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Conditioning Channel Meanders to Well Observations[1]

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Assessment of uncertainty in the performance of fluvial reservoirs often requires the ability to generate realizations of channel sands that are conditional to well observations. For channels with low sinuosity this problem has been effectively solved. When the sinuosity is large, however, the standard stochastic models for fluvial reservoirs are not valid, because the deviation of the channel from a principal direction line is multivalued. In this paper, I show how the method of randomized maximum likelihood can be used to generate conditional realizations of channels with large sinuosity. In one example, a Gaussian random field model is used to generate an unconditional realization of a channel with large sinuosity, and this realization is then conditioned to well observations. Channels generated in the second approach are less realistic, but may be sufficient for modeling reservoir connectivity in a realistic way. In the second example, an unconditional realization of a channel is generated by a complex geologic model with random forcing. It is then adjusted in a meaningful way to honor well observations. The key feature in the solution is the use of channel direction instead of channel deviation as the characteristic random function describing the geometry of the channel.

KEY WORDS: conditional simulation, geologic model, fluvial model, stochastic model, uncertainty.

INTRODUCTION

In many parts of the world, oil is found in reservoirs whose porosity and permeability are largely controlled by the location of paleochannels, and associated facies. These facies are often deposited in a nearly impermeable background, and so connectivity of reservoir between wells and the ability to recover oil is largely determined by the geometry and location of the channel sands. These sands may be observed in only a few locations, but accurate modeling of flow requires the generation of reservoir models that honor the well observations at wells, and are plausible between wells. Because the locations of the channel cannot be observed between wells and because many plausible channels could honor the observations,

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it is usually desirable to generate many realizations, all of which are conditional to the well observations.

Stochastic models for channel geometry range from complex mechanistic models devised by sedimentologists, to purely geostatistical Boolean and indicator models. Typically, the location of a channel center in a Boolean model is described by a principal direction line whose angle and intercept are random variables, and by random variables that describe the deviation of the channel center from the principal direction line. Examples of this approach to modeling channel geometry can be found in the papers by Georgsen and Omre (1993), Georgsen and others (1994), and Deutsch and Wang (1996).

It is important for assessment of uncertainty in reservoir performance that channel realizations be conditioned to well observations. To solve the complete conditioning problem, one must address the three-dimensional aspects and problems associated with the presence of multiple channels. In this paper, I address only the conditioning of a single channel, but one might look at Georgsen and others (1994) to see how this algorithm could be used as part of a more complete package. One efficient method for conditioning the centerline location of a single channel of the type described in the previous paragraph is to use sequential Gaussian simulation to generate a 1D random field of channel deviations along the principal direction line, conditional to known deviations at the well locations (Shmaryan and Deutsch, 1999; Viseur, Shtuka, and Mallet, 1998). Holden and others (1998) and Skorstad, Hauge, and Holden (1999) describe a method that uses the Metropolis–Hastings algorithm for conditioning. In this approach, conditioning points are first drawn near the wells, and then the location of the channel at other locations is found by drawing Gaussian random fields for channel deviations conditioned on the known locations. The Metropolis–Hastings algorithm is used to decide whether or not to accept the proposed reservoir. In conditioning channel geometry to pressure data, Bi, Oliver, and Reynolds (2000) also made use of the fact that the deviation of the channel and width and thickness were single-valued functions of distance along the principal direction line.

The problem with channel simulation approaches that are based on a representation of the channel variables as single valued functions along a line in space is that they are unable to represent channel meanders of high sinuosity as shown in Figure 1. This shape could not be represented as a random deviation from a principal direction line.

In this paper, I will show how the method of randomized maximum likelihood can be used to generate conditional realizations of channels with large sinuosity. In one example, the original (unconditional) realization of a channel is generated by a complex geologic model with random forcing. It is then adjusted in a meaningful way to honor well observations. In the second example, I demonstrate the possibility of generating channels with large sinuosity from a gaussian random field model, and conditioning these to well observations. Channels generated in

Figure 1. Highly sinuous channel meanders (Howard, 1996) for which the standard stochastic fluvial models are not appropriate. Copyright John Wiley & Sons Limited. Reproduced with permission.

the second approach are less realistic, but may be sufficient for modeling reservoir connectivity in a realistic way.

MEANDER GEOMETRY

Langbein and Leopold (1966) proposed that the geometry of a channel meander might be modeled as a random walk in which the change in channel direction

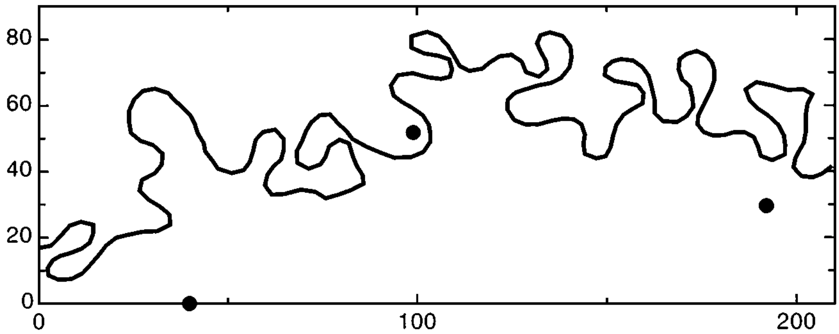

Figure 2. Numerical simulation of a meandering channel (Howard, 1996). Black dots indicate the locations to be used as well observations.

in each increment of length is a normally distributed random variable with zero mean. Then, if the channel is constrained to pass through two locations and the length of the channel is fixed, the most probable shape for the channel is described by $\theta(s) = \sin(s)$, where s is the distance along the channel and θ is the channel direction at that point. Langbein and Leopold (1966) cited a number of channels for which this seemed to be an appropriate mathematical model.

Others have noted that while individual bends in a freely meandering river may be modeled by a sine-generated curve, the actual geometry over several wavelengths is far too complex to be described by such a simple model (Furbish, 1991). In general, authors who wish to generate realistic meanders resort to geologic modeling in which sediment transport, erosion, channel cutoff, and crevasse and levee formation are based on statistically sampled discharge and duration of annual floods. Examples of this type of simulation can be found in Sun and others (1996) or Gross and Small (1998). The meanders in Figure 2 are the result of a simulation by Howard (1996).

The difficulty with this type of modeling is in obtaining a realization that is conditional to well observation. The computational expense of generating a single unconditional realization is simply too great to permit conditioning to be done in a trial-and-error manner, and the stochastic forcing does not seem to adapt easily to a Markov chain Monte Carlo approach. In general terms, the choice for stochastic models of channel meanders seems to be between simple models that can be conditioned to well observations but are not plausible, or complex models that cannot be conditioned. In this paper, I present a method for conditioning realizations from complex stochastic models using the method of randomized maximum likelihood (Oliver, He, and Reynolds, 1996).

The method of randomized maximum likelihood is an approximate method for generating conditional realizations. Qualitatively, the method consists of the following steps: (1) generation of an unconditional realization of the model variables, (2) generation of a realization of the data from $N[d_{obs}, \sigma_D]$, (3) generation of a set of model variables that is as close as possible to the model generated in Step 1 and for which the observations would be as close as possible to those generated in Step 2, and (4) a decision as to whether or not the realization generated is acceptable or not. Although the criterion for this decision could be made rigorous as described in Oliver, He, and Reynolds (1996), an ad hoc acceptance criterion in which realizations that do not adequately match the data are rejected is usually employed. For Gaussian models the choice of a measure of closeness is usually clear, but for non-Gaussian models reparameterization of the model is sometimes required. The realizations will all be plausible and all will honor the data; the approximation is that by using a simplified accept/reject criterion in Step 4 the ensemble of realizations may not be a good representation of the pdf of conditional channel realizations. If the prior for the model variables is Gaussian, and if the data are linearly related to the model variables, the method of randomized maximum likelihood is known to sample correctly by accepting all proposed realizations. If either of the conditions is not met, the sampling is only approximate, although Oliver, He, and Reynolds (1996) showed that even for very nonlinear relationships, the sampling can be quite good. Preliminary work by the authors on single-phase flow problems seems to indicate that the bias introduced by the approximate acceptance test may be small.

One key requirement for the application of randomized maximum likelihood is the identification of variables whose density function is approximately multinormal. As mentioned earlier, Langbein and Leopold (1966) proposed that channel direction might be considered to be a random function of distance along the channel. This suggests that channel direction might be a good candidate for approximation as a Gaussian random variable. We can investigate the reasonableness of this suggestion by digitizing the channel of Figure 2, passing a cubic spline through the points on the path, then resampling the channel direction at equally spaced increments along the path. The series of channel directions obtained in this way is shown in Figure 3 as a function of distance.

Although we do not need to create the unconditional realizations from a Gaussian simulation, we can obtain a measure of distance between models by examining the statistics of the channel direction. Figure 4(A) shows the histogram of channel directions at uniform distances along the channel. The univariate distribution of channel directions is clearly not Gaussian as it lacks extreme values, but it appears to be sufficiently close to Gaussian that a normal score transform appears to be unnecessary for conditioning of the channel. Figure 4(B) shows the variance between channel directions at various lag distances is very well approximated by a Gaussian covariance function $C_\theta(s) = \sigma_\theta^2 \exp[-3(s/a)^2]$ with variance $\sigma_\theta^2 = 4.55$ and range $a = 20.3$.

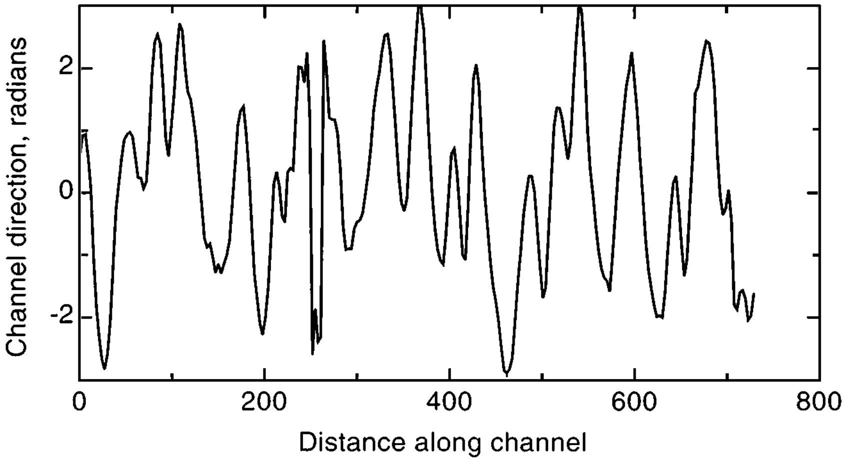

Figure 3. Deviation of the channel direction from the mean for the channel realization in Figure 1.

The channel direction can be written as the sum of a mean direction $\overline{\theta}$ and a fluctuating direction field u_θ, i.e.,

$$\theta(s) = \overline{\theta} + u_\theta(s) \tag{1}$$

where s is the distance along the channel path. Let us assume that the uncertainty in the mean channel direction can be represented as a Gaussian random variable with mean $\mu_{\overline{\theta}}$ and variance $\sigma^2_{\overline{\theta}}$. We will also assume that the fluctuating direction field can be represented as a Gaussian random field with covariance $C_\theta(s - s')$ between the direction at s and the direction at s'. To completely specify the channel centerline geometry, the $x - y$ location of the centerline at one value of s must be specified; so let us specify that $y(0) = y_0$ and $x(0) = 0$, and that the prior

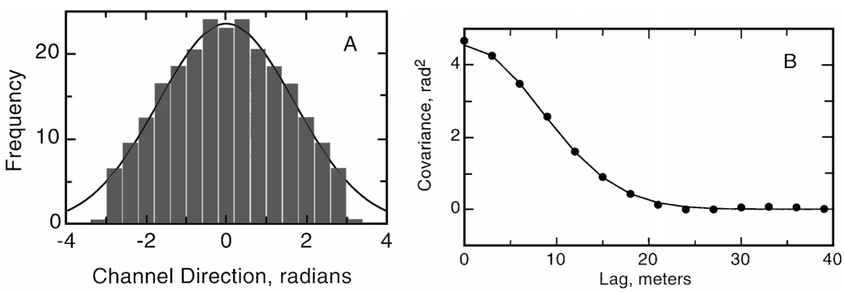

Figure 4. The histogram of channel direction (A) and the covariance of channel direction (B).

uncertainty in y_0 is Gaussian with mean μ_y and variance $\sigma_{y_0}^2$. Our mathematical model of the channel meander geometry is thus specified by two random variables y_0 and $\overline{\theta}$ and a random function $u_\theta(s)$.

We define the model m, the prior model m_{prior}, and the prior model covariance as follows:

$$m = \begin{bmatrix} y_0 \\ \overline{\theta} \\ u_\theta(s) \end{bmatrix} \qquad m_{\text{prior}} = \begin{bmatrix} \mu_y \\ \mu_{\overline{\theta}} \\ \mu_\theta \end{bmatrix} \tag{2}$$

$$C_{\text{M}} = \begin{bmatrix} \sigma_{y_0}^2 & & 0 \\ & \sigma_{\overline{\theta}}^2 & \\ 0 & & C_\theta \end{bmatrix} \tag{3}$$

The $x - y$ location of the channel at any distance s along the channel path is given by the following functions

$$x(s) = \int_0^s \cos\theta(s')\,ds' = \int_0^s \cos[\overline{\theta} + u_\theta(s')]\,ds' \tag{4}$$

$$y(s) = y_0 + \int_0^s \sin\theta(s')\,ds' = y_0 + \int_0^s \sin[\overline{\theta} + u_\theta(s')]\,ds' \tag{5}$$

If the ith channel observation is made in a well located at x_i, y_i, then the data for conditional simulation of channels to N well observations is a $2N$-vector of locations of channel observations, i.e.,

$$d_{\text{obs}} = [x_{\text{obs},1} \quad y_{\text{obs},1} \quad \cdots \quad x_{\text{obs},N} \quad y_{\text{obs},N}]^{\text{T}} \tag{6}$$

These measurements could be considered to be uncertain, either because the actual location of the well is slightly uncertain, or because the position of the channel center with respect to the well is uncertain. We will represent this uncertainty via the data covariance matrix C_{D}.

Because of the simplicity of this model, it cannot capture all of the features of actual channel meanders. In particular, it is likely that a realization of channel from this model may intersect itself if the sinuosity is large. In more sophisticated geological models, channel cutoff can occur, and loops may be abandoned. On the basis of the analysis of complex channels, however, it may be reasonable to assume that the covariance of channel direction is adequate for defining the "distance" between different channel realizations. For arbitrary stochastic models the definition of a small adjustment to the model is not always clear, but for the model I have defined here, the definition is obvious. If one wanted to generate a conditional realization, one would simply solve for a realization m_{c} that would minimize the following objective function

$$S(m) = \frac{1}{2}(m - m_{\mathrm{uc}})^{\mathrm{T}} C_{\mathrm{M}}^{-1}(m - m_{\mathrm{uc}}) + \frac{1}{2}(g(m) - d_{\mathrm{uc}})^{\mathrm{T}} C_{\mathrm{D}}^{-1}(g(m) - d_{\mathrm{uc}}) \quad (7)$$

where m_{uc} is an unconditional realization of the channel model parameters, d_{uc} is a realization of the channel well locations, C_{M}^{-1} is the inverse of the covariance operator (Oliver, 1998), and $g(m)$ represents the relationship between theoretical data and the model variables given by Eqs. (4) and (5). The expectation of d_{uc} is d_{obs}, and the covariance C_{D} is the data error covariance matrix. If the observation errors are small, then it is possible to use d_{obs} in place of d_{uc} in Eq. (7). That model that minimizes Eq. (7) can be thought of as the one that is closest to the unconditional realization in the sense that $(m_{\mathrm{c}} - m_{\mathrm{uc}})^{\mathrm{T}} C_{\mathrm{M}}^{-1} (m_{\mathrm{c}} - m_{\mathrm{uc}})$ is small, and approximately honors the data (i.e., $(g(m_{\mathrm{c}}) - d_{\mathrm{uc}})^{\mathrm{T}} C_{\mathrm{D}}^{-1} (g(m_{\mathrm{c}}) - d_{\mathrm{uc}})$ is small). The real purpose, then, for specifying an approximately Gaussian model and for estimating the covariance was to define the meaning of one channel realization being similar (close in model space) to another channel realization.

In general, the fastest and most efficient method of minimizing the objective function in Eq. (7) is to use information about the sensitivity of the data with respect to the model variables y_0 and $\overline{\theta}$, and the function $u_\theta(s)$. From Eq. (4), it can be easily seen that small changes in the average channel direction and small changes in the fluctuating direction function affect the x location of the channel in the following way.

$$\begin{aligned} \delta x(s_i) &\approx -\delta\overline{\theta} \int_0^{s_i} \sin\theta(s)\, ds - \int_0^{s_i} \sin\theta(s')\, \delta u_\theta(s')\, ds' \\ &= -\delta\overline{\theta} \int_0^{s_i} \sin\theta(s)\, ds - \int_0^{\infty} [1 - H(s_i - s')] \sin\theta(s')\, \delta u_\theta(s')\, ds' \end{aligned} \quad (8)$$

where $H(s)$ is the Heaviside step function. The sensitivity of the y location of the channel at s_i is similar, except that the y location is also sensitive to y_0,

$$\begin{aligned} \delta y(s_i) &\approx \delta y_0 + \delta\overline{\theta} \int_0^{s_i} \cos\theta(s)\, ds + \int_0^{s_i} \cos\theta(s')\, \delta u_\theta(s')\, ds' \\ &= \delta y_0 + \delta\overline{\theta} \int_0^{s_i} \cos\theta(s)\, ds + \int_0^{\infty} [1 - H(s_i - s')] \cos\theta(s')\, \delta u_\theta(s')\, ds' \end{aligned} \quad (9)$$

For convenience, I define the sensitivity matrix or operator G to be the gradient of the theoretical data with respect to the model variables, $G = (\nabla_m g(m)^{\mathrm{T}})^{\mathrm{T}}$. From

Eqs. (8) and (9), the sensitivity matrix is

$$G = \begin{bmatrix} 0 & -\int_0^{s_1} \sin\theta(s)\,ds & -\sin[\theta(s)][1 - H(s - s_1)] \\ 1 & \int_0^{s_1} \cos\theta(s)\,ds & \cos[\theta(s)][1 - H(s - s_1)] \\ \vdots & \vdots & \vdots \\ 0 & -\int_0^{s_N} \sin\theta(s)\,ds & -\sin[\theta(s)][1 - H(s - s_N)] \\ 1 & \int_0^{s_N} \cos\theta(s)\,ds & \cos[\theta(s)][1 - H(s - s_N)] \end{bmatrix} \tag{10}$$

In case it is not obvious, the elements in the first two columns of G are scalars, and the elements in the last column are functions of s.

The minimum of Eq. (7) will occur when the gradient of the objective function with respect to the model variables vanishes. To find the model that minimizes Eq. (7) we need to solve

$$\begin{aligned} \nabla_m S(m) &= C_{\mathrm{M}}^{-1}(m - m_{\mathrm{uc}}) + G^{\mathrm{T}} C_{\mathrm{D}}^{-1}(g(m) - d_{\mathrm{uc}}) \\ &= 0 \end{aligned} \tag{11}$$

for m. In most cases, the number of channel observations will probably be quite small while the model space is infinite dimensional, or at least quite large. A Newton method is an appropriate choice for minimization in this case. For this problem, the following form of the Gauss–Newton equations (McLaughlin and Townley, 1996; Tarantola, 1987) is appropriate:

$$m^{n+1}(s) = m_{\mathrm{uc}}(s) - C_{\mathrm{M}} G^{\mathrm{T}}[C_{\mathrm{D}} + G C_{\mathrm{M}} G^{\mathrm{T}}]^{-1}[g(m^n) - d_{\mathrm{uc}} - G(m^n - m_{\mathrm{uc}})] \tag{12}$$

In order to use Gauss–Newton, we need to compute the product $GC_{\mathrm{M}}G^{\mathrm{T}}$. We begin with the computation of GC_{M}.

$$GC_{\mathrm{M}} = \begin{bmatrix} 0 & -\sigma_\theta^2 \int_0^{s_1} \sin\theta(s)\,ds & -\int_0^{s_1} \sin[\theta(s)] C_\theta(s - s')\,ds \\ \sigma_{y_0}^2 & \sigma_\theta^2 \int_0^{s_1} \cos\theta(s)\,ds & \int_0^{s_1} \cos[\theta(s)] C_\theta(s - s')\,ds \\ \vdots & \vdots & \vdots \\ 0 & -\sigma_\theta^2 \int_0^{s_N} \sin\theta(s)\,ds & -\int_0^{s_N} \sin[\theta(s)] C_\theta(s - s')\,ds \\ \sigma_{y_0}^2 & \sigma_\theta^2 \int_0^{s_N} \cos\theta(s)\,ds & \int_0^{s_N} \cos[\theta(s)] C_\theta(s - s')\,ds \end{bmatrix} \tag{13}$$

The product $GC_{\mathrm{M}}G^{\mathrm{T}}$ is formed by taking the product of Eq. (13) and the transpose of the matrix in Eq. (10). The formula for the general case is fairly complex, and so here I will simply write out the elements of the matrix for a problem with two channel observations.

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{11} = \sigma_\theta^2 \left[\int_0^{s_1} \sin\theta(s)\,ds\right]^2 + \int_0^{s_1} ds \int_0^{s_1} ds' \sin[\theta(s)] \times C_\theta(s-s')\,\sin[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{12} = -\sigma_\theta^2 \int_0^{s_1} \sin\theta(s)\,ds \int_0^{s_1} \cos\theta(s)\,ds - \int_0^{s_1} ds \int_0^{s_1} ds' \sin[\theta(s)] \times C_\theta(s-s')\,\cos[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{13} = \sigma_\theta^2 \int_0^{s_1} \sin\theta(s)\,ds \int_0^{s_2} \sin\theta(s)\,ds + \int_0^{s_1} ds \int_0^{s_2} ds' \sin[\theta(s)] \times C_\theta(s-s')\,\sin[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{14} = -\sigma_\theta^2 \int_0^{s_1} \sin\theta(s)\,ds \int_0^{s_2} \cos\theta(s)\,ds - \int_0^{s_1} ds \int_0^{s_2} ds'\ \sin[\theta(s)] \times C_\theta(s-s')\,\cos[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{22} = \sigma_{y_0}^2 + \sigma_\theta^2 \left[\int_0^{s_1} \cos\theta(s)\,ds\right]^2 + \int_0^{s_1} ds \int_0^{s_1} ds'\ \cos[\theta(s)] \times C_\theta(s-s')\,\cos[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{23} = -\sigma_\theta^2 \int_0^{s_1} \cos\theta(s)\,ds \int_0^{s_2} \sin\theta(s)\,ds - \int_0^{s_1} ds \int_0^{s_2} ds'\ \cos[\theta(s)] \times C_\theta(s-s')\,\sin[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{24} = \sigma_{y_0}^2 + \sigma_\theta^2 \int_0^{s_1} \cos\theta(s)\,ds \int_0^{s_2} \cos\theta(s)\,ds + \int_0^{s_1} ds \times \int_0^{s_2} ds'\ \cos[\theta(s)] C_\theta(s-s')\,\cos[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{33} = \sigma_\theta^2 \left[\int_0^{s_2} \sin\theta(s)\,ds\right]^2 ds + \int_0^{s_2} ds \int_0^{s_2} ds'\ \sin[\theta(s)] \times C_\theta(s-s')\,\sin[\theta(s')]$$

$$[GC_{\mathrm{M}}G^{\mathrm{T}}]_{34} = -\sigma_\theta^2 \int_0^{s_2} \sin\theta(s)\,ds \int_0^{s_2} \cos\theta(s)\,ds - \int_0^{s_2} ds \int_0^{s_2} ds' \sin[\theta(s)] \times C_\theta(s-s')\,\cos[\theta(s')]$$

$$[GC_M G^T]_{44} = \sigma_{y_0}^2 + \sigma_{\bar\theta}^2 \left[\int_0^{s_2} \cos\theta(s)\, ds \right]^2 + \int_0^{s_2} ds \int_0^{s_2} ds' \cos[\theta(s)] \times C_\theta(s - s') \cos[\theta(s')] \tag{14}$$

The matrix is symmetric, and so the lower elements need not be computed.

Computation of the data mismatch vector on the right side of Eq. (12) is again straightforward if the value of s at the observation locations is known. For simplicity, the results are again presented for the case of two observations only.

$$g(m^n) - d_{uc} - G(m^n - m_{uc}) = \begin{bmatrix} x_1(m^n) - x_{1,uc} + (\bar\theta^n - \bar\theta_{uc}) \int_0^{s_1} \sin\theta^n(s)\, ds + \int_0^{s_1} \sin\theta^n(s) \left(u_\theta^n(s) - u_{\theta,uc}(s)\right) ds \\ y_1(m^n) - y_{1,uc} - \left(y_0^n - y_{0,uc}\right) - (\bar\theta^n - \bar\theta_{uc}) \int_0^{s_1} \cos\theta^n(s)\, ds - \int_0^{s_1} \cos\theta^n(s) \left(u_\theta^n(s) - u_{\theta,uc}(s)\right) ds \\ x_2(m^n) - x_{2,uc} + (\bar\theta^n - \bar\theta_{uc}) \int_0^{s_2} \sin\theta^n(s)\, ds + \int_0^{s_2} \sin\theta^n(s) \left(u_\theta^n(s) - u_{\theta,uc}(s)\right) ds \\ y_2(m^n) - y_{2,uc} - \left(y_0^n - y_{0,uc}\right) - (\bar\theta^n - \bar\theta_{uc}) \int_0^{s_2} \cos\theta^n(s)\, ds - \int_0^{s_2} \cos\theta^n(s) \left(u_\theta^n(s) - u_{\theta,uc}(s)\right) ds \end{bmatrix} \tag{15}$$

Finally, we need to compute the vector of functions, $C_M G^T = [GC_M]^T$. Thus, when there are two observations of channel location, we see from Eq. (13) that this becomes

$$C_M G^T = \begin{bmatrix} 0 & -\sigma_{\bar\theta}^2 \int_0^{s_1} \sin\theta(s)\, ds & -\int_0^{s_1} \sin[\theta(s)] C_\theta(s - s')\, ds \\ \sigma_{y_0}^2 & \sigma_{\bar\theta}^2 \int_0^{s_1} \cos\theta(s)\, ds & \int_0^{s_1} \cos[\theta(s)] C_\theta(s - s')\, ds \\ 0 & -\sigma_{\bar\theta}^2 \int_0^{s_2} \sin\theta(s)\, ds & -\int_0^{s_2} \sin[\theta(s)] C_\theta(s - s')\, ds \\ \sigma_{y_0}^2 & \sigma_{\bar\theta}^2 \int_0^{s_2} \cos\theta(s)\, ds & \int_0^{s_2} \cos[\theta(s)] C_\theta(s - s')\, ds \end{bmatrix}^T \tag{16}$$

The equations are easier to understand if we take them one bit at a time. For example, let us use Eq. (12) to compute the new value of y_0 and $\bar\theta$ for the case in which channel observations are available at s_1 and s_2,

$$y_0^{n+1} = y_{0,uc} - \sigma_{y_0}^2 [\,0 \quad 1 \quad 0 \quad 1\,] a \tag{17}$$

$$\bar\theta^{n+1} = \bar\theta_{uc} - \sigma_{\bar\theta}^2 \left[-\int_0^{s_1} \sin\theta^n(s)\, ds \quad \int_0^{s_1} \cos\theta^n(s)\, ds \quad -\int_0^{s_2} \sin\theta^n(s)\, ds \quad \int_0^{s_2} \cos\theta^n(s)\, ds \right] a \tag{18}$$

where

$$a = [C_D + GC_M G^T]^{-1}[g(m^n) - d_{uc} - G(m^n - m_{uc})] \tag{19}$$

is a 4-vector when the channel has been observed at two well locations.

EXAMPLES

Gaussian Random Field

In this first example, I illustrate the possibility of using the Gaussian assumption on channel direction both for generating the unconditional realization of channel meanders, and for conditioning the channel to well observations. Equations 4 and 5 are used to compute the realization of channel location $x(s), y(s)$ from realizations of channel direction $\theta(s)$.

I started by creating an unconditional realization of a random channel direction function with a mean of 0, and a Gaussian covariance function. The realization of the channel (Fig. 5) starts at the origin, although this is clearly not necessary. I assumed for this example that the true channel had been observed at the locations (7, 0.) and (15, 0.). The channel realization must be modified in some reasonable way to make it pass through the observation locations.

In order to apply Eqs. (8) and (9), one must select values of s at the conditioning locations. For this example, I assumed that we want to move the closest point on the channel to the observation location. A nonlinear minimization found

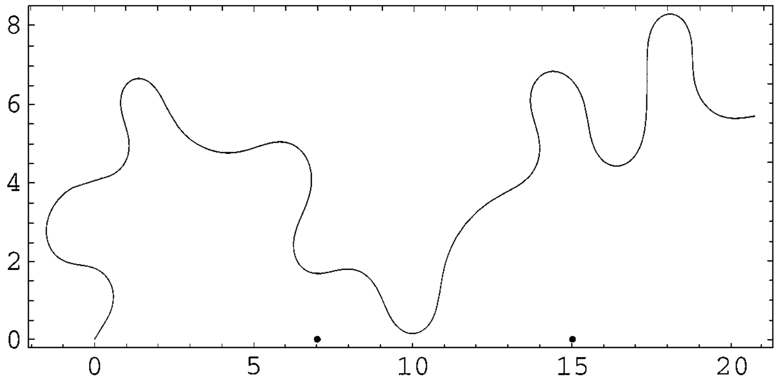

Figure 5. An unconditional Gaussian realization of a meandering channel with well observation locations (black dots).

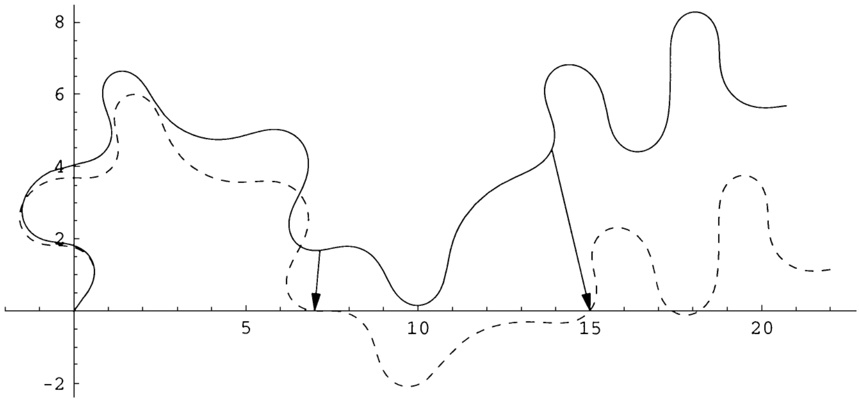

Figure 6. The conditional and unconditional Gaussian realizations of a meandering channel showing the adjustment in location required.

the two nearest values to be at $s_1 = 19.76$ and $s_2 = 28.71$. These locations and the local adjustments are shown by the arrows in Figure 5. Once the values of s have been determined it is possible to compute $GC_M G^T$ with the use of Eq. (15).

Only two Gauss–Newton iterations were required to match the data to four significant digits. The final match (a conditional realization) is shown as the dotted curve in Figure 6. The stochastic character of the curve seems to be largely unchanged; i.e., the conditioning process has not introduced any obvious additional roughness into the channel meander.

Recall that the direction of the channel was modified, after which the new channel location was determined by integrating Eqs. (4) and (5). The actual correction to the channel direction is shown in Figure 7. Note that the magnitude of the correction to the channel direction is generally of the order of 0.2 but reaches a maximum magnitude of 0.55 between the two observation locations. Thus the problem is fairly linear. The second thing to note is that the correction to θ is largely confined to the region between the origin and the last channel observation. This is reasonable as changing the direction downstream of the observation locations should not affect the location upstream (Fig. 8).

Conditioning of a Geological Model

Now we can illustrate the process on a more complex channel. Instead of generating an unconditional realization of a channel from a simple Gaussian process as in the first example, here we choose to condition a channel meander (Fig. 2) that was described in Howard (1996). The exact details of the geological modeling are

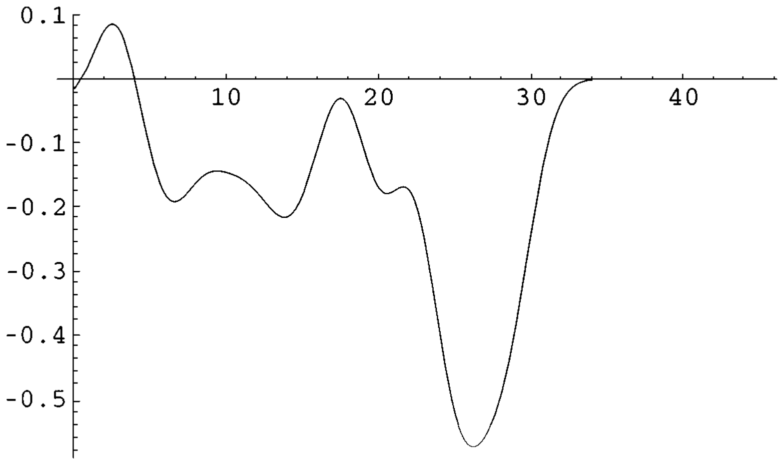

Figure 7. The correction to the channel direction as a function of distance along the channel. Data were recorded at $s \approx 20$ and $s \approx 30$.

not important for this paper; it should suffice to say that erosion and transport of sediment were modeled. We assume that the model contains sufficient parameters to create plausible channels for the location of interest. It is unable, however, to create realizations that are conditional to well observations.

In order to create conditional realizations from a complex model using the randomized maximum likelihood method, we must first identify a parameterization of

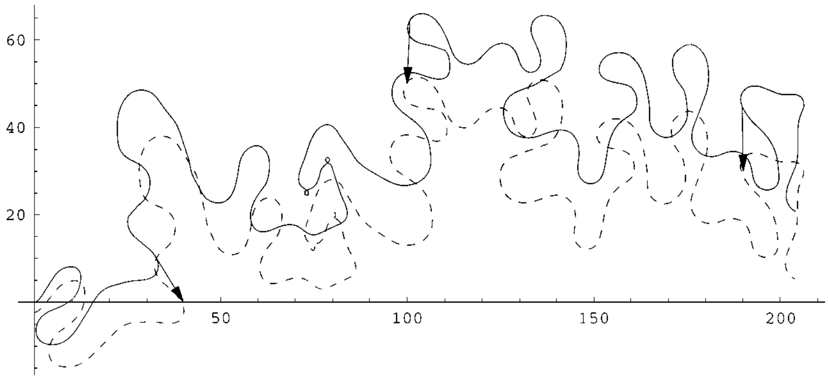

Figure 8. The original channel meander (solid curve) and the realization conditioned to well observations (dashed curve). The heads of the arrows show observation locations and the tails show the locations of unconditional channel that were moved to the observation.

the channel for which the variables can be approximated by Gaussian random fields. We can clearly see that deviation from a principal direction line is not suitable, as it is multivalued. The channel directions, on the other hand, appeared to be suitable. The direction field was shown in Figure 3 and the covariance and histogram were shown in Figure 4. The only use of these moments is to create a measure of the mismatch between two models, i.e., $O(m_1, m_2) = \frac{1}{2}(m_1 - m_2)^{\mathrm{T}} C_{\mathrm{M}}^{-1}(m_1 - m_2)$, so that in the process of honoring data, we can ensure that the conditioned channel is as close as possible to the unconditioned model.

We assume that the channel has been observed at three locations as indicated by the dots in Figure 2. There are several ways to adjust the channel in an attempt to honor the observations. First, the entire channel can be shifted down by adjusting the value of y at the origin. Second, the average orientation of the channel can be adjusted, and third, the channel direction can be adjusted continuously along the channel. The goal is to honor the observations while making the smallest possible adjustment in the least-squares sense. The most effective way to do this is to make all three types of adjustment simultaneously.

DISCUSSION AND CONCLUSIONS

Realistic channel meanders with high sinuosity are difficult to simulate stochastically because, unless processes such as cut-off and fill are modeled, the channel path could intersect itself. As a result, high sinuosity channel meanders tend to be computationally expensive to simulate. It is even more difficult to generate realizations that are conditional to well observations, since channel location is a dynamic function of time. What I have proposed in this paper is that the process of conditioning the realization be separated from the process of generating the unconditional channel meander, so that the method of generation of channels can be somewhat arbitrary. In the process of conditioning the channel, we search for a channel that is as similar as possible to the unconditional realization, while also honoring the well observations. For highly sinuous meander channel, it appears to be inappropriate to use x and y location of the channel center as a measure of similarity. On the other hand, instantaneous channel direction may be sufficiently close to a Gaussian distribution that its covariance can provide a useful penalty term for defining the distance between the channel models. The identification of a variable for which the covariance provides a useful measure of distance is a key feature of the problem solution and is not limited to channel geometry.

The conditioning procedure itself was shown to be quite simple to apply. If the channel directions had been approximated by a piecewise constant function, instead of a cubic spline, all of the matrices would have been finite-dimensional, and the computation would have required simple matrix multiplication instead of integration. The conditional and unconditional channels are visually similar in character. The main difference is that the conditional realization honors the data.

If other observations, such as transient pressures or channel thickness measurements, were available, the problem could be solved much as described by Bi, Oliver, and Reynolds (2000). The primary difference would be to use channel direction to parameterize the location of the channel center instead of deviation from a principle direction line. The conditioning algorithm that was used in this paper is known to sample correctly for problems with Gaussian priors and linear data with normally distributed measurement and observation errors. It has been shown to sample approximately the correct distribution when the relation between the model variables and the data is highly nonlinear. The ability of this algorithm to assess uncertainty accurately is unknown, but for small numbers of realizations the difference may be of little practical consequence.

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