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Modelling the electric field at the seafloor due to a non-uniform ionospheric current

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Abstract

Electric fields occurring at the seafloor during geomagnetic disturbances drive geomagnetically induced currents (GIC) in submarine cables, possibly leading to problems in the operation of the equipment and systems connected to the cable. To be able to estimate the GIC risk on the cables, it is necessary to know the magnitude of the electric field at the seafloor, which can be conveniently determined from the magnetic data recorded at the surface. In this paper, we consider the theoretical models associated with non-uniform source fields existing at high latitudes. The case of a two-dimensional ionospheric current distribution is summarized together with numerical examples, and formulas for the general three-dimensional ionospheric–magnetospheric current system are derived in detail. The seawater is modelled by a highly conducting surface layer, and the basement is characterized by a surface impedance. In the 2D case, the expression for the seafloor electric field is either an inverse Fourier transform over the wave number or a spatial convolution. Corresponding formulas are also obtained in the 3D case, and both expressions are then double integrals. Three-dimensional numerical computations are therefore much more difficult, and they will be the topic of a future paper. This paper demonstrates the applicability of the method to the estimation of the impact of geomagnetic disturbances on submarine cables. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Geomagnetically induced currents (GIC); Space weather; Submarine cables; Geomagnetic disturbances; Ionospheric currents

1. Introduction

Geomagnetic variations accompanied by a geoelectric field are a manifestation of the space weather at the earth's surface. The electric field drives geomagnetically induced currents (GIC) in technological systems, like electric power transmission grids, oil and gas pipelines, telecommunication cables and railway equipment (Boteler et al., 1998). GIC are a potential source of problems to the systems. In power grids, transformers may be saturated due to GIC, resulting in harmful effects, and possibly, even in a collapse of the whole system (Kappenman, 1996). Transformers may also be permanently damaged. In pipelines, problems associated with corrosion and its control occur (Boteler, 2000). Geoelectric fields also exist at the seafloor, driving GIC in submarine cables, thus possibly leading to problems in the operation of the phone system (Lanzerotti et al., 1995).

The voltages experienced by a system and the resulting GIC can be calculated in a straightforward manner if the electric field producing them is known. Many studies have been made about the relation between electric and magnetic fields at the earth's surface, permitting the estimation of the surface geoelectric field from magnetic recordings. Regarding

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seafloor electric fields, magnetic data collected at many surface observatories may be applied to their determination, provided that the relation between the surface magnetic field and the seafloor electric field is known.

Boteler and Pirjola (submitted for publication) discuss the electric and magnetic fields in the seawater and at the seafloor. They derive relations and give numerical examples applicable to the engineering estimation of the GIC risk in submarine cables. They, however, assume that the fields are vertically propagating plane waves, which is not valid at high latitudes close to auroral electrojets (e.g. Mareschal, 1981). Pirjola et al. (2000) consider a more realistic situation by discussing a two-dimensional *E*-polarization case, which results from a line or sheet current (or any other 2D source) above a layered earth. The formula for the seafloor electric field in terms of the surface magnetic field is either an inverse Fourier transform integral over the wave number or, equivalently, a convolution integral in the spatial domain. Numerical examples discussed by Pirjola et al. (2000) contain both model calculations and an application of real magnetic data.

A single frequency is considered by Boteler and Pirjola (submitted for publication) and Pirjola et al. (2000). It is further assumed that the seawater constitutes a surface layer above a basement characterized by a (scalar) surface impedance, which is a function of the wave number. In principle, in the two-dimensional case, the basement may also have a 2D structure with the same strike direction as the source current. To make numerical computations feasible, the basement should however be assumed to be one-dimensional in practice, and the two-dimensionality due to the ionospheric source. A 1D seawater–earth model is an approximation of the real situation, and in fact, many discussions have been arising about errors in the estimation of the conductivity below oceans due to 2D and 3D structures (Heinson and Constable, 1992). This paper concentrates on the non-uniformity of the source field in connection with GIC investigations, so a 1D earth model is a reasonable and acceptable approximation. The theory of the 2D case is summarized in this paper, and numerical examples demonstrate its applicability to studies of the electric fields affecting submarine cables.

Viljanen (1997) indicates that it is not only the main two-dimensional east–west electrojet that significantly contributes to the electric field, but other currents belonging to the electrojet system also play an important role. This gives a reason for extending the seafloor electric field investigations to the case of a three-dimensional magnetospheric–ionospheric current system. The basement below the seafloor is then characterized by a surface impedance tensor, and analogous to the 2D case, the structure of the basement may, in principle, be three-dimensional but in practice, the structure should be considered as one-dimensional, and the three-dimensionality due to the source. In this paper, we go through the theory of the 3D case by deriving theoretical formulas, which couple the two horizontal electric components at the seafloor to the two horizontal magnetic components at the surface. The equations are analogous to those in the 2D case, but the inverse Fourier transforms and the convolution integrals are double, making numerical computations, to be presented in a future paper, time-consuming and more difficult to practice.

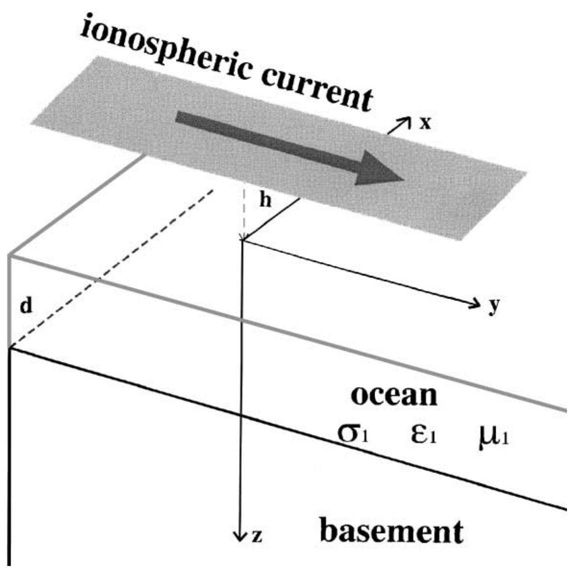


Fig. 1. The geophysical model and the coordinate system. The earth contains a top layer (seawater), whose thickness, conductivity, permittivity and permeability are denoted by d, σ_1, ε_1 and μ_1, respectively, and a seafloor basement. The x-, y- and z-axes are northward, eastward and downward. In the two-dimensional case (Section 2.1), the source is an ionospheric current distribution independent of the y coordinate and parallel to the y-axis. In the figure, the current is plotted as a sheet located at the height h above the sea surface (= the xy plane of the coordinate system). In the three-dimensional case (Section 2.2), the source is any ionospheric–magnetospheric current distribution.

2. Theory

2.1. Two-dimensional model

Throughout this paper, we use the standard coordinate system in which the earth's surface is the *xy* plane with the *x*- and *y*-axes pointing northward and eastward, respectively, and the *z*-axis is vertically downward (see Fig. 1). The ionospheric source, plotted as a sheet current at a height *h* in Fig. 1, is any two-dimensional distribution of the currents flowing parallel to the *y*-axis and independent of *y*. The time dependence, not written explicitly below, is assumed to be in the form $e^{i\omega t}$ (ω = angular frequency). The earth consists of a top seawater layer (thickness *d*, conductivity σ_1, permeability μ_1 and permittivity ε_1) and a basement characterized by a surface impedance $Z_T(b)$ (b = wave number). In principle, the structure of the basement can be two-dimensional with the *y*-axis as the strike direction. In practical computations, however, the structure should be assumed to be one-dimensional, and the two-dimensionality results from the ionospheric source.

Pirjola et al. (2000) have derived the following relations between the seafloor electric field E_y and the surface magnetic field B_x:

as an inverse Fourier transform:

$$E_y(x,d) = \int_{-\infty}^{\infty} F(b,d) B_x(b,0) e^{ibx} db \tag{1}$$

and as a convolution:

$$E_y(x,d) = \int_{-\infty}^{\infty} f(x-x',d) B_x(x',0) dx' = \int_{-\infty}^{\infty} f(x',d) B_x(x-x',0) dx' \tag{2}$$

The kernel functions $F(b,d)$ and $f(x,d)$ are defined by:

$$f(x,d) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(b,d) e^{ibx} db \tag{3}$$

$$F(b,d) = -\frac{Z(b)}{\mu_1} \frac{e^{-\kappa_1 d} + \alpha(b) e^{\kappa_1 d}}{1-\alpha(b)} \tag{4}$$

$$Z(b) = \frac{i\omega\mu_1}{\kappa_1} \tag{5}$$

$$\kappa_1 = \sqrt{b^2 - k_1^2} \tag{6}$$

$$k_1 = \sqrt{\omega^2 \mu_1 \varepsilon_1 - i\omega\mu_1\sigma_1} \tag{7}$$

$$\alpha(b) = \frac{Z_T(b) - Z(b)}{Z_T(b) + Z(b)} e^{-2\kappa_1 d} \tag{8}$$

The Fourier and their inverse transforms of the field components are given by:

$$E_y(b,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_y(x,z)\mathrm{e}^{-ibx}\mathrm{d}x \tag{9}$$

$$E_y(x,z) = \int_{-\infty}^{\infty} E_y(b,z)\mathrm{e}^{ibx}\mathrm{d}b \tag{10}$$

$$B_x(b,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} B_x(x,z)\mathrm{e}^{-ibx}\mathrm{d}x \tag{11}$$

$$B_x(x,z) = \int_{-\infty}^{\infty} B_x(b,z)\mathrm{e}^{ibx}\mathrm{d}b \tag{12}$$

It should be noted that the functions $F(b,d)$ and $f(x,d)$ only depend on the properties of the seawater and the basement, i.e. not on the ionospheric source. Consequently, the functions need to be calculated just once for a given earth structure, and they are applicable to any 2D event. It is mostly a matter of taste whether Eq. (1) or (2) is more usable for the estimation of seafloor electric fields. The function $F(b,d)$ is easier to be calculated than $f(x,d)$ favouring Eq. (1). However, using Eq. (2) avoids the spatial Fourier transform of the magnetic data, and as stated, once calculated, $f(x,d)$ is valid for all events.

The surface magnetic field values used in Eqs. (1) and (2) are the data measured at the sea surface. However, such recordings are seldom available and the magnetic data are usually collected at land observatories. Therefore, as explained by Pirjola et al. (2000), a conversion of the magnetic data from land to the sea surface has to be made first. By assuming that the sea area considered and the land area from which the data are available can be described by surface impedances, the conversion can be included in the formulas in a straightforward manner. Such a surface impedance concept becomes problematic if the observatory is located at a site with an anomalous induction like if it is near a coastline. Therefore, data from inland observatories would be more desirable to be used. However, observing that the models are approximate anyway and avoiding unnecessary complications, we neglect the conversion of land magnetic data to sea surface data in this paper. It should be noted that when using the plane wave model (Boteler and Pirjola, submitted for publication), a land-to-sea conversion is not needed because, independent of the exact conductivity values, the horizontal component of the magnetic field at the surface is twice the same component of the primary field, implying that land and sea surface data are the same.

2.2. Three-dimensional model

The geophysical model is similar to that in the 2D case (Fig. 1). The source current, plotted as an ionospheric sheet in Fig. 1, can be any three-dimensional ionospheric–magnetospheric distribution. The surface impedance Z_T characterizing the basement is a tensor.

$$Z_\mathrm{T} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \tag{13}$$

The surface impedance depends, besides the electrical properties of the basement and the frequency considered, also on the two wave numbers b and q associated with the Fourier transforms of the fields with respect to the x and y coordinates. Analogous to the 2D case, the structure of the basement can, in principle, be three-dimensional but in practical computations, the structure should be assumed to be one-dimensional, and the 3D character is due to the ionospheric–magnetospheric source (Wait, 1981, pp. 44–55). As in Section 2.1, the time dependence term $\mathrm{e}^{i\omega t}$ is not written explicitly.

The Fourier transforms (with respect to x and y) of the horizontal components of the electric and magnetic fields have the following expressions in the ocean (see Pirjola, 1982, p. 83):

$$E_x = -\frac{q}{\eta_1^2} b (D_1 e^{\kappa_1 z} + G_1 e^{-\kappa_1 z}) + \frac{i\omega}{\eta_1^2} \kappa_1 (Q_1 e^{\kappa_1 z} - R_1 e^{-\kappa_1 z}) \tag{14}$$

$$E_y = D_1 e^{\kappa_1 z} + G_1 e^{-\kappa_1 z} \tag{15}$$

$$B_x = -\frac{\mu_1(\sigma_1 + i\omega\varepsilon_1)}{\eta_1^2} \kappa_1 (D_1 e^{\kappa_1 z} - G_1 e^{-\kappa_1 z}) - \frac{q}{\eta_1^2} b (Q_1 e^{\kappa_1 z} + R_1 e^{-\kappa_1 z}) \tag{16}$$

$$B_y = Q_1 e^{\kappa_1 z} + R_1 e^{-\kappa_1 z} \tag{17}$$

where D_1, G_1, Q_1 and R_1 are (integration constant) functions of the wave numbers b and q. The parameters η_1 and κ_1 are defined by:

$$\eta_1 = \sqrt{k_1^2 - q^2} \tag{18}$$

$$\kappa_1 = \sqrt{b^2 + q^2 - k_1^2} = \sqrt{b^2 - \eta_1^2} \tag{19}$$

where k_1 is the propagation constant of the seawater layer defined by Eq. (7). As expected, Eq. (19) reduces to Eq. (6) when $q = 0$. It should be noted that the sign of q has been changed in Eqs. (14)–(17) as compared to the convention used by Pirjola (1982), which was done to make the x and y Fourier transforms similar, i.e. minus signs in the exponents of the forward and plus signs in the inverse transforms.

Analogous to the two-dimensional case, we now want to determine the transfer functions $f_1(b,q)$, $f_2(b,q)$, $g_1(b,q)$ and $g_2(b,q)$ which give the seafloor electric field $E_{x,y}(b,q,d)$ in terms of the surface magnetic field $B_{x,y}(b,q,0)$ as follows:

$$E_x(b,q,d) = f_1(b,q) B_x(b,q,0) + f_2(b,q) B_y(b,q,0) \tag{20}$$

$$E_y(b,q,d) = g_1(b,q) B_x(b,q,0) + g_2(b,q) B_y(b,q,0) \tag{21}$$

Strictly speaking, the B_x and B_y components at $z = 0$ discussed in this connection are the values at $z = 0+$, i.e. in the seawater just below the ocean surface. Assuming (which is reasonable in practice) that μ_1 equals the permeability of the air ($= \mu_0$), B_x and B_y are continuous across the surface. However, in the theoretical case $\mu_1 \neq \mu_0$, the B_x and B_y values at $z = 0+$ must be multiplied by μ_0/μ_1 to obtain B_x and B_y at $z = 0-$, i.e. on the air side of the surface.

The surface impedance tensor in Eq. (13) expresses the relation between the electric and magnetic fields at the seafloor:

$$E_x(b,q,d) = Z_{xx} \frac{B_x(b,q,d)}{\mu_1} + Z_{xy} \frac{B_y(b,q,d)}{\mu_1} \tag{22}$$

$$E_y(b,q,d) = Z_{yx} \frac{B_x(b,q,d)}{\mu_1} + Z_{yy} \frac{B_y(b,q,d)}{\mu_1} \tag{23}$$

Substituting expressions (14)–(17) into Eqs. (22) and (23) permits us to solve the coefficients D_1 and Q_1 in terms of G_1 and R_1. After tedious algebraic calculations, the final result is:

$$D_1 = \beta_1 G_1 + \beta_2 R_1 \tag{24}$$

$$Q_1 = \gamma_1 G_1 + \gamma_2 R_1 \tag{25}$$

where

$$\beta_1 = \frac{e^{-2\kappa_1 d}}{A}(\mu_1\alpha_3 + Z_{yx}(\mu_1\alpha_1^2 - \alpha_2\alpha_3) - \alpha_2(Z_{xx}Z_{yy} - Z_{xy}Z_{yx}) - \mu_1\alpha_1(Z_{xx} - Z_{yy}) - \mu_1 Z_{xy}) \tag{26}$$

$$\beta_2 = -\frac{2\alpha_3 e^{-2\kappa_1 d}}{A}(Z_{yy} + \alpha_1 Z_{yx}) \tag{27}$$

$$\gamma_1 = \frac{2\mu_1\alpha_2 e^{-2\kappa_1 d}}{A}(-Z_{xx} + \alpha_1 Z_{yx}) \tag{28}$$

$$\gamma_2 = \frac{e^{-2\kappa_1 d}}{A}(-\mu_1\alpha_3 + Z_{yx}(\mu_1\alpha_1^2 - \alpha_2\alpha_3) + \alpha_2(Z_{xx}Z_{yy} - Z_{xy}Z_{yx}) - \mu_1\alpha_1(Z_{xx} - Z_{yy}) - \mu_1 Z_{xy}) \tag{29}$$

$$\alpha_1 = -\frac{bq}{\eta_1^2} \tag{30}$$

$$\alpha_2 = \frac{\mu_1(\sigma_1 + i\omega\varepsilon_1)\kappa_1}{\eta_1^2} \tag{31}$$

$$\alpha_3 = \frac{i\omega\mu_1\kappa_1}{\eta_1^2} \tag{32}$$

$$A = -\mu_1\alpha_3 - Z_{yx}(\mu_1\alpha_1^2 + \alpha_2\alpha_3) - \alpha_2(Z_{xx}Z_{yy} - Z_{xy}Z_{yx}) + \mu_1\alpha_1(Z_{xx} - Z_{yy}) + \mu_1 Z_{xy} \tag{33}$$

Wait (1981, p. 48) indicates that for a layered structure, $Z_{xx}=-Z_{yy}$, which would permit simplifications in the equations above.

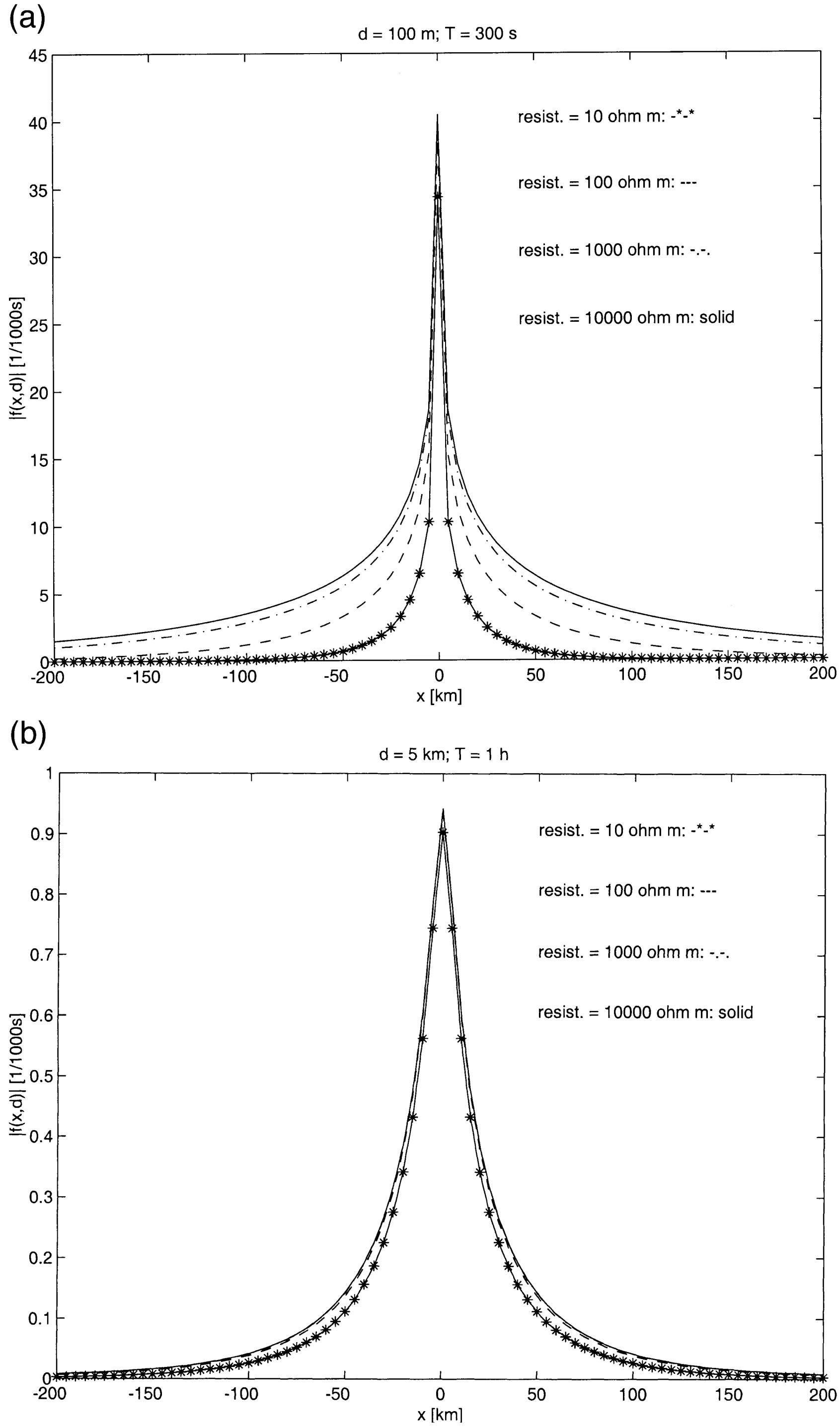
Denoting

$$\beta_3 = \beta_1 e^{2\kappa_1 d}, \quad \beta_4 = \beta_2 e^{2\kappa_1 d}, \quad \gamma_3 = \gamma_1 e^{2\kappa_1 d}, \quad \gamma_4 = \gamma_2 e^{2\kappa_1 d} \tag{34}$$

and substituting Eqs. (24)–(33) into formulas (14)–(17), Eqs. (20) and (21) can be written as:

$$\begin{aligned}&\left(\alpha_1(1+\beta_3) + \frac{\alpha_3\gamma_3}{\mu_1}\right)G_1 e^{-\kappa_1 d} + \left(\alpha_1\beta_4 - \frac{\alpha_3}{\mu_1}(1-\gamma_4)\right)R_1 e^{-\kappa_1 d} \\ &\quad = f_1((\alpha_2(1-\beta_1) + \alpha_1\gamma_1)G_1 + (-\alpha_2\beta_2 + \alpha_1(1+\gamma_2))R_1) + f_2(\gamma_1 G_1 + (1+\gamma_2)R_1)\end{aligned} \tag{35}$$

Fig. 2. Absolute value of the kernel function $f(x,d)$ appearing in the convolution in Eq. (2) as a function of the x coordinate. The resistivity of the seawater is 0.25 Ω m. Four different resistivities (10, 100, 1000 and 10 000 Ω m) of the uniform basement are considered. In (a) and (b), the seawater depth and the period are 100 m, 5 min and 5 km, 1 h, respectively.


(a)
d = 100 m; T = 300 s
resist. = 10 ohm m: -*-*
resist. = 100 ohm m: ---
resist. = 1000 ohm m: -.-.
resist. = 10000 ohm m: solid
|f(x,d)| [1/1000s]
x [km]
(b)
d = 5 km; T = 1 h
resist. = 10 ohm m: -*-*
resist. = 100 ohm m: ---
resist. = 1000 ohm m: -.-.
resist. = 10000 ohm m: solid
|f(x,d)| [1/1000s]
x [km]

$$(1+\beta_3)G_1 \mathrm{e}^{-\kappa_1 d} + \beta_4 R_1 \mathrm{e}^{-\kappa_1 d} = g_1((\alpha_2(1-\beta_1)+\alpha_1\gamma_1)G_1 + (-\alpha_2\beta_2+\alpha_1(1+\gamma_2))R_1) + g_2(\gamma_1 G_1 + (1+\gamma_2)R_1) \tag{36}$$

The transfer functions $f_1(b,q)$, $f_2(b,q)$, $g_1(b,q)$ and $g_2(b,q)$ are now solved from Eqs. (35) and (36) by noting that G_1 and R_1 are independent of each other so that their coefficients can be set to be equal on both sides of the equations. The final results are:

$$f_1 = \mathrm{e}^{-\kappa_1 d}\frac{\left(\alpha_1(1+\beta_3)+\frac{\alpha_3\gamma_3}{\mu_1}\right)(1+\gamma_2)-\gamma_1\left(\alpha_1\beta_4-\frac{\alpha_3}{\mu_1}(1-\gamma_4)\right)}{\alpha_2((1-\beta_1)(1+\gamma_2)+\beta_2\gamma_1)} \tag{37}$$

$$f_2 = \mathrm{e}^{-\kappa_1 d}\frac{(\alpha_2(1-\beta_1)+\alpha_1\gamma_1)\left(\alpha_1\beta_4-\frac{\alpha_3}{\mu_1}(1-\gamma_4)\right)+\left(\alpha_2\beta_2-\alpha_1(1+\gamma_2)\right)\left(\alpha_1(1+\beta_3)+\frac{\alpha_3\gamma_3}{\mu_1}\right)}{\alpha_2((1-\beta_1)(1+\gamma_2)+\beta_2\gamma_1)} \tag{38}$$

$$g_1 = \mathrm{e}^{-\kappa_1 d}\frac{(1+\beta_3)(1+\gamma_2)-\gamma_1\beta_4}{\alpha_2((1-\beta_1)(1+\gamma_2)+\beta_2\gamma_1)} \tag{39}$$

$$g_2 = \mathrm{e}^{-\kappa_1 d}\frac{(\alpha_2(1-\beta_1)+\alpha_1\gamma_1)\beta_4+(\alpha_2\beta_2-\alpha_1(1+\gamma_2))(1+\beta_3)}{\alpha_2((1-\beta_1)(1+\gamma_2)+\beta_2\gamma_1)} \tag{40}$$

To check the formulas derived, let q equal zero. The diagonal terms of the impedance tensor in Eq. (13) are zero (Wait, 1981, p. 48). Eqs. (37) and (40) then reduce to $f_1 = g_2 = 0$. This is an expectable result since the assumption $q = 0$ removes the y dependence of the fields, and thus, also the coupling between E_x and B_x and between E_y and B_y. Eqs. (38) and (39) yield:

$$f_2 = -\mathrm{e}^{-\kappa_1 d}\frac{\alpha_3(1-\gamma_4)}{\mu_1(1+\gamma_2)} \tag{41}$$

$$g_1 = \mathrm{e}^{-\kappa_1 d}\frac{(1+\beta_3)}{\alpha_2(1-\beta_1)} \tag{42}$$

Following the notations in Section 2.1, i.e. $-Z_{yx} = Z_T(b)$, $i\omega\mu_1/\kappa_1 = Z(b)$ and $\exp(-2\kappa_1 d)(Z_T(b)-Z(b))/(Z_T(b)+Z(b)) = \alpha(b)$, Eq. (42) reduces to:

$$g_1(b,0) = -\frac{Z(b)}{\mu_1}\frac{\mathrm{e}^{-\kappa_1 d}+\alpha(b)\mathrm{e}^{\kappa_1 d}}{1-\alpha(b)} \tag{43}$$

which is exactly the same as Eq. (4).

The electric and magnetic components have been considered in the $b-q$ wave number domain so far. Eqs. (20) and (21) can be inverse Fourier-transformed into the $x-y$ space domain:

$$E_x(x,y,d) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(b,q)B_x(b,q,0)\mathrm{e}^{ibx}\mathrm{e}^{iqy}\mathrm{d}b\mathrm{d}q + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2(b,q)B_y(b,q,0)\mathrm{e}^{ibx}\mathrm{e}^{iqy}\mathrm{d}b\mathrm{d}q \tag{44}$$

$$E_y(x,y,d) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(b,q)B_x(b,q,0)\mathrm{e}^{ibx}\mathrm{e}^{iqy}\mathrm{d}b\mathrm{d}q + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_2(b,q)B_y(b,q,0)\mathrm{e}^{ibx}\mathrm{e}^{iqy}\mathrm{d}b\mathrm{d}q \tag{45}$$

Note that Eqs. (44) and (45) imply that the inverse Fourier transforms of the field components from b to x and from q to y do not contain any coefficient related to 2π, so the corresponding forward transforms should have $1/2\pi$ (cf.

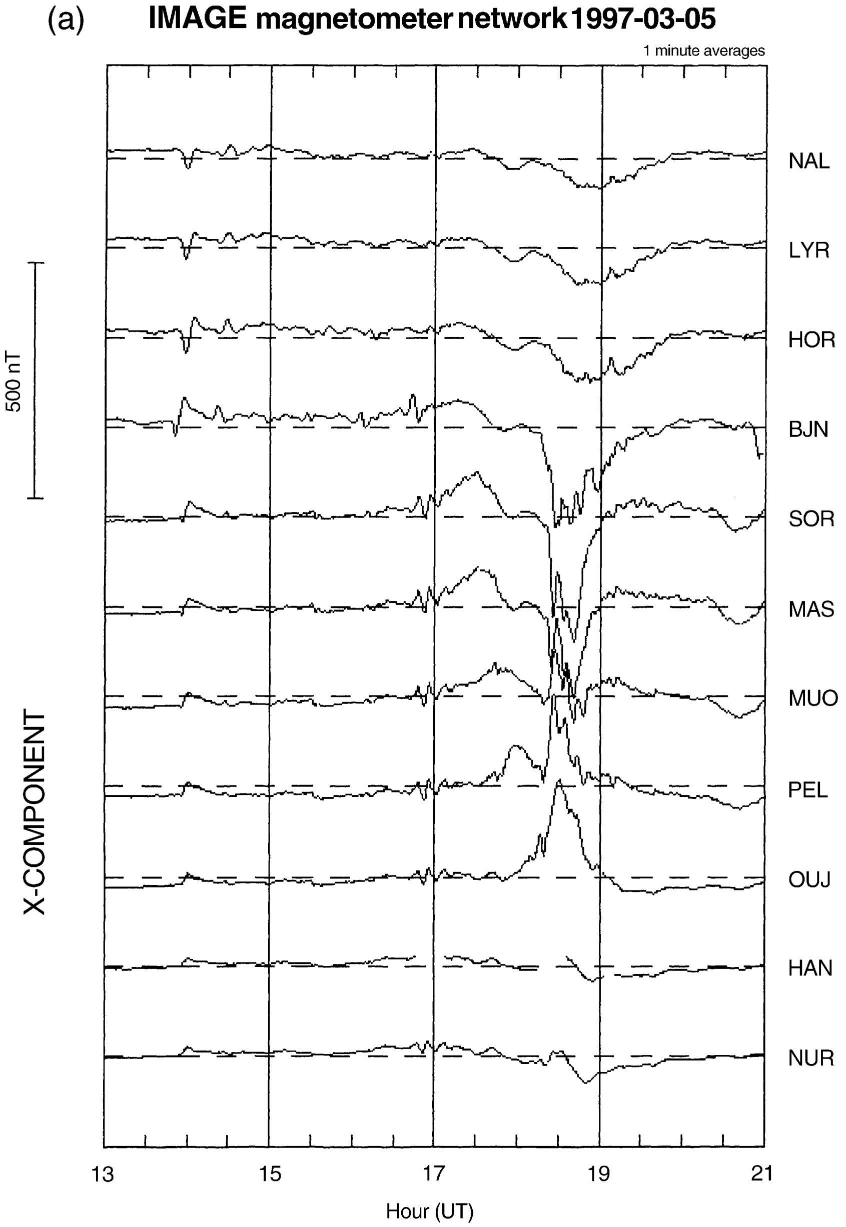


Fig. 3. (a) Magnetic north component (B_x=X) recorded at 11 IMAGE stations (Viljanen and Häkkinen, 1997) during a magnetic disturbance event on March 5, 1997 at 13.00–21.00 UT. The x coordinates of the stations from south to north are −712, −509, −260, 0, 124, 284, 405, 852, 1141, 1273 and 1368 km (with an arbitrarily chosen location of the origin at PEL). (b) Magnetic east component ($B_y = Y$) corresponding to (a). (c) Absolute value, real part and imaginary part of the magnetic component B_x at the PEL station (at $x = 0$) as functions of the period calculated by Fourier-transforming the data shown in (a). (d) Absolute value, real part and imaginary part of the electric component E_y at the seafloor as calculated applying Eq. (2) in which B_x is obtained by Fourier transforming the data presented in (a). E_y, Re(E_y) and Im(E_y) are shown at $x = 0$ as functions of the period. The resistivity and depth of the seawater are 0.25 Ω m and 100 m, respectively. The earth below the sea has a six-layer structure with thicknesses and resistivities 3, 6, 5, 7, 23 and ∞ km and 5000, 500, 100, 10, 20 and 1000 Ω m.

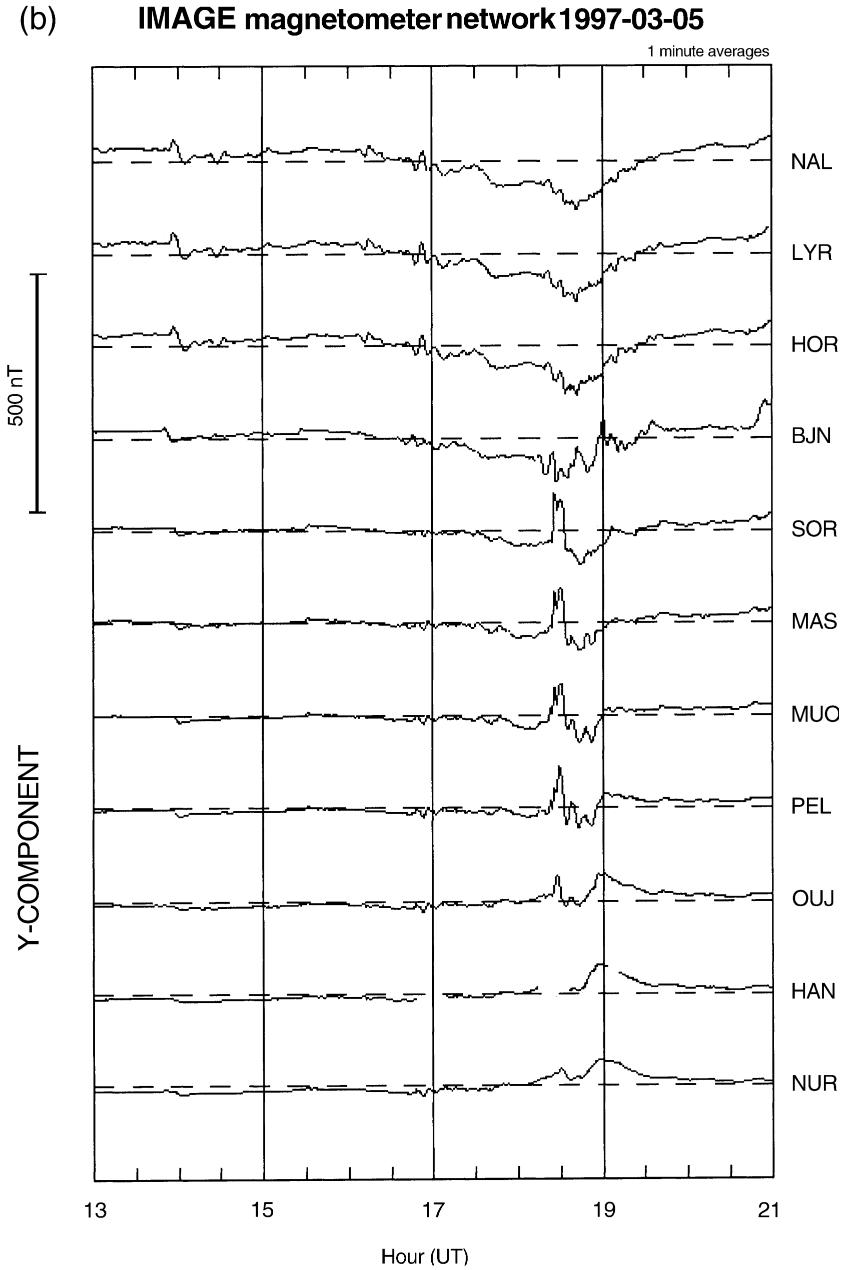
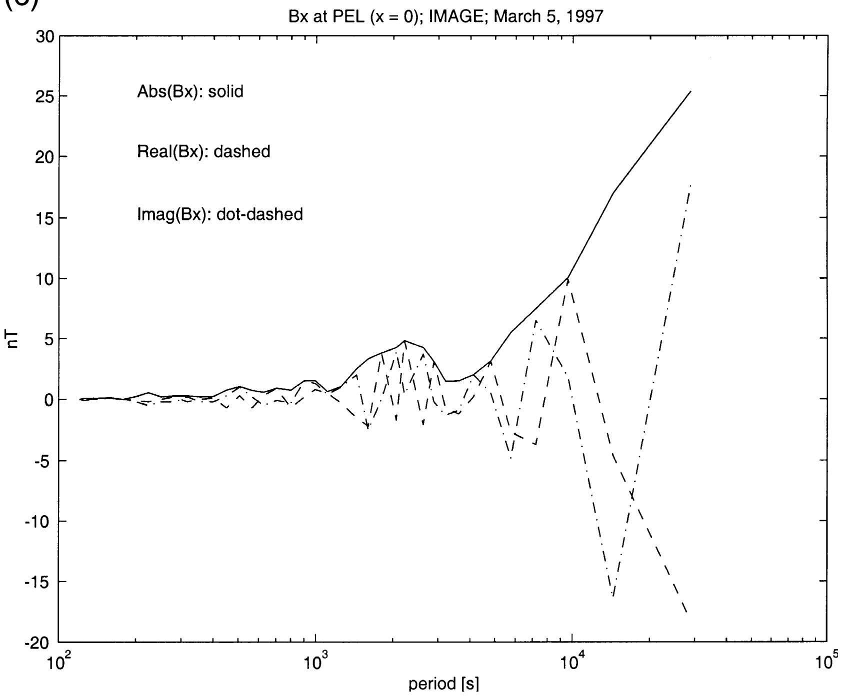


Fig. 3 (*continued*).

Eqs. (9) and (11)). The convolution theorem (Arfken, 1985, p. 811) allows for writing Eqs. (44) and (45) in terms of the surface magnetic field expressed in the $x-y$ domain:

$$E_x(x,y,d) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(x',y')B_x(x-x',y-y',0)\mathrm{d}x'\,\mathrm{d}y' + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2(x',y')B_y(x-x',y-y',0)\mathrm{d}x'\,\mathrm{d}y' \quad (46)$$

(c)



(d)

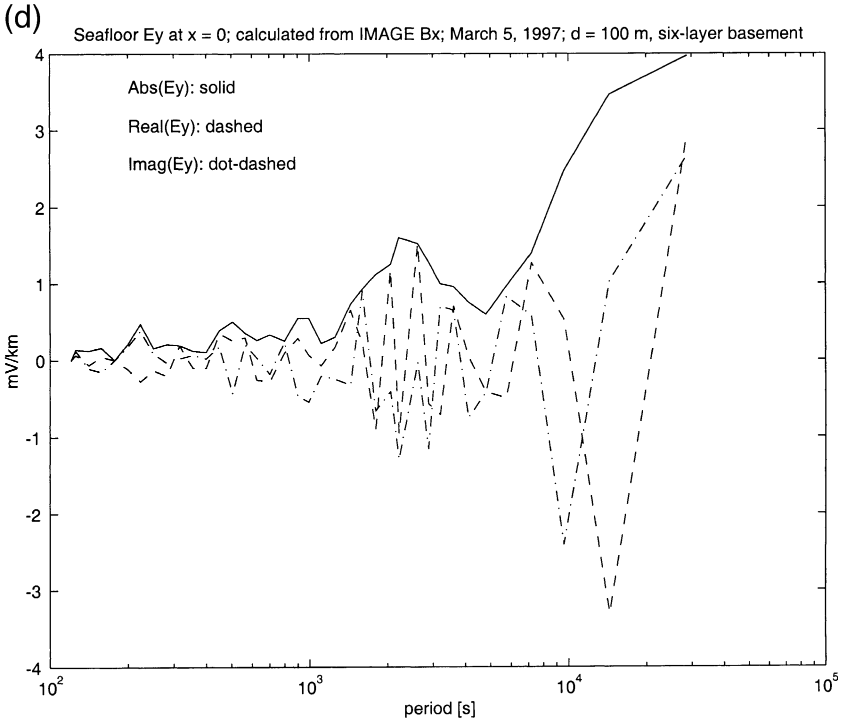


Fig. 3 (*continued*).

$$E_y(x,y,d) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(x',y')B_x(x-x',y-y',0)\mathrm{d}x'\,\mathrm{d}y' + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_2(x',y')B_y(x-x',y-y',0)\mathrm{d}x'\,\mathrm{d}y' \tag{47}$$

The inverse Fourier transforms of $f_1(b,q)$, $f_2(b,q)$, $g_1(b,q)$ and $g_2(b,q)$ are defined as:

$$u(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(b,q)\mathrm{e}^{ibx}\mathrm{e}^{iqy}\mathrm{d}b\mathrm{d}q \tag{48}$$

where $u = f_1$, f_2, g_1, g_2. (Note the coefficient $1/(2\pi)^2$ in Eq. (48), cf. Eq. (3).) As what is known from the convolution theorem, Eqs. (46) and (47) can be written in different forms by interchanging the positions of x' and $x-x'$ or y' and $y-y'$ in the integrals.

Eqs. (44) and (45) or (46) and (47) now permit the calculation of the seafloor electric field, provided that the surface magnetic data are available (at a given angular frequency ω). The data are usually given as functions of x and y rather than in the wave number domain, so Eqs. (46) and (47) seem more practical. Their application, however, requires the computation of the inverse Fourier transforms in Eq. (48) first. It should be noted that the transfer functions $f_1(b,q)$, $f_2(b,q)$, $g_1(b,q)$ and $g_2(b,q)$ expressed by Eqs. (37)–(40) as well as the convolution kernels $f_1(x,y)$, $f_2(x,y)$, $g_1(x,y)$ and $g_2(x,y)$ appearing in Eqs. (46) and (47) only depend on the properties of the seawater and basement, and thus, not on the ionospheric source current. This means that we only have to compute these functions once for a given earth structure, and they are available for all geomagnetic disturbance events.

Since the diagonal elements Z_{xx} and Z_{yy} are odd and the off-diagonal elements Z_{xy} and Z_{yx} are even with respect to b and q, it can be concluded from the equations above that α_1, β_2, β_4, γ_1 and γ_3 are odd while α_2, α_3, β_1, β_3, γ_2 and γ_4 are even. Further, it may be seen that $f_1(b,q)$ and $g_2(b,q)$ are odd and $f_2(b,q)$ and $g_1(b,q)$ are even. This implies that $f_1(x,y)$ and $g_2(x,y)$ are odd, and $f_2(x,y)$ and $g_1(x,y)$ are even in x and y.

Similar to the 2D case, the magnetic data used in Eqs. (44)–(47) refer to the field at the sea surface. Therefore, when using land observatory data, a conversion from land to the sea surface should be made first, and obviously, a procedure analogous to that presented by Pirjola et al. (2000) may be applied in the 3D case, too.

We now only consider the seafloor electric field in terms of the surface magnetic field, but the analogous relations can also be derived between the seafloor and surface magnetic fields or between the seafloor and surface electric fields.

3. Numerical results

We now discuss the two-dimensional case numerically. As pointed out above, the numerical calculations associated with the three-dimensional formulation are difficult and consume a lot of computer time, and they are therefore a topic of another paper.

It is demonstrated by Pirjola et al. (2000) that an increase of the frequency considered makes the kernel function $f(x,d)$ appearing in the convolutions of Eq. (2) more peaked around $x = 0$. In the extreme case, $f(x,d)$ becomes a delta function, implying that $E_y(x,d)$ at a given point x at the seafloor only depends on $B_x(x,0)$ at the same value of x at the sea surface. This is similar to the relation obtained in the plane wave case, and the conclusion is in agreement with the well-known fact that source effects distorting the plane wave formulation in electromagnetic induction studies decrease with an increasing frequency.

Following Pirjola et al. (2000), let us consider a continental shelf ($d = 100$ m) and a deep ocean ($d = 5$ km). In this paper, the conductivity, permeability and permittivity of the seawater are $\sigma_1 = 4\ \Omega^{-1}\ \mathrm{m}^{-1}$, $\mu_1 = \mu_0$ and $\varepsilon_1 = 80\varepsilon_0$. Real resistivity values below oceans are still partly unknown (Heinson and Constable, 1992). We now therefore investigate the effect of the resistivity on $f(x,d)$ by considering uniform basements of 10, 100, 1000 and 10 000 Ω m. Fig. 2a shows $f(x,d)$ for the period $T = 300$ s and for $d = 100$ m. For $d = 5$ km, $T = 300$ s is too small for a period to get any difference between the curves corresponding to the four resistivities. Therefore, Fig. 2b refers to

$T = 3600$ s and $d = 5$ km. Fig. 2 indicates that the effect of the basement resistivity on the kernel function is not very dramatic. This is particularly true when going from 1000 to 10 000 Ω m. A decrease of the resistivity makes $f(x,d)$ more peaked, which is again in agreement with the known facts about the source distortion in electromagnetic induction studies.

Fig. 3a and b depicts the north ($B_x = X$) and east ($B_y = Y$) components of the magnetic field recorded by the IMAGE magnetometers operating in northern Europe and Svalbard (Viljanen and Häkkinen, 1997). The 11 stations denoted by NUR, HAN, OUJ, PEL, MUO, MAS, SOR, BJN, HOR, LYR, NAL roughly constitute a chain parallel to the x-axis, and the corresponding x coordinates are −712, −509, −260, 0, 124, 284, 405, 852, 1141, 1273 and 1368 km (with the origin $x = 0$ set arbitrarily at PEL). The event shown in the figures occurred on March 5, 1997. It is seen that Y variations are smaller than X variations but not negligible. Consequently, the criterion of two-dimensionality is only very roughly satisfied. Anyway, the event can be used for demonstration purposes of the techniques of determining seafloor electric fields from surface magnetic data.

The time variations of B_x presented in Fig. 3a can be Fourier-transformed, and Fig. 3c shows the absolute value and the real and imaginary parts of the Fourier transform as functions of the period at $x = 0$.

Fig. 2a and b shows that the kernel function $f(x,d)$ is not very sensitive to the resistivity of the basement. Therefore, the somewhat arbitrary assumption that the basement consists of six layers with layer thicknesses and resistivities equal to 3, 6, 5, 7, 23 and ∞ km and 5000, 500, 100, 10, 20 and 1000 Ωm is not critical for the results obtained for the seafloor electric field. In fact, the structure chosen corresponds to a model of the resistivity in southern Finland (Viljanen and Pirjola, 1994). We set d to be equal to 100 m.

The function $f(x,d)$ can now be calculated, and substituting it together with the B_x data Fourier-transformed from the time to the frequency domain into Eq. (2) gives us the electric field E_y at the seafloor. Fig. 3d depicts the absolute value and the real and imaginary parts of E_y as functions of the period at $x = 0$. The electric field values in the order of a few millivolts per kilometer are quite reasonable, resulting in voltages of some volts over oceanic distances. However, the event discussed does not at all belong to the largest category, and the important result of this example is the demonstration of the applicability of the method.

4. Summary and concluding remarks

Investigations of ground effects of space weather, i.e. geomagnetically induced currents (GIC) and voltages in technological systems, such as power grids and telecommunication cables, are an important practical application of electromagnetic induction studies. GIC are driven by the electric field induced by a temporal variation of the magnetic field. Since magnetic data are usually available, a theoretical calculation of GIC is possible, provided that the dependence of the electric field on the magnetic field is known. Relations between the two fields at the earth's surface have been thoroughly studied for several decades. It is particularly important to note that in GIC estimations, the electric field is integrated along a system having dimensions of typically several hundreds or thousands of kilometers so small-scale inhomogeneities locally affecting the electric field need not be taken into account.

Oceanic submarine cables constitute a special category of conductors influenced by space weather as they often extend over very large distances, and they are affected by the electric field induced at the seafloor, not by the surface field. Estimating GIC based on surface magnetic recordings thus requires a relation between the surface magnetic and seafloor electric fields. As always in electromagnetic induction studies of the earth, complexities arise when a cable located at a high latitude is investigated since the source field is usually non-uniform. It is shown by Pirjola et al. (2000) that for a two-dimensional source above the earth characterized by a surface impedance, the relation between the seafloor electric field and the surface magnetic field is a spatial convolution integral. Its applicability to determining the electric field from magnetic data is demonstrated by a numerical example in this paper.

The most important conclusion of this paper is that convolution relations can also be derived between the seafloor electric and surface magnetic fields in the situation of a general three-dimensional ionospheric–magnetospheric current distribution. In this case, however, the convolutions are double integrals, making fast and accurate numerical computations more difficult. They are planned to be the topic of a future paper.

Voltages induced in oceanic cables have been measured for a long time, and this paper provides techniques applicable to their theoretical calculations. Future comparisons between measured and computed data will verify the validity of the techniques, and possibly, also yield new information about the geophysical parameters involved, like the conductivity structure of the basement. A theoretical calculation of the seafloor electric field and of the resulting GIC and voltages in a submarine cable can be applied in evaluating space weather risks on systems connected to the cable.

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