}$  if  $I$  is in  $mm h^{-1}$  and  $D$  is in  $cm$ ) as

$$KE_{\text{time}} = 1288.17\mu^{-1.34}I^{1+1.34\beta}. \tag{28}$$

This equation points out that the proposed  $KE_{\text{time}}-I$  relation will be governed by only two parameters, which are those of the DSD formulation. These parameters,  $\beta$  and  $\mu$ , are linked to the type of micro-physical process predominant in the raindrop growth, or, in a more plain way, to what has been called the ‘type of rainfall’ (see for instance Sempere-Torres et al., 2000).

We can now use the values for  $\beta$  and  $\mu$  obtained from previous studies (see compilation in Sempere-Torres et al., 1994) to interpret the  $KE_{\text{time}}-I$  relations.

The  $\beta$  value can vary from 0, which corresponds to the equilibrium raindrop population first described by Zawadzki and De Agostinho Antonio (1988) and theoretically derived by List et al. (1987), to 0.35, which is the maximum reported in the literature (although it is usually less than 0.3). Values around 0.12 and 0.15 are related to convective storms and the value  $\beta = 0.21$  of Marshall and Palmer (1948) is commonly associated with stratiform, widespread rain.

Using Eq. (25) all parameter values can be summarised in Table 2.

The possible  $KE_{\text{time}}-I$  relations will thus lay in the range of power laws with exponents varying between 1 and 1.4. For intense rains, values of 1.1 to 1.2 would be suitable, showing a tendency to become 1 if the number of interactions between the drops increases, allowing the population to approach the equilibrium distribution.

In this case,  $KE$  will be strictly linear with  $I$  (as any other rain bulk variable will be, following what was demonstrated theoretically by List et al. (1987)). Hence, during high intensity periods (normally related to convective rain) the  $KE-I$  relation will be close to

Table 2

Range of values for the exponent of the  $KE_{\text{time}}-I$  power law relation deduced from  $\beta$  values obtained from previous studies

$\beta$ Value	$\mu$ Value	Type of rain	$KE_{\text{time}}-I$
0	–	Equilibrium raindrop population	$AI^1$
0.12–0.15	30	Convective rain	$AI^{1.2}$
0.21	40	Stratiform rain	$AI^{1.3}$
0.3	–	Maximum value	$AI^{1.4}$

linear, and a linear fit will reproduce the  $KE-I$  relation well, especially for high rain intensities. This explains what has been observed in some of the studies commented above.

Regarding  $\mu$ , the studies previously published show that values between 30 and 40 are to be expected. The value of 30 can be taken as more representative for convective storms (Sempere-Torres et al., 1994, 1998; Salles et al., 1999), the value of 40 (41 in the case of Marshall and Palmer, 1948) for stratiform rain, and the value of 50 for drizzle (Joss and Waldvogel, 1969). Using Eq. (27) these values lead to values of  $A$  (to obtain  $KE_{\text{time}}$  in  $J m^{-2} h^{-1}$ ) between 13.5 for convective rain ( $\mu = 30$ ) and 9.2 for widespread rain ( $\mu = 40$ ), which is essentially what has been found in previous studies (see Table 1).

## 5. Conclusions

Most soil erosion models use the rain kinetic energy as an erosivity parameter. For historical reasons, a strong emphasis has been put on the volume-specific rain kinetic energy ( $KE_{\text{mm}}$ ). Direct measurements of the rain kinetic energy are not widely available. Therefore, empirical relationships between the widely measured rain intensity  $I$  and  $KE_{\text{mm}}$  have been proposed. The literature review reported in this study illustrates the diversity of the selected mathematical functions used to link  $I$  with  $KE_{\text{mm}}$ .

Using a definition similar to that of  $I$ ,  $KE$  can also be expressed in units of energy per unit time and per unit area (time specific kinetic energy,  $KE_{\text{time}}$ ).  $KE_{\text{time}}$  is more appropriate when using DSD data collected with automatic measuring devices. Using such data, we show that relating  $KE_{\text{mm}}$  and  $I$  is a perfect example of a spurious self-correlation. Therefore, the  $KE_{\text{time}}$  expression is preferred over  $KE_{\text{mm}}$  when establishing

an empirical relationship between  $KE$  and  $I$ . Reported analyses in the literature use power law or linear functions between  $KE_{\text{time}}$  and  $I$ . A discussion based on reported DSD models demonstrates that the most evident function linking  $KE_{\text{time}}$  and  $I$  is the power law. There is evidence that the constants of the power law are related to rain type, geographical location and measuring technique. A general function capturing all these dependencies is proposed.

For the future, we agree with Parsons and Gadian (2000) who concluded that there is a need to provide more detailed information on the DSD and not only on a global parameter such as  $I$  or the median-volume diameter in order to derive a suitable rain erosivity index. Despite the identification of a functional relationship between  $I$  and  $KE_{\text{time}}$ , variations in the data related to various parameters (e.g. rain type, altitude, climate, method of measurement) are still existing and more research to explain this variation is still needed. However, it is worth noting that the lack of information occurs essentially for the larger  $I$  values. This is a consequence of the low frequency of extreme rain events and this aspect deserves a more thorough examination.

## Acknowledgements

This study was initiated when the first author worked at the Laboratory for Experimental Geomorphology (K.U. Leuven) as part of a Community Training Project funded by the European Commission under the Training and Mobility of Researchers programme (contract no. ERBFMBICT611631).

## Appendix A. Summary of the DSD general scaling formulation

The work of Sempere-Torres et al. (1994) shows that any DSD used up to now in hydrometeorological studies is a particular case of a general scaling formulation where the number of drops per unit of air volume in the size range  $D$  to  $D + \Delta D$ ,  $N(D, \Psi)\Delta D$ , depends on  $D$  and on the reference variable  $\Psi$  as

$$N(D, \Psi) = \Psi^{\alpha\psi} g(D\Psi^{-\beta\psi}). \quad (\text{A1})$$

In this general expression  $\Psi$  can be any integral

rainfall variable, although  $I$  has generally been used. For a given  $\Psi$ ,  $\alpha_\Psi$  and  $\beta_\Psi$  are constants (they do not have any functional dependence on  $\Psi$ ) and  $g$  is a function which is independent of the value of  $\Psi$  and which is called the general distribution function.

The form of this scaling law is based on two hypotheses and experimental evidences:

1. Any DSD can be written as a function of the drop Diameter,  $D$ , a single rainfall bulk variable,  $\Psi$ , and a number of constant parameters (not depending on the value of  $D$  nor that of  $\Psi$ ).
2. The dependence of  $N(D, \Psi)$  on  $\Psi$  and  $D$  can be separated into two independent terms:  $N(D, \Psi) = f(\Psi)g(D/D^*)$ , where  $D^*$  can be any characteristic diameter (i.e. the quotient between the DSD's integral moments of order  $n + 1$  and  $n$ ). An interpretation of these functions in terms of the raindrop size p.d.f. and the total number of drops is provided in the Appendix of Sempere-Torres et al. (1998) or in Porrà et al. (1998).

Sempere-Torres et al. (1994) showed that all the previous studies and experimental evidence lead to the selection of a power relation for  $f(\Psi)$ . From this, it can be deduced that any integral moment of the DSD  $M_n$  can be expressed as a power function of  $\Psi$

$$M_n = \int_{D_{\min}}^{D_{\max}} D^n N(D, \Psi) dD = a \Psi^b, \tag{A2}$$

and that any characteristic diameter becomes

$$D_n^* = \frac{M_{n+1}}{M_n} = c_n \Psi^{\beta_\Psi}, \tag{A3}$$

following the general scaling law formulation given in Eq. (A1).

In our case we are interested in relating the flux density of kinetic energy at the ground ( $KE_{\text{time}}$ ) with the flux of water at ground (the rain intensity,  $I$ ). Thus, we can express  $KE_{\text{time}}$  as an integral moment of the DSD, making the choice of using  $I$  as the reference variable (the most common choice in the literature), which leads us to Eq. (15):  $N(D, I) = I^\alpha g(DI^{-\beta})$ .

As our selected reference variable  $I$  can itself be expressed as an integral moment of the  $N(D, I)$ , the number of degrees of freedom of Eq. (15) can be reduced by two if the self-consistency requirement proposed by Bennet et al. (1984) is imposed. That

is, for any value of the reference variable  $I$ , we should be able to get the same value when calculating it from the DSD (if not the formulation will not be consistent and will lead to errors). This can be expressed as

$$I = 0.6\pi 10^6 \int_{D_{\min}}^{D_{\max}} N(D, I) D^3 V_t(D) dD$$

$$= I^\alpha A_I \int_{D_{\min}}^{D_{\max}} g(DI^{-\beta}) D^3 V_t(D) dD. \tag{A4}$$

Sempere-Torres et al. (1994) showed that the only way to guarantee the self-consistency of this formulation is allowing  $V_t(D)$  to be a power law of  $D$

$$V_t(D) = a_0 D^{b_0}. \tag{A5}$$

Of course any power law can be used (see Uijlenhoet and Stricker (1999a) for a wider discussion of the self-consistency aspects), but if the expression provided by Atlas and Ulbrich (1977) is used (where  $a_0 = 17.67$  and  $b_0 = 0.67$ , with  $D$  expressed in cm and  $V_t$  in  $m\ s^{-1}$ ), the self-consistency equation (A4) leads to (see Sempere-Torres et al., 1994, 1998):

$$\alpha = 1 - 4.67\beta, \tag{A6}$$

$$1 = A_I \int_{x_{\min}}^{x_{\max}} g(x) x^{4.67} dx, \tag{A7}$$

with  $x = DI^{-\beta}$ . Thus  $\alpha$  and  $\beta$  are related and whatever would be the choice for the general function  $g$ , it should verify the constraint given by Eq. (A7).

There has been some misunderstanding in using this general formulation (see Uijlenhoet and Stricker (1999b) for a discussion example). One of the usual points of confusion leading to criticism is the use of a power relation between  $V_t$  and  $D$  (see for instance Ulbrich and Atlas, 1998). However, the use of this functionality is not a hypothesis of the formulation but a requirement for its self-consistency, something which is analytically related to the choice of power functions to relate rainfall bulk variables. That means, if the use of a non-power  $V_t(D)$  relation is thought to be more sound, its use should be consistent with a non-power choice for  $f(\Psi)$ , and all this will lead to non-power relations between DSD moments (thus between rainfall bulk variables).

However, all previous studies (including those that criticize the use of a power terminal velocity relationship) confirm this choice and, as stated by Uijlenhoet

and Stricker (1999a) ‘there exists an impressive body of experimental evidence confirming the existence of power law relationships between various rainfall related variables’. If this is accepted, or a power function is used, the conservation of the consistence will lead to a power law for  $V_t(D)$ .

It is true that other non-power relations have been proposed in the literature (e.g. Atlas et al., 1973; Beard, 1976; Uplinger, 1981), and that they are supposedly more accurate. However, they rely essentially on the same source of experimental data (normally they are fits to the Gunn and Kinzer (1949) or Foote and du Toit (1969) data), and they are assuming that even in real rain all the newly formed drops (i.e. those formed by breakup or coalescence for instance) are instantaneously falling at the terminal fall speed in stagnant air corresponding to their characteristic diameter. Therefore, in our opinion the power relation should be interpreted as an *effective* terminal fall velocity instead of a fit to the laboratory data.

Finally, it is worth noting that the power law relationships proposed in this paper concern DSD moments of order 3.67 and 5.01. Those criticising power law terminal fall velocity relations are in fact criticising the goodness-of-fit of power law  $KE_{\text{time}}-I$  relations. According to our experience, however, this is something which can be hardly improved upon.

## References

- Atlas, D., Ulbrich, C.W., 1977. Path and area integrated rainfall measurement by microwave attenuation in the 1–3 cm band. *J. Appl. Meteorol.* 16, 1322–1331.
- Atlas, D., Srivastava, C., Sekhon, S., 1973. Doppler radar characteristics of precipitation at vertical incidence. *Rev. Geophys. Space Phys.* 11, 1–35.
- Beard, K.V., 1976. Terminal velocity and shape of cloud and precipitation drops aloft. *J. Atmos. Sci.* 33, 851–864.
- Bennet, J.A., Fang, D.J., Boston, R.C., 1984. The relationship between  $N_0$  and  $A$  for Marshall–Palmer type raindrop-size distributions. *J. Clim. Appl. Meteorol.* 23, 768–771.
- Best, A.C., 1950. The size distribution of raindrops. *Quart. J. R. Meteorol. Soc.* 76, 16–36.
- Bollinne, A., Florins, P., Hecq, P., Homerin, D., Renard, V., Wolfs, J.L., 1984. Etude de l’énergie des pluies en climat tempéré océanique d’Europe Atlantique. *Z. Geomorph. N.F.*, 27–35.
- Brandt, C.J., 1989. The size distribution of throughfall drops under vegetation canopies. *Catena* 16, 507–524.
- Brandt, C.J., 1990. Simulation of the size distribution and erosivity of raindrops and throughfall drops. *Earth Surf. Process.* 15, 687–698.
- Brown, L.C., Foster, G.R., 1987. Storm erosivity using idealized intensity distributions. *Trans. ASAE* 30 (2), 379–386.
- Carter, C.E., Greer, J.D., Braud, H.J., Floyd, J.M., 1974. Raindrop characteristics in south central United States. *Trans. ASAE*, 1033–1037.
- Cerdà, A., 1997. Rainfall drop size distribution in the Western Mediterranean basin, València, Spain. *Catena* 30, 169–182.
- Cerro, C., Bech, J., Codina, B., Lorente, J., 1998. Modeling rain erosivity using disdrometric techniques. *Soil Sci. Soc. Am. J.* 62, 731–735.
- Coutinho, M.A., Tomás, P.P., 1995. Characterisation of raindrop size distributions at the Vale Formoso Experimental Erosion Center. *Catena* 25, 187–197.
- Doelling, I.G., Joss, J., Riedl, J., 1998. Systematic variations of  $Z-R$  relationships from drop size distributions measured in northern Germany during seven years. *Atmos. Res.* 47–48, 635–649.
- Elwell, H.A., 1978. Modelling soil losses in Southern Africa. *J. Agri. Engng* 23, 117–127.
- Feingold, G., Levin, Z., 1986. The lognormal fit of raindrop spectra from frontal convective clouds in Israel. *J. Clim. Appl. Meteorol.* 25, 1346–1363.
- Foote, G.B., du Toit, P.S., 1969. Terminal velocity of raindrops aloft. *J. Appl. Meteorol.* 8, 249–253.
- Govers, G., 1991. Spatial and temporal variations in splash detachment: a field study. *Catena* 20 (Suppl.), 15–24.
- Gunn, R., Kinzer, G.D., 1949. The terminal velocity of fall for water droplets in stagnant air. *J. Meteorol.* 6, 243–248.
- Hudson, N.W., 1961. An introduction to the mechanics of soil erosion under conditions of sub-tropical rainfall. *Trans. Rhodesian Sci. Assoc.*, 15–25.
- Hudson, N.W., 1965. The influence of rainfall mechanics on soil erosion. MSc Thesis, Cape Town.
- Jayawardena, A.W., Rezaur, R.B., 2000a. Measuring drop size distribution and kinetic energy of rainfall using a force transducer. *Hydrol. Process.* 14, 37–49.
- Jayawardena, A.W., Rezaur, R.B., 2000b. Drop size distribution and kinetic energy load of rainstorms in Hong Kong. *Hydrol. Process.* 14, 1069–1082.
- Joss, J., Waldvogel, A., 1967. Ein spektrograph für Niederschlagsstropfen mit automatischer auswertung (A spectrograph for automatic measurement of rainfalls). *Geofis. Pura Appl.* 68, 240–246.
- Joss, J., Waldvogel, A., 1969. Raindrop size distribution and sampling size errors. *J. Atmos. Sci.* 26, 566–569.
- Kenney, B.C., 1982. Beware of spurious self-correlations! *Water Resour. Res.* 18 (4), 1041–1048.
- Kinnell, P.I.A., 1973. The problem of assessing the erosive power of rainfall from meteorological observations. *Soil. Sci. Soc. Am. Proc.* 37, 617–621.
- Kinnell, P.I.A., 1981. Rainfall intensity–kinetic energy relationship for soil loss prediction. *Soil. Sci. Soc. Am. Proc.* 45, 153–155.
- Kinnell, P.I.A., 1987. Rainfall energy in Eastern Australia: intensity–kinetic energy relationships for Canberra, A.C.T. *Aust. J. Soil Res.* 25, 547–553.
- Laws, J.O., 1941. Measurements of the fall-velocity of water-drops and raindrops. *Trans. Am. Geophys. Union* 22, 709–721.

- Laws, J.O., Parsons, D.A., 1943. Relation of raindrop size to intensity. *Trans. Am. Geophys. Union* 24, 452–459.
- List, R., Donaldson, N.R., Stewart, R.E., 1987. Temporal evolution of drop spectra to collisional equilibrium in steady and pulsating rain. *Journal of the Atmospheric Sciences*, 44 (2), 362–372.
- Madden, L.V., Wilson, L.L., Ntahimpera, N., 1998. Calibration and evaluation of an electronic sensor for rainfall kinetic energy. *Am. Phytopathol. Soc.* 88 (9), 950–959.
- Marshall, J.S., Palmer, W.M., 1948. The distribution of raindrop with size. *J. Meteorol.* 5, 165–166.
- McGregor, K.C., Mutchler, C.K., 1976. Status of the *R* factor in northern Mississippi, soil erosion: prediction and control. *Soil Cons. Soc. Am.*, 135–142.
- McIsaac, G.F., 1990. Apparent geographic and atmospheric influences on raindrop sizes and rainfall kinetic energy. *J. Soil Water Cons.* 45, 663–666.
- Mihara, Y., 1951. Raindrop and soil erosion. *Bull. National Inst. Agric. Sci. (Japan) A* (1), 1–51.
- Morgan, R.P.C., 1995. Soil erosion and conservation. 198 pp.
- Morgan, R.P.C., Quinton, J.N., Smith, R.E., Govers, G., Poesen, J.W.A., Auerswald, K., Chisci, G., Torri, D., Styczen, M.E., 1998a. The European soil erosion model (EUROSEM): a dynamic approach for predicting sediment transport from fields and small catchments. *Earth Surf. Process. Landforms* 23, 527–544.
- Morgan, R.P.C., Quinton, J.N., Smith, R.E., Govers, G., Poesen, J.W.A., Auerswald, K., Chisci, G., Torri, D., Styczen, M.E., Folly, A.J.V., 1998b. The European soil erosion model (EUROSEM): documentation and user guide. Cranfield University.
- Mualem, Y., Assouline, S., 1986. Mathematical model for raindrop size distribution and rainfall kinetic energy. *Trans. ASAE* 29 (2), 494–500.
- Onaga, K., Shirai, K., Yoshinaga, A., 1988. Rainfall erosion and how to control its effects on farmland in Okinawa. In: Rimwanich, S. (Ed.), *Land Conservation for Future Generation*. Department of Land Development, Bangkok, pp. 627–639.
- Park, S.W., Mitchell, J.K., Bubenzer, G.D., 1980. An analysis of splash erosion mechanics. ASAE 1980 Winter Meeting, paper no. 80-2502, Chicago, USA.
- Parsons, A.J., Gadian, A.M., 2000. Uncertainty in modelling the detachment of soil by rainfall. *Earth Surf. Process. Landforms* 25, 723–728.
- Poesen, J., 1985. An improved splash transport model. *Z. Geomorph. N.F.* 29 (2), 193–211.
- Porrà, J.M., Sempere-Torres, D., Creutin, J.D., 1998. Modeling of drop size distribution and its applications to rainfall measurements from radar. In: Bandorff-Nielsen, O.E., Gupta, V.K., Pérez-Abreu, V., Waymire, E. (Eds.), *Stochastic Methods in Hydrology*. Advanced Series on Statistics Science and Applied Probability Series World Scientific Corporation, New Jersey, pp. 73–84.
- Pruppacher, H.R., Klett, J.D., 1998. *Microphysics of Clouds and Precipitation*. Kluwer Academic Publishers, London, 954 pp.
- Renard, V., 1983. Etude de l'énergie cinétique des pluies et observation de l'érosion dans les sols limoneux de moyenne Belgique, Unpublished MSc Thesis, Department of Geography, University of Liège, Liège (Belgium).
- Renard, K.G., Foster, G.R., Weesies, G.A., McCool, D.K., Yoder, D.C., 1997. Predicting soil erosion by water: a guide to conservation planning with the Revised Universal Soil Loss Equation (RUSLE). USDA, 404 pp.
- Rosewell, C.J., 1986. Rainfall kinetic energy in eastern Australia. *J. Clim. Appl. Meteorol.* 25, 1695–1701.
- Salles, C., Sempere-Torres, D., Creutin, J.D., 1999. Characterisation of raindrop size distribution in Mediterranean climate: analysis of the variations on the *Z-R* relationship. Proceedings of the 29th Conference on Radar Meteorology. AMS, Montreal, Canada, pp. 671–673.
- Sempere-Torres, D., 1994. La lluvia como agente erosivo: formación distribución, erosividad e interceptación. *Ingeniería Hidráulica en México*, IX: 5–18.
- Sempere-Torres, D., Salles, C., Creutin, J.D., Delrieu, G., 1992. Quantification of soil detachment by raindrop impact: performances of classical formulae of kinetic energy in Mediterranean storms. In: Bogen, J., Walling, D.E., Day, T. (Eds.), *Erosion and sediment transport monitoring programmes in river basins*. IAHS Publ. no. 210, Oslo, pp. 115–124.
- Sempere-Torres, D., Porrà, J.M., Creutin, J.-D., 1994. A general formulation for raindrop size distribution. *J. Appl. Meteorol.* 33 (12), 1494–1502.
- Sempere-Torres, D., Porrà, J.M., Creutin, J.-D., 1998. Experimental evidence of a general description for raindrop size distribution properties. *J. Geophys. Res.* 103 (D2), 1785–1797.
- Sempere-Torres, D., Sánchez-Diezma, R., Zawadzki, I., Creutin, J.D., 2000. Identification of stratiform and convective areas using radar data with application to the improvement of DSD analysis and *Z-R* relations. *Phys. Chem. Earth* 25, 985–990.
- Smith, J.A., De Veaux, R.D., 1992. The temporal and spatial variability of rainfall power. *Environmetrics* 3 (1), 29–53.
- Steiner, M., Smith, J.A., 2000. Reflectivity, rain rate, and kinetic energy flux relationships based on raindrop spectra. *J. Appl. Meteorol.* 39 (11), 1923–1940.
- Tracy, F.C., Renard, K.G., Fogel, A.M., 1984. Rainfall energy characteristics for southern Arizona. *Proc. Am. Soc. Civil Engng. Irrig. Drain Div. Specialty Conf. Flagstaff, A.Z.*, 559–566.
- Uijlenhoet, R., Stricker, J.N.M., 1999a. Dependence of rainfall interception on drop size. A comment. *J. Hydrol.* 217, 157–163.
- Uijlenhoet, R., Stricker, J.N.M., 1999b. A consistent rainfall parameterization based on the exponential raindrop size distribution. *J. Hydrol.* 218, 101–127.
- Ulbrich, C.W., 1983. Natural variations in the analytical form of the raindrop size distribution. *J. Clim. Appl. Meteorol.* 22, 1764–1775.
- Ulbrich, C.W., 1985. The effects of drop size distribution truncation on rainfall integral parameters and empirical relations. *J. Clim. Appl. Meteorol.* 24, 580–590.
- Ulbrich, C.W., Atlas, D., 1998. Rainfall microphysics and radar properties: analysis methods for drop size spectra. *J. Appl. Meteorol.* 37, 912–923.
- Uplinger, C.W., 1981. A new formula for raindrop terminal velocity. Proceedings of the 20th Conference on Radar Meteorology. American Meteorological Society Boston, USA, 389–391 pp.
- Usòn, A., Ramos, M.C., 2001. An improved rainfall erosivity index

- obtained from experimental interrill soil losses in soils with Mediterranean climate. *Catena* 43, 293–305.
- Wischmeier, W.H., Smith, D.D., 1958. Rainfall energy and its relationship to soil loss. *Trans. AGU* 39, 285–291.
- Wischmeier, W.H., Smith, D.D., 1978. *Predicting Rainfall Erosion*. Agriculture Handbook No. 537. United States Department of Agriculture, Washington, DC.
- Zanchi, C., Torri, D., 1980. Evaluation of rainfall energy in central Italy. In: De Boodt, M., Gabriels, D. (Eds.). *Assessment of Erosion*. Wiley, Toronto, pp. 133–142.
- Zawadzki, I., De Agostinho Antonio, M., 1988. Equilibrium rain-drop size distributions in tropical rain. *J. Atmos. Sci.* 45, 3452–3459.