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Estimation of formation hydraulic properties accounting for pre-test injection or production operations

D.B. Silin, C.-F. Tsang*

Earth Sciences Division, Lawrence Berkeley National Laboratory, University of California, 1 Cyclotron Road, MS90-1116, Berkeley, CA 94720, USA

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Abstract

We propose to use regular monitoring data from a production or injection well for estimating the formation properties around the wellbore without interrupting the operations. Thus, instead of shutting-in the well for a substantial time period, we propose to select a portion of the pumping data over a certain time interval and then derive our conclusions from these data. A distinctive feature of our analysis is that we introduce an auxiliary parameter to account for the possible after-effects of pumping that preceded the test interval and, consequently, the non-uniform initial pressure conditions. We demonstrate that those effects influence not only the analysis of regular operations data, but also the analysis of a traditional pressure drawdown or pressure buildup test with a prior shut-in period. We show that phenomena usually attributed to wellbore storage or skin effects can be at least partially interpreted through the parameter we introduce. Unlike some traditional methods, our analysis utilizes almost the entire test time period for curve-fitting. It turns out that it produces good data matching even if the test period is short and the frequency of measurements is low.

Another distinctive feature of the present approach is that the parameter estimation problem is reduced to a combination of quadratic criterion minimization and a search for the minimum of a one-variable function. Because we can obtain the solution to the quadratic problem analytically, we significantly simplify the problem and dramatically reduce the amount of computations required. © 2002 Published by Elsevier Science B.V.

Keywords: Hydraulic conductivity; Pumping test; Parameters estimation; Radial transient flow equation; Skin effect

1. Introduction

A pumping or injection well test is common practice in hydrology and petroleum engineering. It is performed to estimate the formation hydraulic properties in the vicinity of the well. Normally, such a test requires either maintaining a constant pumping rate or a full shut-in of the well for a substantial period of time prior to the beginning of the test. Here we propose to use regular monitoring data from a production or injection well for estimating the formation properties in the vicinity of the wellbore without interrupting the operations. Thus, instead of shutting-in the well for a substantial time period, we propose to select a portion of the regular pumping data over a certain time interval and then derive our conclusions from these data. A new feature of the proposed approach is that we introduce an additional parameter, an effective pre-test pumping rate, to account for the non-uniform pressure distribution at the beginning of testing time interval.

The theoretical background for well test analysis

* Corresponding author. Tel.: +1-510-586-6726; fax: +1-510-486-5686.

E-mail address: cftsang@lb.gov (C.-F. Tsang).

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was established in the work of Theis (1935). Since then, a substantial number of publications on the subject have appeared. We do not present an exhaustive reference list here, since the reader can find these references in books by Earlougher (1977), Horne (1990), Matthews and Russell (1967), Raghavan (1993), Sabet (1991) and Streltsova (1988), and in the survey by Ramey (1992). The bibliography of well test literature published in Russian by mid-1970s is presented in Kulpin and Miasnikov (1974).

The traditional methods for estimating formation transmissivity and other parameters are based on graphical matching of measured data (Bear, 1979; Earlougher, 1977; Ramey, 1992). These approaches involve simple analytical solutions of flow equations that are valid for constant pumping rates and a uniform initial pressure distribution around the wellbore. Neither one of these conditions is met when we analyze the regular operations data. The uniformity of initial pressure distribution does not take place even in the case of well test performed at constant pumping rates with a preceding shut-in period. This fact makes data matching difficult at early test times, that was pointed out by many authors (Barenblatt et al., 1990; Kulpin and Miasnikov, 1974).

Variable pumping rates can be incorporated into well test analysis through Duhamel convolution integral (Kulpin and Miasnikov, 1974; Raghavan, 1993; Streltsova, 1988), and algorithms and procedures based on this approach are well-known (McEdwards and Benson, 1981). In this paper, we propose a method to account for non-uniform initial pressure distribution. For our analysis, we use the well-known solution to the radial flow equation for a variable injection rate, and we introduce an auxiliary parameter estimating an effective pumping rate before the test.

In numerous practical cases presented in the literature, the poor matching of the measured data by a computed curve at early points of the testing period is usually explained by assuming formation damage around the well (skin effect) and wellbore storage effects (Earlougher, 1977). We demonstrate through synthetic and practical examples that deviation of the matching curve from the data will occur if the pre-test pressure distribution around the well is not uniform. Moreover, this deviation is very similar to that usually interpreted with skin and wellbore storage effects.

The efficiency of our approach is immediately confirmed by examples. For instance, we applied the procedure described later to analyze a data set from a pressure drawdown test performed at an injecting well at a site in Ohio, USA. We analyzed only a part of the original data by removing some early-time data points. Thus, on the removed early data interval the injection rate was known. Then, we processed the remaining data with our parameters' estimation procedure. The data curve matched very well. Moreover, the injection rate on the removed interval was estimated with remarkably high accuracy. At the same time, the skin coefficient that was evaluated was much smaller than the one obtained independently via traditional Horner plot analysis. Hence, it is extremely important to account for pre-test pumping in order to correctly separate the consequences of the different effects and to obtain correct estimates of the formation properties.

The paper is organized as follows. To make the presentation self-contained, in Section 2, we briefly overview the theoretical background of traditional pumping test analysis. In Section 3, we perform an error analysis to estimate the consequences of ignoring the pre-test pumping. In Section 4, we produce a modified approximate radial flow solution incorporating a new parameter: an effective pre-test pumping rate. In Section 5, we discuss short-term transient effects. In Section 6, we formulate the parameters estimation problem. We develop an efficient estimation procedure combining analytical calculations with numerical minimization of a one-variable function. We verify our method with a field example. In Section 7, we present our summary and conclusions on the proposed pumping test procedure.

2. The background

In this section, we discuss the analysis of pressure and injection rate logs for the case of pumping into a vertically confined horizontal layer. We also discuss the importance of wellbore storage and skin effects in comparison to the impact of pre-test pumping. The same analysis can be applied to analyze formation properties around a producing well.

By pumping, in this and the following sections, we understand both injection and withdrawal of fluids.

Since we mostly focus at injecting wells, the pumping rate is assumed positive if the fluid is injected into the formation, and negative otherwise. We assume that during the entire test period the injected fluid has constant temperature, density, and viscosity.

The radial transient flow of liquid from an injecting well into a formation is characterized by a parabolic equation

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c}{k}\frac{\partial p}{\partial t} \tag{1}$$

along with the following initial and boundary conditions:

$$p(r, t_0) = p_0 \qquad \text{(initially uniform pressures)} \tag{2}$$

$$p(\infty, t) = p_0, \quad t \geq t_0 \text{ (constant initial pressure at infinity)} \tag{3}$$

$$\lim_{r \to +0} r\frac{\partial p(r,t)}{\partial r} = -\frac{\mu}{2\pi k H}Q(t) \tag{4}$$

(given variable flow rate $Q(t)$ at the wellbore)

(Earlougher, 1977; Kulpin and Miasnikov, 1974). Here $p(t, r)$ is the fluid pressure at the time t and distance r from the well, ϕ is the porosity of the formation near the wellbore, μ, the viscosity of the fluid, k, the permeability, and the coefficient c characterizes the compressibility of water and rock (Shchelkachev, 1959; Barenblatt et al., 1990). In Eq. (4), H is the thickness of the injection layer. The solution to the initial and boundary value problem (1)–(4) can be obtained using Duhamel convolution integral (Carslaw and Jaeger, 1959; Tikhonov and Samarskii, 1963) and is given by

$$p(t) = p_0 + A\int_0^t \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)\mathrm{d}\tau \tag{5}$$

where A and B are defined by $A = \mu/(4\pi kH)$ and $B = (\phi\mu c r_w^2)/(4k)$. Coefficients A and B are introduced to simplify further calculations and are expressed in transmissivity T and storativity S by $A = 1/(4\pi T)$, $B = (Sr^2)/(4T)$. Conversely, transmissivity and storativity can be expressed in A and B by $T = 1/4\pi A$, $S = 2B/\pi r^2 A$.

In a pressure fall-off test, a period of injection at a constant rate, say Q_0, from initial time $t_0 = 0$ is followed by a shut-in period starting at $t = t_1$ during which the injection rate is either equal or close to zero. Assuming *no pumping before the test* and therefore, a *uniform* initial pressure distribution (as defined in Eq. (2)), one obtains from Eq. (5):

$$p(t) = p_0 + A\int_0^{t_1} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q_0\,\mathrm{d}\tau, \qquad t \geq t_1 \tag{6}$$

Integration in Eq. (6) yields

$$p(t) - p_0 = AQ_0\left(-\mathrm{Ei}\left(-\frac{B}{t-t_1}\right) + \mathrm{Ei}\left(-\frac{B}{t}\right)\right) \tag{7}$$

If both $B/(t - t_1)$ and B/t do not exceed 0.01 (Abramowitz and Stegun, 1965; Bear, 1979), then Eq. (7) is usually approximated by

$$p(t) - p_0 \approx AQ_0 \ln\left(\frac{t}{t-t_1}\right) \tag{8}$$

Hence, the transmissivity coefficient A can be estimated from the slope of the plot of $p(t) - p_0$ versus $\ln(t/(t - t_1))$, which must be, from Eq. (8), a straight line. This is a brief description of the Horner (1951) method.

The argument above highlights several typical assumptions usually made in traditional well test analysis. In particular, the initial condition (2) implies a uniform initial pressure distribution in the aquifer. Therefore, the consequences of pumping before the test period are neglected. Clearly, this initial pressure uniformity assumption is not valid if the test is performed on a well that has been pumped earlier. Examples of pumping test analysis presented in literature (Bear, 1979; Earlougher, 1977; Matthews and Russell, 1967; Ramey, 1992) confirm that there is an on-set period of time in which the quality of data matching is poor. This is usually attributed exclusively to wellbore storage and skin effects. However, we show later that the on-set time period can also be explained by the non-uniformity of the initial pressure distribution prior to the test.

To illustrate how pre-test pumping affects the pressure curve, consider a synthetic example. Assume that 400 days of injection at a constant rate of approximately 272.5 m^3/day is followed by a 408-h (17-day) shut-in period. The pressures computed by

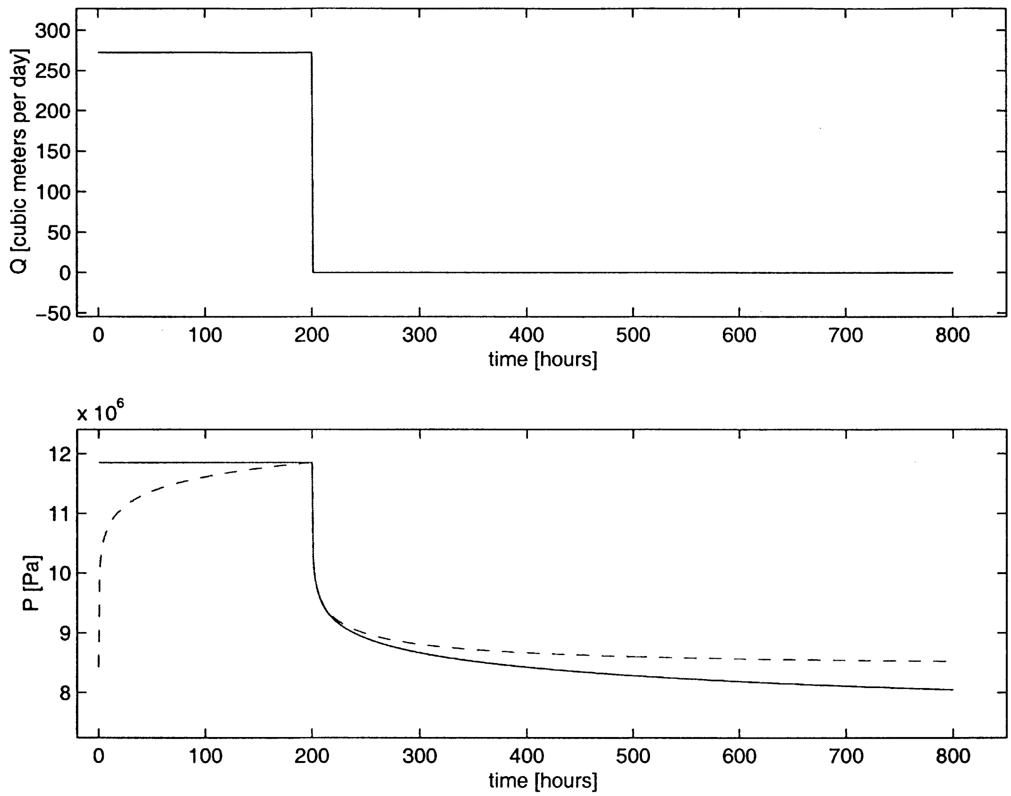

Fig. 1. The upper graph shows injection rate in gallons per minute versus time in hours during a fall-off test. On the lower graph, the solid line plots the pressures whereas the dashed line plots the computed pressures assuming no pumping prior to the test (1 psi ≈ 6894.7 Pa).

solving the boundary-value problem (1)–(4) for the entire 417-day interval will be our 'measured' data. Let us assume that we were not provided with information during the first 392 days after the start of constant pumping and that we set the time of 'data measurement' to begin at 8 days (192 h) before shut-in. Then, accounting only for the 8 days of pre-shut-in pumping, the solution will be different from the measured one. In Fig. 1, both pressures from the 8-day solution and the measured data are plotted from

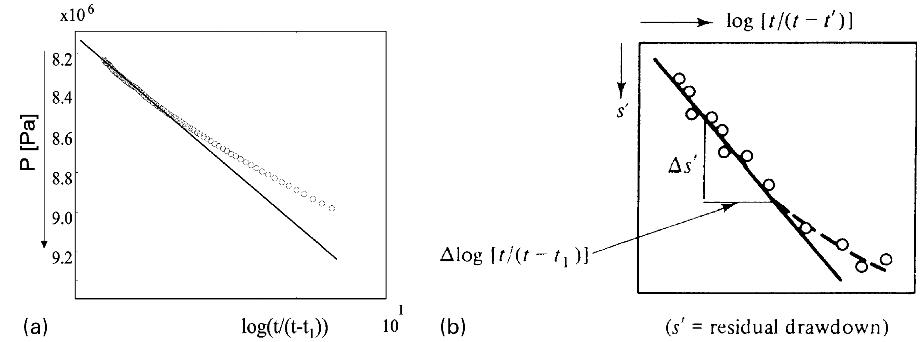

Fig. 2. Horner plot fitting of the fall-off pressures. Plot 2(a) is synthetic whereas plot 2(b) (Fig. 11-3 from Bear (1979)) shows an example of a recovery test.

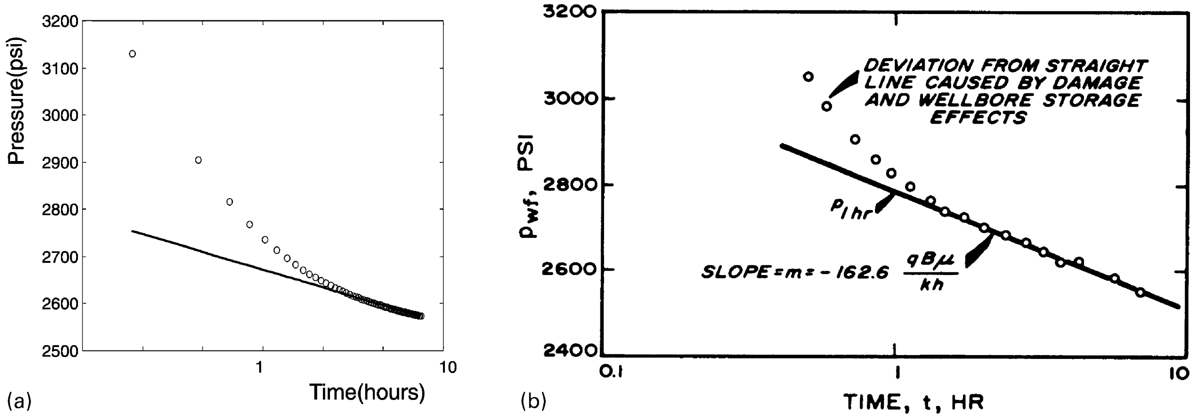

Fig. 3. Synthetic pumping test data (a) and real pumping test plot from Earlougher (1977) (b) (1 psi ≈ 6.8947 × 10^3 Pa). On both plots, pressure (in psi) is plotted versus time (in hours) in semi-log scale.

the 393rd day of pumping through the end of shut-in period. We set the initial condition for the 'calculated' pressures in such a way that both pressures are equal at the beginning of the shut-in interval.

Now, if we plot the pressures on shut-in interval versus $\ln(t/(t - t_1))$, and then do the best fitting of the measured pressures by a linear function on a small later time interval, we get the result shown in Fig. 2(a). The circles are the synthetic data points and the straight line is the fitting by a linear function. The fitting is poor in the right part of the plot, i.e. for times close to t_1. The character of deviation from the straight line shown on the synthetic plot, Fig. 2(a), is remarkably similar to the plot provided in Bear (1979) as an example of a recovery test. Since the plot shown in Fig. 2(b) from the book by Bear (1979) is qualitative and has no units along the axes, our plot on Fig. 2(a) also is presented in the qualitative form.

Another example is presented in Fig. 3, where we present side-by-side a traditional matching of our

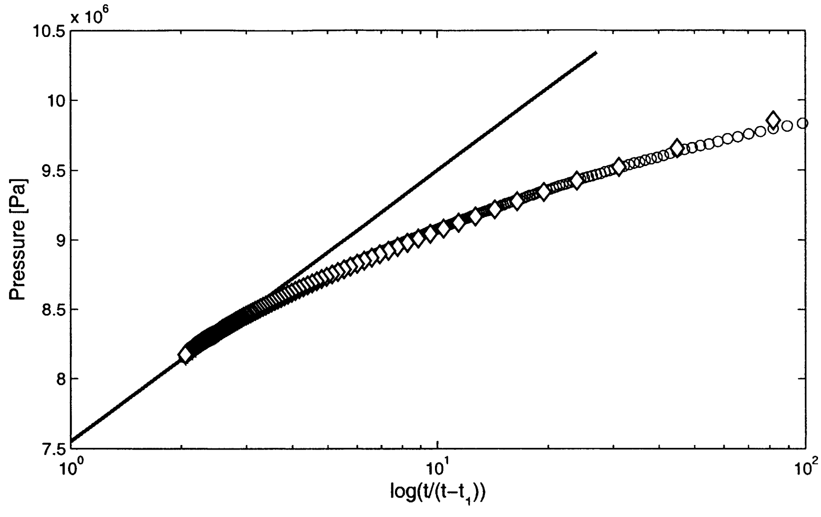

Fig. 4. Horner plot analysis of data from an Ohio site. The circles (coalescing into a thick solid line) correspond to the data and the straight line is the result of Horner analysis performed by a service company. The deviation of the curve from the straight line is attributed to wellbore storage and skin effects. The synthetic data are produced assuming zero skin coefficient but accounting for pre-test pumping and are plotted in the same scale as diamonds.

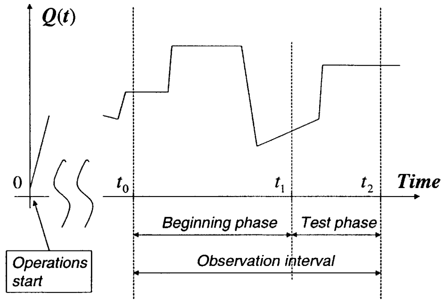

Fig. 5. The scheme of a test interval.

synthetic data in semi-log plot and the data matching for a real test as given in Earlougher (1977). The synthetic data was produced by solving boundary-value problem (1)–(4) assuming constant rate production, followed by 350 h of well shut-in and then test pumping at the rate of 272.5 m^3/day. The similarity between the two plots is clear. Note that in Earlougher (1977), the deviation of the matching straight line from the data points is attributed exclusively to wellbore storage and formation damage effects, whereas our calculations for this particular example assume neither of these effects.

One more example confirming the importance of accounting for pumping before the test is presented in Fig. 4. Horner plot (diamonds) of synthetic data remarkably mimics the Horner plot of real measured injection pressure data (circles), see Section 6 for further details. The synthetic data were generated assuming no wellbore storage or skin effect; only the after-effects of the pumping before the test were accounted for.

To summarize, accounting for pumping that occurred before a well test including a period of shut-in and then pumping is important in analyzing well test data. In Section 4, we propose a parameter estimation method that accounts for the unknown or uncertain flow rate before the test.

3. Error estimate

In this section, we rigorously analyze the model and estimate the error introduced by neglecting the effects of pumping that occurred before the test. Based on this analysis, a modified solution to be used for a new parameter estimation method will be developed and analyzed in Section 4.

Denote by t the elapsed time from the beginning of pumping at the well to the moment when data begins to be available. The injection pressures and rates are measured on a time interval $[t_0, t_2]$ where only the value of the difference $t_2 - t_0$ is known. The only available information about the time t_0 is that estimate $t_0 \gg t_2 - t_0$ holds true. Select a time t_1 such that $t_0 < t_1 < t_2$. Let us estimate the influence of the pumping prior to the beginning of measurements t_0 on pressures over the time period $[t_1, t_2]$ (Fig. 5).

Comparison between the solution (5) evaluated at t_1 and any t ($t_1 < t < t_2$) yields

$$p(t) = p(t_1) - A\int_0^{t_1} \frac{\exp\left(-\dfrac{B}{t_1-\tau}\right)}{t_1-\tau} Q(\tau)\mathrm{d}\tau + A\int_0^{t} \frac{\exp\left(-\dfrac{B}{t-\tau}\right)}{t-\tau} Q(\tau)\mathrm{d}\tau, \quad t_1 \le t \le t_2 \tag{9}$$

Eq. (9) expresses the pressures on the time interval $[t_1, t_2]$ through $p(t_1)$, whose value is available through measurements. Since the injection rate on the interval $[0, t_0]$ is not known, we replace Eq. (9) by

$$p(t) = p(t_1) - A\int_{t_0}^{t_1} \frac{\exp\left(-\dfrac{B}{t_1-\tau}\right)}{t_1-\tau} Q(\tau)\mathrm{d}\tau + A\int_{t_0}^{t} \frac{\exp\left(-\dfrac{B}{t-\tau}\right)}{t-\tau} Q(\tau)\mathrm{d}\tau + R(t) \tag{10}$$

where

$$R(t) = A\int_0^{t_0} \left(\frac{\exp\left(-\dfrac{B}{t-\tau}\right)}{t-\tau} - \frac{\exp\left(-\dfrac{B}{t_1-\tau}\right)}{t_1-\tau} \right) Q(\tau)\mathrm{d}\tau, \quad t_1 \le t \le t_2 \tag{11}$$

The remainder term $R(t)$ can be estimated in the following way. Let

$$Q_{\rm ave}(t) = R(t)/A \int_0^{t_0} \left(\frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau} - \frac{\exp\left(-\frac{B}{t_1-\tau}\right)}{t_1-\tau} \right) {\rm d}\tau \quad (12)$$

so that $Q_{\rm ave}(t)$ is an effective injection rate for the interval $(0, t_0)$ evaluated at time $t \geq t_1$. Note, that $Q_{\rm ave}(t)$ in Eq. (12), in general, depends on t. Let $Q_0 = \max_{[t_1,t_2]} Q_{\rm ave}(t)$. Clearly, Q_0 does not exceed the maximal injection rate on $[0, t_0]$. From Eq. (12), we get:

$$R(t) \leq -AQ_0\left({\rm Ei}\left(-\frac{B}{t}\right) - {\rm Ei}\left(-\frac{B}{t-t_0}\right) - {\rm Ei}\left(-\frac{B}{t_1}\right) + {\rm Ei}\left(-\frac{B}{t_1-t_0}\right)\right) \quad (13)$$

Using estimates

$${\rm Ei}\left(-\frac{B}{t_1}\right) - {\rm Ei}\left(-\frac{B}{t}\right) = \int_{t_1}^{t} \frac{\exp\left(-\frac{B}{\eta}\right)}{\eta} {\rm d}\eta \leq \exp\left(-\frac{B}{t_2}\right)\frac{t_2-t_1}{t_1} \leq \frac{t_2-t_1}{t_1} \quad (14)$$

and

$${\rm Ei}\left(-\frac{B}{t_1-t_0}\right) - {\rm Ei}\left(-\frac{B}{t-t_0}\right) = \int_{t_1-t_0}^{t-t_0} \frac{\exp\left(-\frac{B}{\eta}\right)}{\eta} {\rm d}\eta \leq \exp\left(-\frac{B}{t_2-t_1}\right)\frac{t_2-t_1}{t_1-t_0} \quad (15)$$

we get the following inequality:

$$R(t) \leq AQ_0\left(\frac{t_2-t_1}{t_1} + \exp\left(-\frac{B}{t_2-t_1}\right)\frac{t_2-t_1}{t_1-t_0}\right) \quad (16)$$

Inasmuch as $t_2 - t_1 \ll t_1$, the principal term on the right-hand side of estimate (16) is the second one. In practically important cases, the exponential is close to one. Thus, the main criterion of error is the ratio $(t_2 - t_1)/(t_1 - t_0)$. For example, the relative error will be guaranteed not to exceed 10% if the matching interval $[t_1, t_2]$ is 10 times less than the preceding part of the whole observation interval $[t_0, t_1]$.

However, constraints exist on how small the interval $[t_1, t_2]$ can be. The measurements inevitably incorporate random errors. The impact of error is greater on a smaller interval than on a larger one for at least two reasons. First, for a smaller interval, we have fewer sample data points, and averaging of errors may be insufficient. Second, over a smaller interval the measurement error can be comparable or even larger than the variation of the pressure. Hence, changes in pressure would be unobservable. Also, in our selection of the interval $[t_1, t_2]$ we need to comply with the remarks in Section 5 (i.e. we must have $t_2 - t_1 \gg 4B$).

4. Modified solution

In the Section 3, we estimated the remainder term in Eq. (16). Here, we enhance our analysis and modify Eq. (10) by introducing an effective pre-test pumping rate parameter. As a consequence, the remainder term estimate (16) is improved. We emphasize that we use the same solution (5) to radial flow equation (1) that is usually employed in the analysis of a pumping test.

Note that the significance of accounting for the pumping prior to test was pointed out in earlier works (Barenblatt et al., 1990; Kulpin and Miasnikov, 1974). However, the distinctive feature of our approach is that we propose a constructive procedure including estimation of the effective pre-test injection rate parameter. Thus, we use the pre-test pumping rate as a fitting parameter and recover an estimate of its value.

To explain the idea, let us assume a constant injection rate Q_{-1} on the interval $[0, t_0]$. Then Eq. (9) can be formally transformed into the following:

$$p(t) = p(t_0) + AQ_{-1}\mathrm{Ei}\left(-\frac{B}{t_0}\right) - A\int_0^{t_0} \frac{\exp\left(-\frac{B}{t_0-\tau}\right)}{t_0-\tau}(Q(\tau)-Q_{-1})\mathrm{d}\tau + A\int_0^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)\mathrm{d}\tau \quad (17)$$

The last integral on the right-hand side of Eq. (17) can be rewritten as

$$\int_0^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)\mathrm{d}\tau = Q_{-1}\left(\mathrm{Ei}\left(\frac{B}{t-t_0}\right) - \mathrm{Ei}\left(-\frac{B}{t}\right)\right) + \int_0^{t_0} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}(Q(\tau)-Q_{-1})\mathrm{d}\tau + \int_{t_0}^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)\mathrm{d}\tau \quad (18)$$

Hence, from Eq. (17) we obtain

$$p(t) = p(t_0) - AQ_{-1}\mathrm{Ei}\left(\frac{B}{t-t_0}\right) + A\int_{t_0}^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)\mathrm{d}\tau + R_*(t) \quad (19)$$

where

$$R_*(t) = A\int_0^{t_0}\left(\frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau} - \frac{\exp\left(-\frac{B}{t_1-\tau}\right)}{t_1-\tau}\right) \times (Q(\tau)-Q_{-1})\mathrm{d}\tau - AQ_{-1}\left(\mathrm{Ei}\left(-\frac{B}{t}\right) - \mathrm{Ei}\left(-\frac{B}{t_0}\right)\right) \quad (20)$$

Similar to estimate (14), we obtain

$$\mathrm{Ei}\left(-\frac{B}{t_0}\right) - \mathrm{Ei}\left(-\frac{B}{t}\right) \le \frac{t_2-t_0}{t_0} \quad (21)$$

Hence, the last term in Eq. (20) is negligibly small as $t_0 \gg t_2 - t_0$. The magnitude of the first term on the right-hand side of Eq. (20) can be minimized by an appropriate choice of Q_{-1}. For instance, if injection rate $Q(t)$ is constant on the whole interval $[0, t_0]$, i.e. $Q(t) = Q_*$, then by putting $Q_{-1} = Q_*$ the first term in Eq. (20) cancels and the equation reduces to

$$R_*(t) = AQ_{-1}\left(\mathrm{Ei}\left(-\frac{B}{t_0}\right) - Ei\left(-\frac{B}{t}\right)\right) \quad (22)$$

If the injection rate is constant $Q(t) = Q_*$ only over some part $[t_{-1}, t_0]$ of the entire interval $[0, t_0]$, then by putting $Q_{-1} = Q_*$ and using an argument analogous to the one we applied to derive estimate (16), we obtain

$$R_*(t) \le AQ_0\left(\frac{t_2-t_1}{t_1} + \exp\left(-\frac{B}{t_2-t_1}\right)\frac{t_2-t_1}{t_1-t_{-1}}\right) \quad (23)$$

Here

$$Q_0 = \max_{0\le t\le t_{-1}} Q(t).$$

Clearly,

$$\max_{0\le t\le t_{-1}} Q(t) \le \max_{0\le t\le t_0} Q(t)$$

and $t_1 - t_{-1} \ge t_1 - t_0$, and, therefore the remainder term estimate (23) refines the earlier estimate (16). Thus, we replace Eq. (9) with an approximate equality

$$p(t) \approx p(t_0) + AQ_{-1}\mathrm{Ei}\left(-\frac{B}{t-t_0}\right) + A\int_{t_0}^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)\mathrm{d}\tau \quad (24)$$

In many practical situations, the hydraulic conductivity around the wellbore can be considerably different from that of the bulk formation. For example, the hydraulic conductivity around the well can be artificially increased by acidization (Bidaux and Tsang, 1991). To account for such a phenomenon in well-test analyses, the concept of skin effect was introduced (Earlougher, 1977). The skin effect is then usually handled by adding an additional pressure drop proportional to the injection rate into the solution (5), where the coefficient of the proportionality includes a dimensionless skin factor. For our modified solution (24), we obtain

$$p(t) \approx p(t_0) + AQ_{-1}\mathrm{Ei}\left(-\frac{B}{t-t_0}\right) + A\int_{t_0}^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau} Q(\tau)\mathrm{d}\tau + sAQ(t), \quad (25)$$

$$t_1 \leq t \leq t_2$$

where s is the skin factor.

In summary, given an injection rate $Q(t)$, the (downhole) pressures on the interval $[t_1, t_2]$ are controlled by five parameters: $p(t_0)$, Q_{-1}, A, B and s. In Section 6, we discuss how these parameters can be estimated.

5. Remarks on short-term transient effects

In our solution, we have assumed zero wellbore radius. It has been shown that in some situations, e.g. in slug test analysis (Karasaki, 1990) it is important to account for the finite wellbore radius. In such a case, the solution to the radial flow Eq. (1) is more complicated than the solution given by Eq. (5). It can be obtained through the inverse Laplace transform of a combination of Bessel functions (van Everdingen and Hurst, 1949).

A rigorous asymptotic analysis of the influence of wellbore radius on the distribution of pressures is performed in Barenblatt et al. (1990). In particular, it is demonstrated there that after a sufficiently long time, the pressure distribution does not depend on the wellbore radius and, therefore, we can use the asymptotic solution that coincides with the solution defined by Eq. (5). In our context, sufficiently large t means that $t \gg (\phi\mu c r_{\mathrm{w}}^2)/k$, where r_{w} is the wellbore radius. In our notations, this condition can be rewritten as $t \gg 4B$. Hence, if the characteristic time scale of injection rate variation is substantially larger than $4B$, then the solution represented by Eq. (5) is appropriate. In a review of real data from an injection well (Section 6), we have found that the coefficient B obtained as the result of data fitting was of the order of minutes. Thus, a characteristic injection variation time of a few hours is a reasonable lower bound. With this condition, in our analysis we can use intermediate asymptotics at $r_{\mathrm{w}} \to 0$. In other words, instead of dealing with the solution expressed through the inverse Laplace transform of a combination of Bessel functions, we can use a simpler solution from Eq. (5). The high quality of fitting real data verifies the validity of such approach.

Another important issue impacting the processing of early-time pumping data is wellbore storage effect. In a short-time transient pressure variation, the flow rate at the bottomhole and at the tubing head may differ because of the compressibility of the fluid in the tubing, afterflow, etc. The time scale over which the wellbore storage effect is important is estimated by Matthews and Russell (1967) to be several minutes.

Accounting for wellbore effects is especially important when the bottomhole pressures and injection rates are calculated from the measurements at the wellhead. The accounting for wellbore storage can be done straightforwardly using methods described in detail by Matthews and Russell (1967) and Earlougher (1977). In our studies here, we focus our attention on the bottomhole pressures and assume that they are already calculated, with due correction for wellbore storage effects, see Eq. (2.18) in Earlougher (1977).

6. Parameters estimation and a case example

In this section, our purpose is to estimate parameters A and B. We use Eq. (25) and propose an estimation procedure based on minimization of a quadratic matching criterion. Three other parameters in Eq. (25), Q_{-1}, $p(t_0)$ and s are also fitting parameters to be estimated by our procedure.

Mathematically, we are looking for the minimum of the functional

$$J = \frac{1}{2}\int_{t_1}^{t_2} w_{\mathrm{P}}(t)(p(t) - p_*(t))^2 \mathrm{d}t \quad (26)$$

where $p_*(t)$ is the pressure measured on $[t_1, t_2]$, $w_{\mathrm{P}}(t)$ is a positive weight function, and $p(t)$ is defined by Eq. (25). Although the measurements of pressures and pumping rate is available on entire interval $[t_0, t_2]$, the matching is performed over a smaller interval $[t_1, t_2]$, which we call the matching test interval. The weight function $w_P(t)$ usually reflects the reliability of the data obtained at different times. A higher weight value is assigned on those portions of the matching interval where the measurements are trustworthier. In an example below, we put $w_P(t)$ identically equal to one due to the uniform quality of the data set we analyzed. More sophisticated function $w_P(t)$ can be incorporated straightforwardly if needed.

The functional (26) depends on four parameters: $p(t_0)$, Q_{-1}, A, B and s through Eq. (25), i.e. $J = J(p(t_0), Q_{-1}, A, B, s)$. Although $p(t_0)$ is the pumping pressure at $t = t_0$ and its value is available, we keep it as an estimation parameter. We do so because any single measurement is subject to error, so that at $t = t_0$ the measurement produces not exactly $p(t_0)$, but $p(t_0) + \varepsilon$, where ε is a measurement error. If we substitute $p(t_0) + \varepsilon$ instead of $p(t_0)$ in Eq. (25), then this error ε will be present in the right-hand side for every t, therefore, it will affect the quality of fitting. Allowing a slight variation of $p(t_0)$ in the course of fitting the data over the entire matching interval can actually correct the measurement error at one particular point, because in this case the result will be affected by the average error, which is, presumably, negligible for a large data set.

Now we could straightforwardly apply a gradient descent method for minimization of criterion (26) depending on five scalar variables. Further analysis, however, allows us to considerably simplify the problem. Let us fix parameter B; then the minimization with respect to the other four parameters can be performed analytically. Let us denote

$$Z_1 = p(t_0), \qquad Z_2 = AQ_{-1}, \qquad Z_3 = A \qquad Z_4 = As \tag{27}$$

Clearly, given Z_1, Z_2, Z_3 and Z_4, we can determine $p(t_0)$, Q_{-1}, A and s, and vice versa. Substitution of Eq. (27) into Eq. (24) and then into Eq. (26) yields

$$J = \frac{1}{2}\int_{t_1}^{t_2} w_P(t)(Z_1 + g(B;t)Z_2 + \varphi(B;t)Z_3 + \psi(B;t)Z_4 - p_*(t))^2 \mathrm{d}t \tag{28}$$

where

$$g(B;t) = \mathrm{Ei}\left(-\frac{B}{t-t_0}\right),$$

$$\varphi(B;t) = \int_{t_0}^{t} \frac{\exp\left(-\dfrac{B}{t-\tau}\right)}{t-\tau} Q(\tau)\mathrm{d}\tau, \tag{29}$$

$$\psi(B;t) = Q(t)$$

For a fixed B, the functional (28) is quadratic with respect to Z_1, Z_2, Z_3, and Z_4. Consequently, its minimum can be explicitly calculated. Indeed, the necessary and sufficient condition for minimum with respect to Z_1 is obtained by equating to zero the derivative of the functional J in Eq. (28):

$$\int_{t_1}^{t_2} w_P(t)\mathrm{d}tZ_1 + \int_{t_1}^{t_2} w_P(t)g(B;t)\mathrm{d}tZ_2 + \int_{t_1}^{t_2} w_P(t)\varphi(B;t)\mathrm{d}tZ_3 + \int_{t_1}^{t_2} w_P(t)\psi(B;t)\mathrm{d}tZ_4 = \int_{t_1}^{t_2} w_P(t)p_*(t)\mathrm{d}t \tag{30}$$

The minima with respect to Z_2, Z_3 and Z_4 can be

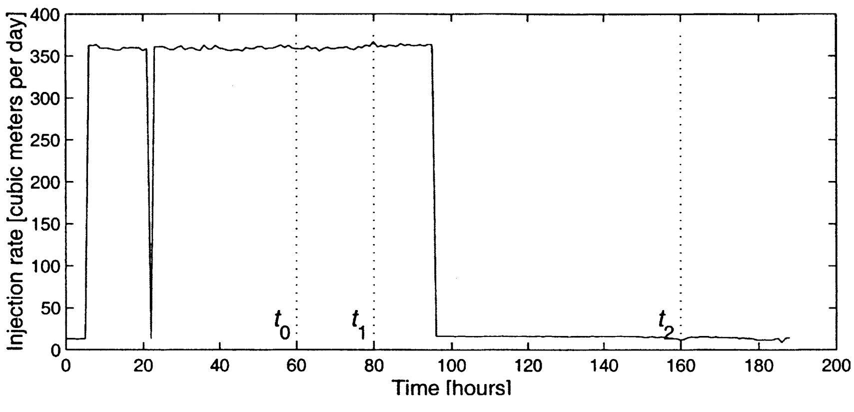

Fig. 6. Example of analysis of injection data: injection rate versus cumulative time.

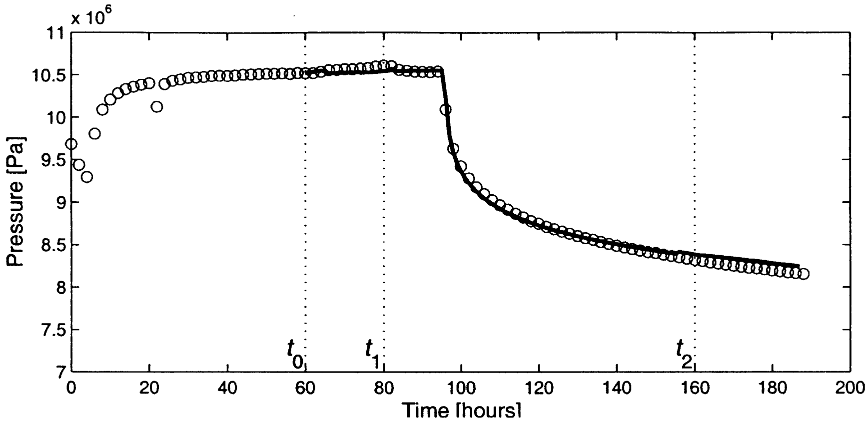

Fig. 7. Example of fitting pressure curve: the circles are the data points and the solid line is the fitting curve. The estimated skin factor is equal to −0.07.

analyzed in a similar way. Thus, we obtain a system of four linear equations similar to Eq. (30) with four unknown variables. Such a system can be solved analytically, for example, using the Cramer's rule.

As a result, we express Z_1, Z_2, Z_3 and Z_4 through B analytically: $Z_i = Z_i(B)$, $i = 1, 2, 3, 4$, and the criterion (28) reduces to a function of one variable

$$f(B) = \frac{1}{2}\int_{t_1}^{t_2} w_P(t)(Z_1(B) + g(B;t)Z_2(B) + \varphi(B;t)Z_3(B) + \psi(B;t)Z_4(B) - p_*(t))^2 \mathrm{d}t \quad (31)$$

which can then be minimized numerically via a simple procedure, such as a Golden Section method.

Note that the minimization procedure we propose here does not include an iterative gradient descent method and allows us to avoid dealing with stiffness of the problem coming from large variations in magnitudes of the coefficients.

Kulpin and Miasnikov (1974) proposed a procedure of estimating the storativity and transmissivity coefficients from analysis of a multiple flow rate test. Their procedure does not use logarithmic or Horner time scale and in this respect is similar to the one we described earlier. However, Kulpin and Miasnikov, in their calculations, neither include a parameter characterizing the pumping rate prior to the test, nor estimate the consequences of neglecting it. Moreover, in their calculations they assume that this rate is equal to zero (Kulpin and Miasnikov, 1974, p. 43). In our approach, the pre-test pumping rate is one of the fitting parameters and in Sections 3 and 4 we estimate how the calculations are affected if this parameter is ignored. The practical importance of this parameter is demonstrated on data from a real practical field case.

Let us consider an application of the method described earlier to the analysis of data from an injecting well in Ohio, USA. The computations have been performed using our code Operation Data Analysis (ODA), which incorporates the method described earlier.

The injection well is located in a formation consisting principally of finely crystalline dolomite, sandy and argillaceous dolomites. The confining zone formation consists of finely crystalline limestone. The wellbore perforation intervals in the well were located at depths between 1669 and 1764.5 m. The base of the packer was set at 1661.7 m depth. The perforation is in protective casing of 17.8 cm (7 in.) diameter. Core analysis of injection interval indicates porosity varying between 0.5 and 5%. The nearest other injection well is located at a distance of about 700 m.

The test was performed as follows. After a short shut-in of about 4.5 h, pumping was conducted at an approximately constant rate of 360 m^3/day for approximately 90 h except a short break at about 15 h after the pumping started. Then the well was shut in for approximately 100 h, see Fig. 6. Information about operations prior to the present test is not available. For our analysis, we selected only a part of

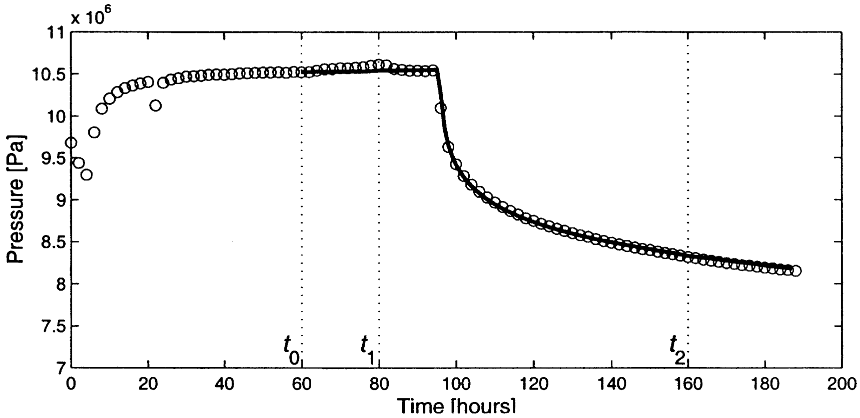

Fig. 8. Example of fitting pressure curve: the circles are the data points and the solid line is the fitting curve. The skin effect is neglected.

the data from about 90-h time interval starting after approximately 60 h since the beginning of data acquisition. Thus, in term of time elapsed from the beginning of measurements, we get $t_0 \approx 60\ h$, $t_2 \approx 150\ h$, and we selected $t_1 \approx 90\ h$. Due to this artificial truncation of data, we know that the injection rate was fluctuating between 354.3 and 362.5 m^3/day with a short rate temporary drop-down about 35 h before t_0. The injection pressure and rate were measured every minute. For our analysis, we selected only data points at intervals of about half an hour, so that the analyzed data file is much smaller than the entire set of measurements. We performed data fitting two times: assuming a skin effect and assuming zero skin effect. The results are presented, respectively, in Figs. 7 and 8. In both cases, the matching curve and data curve almost coincide over the entire time interval. The estimated skin factor is equal to -0.07, i.e. is very close to zero. Remarkably, the pre-test injection rate was estimated very close to the measured value at 360.3 m^3/day with account for skin effect and at 362.5 m^3/day with zero skin effect. This fact confirms the physical sensibility of the parameter we introduced. The estimated coefficients A and B, and consequently the transmissivity and storativity, are very close to each other in both cases. Moreover, variations of t_0, t_1 and t_2 did not lead to substantial changes of the results, although the storativity coefficient varied more than the transmissivity. Note, that for Horner plot analysis only a small interval of data at later time can be used.

Coefficient B, having dimension of time, was estimated at 5.5 min if skin effect is accounted for and at 8.4 min otherwise. The transmissivity estimate that came with the data was obtained by Horner plot analysis. It is estimated at 3.68 [d-ft/cp] with skin factor $s \approx -4.4$. Our estimate of transmissivity is 2.78 [d-ft/cp] with skin factor $s \approx -0.07$, Fig. 7. In kinematic transmissivity units, the results are 9.2 and 6.9 m^2/day, respectively. If the skin effect is ignored, we get $T = 2.6$ [d-ft/cp] (6.5 m^2/day; Fig. 8). We think that the difference between our estimates and the other estimate is the result of the high sensitivity of Horner plot method with respect to the selection of 'straight' segment of the curve, see Fig. 4. The result obtained by our method is accurately reproduced even with a wide range of choices of values of t_0, t_1 and t_2. Overall, the Horner analysis yields a significantly larger negative value of skin factor. The value of $s = -4.4$ from Horner analysis compared with ours $s \approx 0$ indicates probably an erroneous picture of the skin effect in the vicinity of the wellbore.

7. Conclusions

In this paper, we have revisited methods of well test analysis. We use regular pumping operation logs instead of specially designed tests in order to monitor the formation hydraulic properties in the vicinity of wellbore. In our analysis, we select a section of pumping data that we further divide into two parts: the observation interval and the matching or test interval. We emphasize the importance of appropriate accounting for the after-effect of the pumping done prior to the testing. For this purpose, in addition to traditional transmissivity and storativity coefficients, we introduce a new parameter which characterizes an effective pumping before the test and use this parameter in the data fitting procedure. Some phenomena usually attributed exclusively to wellbore storage and skin effects can also be explained at least partially through this parameter. Its importance is crucial for analyzing regular pumping operations because there is no shut-in period and, therefore, the immediate history of operations prior to the test period needs to be accounted for.

We demonstrate that accounting for pumping before the test may be important even in traditional well test analysis. Our method produces results, which are stable with respect to variations in selection of test period and will yield probably more accurate estimate of the skin parameter. Moreover, our approach requires a shorter test time interval and less frequent measurements on this interval.

In conclusion, the following procedure of estimating formation properties in the vicinity of the wellbore using operation data is proposed. First, a test interval in the recorded operations data has to be specified (cf. interval $[t_0, t_2]$ above; see Fig. 5). The choice is case-specific, dependent on the period of available data and on the frequency of sample points. In test examples, we have found that an interval of about 100 h with measurements performed every half an hour is good enough. We split the whole observation interval into two parts: the beginning phase and the test phase (intervals $[t_0, t_1]$ and $[t_1, t_2]$ above). This splitting is conventional rather than physical. The injection rate should not be constant over the entire observation interval since the functional (26) is almost insensitive to variations in parameters if there is no variation of injection rate. Of course, the range of pressure variation during the matching interval has to substantially exceed the measurement error.

A similar analysis can be applied to testing fractured wells, wells injecting or pumping from fissured formation, wells in bounded reservoirs, etc. We will investigate those cases in future publications.

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