



An efficient method of discharge measurement in tidal streams

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Abstract

The efficient method presented herein is based on the regularity of water flow in an open channel, which is maintained by nature at a constant ratio of mean to maximum velocities at a channel section. This ratio is a function of a parameter of a probability distribution that is equivalent to velocity distribution in the physical space. The maximum velocity can be determined quickly by measuring only a few velocities from a single vertical axis. As well as the ratio, the location of the sampling vertical axis on which the maximum velocity occurs tends to remain invariant with time. The mean velocity in a section can be rapidly determined by estimating the product of the maximum velocity multiplied by the ratio. The cross-sectional area of an open channel can be determined by the relation between gage height and area. Thus, the discharge in an open channel may be estimated as the product of mean velocity multiplied by the cross-sectional area. This efficient method can be used with any current meter to reduce the time and cost of discharge measurement in open channel flows under tidal effect. The available data of the Tanshui River downstream reach of which is in an estuarine area is used to illustrate the accuracy and reliability of the method. The results show that the simple but reliable method is capable of estimating discharge in tidal streams. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Discharge measurement; Entropy; Estuarine; Maximum velocity; Tidal streams

1. Introduction

One of the requirements for understanding the characteristics of a tidal stream (e.g. the ecosystem and transport of dissolved matter) is to have the knowledge of water flow. Obtaining knowledge on discharge can help the hydrologist to have better understanding of the hydrological processes in a tidal stream and to develop disaster mitigation programs. Unlike water flowing downstream in a river without

tidal effect, the flow direction of a large tidally effected river usually changes diurnally. Typically, water flows downstream during ebb tide, and upstream during flood tide. Because of the specific geological and geographic structure, the hydrological systems in open channel flows under tidal effect, which are unique with a continuous discharge under physical and chemical processes exhibited by the interaction of the fresh river water and the saline seawater. The most obvious factors, which have a profound influence on the discharge, include geological shape of the estuary, astronomical tide, wind, salinity, river discharge, storm surge and others. The shape of the estuary reflects on the formation of

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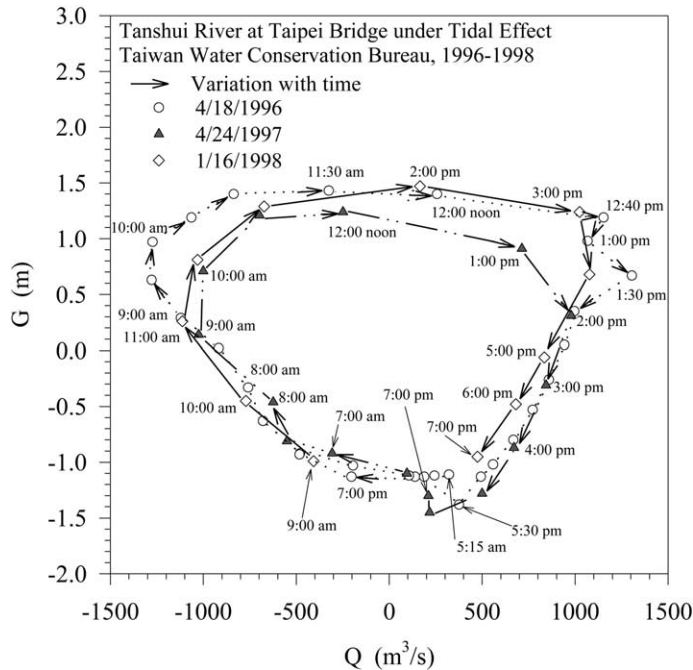


Fig. 1. Stage–discharge relation of the Tanshui River at the Taipei Bridge under tidal effects.

physical and chemical processes and in maintaining the hydrological processes (Mays, 1996). The astronomical tide, which is forced by the variation in gravitational forces resulting mainly from the orbital movement of moon and the sun, strongly dominates tidal river flow. The wind generates water-surface waves. The circulation of water is driven by the density differences, which depends on the salinity (Dyer, 2000). The drainage of fresh water from a watershed causes the salinity of tidal streams to be less effective than the open ocean and affects the hydrography of the estuary. Storm surges accompanied by tropical storms will drastically change the hydrography and make urban drainage difficult. Consequently, the hydrodynamic processes of tidal streams are manifestly complex.

Estimation of discharge, especially in tidal streams, is always one of the major issues of flood management in Taiwan. The water discharge and stage are needed as the boundary condition for flood forecasting after heavy rainfalls brought by tropical storms. The preferred method of measuring discharge in a large tidally affected stream is the moving-boat method (Rantz et al., 1983b; International Organiz-

ation for Standardization, 1979). However, the flow conditions of tidal stream are rarely steady or uniform that prevents us from estimations. It makes stage–discharge relations complex. Fig. 1 describes the time variations of water stage and discharge and the pattern of stage–discharge measured at the Taipei Bridge during each of the three measurement periods of 1996–1998. The positive and negative values of Q indicate the ebb and flood flow. Unlike the stream without tidal effects, the discharge in a tidal stream varies with tidal cycle, which hampers the measurement of discharge. Because the orbit of the moon about the earth is approximate 12 h, every day has two looped curves. The flow conditions of a tidal stream change from time to time, thus the individual conditions of a tidal stream have individual energy slopes. Each looped curve can only display an individual case. To establish the stage–discharge relation will therefore be difficult. Many of the concepts or principles derived from other water-courses have been applied for precise estimation of discharge. Both theoretical and empirical approaches, requiring a recording stage gage at each end of a long reach of stream, can be used to obtain the discharge in

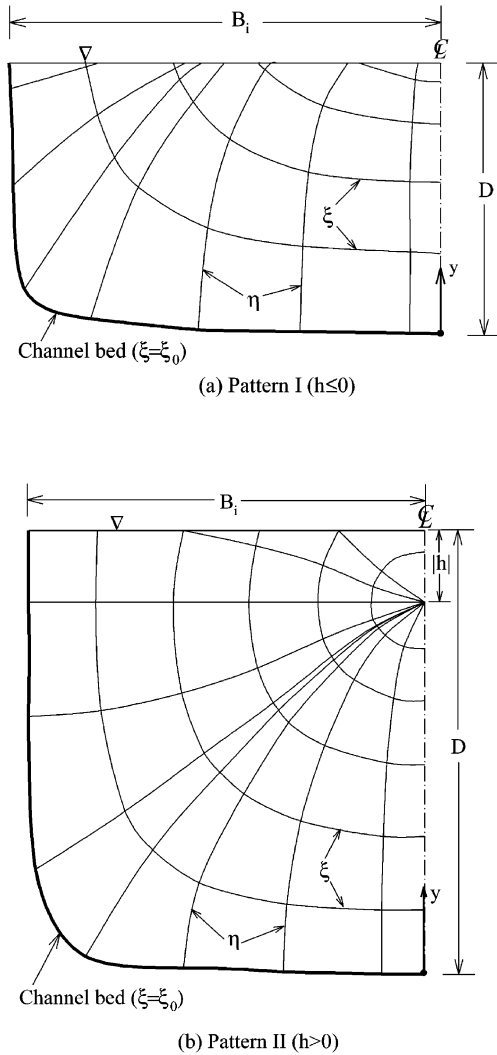


Fig. 2. Velocity distribution pattern in ξ - η coordinate system; (a) $h < 0$; (b) $h \geq 0$.

tidal streams (Rantz et al., 1983b). The theoretical approach is based on the following pair of differential equations:

$$\frac{Q^2}{K^2} = -\frac{\partial H}{\partial x} - \frac{1}{g} \frac{\partial \bar{u}}{\partial t} \tag{1}$$

$$\frac{\partial Q}{\partial x} = -B \frac{\partial d}{\partial t} \tag{2}$$

where Q is the discharge, K , the conveyance of the cross-section, H , the total energy head, x , the distance along the channel, g , the gravity, \bar{u} , the mean velocity of the cross-section, t , the time, B , top width of the channel, and d is the water-surface elevation. Four empirical methods including method of cubatures, rating-fall method, tide-correction method, and coaxial graphical-correlation method were also developed to rate tidal reaches (Parker et al., 1955; Pillsbury, 1956). One of the major disadvantages of using these methods is that the parameters are usually difficult to obtain from the observed data. In addition, enough discharge measurements during rises and subsequent recessions must be made at either ends of the reach to calibrate the parameters and to check on the computed discharge. In large tidally affected rivers and estuaries, it is time-consuming to measure discharge. Owing to the shortage of data, impracticality and unfeasibility, the application of such sophisticated methods in Taiwan has so far been unsuccessful.

Until recently, a key limitation in understanding flow in tidal streams is the difficulty and expense that is associated with using conventional methods. Discharge measurement in tidal streams has to be completed quickly, due to the fact that flow conditions vary rapidly. Although some new instruments have been applied to measure discharge in tidal areas (Grubbs and Pittman, 1997; Simpson and Bland, 2000); it is not yet convincingly accomplished. In order to measure discharge accurately in tidal streams, an efficient method is essential required, which should be quick, simple and requiring only a small number of velocity samples. The proposed method is based on the constant ratio of mean to maximum velocities (Chiu and Chen, 1998). The river tends to adjust all the factors that influence the flow of the cross-section, to allow the ratio of the mean and maximum velocities to remain a constant. Basically, this method applies the velocity area principle:

$$Q = \bar{u}A \tag{3}$$

where A is the cross-sectional area. In the following sections, the theories and formulae of estimating \bar{u} and A are described first. It is then implemented for estimating the discharge of the Tanshui River in Taiwan.

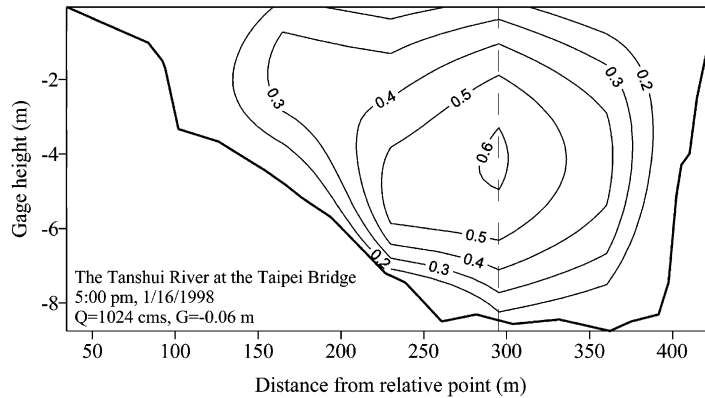


Fig. 3. Channel section, isovels and y-axis in the Tanshui River at the Taipei Bridge.

2. Relation between mean and maximum velocities in a cross-section

A velocity distribution equation, which can describe the maximum velocity occurring below the water surface, is derived (Chiu, 1989)

$$\frac{u}{u_{\max}} = \frac{1}{M} \ln \left[1 + (e^M - 1) \frac{\xi - \xi_0}{\xi_{\max} - \xi_0} \right] \quad (4)$$

where u_{\max} is the maximum velocity in a channel section, M , a parameter, ξ , the isovel in Fig. 2 (Chiu and Chiou, 1988), u , the velocity at ξ , ξ_{\max} and ξ_0 are the maximum and minimum values of ξ at which $u = u_{\max}$ and $u = 0$, respectively. The ξ - η coordinate system can describe the velocity distribution by a set of isovels (ξ), and allows the relation of u and ξ to be one to one. Thus points on an isovel have the same velocity. The set of isovels ξ along the y -axis, which is the vertical with maximum velocity in the cross-section, is

$$\xi = \frac{y}{D-h} \exp \left(1 - \frac{y}{D-h} \right) \quad (5)$$

where y is vertical distance from channel bed, D the water depth, and h indicates the location of u_{\max} . When u_{\max} occurs below water surface $h > 0$. If $h \leq 0$, u_{\max} occurs on the water surface.

Based upon the concept of probability, the probability of velocity less than u is the area of the isovel between ξ_0 and ξ divided by the total area.

$$\frac{\xi - \xi_0}{\xi_{\max} - \xi_0} = \int_0^u p(u) du \quad (6)$$

To derive the ratio of mean and maximum velocities of a cross-section, the concept of maximizing information entropy (Wu, 1997) is applied and the following two conditions needs to be satisfied.

$$\int_0^{u_{\max}} p(u) du = 1 \quad (7)$$

$$\int_0^{u_{\max}} up(u) du = \bar{u} \quad (8)$$

Therefore, \bar{u}/u_{\max} of a cross-section in an open channel flow can be derived as the following function of M :

$$\frac{\bar{u}}{u_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} = \Phi \quad (9)$$

The details of the solution from Eqs. (6)–(8) are given in Appendix A. Eq. (9) indicates that the relation between mean and maximum velocities in a cross-section is a linear relation passing through origin. It is a natural law of open channel flows that the ratio of the mean and maximum velocities of flow in a channel section is constant (Chiu, 1996). The ratio, Φ , characterizes the flow pattern at a channel section. Different cross-sections have different ratios. All factors of the flow system in a channel section affect the velocity distribution. The adjustment of any factor can change the velocity distribution (M , u_{\max} , and h), in order to keep the ratio to be constant. Natural factors, such as energy slope and channel roughness of the flow system allow the ratio to approach a constant; therefore, Φ of a cross-section is not affected by the discharge or the water level. Consequently, mean

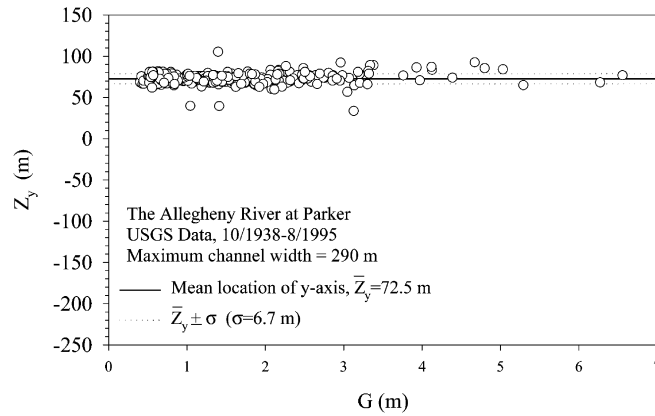


Fig. 4. Location of y-axis.

velocity can be estimated by maximum velocity and the constant ratio Φ .

3. Measurement or estimation of maximum velocity

Looking for the location of y-axis (z_y) in an open channel is difficult. Fortunately, the velocity used to determine the discharge reveals z_y . Fig. 3 shows isovels plotted by using velocity data collected by the Taiwan Water Conservancy Agency at the Taipei Bridge. Such a figure can be used to indicate z_y (dash line in Fig. 3). It is obvious that y-axis is located at the middle of the channel section if the channel is straight and has a symmetrical cross-sectional shape. In natural channel z_y is still very stable when the velocity does not approach zero. z_y does not vary with the fluctuation in discharge and water depth if the channel bed does not change drastically. Fig. 4 shows the relation of z_y to gage height. The data was collected by USGS at different discharges and gage heights during 1938 and 1995. It shows that z_y is fairly stable and invariant with time, gage height, and discharge.

A slight shift of y-axis will not have much effect on the maximum velocity (Chen, 1998). Therefore, u_{max} can be measured or estimated at the mean location of y-axis, denoted as \bar{z}_y , by any kind of current meter, such as Price AA and acoustic Doppler current profiler (ADCP). If many velocity samples are available, u_{max} can be determined by Eqs. (4) and (5) with regression method. If the velocities on y-axis are sampled by two-point method (Rantz et al.,

1983a), the following equations are used to determine u_{max} .

$$u_{0.2} = \frac{u_{max}}{M} \ln \left[1 + (e^M - 1) \frac{\xi_{0.2}}{\xi_{max}} \right] \tag{10}$$

$$u_{0.8} = \frac{u_{max}}{M} \ln \left[1 + (e^M - 1) \frac{\xi_{0.8}}{\xi_{max}} \right] \tag{11}$$

$$\bar{u}_y = \frac{u_{max}}{M} \int \ln \left[1 + (e^M - 1) \frac{\xi}{\xi_{max}} \right] d\xi \tag{12}$$

$$\xi_{0.2} = \frac{0.8D}{D-h} \exp \left(1 - \frac{0.8D}{D-h} \right) \tag{13}$$

$$\xi_{0.8} = \frac{0.2D}{D-h} \exp \left(1 - \frac{0.2D}{D-h} \right) \tag{14}$$

$$\bar{u}_y = \frac{u_{0.2} + u_{0.8}}{2} \tag{15}$$

where $u_{0.8}$ and $u_{0.2}$ are the velocities observed at 0.2 and 0.8 of the depth below the water surface.

4. Estimation of cross-sectional area

Sounding equipments, such as sounding weights and sonic sounder, are mechanically used to determine the water depths in the cross-section. The subsection area is then computed by two water depths and the width of the subsection. The cross-sectional area is the summation of the subsection areas. To determine the cross-sectional area by the conventional methods is time-consuming. However, both estimations of mean velocity of cross-section and cross-sectional area have to be done quickly when

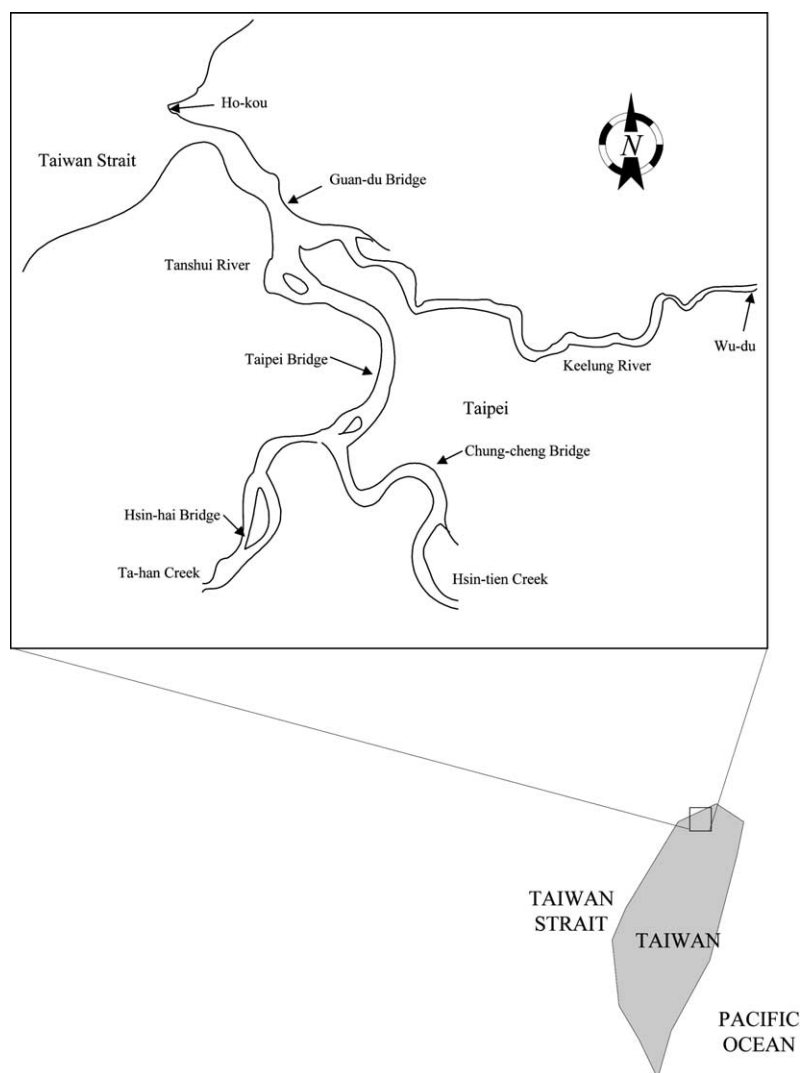


Fig. 5. Locations of the Tanshui River and study sites.

measuring discharge in a large tidally effected stream. For a stable channel without scour or sediment deposition, the cross-sectional area required in computing discharge can be determined by the stage–area relation, as

$$A = a(G - b)^c \quad (16)$$

where G is gage height of water surface; a , and c are coefficients that can be determined from the data; b is gage height of effective zero area. It is normally reliable and, also, easy to use since the cross-sectional area can be determined from the gage height.

5. Description of study sites

To illustrate the practical applications, the Tanshui River is considered. The study sites are the mouth of the Tanshui River at Taipei in the estuary. As shown on the location map in Fig. 5, the downstream reach of the Tanshui River is situated near the City of Taipei. It is the third largest river in Taiwan that drains 1183 km² into Taiwan Strait and its downstream reaches approximately 25 km from the river mouth when it is under tidal effects. The cross-sections studied, including the Taipei Bridge, the Guan-du

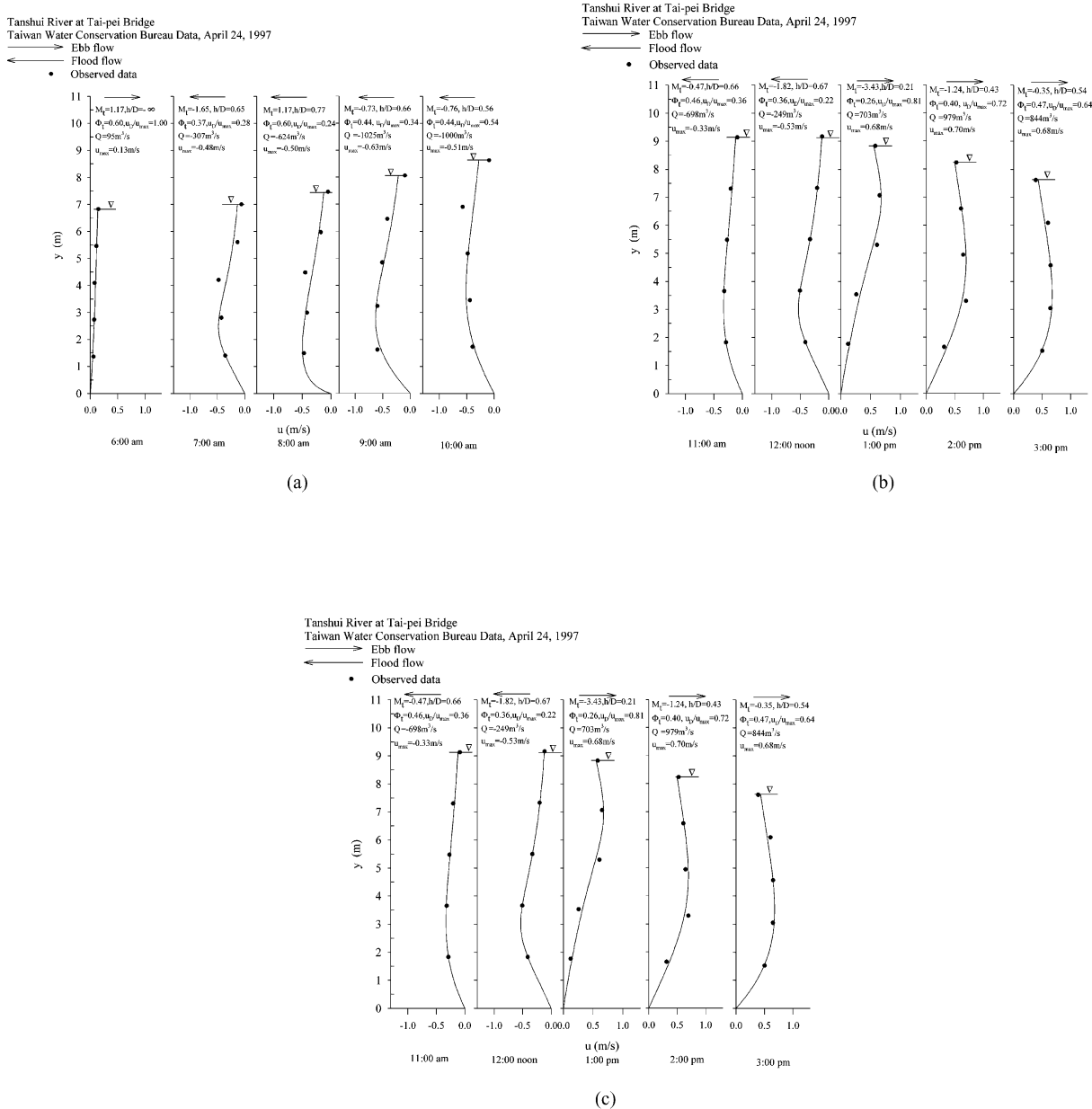
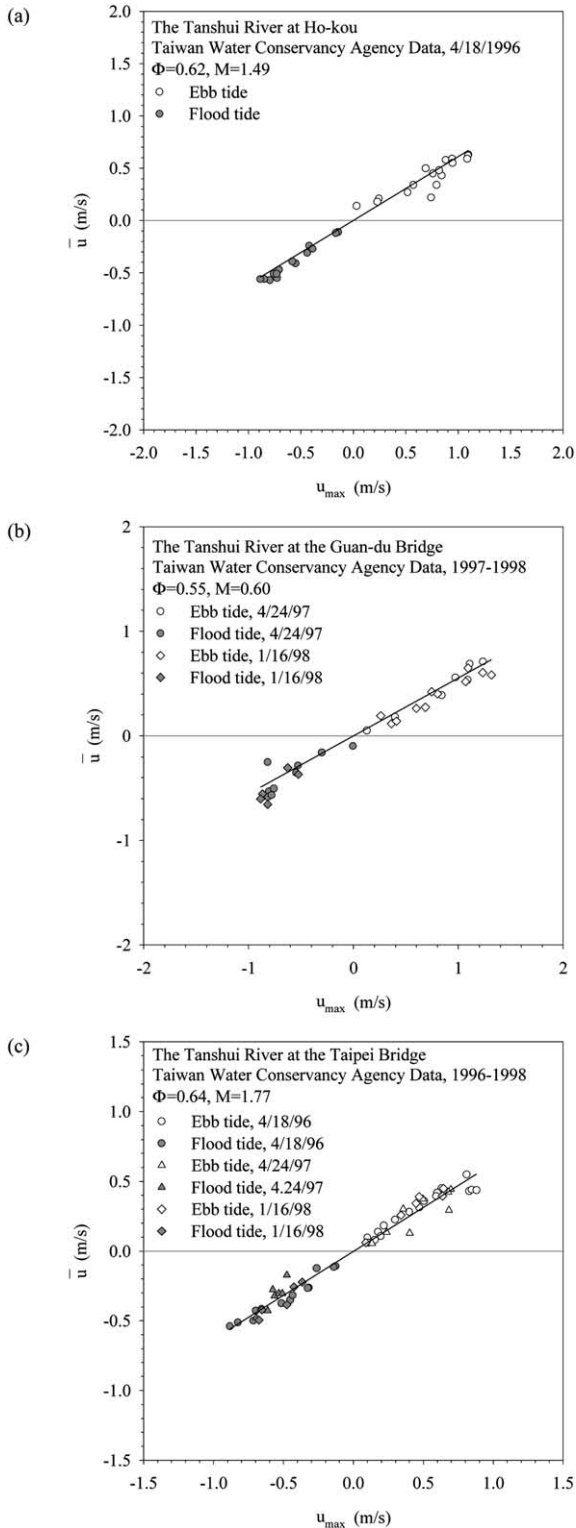


Fig. 6. Time variation of velocity profile in the Tanshui River at the Taipei Bridge; (a) 6:00 to 10:00 a.m.; (b) 11:00 a.m. to 3:00 p.m.; (c) 4:00 to 8:00 p.m.

Bridge, and Ho-kou, are a few hundred meters downstream from the bridges that go across the tidal stream. The stream is several hundred meters at the cross-sections. The Taiwan Conservancy Agency conducts discharge measurement only once a year in

different sites by using a conventional current meter method. A number of boats are spread out and situated across a river section. From each of these boats a current meter is lowered every 30 or 60 min to continuously measure as quickly as possible the



velocities of flow at five different points which are uniformly spaced over the entire depth. The vertical-velocity curve method (Bureau of Reclamation, 1997) is used to determine the mean vertical velocities. The geometrical shapes of the channel bed are measured using the sonic sounder method, and the gage heights are recorded by automatic water stage recorders. Then the midsection method (Herschey, 1999), which is the summation of the products of the segment areas of the stream cross-section multiplied by the mean velocities of the respective segment, is applied to calculate the discharges. Obviously, such a procedure is very difficult, labor intensive and costly. Fig. 6 shows the velocity profiles of the Tanshui River at the Taipei Bridge at various time points. The velocity profiles at 6:00 a.m. and between 1:00 and 7:00 p.m. were measured during the ebb tide. The flow direction changes at about 6:00 a.m., 12:00 noon, and 7:00 p.m., independently. The point velocities (circles in the figure) during the ebb tide can be as well as those during the flood tide adequately described by using Eq. (4) as the solid line in the figure. It also indicates the complexity of tidal flow and makes the discharge measurement difficult.

6. Method validation

Let Q_{obs} and Q_{est} denote the observed and estimated discharges and \bar{Q}_{obs} , \bar{Q}_{est} denote the mean of observed and estimated discharges, respectively. The performance of this method is cross-checked on correlation coefficient and root-mean-square error. The correlation coefficient, given by

$$\rho = \frac{\sum(Q_{obs} - \bar{Q}_{obs})\sum(Q_{est} - \bar{Q}_{est})}{\sqrt{\sum(Q_{obs} - \bar{Q}_{obs})^2\sum(Q_{est} - \bar{Q}_{est})^2}} \quad (17)$$

indicates the strength of association of observed and estimated discharges. It implies perfect matching when ρ is unity. The root-mean-square error for approaching for an assessment of the performance is

Fig. 7. $\bar{u} - u_{max}$ relation of the Tanshui River; (a) Ho-kou; (b) the Guan-du Bridge; (c) the Taipei Bridge.

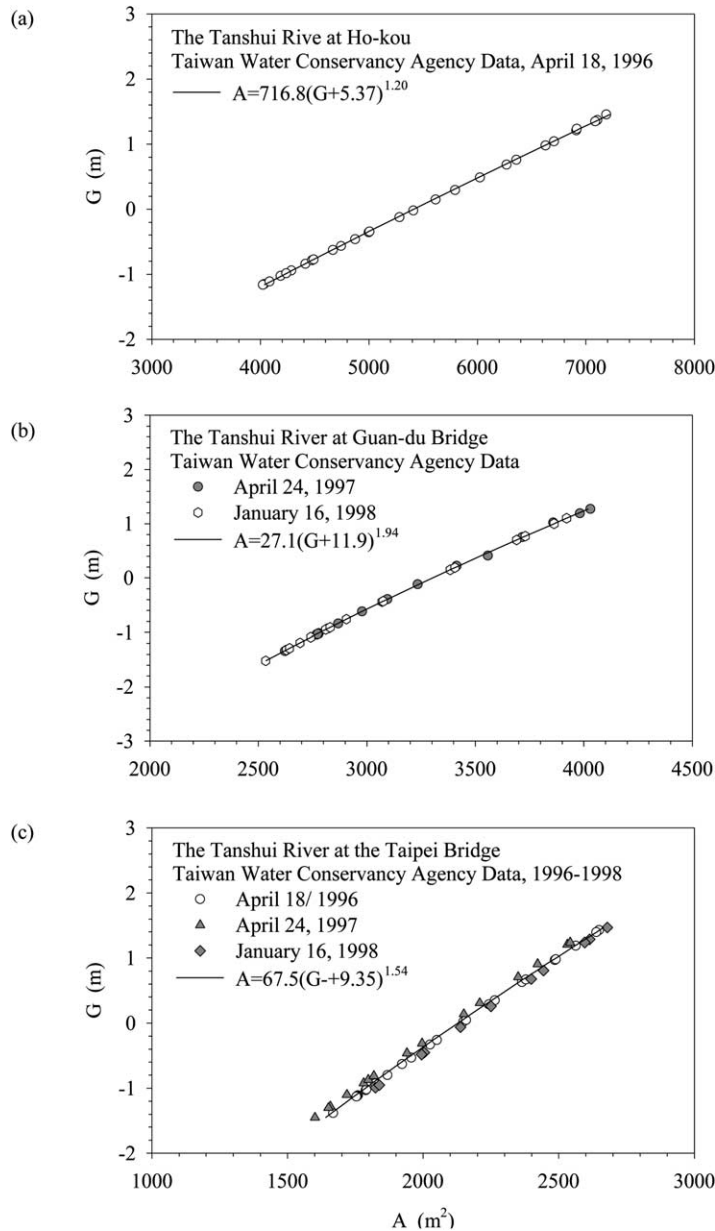


Fig. 8. G–A relation of the Tanshui River; (a) Ho-kou; (b) the Guan-du Bridge; (c) the Taipei Bridge.

given by

$$RMSE = \sqrt{\frac{\sum(Q_{obs} - Q_{est})^2}{N}} \quad (18)$$

where N is the number of observations. RMSE measures the closeness of the estimated and observed

discharges. The smaller RMSE is, the better the performance of the efficient method.

The available data on Ho-kou (1996), the Guan-du Bridge (1996–1997), and the Taipei Bridge (1996–1998) has been used in this study. Fig. 7 shows the relations between \bar{u} and u_{max} of the Tanshui River at Ho-kou, the Guan-du Bridge, and the Taipei Bridge.

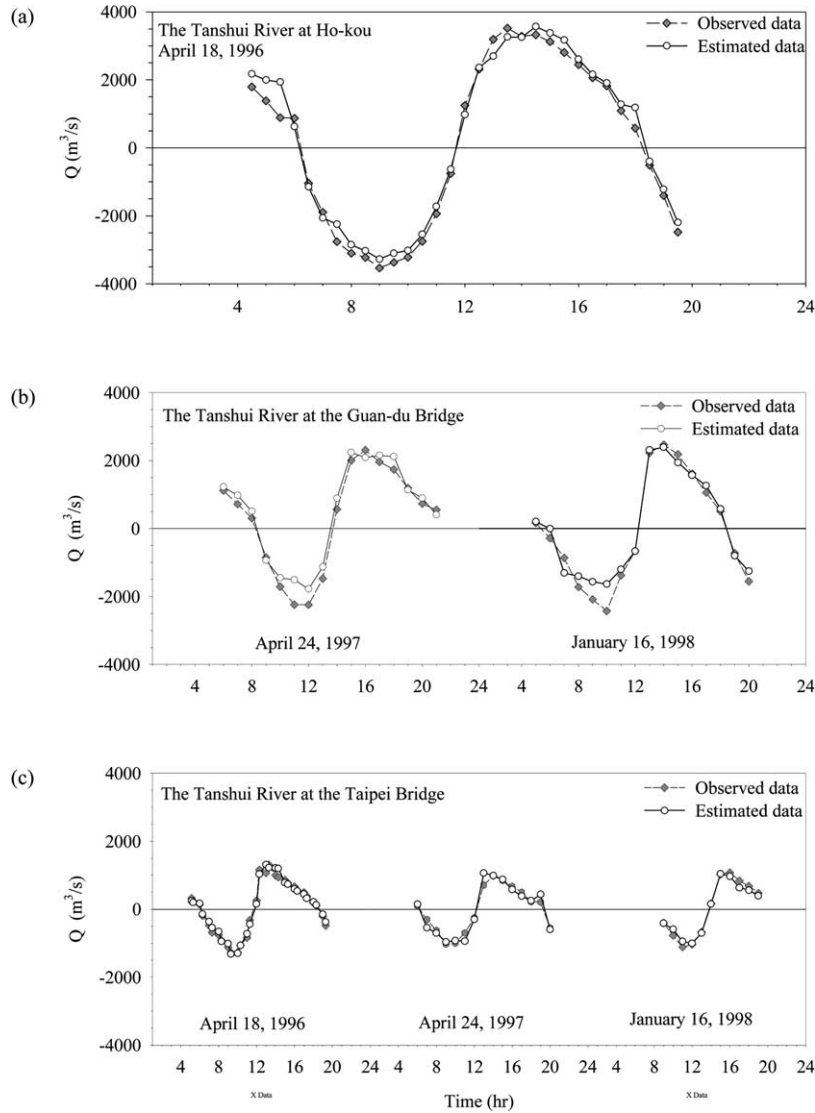


Fig. 9. Observed and estimated hydrographs of the Tanshui River; (a) Ho-kou; (b) the Guan-du Bridge; (c) the Taipei Bridge.

The positive and negative values of \bar{u} and u_{\max} depending on whether the flow is in ebb tide or flood tide periods reflect the flow directions. The u_{\max} is determined on the mean location of y -axis and the \bar{u} is obtained as Q_{obs}/A . Φ is fairly a constant and stable at 0.62, 0.55, and 0.64 for Ho-kou, the Guan-du Bridge, and the Taipei Bridge. The good linear relationship between \bar{u} and u_{\max} indicates Φ at a cross-section is a constant and stable in a wide range of discharges, water levels and sediment concentrations. The relationship between the cross-sectional area and the

gage height for the Tanshui River at Ho-kou, the Guan-du Bridge, and the Taipei Bridge are shown in Fig. 8. It shows that the cross-section areas of these three gaging stations can easily be determined by the gage heights. Thus, at Ho-kou, the Guan-du Bridge, and the Taipei Bridge the discharge can be determined as $0.62u_{\max}A$, $0.55u_{\max}A$, and $0.64u_{\max}A$, respectively. Fig. 9 shows the observed and estimated hydrographs of the Tanshui River and provides a visual impression of the performance of this method. The flows manifest themselves within a semidiurnal

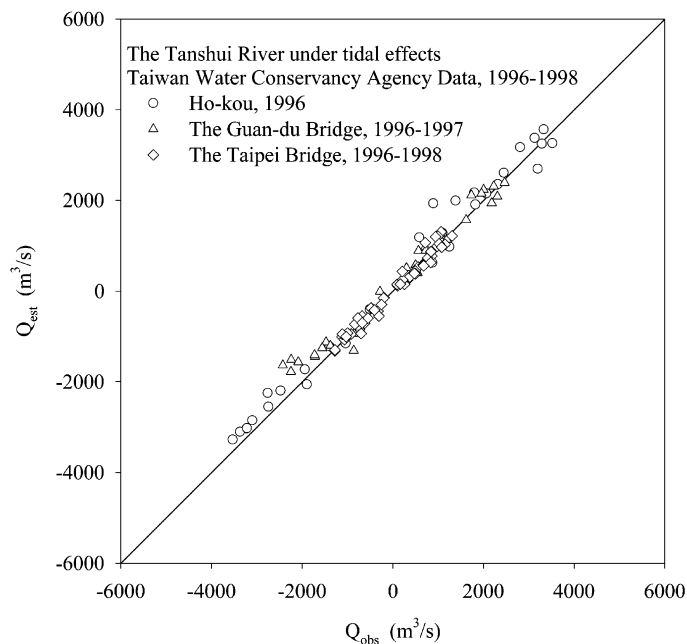


Fig. 10. Accuracy of estimated discharges in the Tanshui River under tidal effects.

period by a rise and a fall of the water stage and a reversal of the current direction. However, the observed hydrographs are well simulated. Accuracy and reliability of this method is exhibited in Fig. 10 that compares estimated discharges with those obtained by the conventional method. All the data points nicely fall onto the line of agreement. It indicates that the discharges estimated by this method agree quite well with the observed discharges. In addition, Table 1 summarizes the results and shows the efficient performance of this method. All correlation coefficients are very close to unity, and all

RMSEs are relatively smaller. It demonstrates that this method can be successfully applied to estimate discharges in tidal streams.

7. Summary and conclusions

In order to measure discharges accurately in a tidal stream, a quick and easy method is preferred. An efficient method to estimate discharge is based on the product of mean velocity of a cross-section multiplied by cross-sectional area. The ratio of the mean and

Table 1
Summary of results

	Ho-kou	Guan-du Bridge	Taipei Bridge
Maximum ebb flow (m ³ /s)	3517	2393	1305
Maximum flood flow (m ³ /s)	−3534	−1779	−1278
Maximum gage height (m)	1.45	1.27	1.47
Minimum gage height (m)	−1.16	−1.52	−1.45
Φ/M	0.62/1.49	0.55/0.60	0.64/1.77
a	716.80	27.10	67.50
b	5.37	11.90	9.35
c	1.20	1.94	1.54
ρ	0.993	0.987	0.987
RMSE (m ³ /s)	339	308	119

maximum velocities of a cross-section, which characterizes the flow pattern, approaches a constant. Therefore, the mean velocity can be obtained by the constant ratio and the maximum velocity. The maximum velocity can be determined by the velocity distribution equation with a few velocity samplings. Owing to the location of maximum velocity occurring stably, it can be sampled at the mean location of y-axis. This proves that with the present method the mean velocity can be determined quicker than using the conventional methods. The cross-sectional area can be easily estimated from the relationship between gage height and area. The method presented herein is faster with more accuracy for discharge measurement in tidal streams. It drastically reduces the time and cost of discharge measurement with preciseness. It's most important utility will be in tidal streams, for which data are scarce and hence essentially needed for practical purposes such as flood forecasting and design of flood control structures. In addition, discharge measurement in tidal steams can be automated and hence now appears to be more realistic. The data of the Tanshui River under tidal effects is used to demonstrate the proposed method. The results provide evidence that this efficient method can offer good performance in measuring discharge in tidal streams.

Acknowledgments

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Appendix A

In the present study, the solution of Eq. (9) is based upon maximizing information entropy. Mathematically, information entropy is expressed as (Shannon, 1948)

$$H(x) = - \sum_j p(x_j) \ln p(x_j) \tag{A1}$$

which represents the average information content. The probabilistical information entropy is defined as

$$H(u) = - \int_0^{u_{\max}} p(u) \ln p(u) du \tag{A2}$$

in which $p(u)$ is density function that must satisfy the following condition:

$$\int_0^{u_{\max}} p(u) du = 1 \tag{A3}$$

Also, $p(u)$ must satisfy the following condition:

$$\int_0^{u_{\max}} up(u) du = \bar{u} \tag{A4}$$

If the entropy is maximized subject to the constraints in Eqs. (A3) and (A4) by the method of Lagrange multipliers, the probability density function can be obtained as

$$p(u) = e^{(\lambda_1 - 1)} e^{\lambda_2 u} \tag{A5}$$

in which λ_1 and λ_2 are coefficients. Substitution of Eq. (A5) into Eq. (A3) yields

$$\int_0^{u_{\max}} e^{(\lambda_1 - 1)} e^{\lambda_2 u} du = 1 \tag{A6}$$

or

$$e^{\lambda_1 - 1} = \frac{\lambda_2}{e^{\lambda_2 u_{\max}} - 1} \tag{A7}$$

By defining $M = \lambda_2 u_{\max}$ and substituting Eq. (A7) with Eq. (A5), the probability density function can be expressed as:

$$p(u) = \frac{M}{u_{\max}(e^M - 1)} e^{(M/u_{\max})u} \tag{A8}$$

The cumulative distribution function of u can be obtained as

$$\int_0^u p(u) du = \frac{e^{M(u/u_{\max})} - 1}{e^M - 1} \tag{A9}$$

From Eq. (A9) and $0 \leq \xi \leq 1$, the velocity distribution equation can be obtain as:

$$\frac{u}{u_{\max}} = \frac{1}{M} \ln \left[1 + (e^M - 1) \frac{\xi - \xi_0}{\xi_{\max} - \xi_0} \right] \tag{A10}$$

Substitution of Eq. (A8) into Eq. (A4) gives

$$\int_0^{u_{\max}} u \frac{M e^{M(u/u_{\max})}}{u_{\max}(e^M - 1)} du = \bar{u} \quad (\text{A11})$$

or

$$\frac{\bar{u}}{u_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} = \Phi \quad (\text{A12})$$

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