PERGAMON

Journal of Geodynamics 33 (2002) 3–20

JOURNAL OF GEODYNAMICS

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Dedicated gravity field missions—principles and aims

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Abstract

Current knowledge of the Earth's gravity field and its geoid, as derived from various observing techniques and sources, is incomplete. Within a reasonable time, substantial improvement can only come by exploiting new approaches based on satellite gravity observation methods. For this purpose three satellite missions will be realised, starting with CHAMP in 2000, followed by GRACE in 2002 and GOCE in 2004. Typical for all three missions is their extremely low and (almost) polar orbit, continuous and three-dimensional tracking by GPS and their ability to separate non-gravitational from gravitational signal parts. A further amplification of the gravity signal is achieved by inter-satellite tracking between two low orbiters in the case of GRACE and by gravity gradiometry in the case of GOCE. The rationale of GOCE will be discussed in more detail. The missions have a wide range of applications in solid Earth physics, oceanography, ice research, climatology, geodesy and sea level research. © 2002 Elsevier Science Ltd. All rights reserved.

1. Science case

For a better understanding of the physics of the interior of the Earth, of the dynamics of the oceans and of the interaction of continents, ice and ocean in sea-level studies (as well as for better orbits and height systems in science and engineering), it is necessary to significantly improve our knowledge of the gravity field of the Earth, both in terms of accuracy and spatial resolution. This need has been emphasised over the years by various international bodies and in particular by the

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PII: S0264-3707(01)00050-3

International Union of Geodesy and Geophysics (IUGG), and two of its associations, the International Association of Geodesy (IAG) and the International Association for the Physical Sciences of the Oceans (IAPSO). Only by means of satellites can this be achieved globally, homogeneously and within a reasonable time period. However, in order to meet the accuracy and resolution requirements for the gravity field by means of a space mission, one has to deal with the attenuation of the gravity field at satellite altitude.

The gravity field plays a peculiar dual role in Earth sciences. On the one hand, by comparing the actual field with that of an idealised Earth body (e.g. an idealised Earth in hydrostatic equilibrium) deviations, called gravity anomalies, can be derived. These indicate the state of mass imbalance in the Earth's interior and provide important insights into the dynamics of the planet. Gravity anomalies are one of only three means available to look into the structure of the Earth's interior, the other two being the analysis of the propagation of seismic waves and magnetometry. In this sense, the gravity field provides a mirror into the Earth's interior.

On the other hand, the geoid (i.e. the equi-potential surface at mean sea-level of a hypothetical ocean at rest) serves as the reference surface for all topographic features, whether they belong to land, ice or ocean. The geoid is defined purely by the Earth's gravity field and its accuracy will benefit from any improvement in the latter's precision and spatial resolution. When mountain topography is measured (e.g. by remote sensing) the accuracy with which the geoid is determined plays no significant role. However, in all cases involving small height differences, such as in engineering and geodesy, or in studies of ice motion, sea-level changes or ocean circulation, requirements for an accurate knowledge of the geoid, in terms of precision and resolution, are extremely high.

Three brief examples are used below to demonstrate the expected impact of the GOCE mission on studies of the interior structure of the Earth, absolute ocean circulation and the unification of height systems.

1.1. Solid-Earth physics

Assume a gravity anomaly model of the Alps derived from terrestrial measurements (Fig. 1, upper panel). Then, given a digital terrain model (DTM) of the Alps and an appropriate model of rock densities, one can calculate the terrain's gravity effect and subtract it from the given anomaly model (Fig. 1, panels of the second row). This is like generating a gravity-anomaly field map for the Earth without topography. Surprisingly, the remaining field (Fig. 1, lower panel) is only slightly smoother and it is even larger in amplitude than the original anomaly field. The well-known explanation of this observation is isostasy, i.e. the concept of dynamic support or static compensation of all visible topography and of the mass deficit of the oceans.

The remaining anomaly field is an expression of the state of mass imbalance and ultimately of the density anomaly structure in the Earth's lithosphere and mantle. It is directly related to a large variety of solid-Earth processes, currently not well understood. However, the translation of the gravity anomaly to the density anomaly field is an inverse problem. Nowadays seismic tomography is providing excellent three-dimensional views of the seismic velocity anomaly field in the Earth's mantle and in the lithosphere in some selected regions. Again, there is no easy way to translate the anomalous velocity into the corresponding anomalous density structure (see Ricard and Froidevaux, 1990).

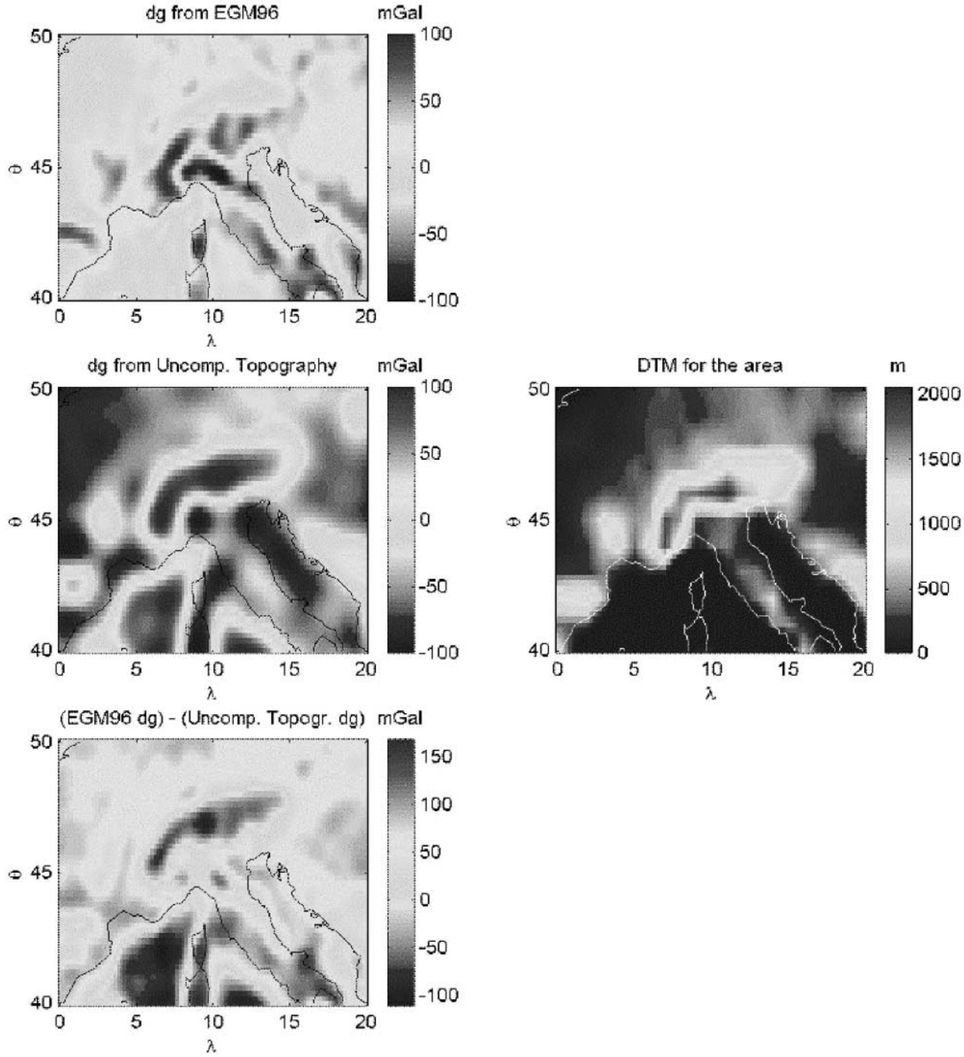

Fig. 1. Observed anomalous gravity field of the Alps (upper panel), terrain model of the same region (middle panel, right), synthetic anomalous gravity field as derived from the terrain model (middle panel, left); difference between observed and synthetic gravity field reveals inhomogeneous density structure in the lithosphere (lower panel).

The combination of the two, i.e. of the residual gravity anomaly field and three-dimensional seismic tomography, supported by information from deformation and displacement measurements made at the Earth's surface, by laboratory research on the physical/chemical properties of mantle material and of magnetic anomalies in the crust and lithosphere, is of tremendous promise. It will significantly improve our understanding, in particular, of the continental lithosphere and of the interaction of the upper mantle with the lithosphere. The key objective is not the gravity anomaly field itself, but the anomalous density structure derived from it. Examples are given in (DiDonato et al., 1999) and in (Negredo et al., 1999).

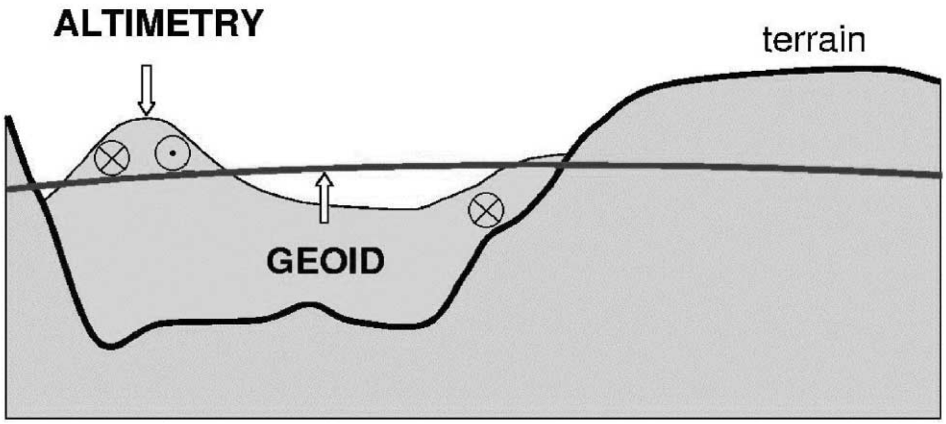

Fig. 2. The geoid defines the idealised ocean surface at rest. Its deviation from the actual mean ocean surface as derived from altimetry is the dynamic ocean surface topography.

1.2. Absolute ocean circulation

Given the observations for GOCE in terms of gravity field determination, the shape of the marine geoid in radial direction can be determined with cm precision down to length scales of 100–200 km. Satellite radar altimetry is determining the actual ocean surface with approximately the same precision. The deviation of this mean ocean surface, as obtained from altimetry, from the geoid is the ocean steady-state dynamic topography. As most ocean currents on long time scales are in geostrophic balance, dynamic ocean topography can be directly translated into ocean surface circulation. The principle is briefly illustrated in Fig. 2.

With data from past, present and future satellite altimetry missions, this method is able to provide long-term global determinations of absolute ocean surface circulation. Furthermore, ocean surface circulation determines the mean transport of heat and mass by the ocean. Despite the fact that these are currently not properly quantified, they are recognised to be important elements of climate. It is referred to (Ganachaud et al., 1997; LeGrand and Minster, 1999; LeProvost et al., 1999).

1.3. Geodesy

On land and ice sheets, the difference between ellipsoidal heights (as measured by the global positioning system (GPS)) and the geoid obtained from GOCE gives 'pseudo-levelled' or orthometric heights. This differentiation opens the prospect of an extremely efficient and accurate method of height determination for science, mapping and engineering. In addition, all existing height systems could be transferred to one global reference level. Also orbit determination and inertial navigation will benefit from of a global, accurate and detailed knowledge of the Earth's gravity. See for example (Schwartz, 1981; Schwarz and Sideris, 1987; Xu and Rummel, 1991; Arabelos and Tscherning, 1999).

2. State-of-the-art of gravity field determination

2.1. Theoretical foundation

Gravitational acceleration as expressed by Newton's fundamental law of gravitation is a three-dimensional vector field. Its dominating feature reflects the almost spherical shape of the Earth, the well known 9.8 m/s^2. The main deviations from a spherical field reflects the Earth's rotation and oblateness. Here in this section the focus of interest is in the important effect of much smaller deviations due to the gravitational attractions of a wide range of mass inhomogeneities at the Earth's surface and in its interior.

For global gravity field analysis, the Earth's gravitational potential is represented by a spherical harmonic series (cf. Heiskanen and Moritz, 1967):

$$\begin{aligned} V(r,\theta,\lambda) &= \frac{GM}{R}\sum_{\ell=0}^{\infty}\left(\frac{R}{r}\right)^{\ell+1}\sum_{m=0}^{\ell}\bar{P}_{\ell m}(\sin\theta)(\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda) = \\ &\frac{GM}{R}\sum_{\ell=0}^{\infty}\left(\frac{R}{r}\right)^{\ell+1}\sum_{m=-\ell}^{\ell}K_{\ell m}Y_{\ell m}(\theta,\lambda) \end{aligned} \tag{1}$$

with $\bar{P}_{\ell m}$ and $Y_{\ell m}$ the real and complex valued spherical harmonics of degree ℓ and order m, respectively, GM the gravitational constant (G) times mass (M) of the Earth and R the Earth's mean radius. In satellite applications, $\{r, \theta, \lambda\}$ are the spherical co-ordinates of the spacecraft. With $r = R + h$ and h the altitude of the satellite, the factor $(R/r)^{\ell+1}$ describes the field attenuation with altitude. The series coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ (or in complex form $K_{\ell m}$) are to be determined. They are the fundamental gravity field unknowns. The infinite series is usually truncated at the maximum resolvable degree $\ell = L$, which can be translated into a corresponding spatial-scale (half wavelength given in km) D with

$$D = 20000/L. \tag{2}$$

The series coefficients allow the determination of geoid heights (measured in metres above an adopted reference ellipsoid) with:

$$N(\theta,\lambda) = R\sum_{\ell=2}^{L}\sum_{m=0}^{\ell}\bar{P}_{\ell m}(\sin\theta)\left[\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right] \tag{3}$$

and of gravity anomalies (measured in mgal) by

$$\Delta g(\theta,\lambda) = \gamma\sum_{\ell=2}^{L}(\ell-1)\sum_{m=0}^{\ell}\bar{P}_{\ell m}(\sin\theta)\left[\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right] \tag{4}$$

where γ is mean gravity.

Alternatively, the gravitational potential can be expressed in a system of orbit elements (for a circular orbit) as

$$V(r,u,\Lambda)=\frac{GM}{R}\sum_{\ell=0}^{L}\left(\frac{R}{r}\right)^{\ell+1}\sum_{m=-\ell}^{+\ell}\sum_{k=-\ell}^{+\ell}K_{\ell m}F_{\ell mk}(I)\exp\left[i(ku+m\Lambda)\right] \tag{5}$$

with $F_{\ell mk}$ being the inclination functions (Kaula, 1966), I the orbit inclination, $\Lambda=\Omega-\theta_G$ the difference in longitude between the Greenwich meridian and the longitude of the ascending node, and u the argument of latitude of the satellite in the orbit plane.

From the gravitational potential, any other gravity function can be deduced quite easily. This includes geoid heights [Eq. (3)], gravity anomalies [Eq. (4)] and the gravitational acceleration vector. For gradiometry, second-order derivatives (with respect to the three spatial directions), the so-called gravitational gradients, are of particular interest. The nine second-order derivatives form a symmetric 3×3 matrix where the trace (diagonal) is zero in empty space. The radial component of the gravitational gradient can be expressed as:

$$V_{zz}=\frac{\partial^2 V}{\partial z^2}=\frac{GM}{R}\sum_{\ell=0}^{L}\frac{(\ell+1)(\ell+2)}{R^2}\left(\frac{R}{r}\right)^{\ell+3}\sum_{m=-\ell}^{+\ell}\sum_{k=-\ell}^{+\ell}K_{\ell m}F_{\ell mk}(I)\exp\left[i(ku+m\Lambda)\right]. \tag{6}$$

In this expression it is important to note that the 'differentiation factor' $(\ell+1)(\ell+2)$ can counteract the attenuation factor $(R/r)^{\ell+3}$. The corresponding expressions for orbit perturbations Δx, Δy and Δz in the along-track, cross-track and radial directions and for all second derivatives of the gravitational potential are summarised in Table 1.

In the case of Δy, V_{xy} and V_{yz} a modified inclination function has to be used (Sneeuw, 1994). The parameter $\beta=(k\dot{u}+m\dot{\Lambda})/n$ is the normalised frequency, with n the mean orbit frequency.

In gravity field studies the average signal strength (i.e. the power spectral density (PSD)), is expressed in terms of degree variances c_ℓ, where

$$c_\ell=\sum_{m=0}^{\ell}\left[\bar{C}^2_{\ell m}+\bar{S}^2_{\ell m}\right]=\sum_{m=-\ell}^{\ell}|K_{\ell m}|^2 \tag{7}$$

or in terms of their square roots, the root-mean-square (RMS) value per degree. It can be shown that on the Earth's surface the degree variances follow the rule of thumb, according to Kaula (1966):

$$c_\ell=1.6\frac{10^{-10}}{\ell^3}\,(\text{dimensionless}) \tag{8}$$

i.e. the field strength tapers off with $1/\ell^3$. At satellite altitude, this attenuation effect is increased by the $(R/r)^{\ell+1}$ term. For high-resolution gravity field determination by satellite the main goal is to counteract this attenuation term.

Table 1
Sensitivity coefficients that relate observable orbit perturbations and gradiometric components to the unknown spherical harmonic coefficients [e.g. compare to Eq. (6)]. The expressions are given for the along-track, cross-track and radial perturbations, Δx, Δy and Δz, respectively, that can be measured by SST, and for the second derivatives V_{xx}, V_{yy}, V_{zz}, V_{xy}, V_{xz}, V_{yz}, measurable by satellite gradiometry. Each of these quantities exhibits a characteristic 'view' on the Earth's gravitational field. Also shown (in the two right columns) are the order of magnitudes of the gradiometer components, i.e. the average size (DC value in Eötvös) of each of the components and the average ratio of the individual signal spectral powers with respect to that of the dominant radial component V_{zz}

Δx	$i\dfrac{2(\ell+1)\beta - k(\beta^2+3)}{\beta^2(\beta^2-1)n^2R}$	$\left(\dfrac{R}{r}\right)^{\ell+2}$		
Δy	$\dfrac{1}{(1-\beta^2)n^2R}$	$\left(\dfrac{R}{r}\right)^{\ell+2}$		
Δz	$\dfrac{(\ell+1)\beta-2k}{\beta(\beta^2-1)n^2R}$	$\left(\dfrac{R}{r}\right)^{\ell+2}$		
V_{xx}	$\dfrac{-(k^2+\ell+1)}{R^2}$	$\left(\dfrac{R}{r}\right)^{\ell+3}$	−1400	3/8
V_{yy}	$\dfrac{k^2-(\ell+1)^2}{R^2}$	$\left(\dfrac{R}{r}\right)^{\ell+3}$	−1400	3/8
V_{zz}	$\dfrac{(\ell+1)\ (\ell+2)}{R^2}$	$\left(\dfrac{R}{r}\right)^{\ell+3}$	+2800	1
V_{xy}	$\dfrac{ik}{R^2}$	$\left(\dfrac{R}{r}\right)^{\ell+3}$	Small	1/8
V_{xz}	$\dfrac{-ik\ (\ell+2)}{R^2}$	$\left(\dfrac{R}{r}\right)^{\ell+3}$	≈10	1/2
V_{yz}	$\dfrac{-(\ell+2)}{R^2}$	$\left(\dfrac{R}{r}\right)^{\ell+3}$	small	1/2

2.2. Available gravity data

Presently three gravity data sources are available.

(a) *Mean gravity anomalies*, taken typically over areas of 100 x 100 km^2 or 50 x 50 km^2, are derived from terrestrial gravimetry in combination with height measurements and from shipborne gravimetry. Their accuracy depends on data density and the precision of the height and gravity measurements. Before the late 1980s, mean values of acceptable accuracy were available only for North America, Western Europe and Australia. In recent years, due to an enormous effort to encourage data exchange, the situation has significantly improved.

The Bureau Gravimétrique International (BGI) and the National Imaging and Mapping Agency (NIMA) are collecting, screening and editing gravity material on a worldwide basis. Airborne gravimetry has been applied with success to some selected areas such as parts of Antarctica and Greenland (e.g. Brozena and Peters, 1994). However, due in particular to the sparseness of data in some large continental areas and the generally poor quality of older sea gravimetry data, severe inconsistencies remain and the geoid precision does not drop much below approximately 50 to 80 cm in most parts of the World. A global map of currently available mean gravity anomalies is shown in Fig. 3.

(b) In ocean areas, *satellite altimetry* can in some sense be regarded as a direct geoid measuring technique. However, after removing time-varying effects, such as tides, by averaging repeated measurements, the resulting stationary sea-surface still deviates from the geoid due to dynamic ocean topography. In fact, this difference, the mean-sea-surface topography, will be seen to be of key importance in oceanography.

For geophysical investigations, the distinction between mean-sea-surface and the geoid is for many applications of minor importance. As a result, altimetry has been applied to the study of the oceanic lithosphere with enormous success. However, the distinction is essential for oceanography and sea-level studies. In these investigations, a geoid determined completely independently of altimetry is essential.

(c) For more than three decades now, several institutions have determined geopotential models from *satellite orbit analysis*. These are derived from the combined analysis of orbits of a large number of mostly non-geodetic satellites with different orbit elements. They exploit a variety of tracking techniques but primarily laser and Doppler measurements. These models are presented as sets of coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ of a spherical harmonic expansion of the field and they provide information on the long wavelength part of the spectrum only. A representative example of one of the best currently available geopotential models, based purely on satellite orbit analysis (no altimetry, no terrestrial surface gravity), is the GRIM-4S gravity field model (Schwintzer et al., 1997). It is complete to degree $\ell = 72$ and order $m = 72$. This corresponds to a spatial half wavelength of $D = 300$ km.

A 'stabilisation technique' has had to be employed to obtain a solution for the complete set of coefficients. However, for some groups of coefficients from this model, the error estimates

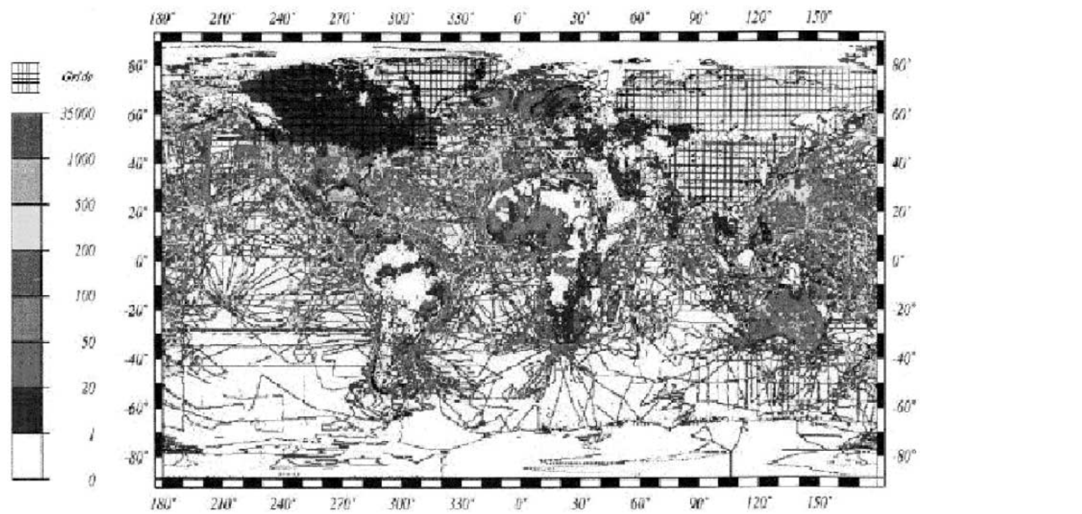

Fig. 3. Distribution of presently available gravity measurements in the database of the Bureau Gravimétrique International spanning the last half century. Over some areas, only grid values are available (i.e. no detailed survey data). Greenland was recently mapped by airborne gravimetry.

approach 100% of the expected size of the terms, particularly above degree 36 (or for half wavelength $D=560$ km). Intrinsic limitations prohibit further significant improvements in resolution by this approach. A better de-correlation of the individual coefficients is feasible in the future by employing new tracking concepts such as DORIS and, in particular, space-borne GPS that allow (almost) uninterrupted tracking.

Combined models of these three data sources exist, of which the best is the EGM96 (Lemoine et al., 1998). Neither of the above three data sources nor their combination can meet the requirements from solid-Earth physics, oceanography and geodesy, not even to a limited extent. This is why one can say with good reason that "... the gravity field over land areas on Earth is less well known than is that of Venus" (McKenzie, 1994). The solution must, therefore, come from dedicated gravity field mapping by satellite.

3. High-resolution gravity field determination from space

As explained above, the traditional techniques of gravity field determination have reached their intrinsic limits. Any advances must rely on space techniques because only they provide global, regular and dense data sets of high and homogeneous quality. It may seem almost paradoxical to obtain the measurements from several hundreds of kilometres up in space, away from the attracting mass anomalies, which one would like to identify and discriminate. This point will be returned to later.

If it is decided to deduce gravity from space, the question is why not extend the traditional method of orbit analysis (see Section 2.2). In that technique the satellite, in its orbital motion around the Earth, is considered as a test mass in free fall in the Earth's gravitational field and from this motion the gravitational field is deduced. However, there are two limitations in the method. The first arises because satellites can be tracked from the ground only over short intervals and, as a consequence, the gravity signal 'printed onto the orbit' can only be extracted where it produces an orbit signal of large size such as at or close to orbit resonances. The second occurs because satellite motion is not determined by gravitation alone but disturbed by several types of surface forces of non-gravitational origin. These disturbances corrupt our present gravity models. From an appreciation of these two limitations, three fundamental criteria for any future dedicated satellite gravity mission arise:

- Uninterrupted tracking in three spatial dimensions
- Measurement or compensation of the effect of non-gravitational forces
- Orbit altitude as low as possible

All three criteria can be met by exploiting the concept of satellite-to-satellite tracking in the high-low mode (SST-*hl*). Thereby a low earth orbiter (LEO) is equipped with a receiver of the US Global Positioning System (GPS) and the Russian counterpart (GLONASS) and with a three-axis accelerometer (see Fig. 4a). The receiver 'sees' 12 or more GPS and GLONASS satellites at any time. Their ephemerides are determined very accurately by the large network of ground stations that participate in the International GPS Service (IGS). Taking their orbits and the GPS/GLONASS measurements of the LEO (pseudo-range and carrier phase), the orbit of the LEO can be monitored to cm-precision without interruption and in three dimensions.

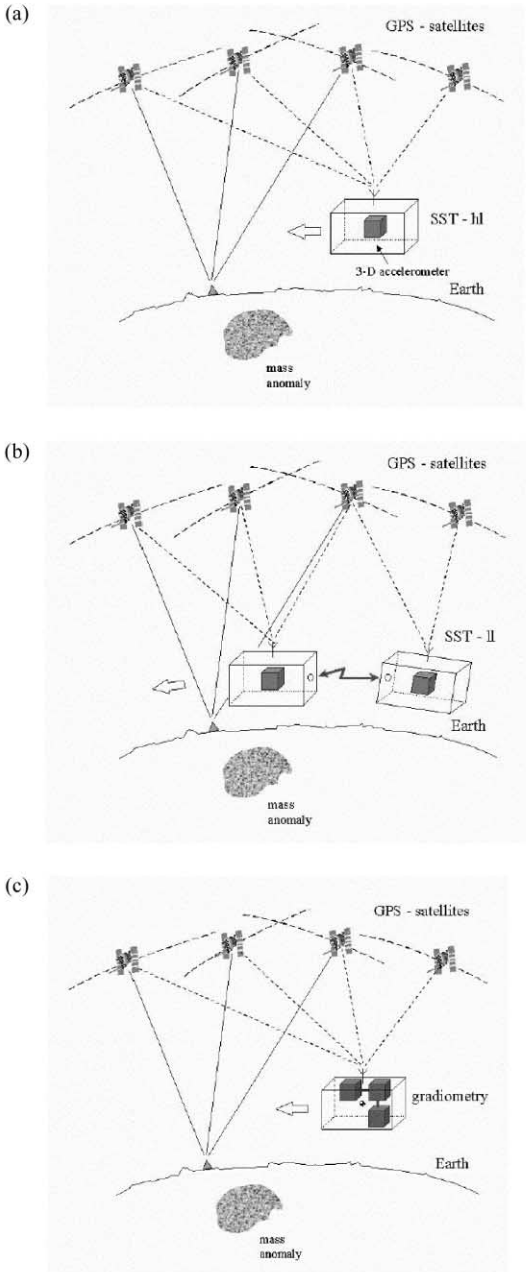

Fig. 4. (a) Concept of satellite-to-satellite tracking in the high-low mode (SST-*hl*). A low Earth orbiter is tracked by the high orbiting GPS and GLONASS satellites, relative to a net of ground stations. Non-gravitational forces on the low orbiter are measured by accelerometry. 4 (b) Concept of satellite-to-satellite tracking in the low-low mode (SST-*ll*) combined with SST-*hl*. The relative motion between two low orbiters following each other in the same orbit at a distance of few hundred kilometres is measured by an inter-satellite link. 4 (c) Concept of satellite gradiometry combined with SST-*hl*. The second-order derivatives of the gravitational potential of the Earth are measured in a low orbiting satellite by differential accelerometry.

In addition, the accelerometer, placed at the satellites' centre of mass measures the non-gravitational forces. The effect of the latter can then be taken into account computationally, or can be compensated for by a drag-free control mechanism. A first satellite of this type (without drag-free control) is the German CHAMP (Reigber et al., 1996) that has been launched in 2000.

However, even with this configuration and with an altitude as low as 300 or 400 km, the problem of gravity field attenuation prohibits the attainment of really high-spatial-resolution. Thus, a fourth criteria enters:

- Counteract gravity field attenuation at altitude

The classical approach of highlighting the effect of small-scale features in physics is differentiation. Two alternative concepts of differentiation can be conceived. Either one applies satellite-to-satellite tracking in the low-low mode (SST-*ll*) or satellite gradiometry; both still combined with SST-*hl*. In the case of SST-*ll* (see Fig. 4b), two spacecraft in essentially the same orbit and a distance of somewhere between 100 and 400 km apart, 'chase each other'. The relative motion between the two satellites is measured with the highest possible precision. Again the effect of non-gravitational forces on the two spacecraft can either be compensated for or be measured. The quantity of interest is the relative motion of the centres of mass of the two satellites, which has to be derived from the inter-satellite link together with the measured acceleration and attitude data. The first experiment of this type will be the US-German mission GRACE, (GRACE, 1998).

The alternative to SST-*ll* is to apply satellite gradiometry as proposed for GOCE. Satellite gradiometry is the measurement of acceleration differences, ideally in all three spatial directions, between the test-masses of an ensemble of accelerometers inside one satellite (see Fig. 4c). The measured signal is the difference in gravitational acceleration at the test-mass locations inside the spacecraft, where of course the gravitational signal stems from all the attracting masses of the Earth, ranging from mountains and valleys, via ocean ridges, subduction zones, mantle inhomogeneities down to the core-mantle-boundary topography. The technique can resolve all these features as they appear in the gravity field. The measured signals correspond to the gradients of the component of gravity acceleration or, in other words, to the second derivatives of the gravitational potential. Non-gravitational acceleration of the spacecraft (for example due to air drag) affects all accelerometers inside the satellite in the same manner and ideally drops out when taking the differences. Rotational motion of the satellite does affect the measured differences, but can be separated from the gravitational signal by separating the measured 3×3 matrix of second derivatives into a symmetric and an anti-symmetric part. Again, a low orbit implies relatively high signals.

Generally speaking, one can now argue that the basic observable in the discussed three cases (namely SST-*hl*, SST-*ll* and satellite gravity gradiometry (SGG)) is gravitational acceleration. With the orbits of the high-orbiting GPS and GLONASS satellites assumed to be known with high accuracy, the case of SST-*hl* corresponds to an 'in situ' 3-D position, velocity or acceleration determination of a LEO. For SST-*ll*, the principle corresponds to the line-of-sight measurement of the range, range rate or acceleration difference between the two low-orbiting satellites. Finally, in the case of satellite gradiometry, the measurement is of acceleration differences in 3-D over the short baseline of the gradiometer instrument. In short, therefore the principles are:

SST-*hl*: 3-D accelerometry corresponds to gravity acceleration

SST-*ll*: inter-satellite link corresponds to acceleration differences between two LEOs
SGG: gradient of gravity components corresponds to the acceleration gradient.

Thus in a mathematical sense it is the transition from the first derivative of the gravitational potential (SST-*hl*), via the difference of first derivatives over a long baseline (SST-*ll*) to the second derivative (gradiometry). The guiding parameter that determines sensitivity with respect to spatial-scales of the Earth's gravity field is the distance between the test masses, being almost infinity for SST-hl and almost zero for gradiometry.

The mathematical concept is illustrated by the spectral scheme of Fig. 5. The sensitivity parameters (eigenvalues) connecting orbit acceleration $\Delta\ddot{x}$, velocity $\Delta\dot{x}$ and position perturbation Δx describe orbit perturbation theory (n being orbit mean angular velocity, k running from 0 to L). The fundamental problem of any satellite gravity mission is the amplification of the errors by the factor $(r/R)^{\ell+1}$ when transferring the measured "signal + noise" from satellite altitude to the Earth's surface. This effect is minimised by:

- flying the test mass as low as possible, and
- not just measuring V or its gradient (= SST-*hl*), but rather its second order derivatives (= gradiometry).

More details are provided by Rummel (1997).

4. GOCE mission rationale

GOCE will be the first gravity gradiometry satellite mission. It is specifically designed for the determination of the stationary gravity field—geoid and gravity anomalies—to high accuracy and spatial resolution. It is capable of meeting all of the fundamental criteria described in Section 3, namely:

- It will be continuously tracked in three dimensions by the systems of GPS and GLONASS satellites, relative to the dense ground network of IGS stations.
- It will control drag forces and eliminate remaining residual effects by differential measurement, the so-called common mode rejection (CMR) principle. Rotational motion will also be controlled, and remaining rotational effects will be determined by a novel combination of measured off-diagonal gradient components and star sensors (Aguirre-Martinez, 1999).
- It will fly in an extremely low and almost polar orbit (sun-synchronous).
- It will efficiently overcome the problem of attenuation of the gravity field at altitude by the principle of gradiometry.

In addition, gradiometry has the unique and important ability of being able to measure the gravity field in three spatial dimensions independently and without any preferred direction. It therefore permits observations of the gravitational field of the Earth in three complementary 'illuminations with no directional bias. This indeed suggests that to avoid aliasing of any component of the gravity field into another component, the main components need to be measured individually as can be obtained with GOCE.

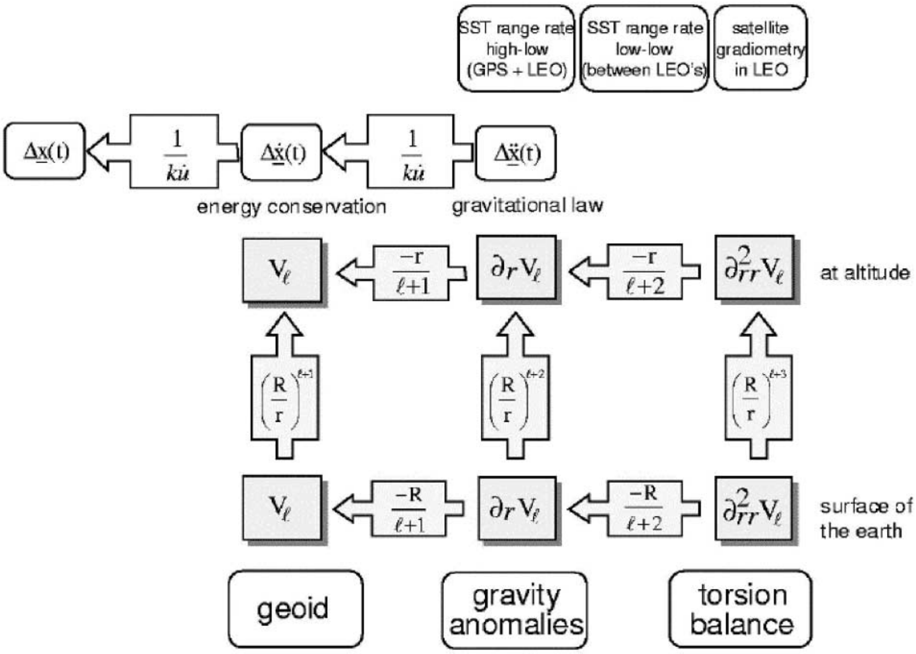

Fig. 5. The three fundamental gravity quantities are the potential (geoid), and its first (gravity anomaly) and second derivatives (corresponding to torsion balance measurements), here as a function of spherical harmonic degree V_ℓ, $\partial V_\ell/\partial r$, $\partial^2 V_\ell/\partial r^2$. These three quantities are damped by a factor $(R/r)^\ell$ at satellite altitude. Measuring gravity gradients balances the attenuation by the factor $(\ell+1)(\ell+2)$.

5. Complementary space missions and airborne projects

In this section a comparison of the GOCE mission capabilities with other spaceborne gravity field missions and airborne gravity observations is first provided, followed by a brief review of the complementarity of information from other satellite missions.

The expected performances of CHAMP, GRACE and GOCE are compared in Fig. 6 (further details of this comparison are reported by Balmino et al., 1998). The figure shows the signal degree RMS values according to Kaula's rule [Eq. (8)] and the noise degree RMS of the best available satellite gravity model. Signal and noise line intersect somewhere between $\ell = 20$ and $\ell = 30$ ($D \approx$1000–660 km).

CHAMP is to be seen twofold. First, it serves as a proof-of-concept mission, as it will be the first time that uninterrupted three-dimensional high-low tracking has been combined with 3-D accelerometry. Second, it will significantly de-correlate the available gravity field models but also increase their accuracy and spatial resolution (see line SST-*hl* in Fig. 6). It will therefore make current models much more reliable.

GRACE will be the first SST-*ll* mission. It will improve the accuracy of the spherical harmonic coefficients at long and medium spatial-scales by up to three orders of magnitude. This will allow the measurement of the temporal variations in the gravity field, such as those due to

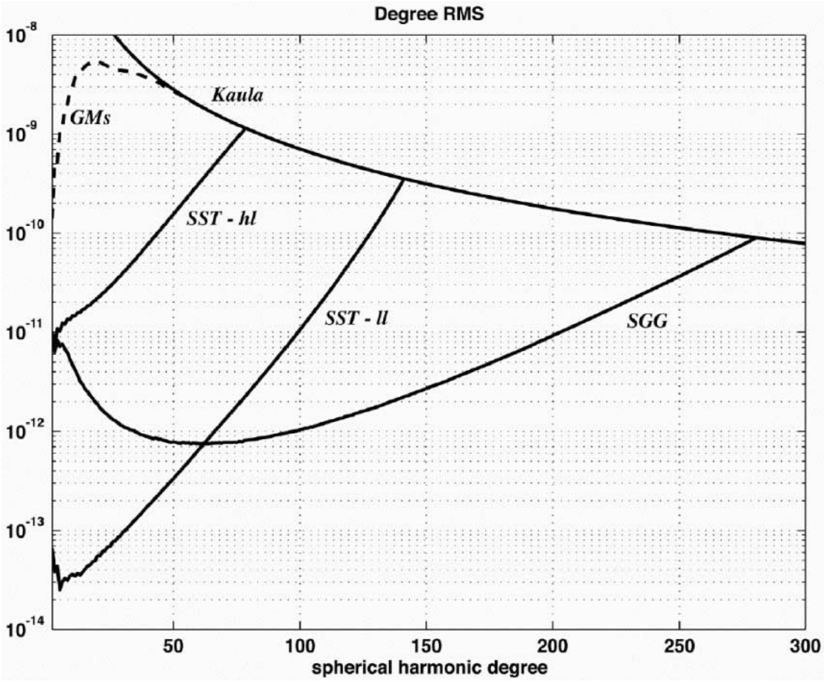

Fig. 6. Representative error degree variance spectra of the gravity mission concepts SST-*hl*, SST-*ll* and satellite gradiometry in comparison with one of the best currently available satellite gravity models (GMs) and with the signal degree variances of the gravity field (Kaula). The high precision of SST-*ll* at long and medium length scales and the high-spatial-resolution of gradiometry are apparent here.

bottom-pressure variations, seasonal and annual variations in groundwater and soil-moisture levels, changes in the masses of the Antarctic and Greenland ice sheets or atmospheric pressure changes (see NRC, 1997). One could refer to this as a 'spyglass effect'. The high slope of the noise line of GRACE (Fig. 6) suggests that any decrease or increase of mission performance has little effect on its spatial resolution, but a large effect on its ability to resolve temporal variations.

The noise line of GOCE, on the other hand, is much flatter, leading to a much higher spatial resolution. One could refer to it as 'extended spectral window effect' (Fig. 6). By employing gradiometry, the noise line is roughly decreased by a factor '*l*-squared' as compared to the case of SST-*hl*. Here an increase in mission performance has only a minor effect in terms of temporal resolution, but a large effect on its ability to resolve spatial variations. One can expect that gravity signatures as short as 65 km will be resolved with GOCE. Thus, it is concluded that the two missions, GRACE and GOCE, are complementary, with GRACE focusing in particular on the temporal variations of the gravity field and GOCE on attaining maximum spatial resolution.

These various complementary satellite gravity field concepts as well as those from other satellite mission and airborne observations are summarised in Table 2. An airborne gravimetric sensor, for instance, measures the sum of gravity and aircraft acceleration. However, by exploiting differential GPS, aircraft acceleration can be determined and separated from the gravity signal. Thus, although still affected by various systematic error sources, airborne gravimetry has been

Table 2
Overview of the complementary space and in situ data to be used in combination with GOCE data

Complementary data	
Proof-of-concept of SST-*hl* combined with 3-D accelerometry	CHAMP
Temporal variations of Earth gravity field	Available models (tides, atmospheric pressure, ocean variability) and results from GRACE
Gravity field at polar caps and small-scale gravity information in some regions	Available and planned airborne and terrestrial gravimetry data
Solid-Earth physics	Topographic models (DTMs) and seismic tomography as primary data sets, lithospheric magnetic field from ØERSTED and CHAMP and planned magnetometry satellite missions
Oceanography	Data sets from past, current and future ocean altimetry (GEOSAT, T/P, ERS-1 & 2, Envisat, Jason etc.)
Ice research	Ice altimetry (ICESAT, CRYOSAT) and INSAR
Geodesy	Current and future global satellite positioning and navigation systems (SLR, VLBI, DORIS, GPS, GLONASS, GNSS-2)
Sea-level	Global tide-gauge network (GLOSS), GPS/DORIS, satellite ocean and ice altimetry and GPS

shown to be able to produce very useful results. Projects at present concentrate on the polar regions, where satellite gravity missions may leave small gaps due to their non-polar inclinations (Brozena et al., 1997 or Wei and Schwarz, 1997).

Past, present and future altimeter missions, such as GEOSAT, TOPEX/ POSEIDON, ERS-1 and ERS-2, in the near future Envisat, Jason and the planned ice altimeter missions ICESAT and CRYOSAT as well as SAR interferometry (INSAR) are all important complements to GOCE for its application to oceanography and ice-sheet research. In ocean areas, the difference between the quasi-stationary ocean surface (freed from all time-varying effects) and the geoid yields steady-state ocean circulation. Over ice sheets, taking an approach similar to that applied to the determination of sea-floor bathymetry from altimetry, the combination of ice-sheet topography (or elevation) and the measured gravity anomaly field would permit better determination of bedrock topography and, therefore, provide important new input to the study of the dynamics of continental ice sheets.

6. Summary

Insufficient knowledge of the Earth's global gravity field is presently the weak link in the realisation of a global integrated geodetic/geodynamic observing system, which combines the three geodetic components:

- geometry and surface deformation (GPS, differential INSAR, ocean and ice altimetry),
- the Earth rotation (VLBI, satellite and lunar laser ranging, GPS, DORIS), and
- the Earth's gravity field.

Once all three components have attained the same level of accuracy and spatial/temporal resolution, their combination will allow the monitoring and modelling of a large variety of geodynamic, ice and ocean processes as well as their interactions. Missions such as GRACE and GOCE will make this observing system complete.

Dedicated satellite gravity field missions are clearly timely and well justified in the context of the current scientific understanding, ongoing and planned international activities and the potential delta that they will provide. In particular, they will address science objectives associated with the determination of the gravity anomaly field and the geoid, the importance of which have long been emphasised in the strategic programme and in the literature (e.g. Williamstown Report, 1969; ESA, 1978, 1986, 1991; NASA, 1987; Mueller and Zerbini, 1989; Blaser et al., 1996), in resolutions by scientific unions and their associations (IUGG, IAG, IAPSO), and international programmes such as WCRP, WOCE and CLIVAR.

The extremely high accuracy of gravity field and geoid recovery that is possible with GRACE is achieved by employing the principle of very precise inter-satellite distance measurement between two low orbiting satellites; on the other hand, the extremely high spatial resolution that is possible with GOCE is achieved by employing the principle of gravity gradiometry, for the first time, in a satellite.

Acknowledgements

This is a modified version of Chapter 2 of "Gravity Field and Steady-State Ocean Circulation Mission, The Four Candidate Earth Explorer Core Missions, ESA, SP-1233(1), Noordwijk, 1999". The assistance of the other members of the GOCE mission advisory group, C. LeProvost, R. Sabadini, H. Sünkel and C.C. Tscherning is gratefully acknowledged. Thanks go also to J. Müller, H. Oberndorfer and N. Sneeuw of IAPG, Techn. Univ. München.

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