Earth and Planetary Science Letters 202 (2002) 49–60
EPSL
www.elsevier.com/locate/epsl

Mode coupling in a viscoelastic self-gravitating spherical earth induced by axisymmetric loads and lateral viscosity variations

Patrick Wu *

Department of Geology and Geophysics, The University of Calgary, Calgary, AB, Canada T2N 1N4

Received 21 February 2002; received in revised form 8 May 2002; accepted 1 June 2002

Abstract

The importance of mode coupling in a viscoelastic self-gravitating spherical earth with lateral variations in linear or nonlinear rheology is investigated with a finite-element model coupled to Poisson's equation. Both the lateral viscosity variations and the harmonic load are axisymmetric. The effects of self-gravitation, viscosity contrast and the location of the abrupt lateral change in viscosity in a linear mantle are studied and are found to be significant in determining the strength of mode coupling. It is demonstrated that a larger number of harmonics than generally assumed is required to give accurate description of the induced deformation. Mode coupling is also found to be important in a nonlinear mantle especially when the harmonic degree l is large.

Keywords: glacial rebound; Mantle dynamics; rheology; viscoelasticity; lateral heterogeneity

1. Introduction

It is well known that for the case of a non-rotating spherically symmetric earth, a harmonic load induces deformation of the same harmonic degree l and order m. However, in the presence of lateral viscosity variation or nonlinear rheology (as we shall show below), not only is degree l and order m excited – modes of other degrees and orders are excited as well. This is called mode coupling.

In the spectral formulation of loading problems with lateral viscosity variation or nonlinear rheology, it is important that the amplitudes of the coupled modes be taken into account properly, otherwise the shape and amplitude of the induced deformations cannot be computed accurately (see discussion of Fig. 7 below). However, mode coupling makes the spectral formulation of loading problems more complex because the amplitudes of all the coupled modes are required as 'known' in the formulation and thus the problem must be solved iteratively [1]. For example, D'Agostino et al. [2] solved the surface loading problem with the iterative approach which assumes that the radial dependence of viscosity at mid-mantle is much larger than the lateral variations. Their computation is for a non-self-gravitating earth where lateral viscosity variation is due to a lithospheric craton and mode coupling was assumed to exist only up to a maximum order of 10. However, it is not clear how many modes are involved in the coupling, what causes the modes to couple and

* Tel.: +1-403-220-7855; Fax: +1-403-284-0074.
E-mail address: ppwu@ucalgary.ca (P. Wu).

0012-821X/02/$ – see front matter
PII: S0012-821X(02)00750-1

what affects the strength of coupling. The purpose of this paper is to study mode coupling in the modeling of the glacial isostatic adjustment (GIA) process.

What motivates this study is the recent interest in mapping lateral heterogeneities of the mantle and in understanding how lateral viscosity variations affect the observations of GIA. Seismic tomography [3,4] reveals that lateral variation of S-wave velocity is about 4% near the surface but decreases to about 2% below 400 km depth. These lateral velocity variations can be due to lateral changes in temperature, chemical composition, non-isotropic pre-stress [5] or some combination of them. Since mantle creep law also depends on the chemistry, the pressure and is thermally activated, lateral velocity variations imply lateral viscosity variations. It has been estimated that the observed 2% variation in S-wave velocity may imply viscosity variations of about 2–4 orders of magnitude (e.g. [6,7])! Recently, a number of studies have investigated the effects of lateral viscosity variation on the observations of GIA [2,6–15]]. However, except for the more recent works [14,15], the earth models used are flat or non-self-gravitating and only Kaufmann and Wolf [11] studied mode coupling. Using sinusoidal lateral viscosity variation in a flat earth, Kaufmann and Wolf [11] found that mode coupling involves only two wave numbers – one corresponding to the load and the other to the lateral viscosity variation – with the magnitude of the latter mode being dependent on the magnitude of lateral viscosity variation. However, sinusoidal variations in viscosity are unrealistic. For a sharp change in lateral viscosity, one can theoretically estimate how many modes are coupled – but the Fourier transform of a step function is complex. Thus, it is unclear how many modes are involved or what are their relative strengths if there is a sharp change in lateral viscosity especially when the earth is spherical and self-gravitating. Recently, spectral models that can handle lateral viscosity variations in a self-gravitating spherical earth have been developed (e.g. [14,15]), but a study of mode coupling with a self-gravitating earth has not been published.

Since Kaufmann and Wolf [11] only used linear rheology in their study, it is unclear what will happen when rheology in the mantle becomes nonlinear. Creep experiments of mantle rocks indicate that both linear (diffusion) and nonlinear (dislocation) creep laws can operate in the mantle. Depending on mantle conditions, either can become the dominant creep mechanism (e.g. [16,17]). Recent modeling of the GIA process in and around Laurentia [18–22] indicates that the creep law in the top 300 km of the upper mantle and in the lower mantle may be nonlinear. The important point is that lateral viscosity variations can be induced if mantle rheology is nonlinear [1]. This is because the effective viscosity for nonlinear rheology is dependent on the stress level (see Eq. 1 below), and when the surface (or internal) load magnitude changes laterally, so does the stress level and effective viscosity. Thus, for a harmonic load with degree l and order $m = 0$ (axisymmetric load), one would expect that the induced lateral viscosity variation in a uniform nonlinear mantle should also be described by the same harmonic degree l and order m. Because of this, it is unclear from the work of Kaufmann and Wolf [11] whether there is any mode coupling when mantle rheology is nonlinear.

A related question is whether the poloidal and toroidal modes are coupled in the GIA process. For geoid studies with lateral viscosity variations, these modes are found to be coupled [1,23]. For GIA studies with an axisymmetric load, D'Agostino et al. [2] assumed that there is no excitation of the toroidal and non-zonal poloidal modes. Although this assumption is reasonable for a linear earth [24], its validity does not appear to have been tested for nonlinear rheology.

In the present study, mode coupling on a spherical, self-gravitating, viscoelastic earth due to axisymmetric harmonic loads is studied. Both linear and nonlinear rheologies will be considered. Wu [22] has demonstrated that the effect of self-gravitation (SG) is important, especially for nonlinear rheology. Thus SG will be turned on or off in this paper to study its effects on mode coupling. Unlike the studies by Kaufmann and Wolf [11] and D'Agostino et al. [2], where the spectral theory is employed and the amplitude of lateral viscosity variation cannot be arbitrarily large, the finite element (FE) method is used here. The advantage of the FE method is that lateral viscosity variations can be arbitrarily large. Furthermore, there is no need to assume that there is no excitation of the toroidal and non-zonal poloidal modes, or to set a limit to the maximum order of coupling (although in reality the maximum degree and order resolved by the FE grid is dependent on the number of FEs on the spherical surfaces, which is limited by the computer resources available). Furthermore, the FE method is suited for the study of nonlinear rheology because the principle of superposition (assumed in the spectral method or linear perturbation theory) no longer applies. However, the inclusion of SG in a non-spectral FE code is not trivial. Recently, however, I have successfully coupled Poisson's equation to the FE code (see Wu [25] for details).

The purpose of this study is: (1) to study the effects of SG on mode coupling; (2) to verify that the excitation of the toroidal and non-zonal poloidal modes is negligible when mantle rheology is linear or nonlinear provided that the load and lateral viscosity variation are both axisymmetric; (3) to study the physical cause for the modes to become coupled together; (4) to determine the extent of mode coupling when there is lateral viscosity variation; (5) to demonstrate that there is mode coupling when mantle rheology is nonlinear.

2. The model

A FE earth model coupled with Poisson's equation is used for the calculations here [25]. The earth model is assumed to be a non-rotating, spherical, self-gravitating, incompressible, Maxwell earth. The rheology in the mantle is assumed to be described by the steady state creep law: $\partial_t \varepsilon_{ij}{}^{C} = A^* \sigma'^{\,n-1}_{E} \sigma'_{ij}$ where ε^{C}_{ij} are the ijth components of the creep strain, A^* is the creep parameter, n is the stress exponent, taken to be 3 for power-law and 1 for linear rheology. Here, $\sigma'_{E} = \sqrt{(1/2)\sigma'_{ij}\sigma'_{ij}}$ is the equivalent deviatoric stress, σ'_{ij} are the deviatoric stress components and the effective viscosity is given by [26]:

$$\eta_{\text{eff}} = \frac{1}{3A * \sigma'^{(n-1)}_{E}} \tag{1}$$

For nonlinear rheology, Eq. 1 shows that large stress levels will lead to small effective viscosities. Since the stress induced by surface load varies in both space and time (e.g. figure 4 in Wu [26]), the effective viscosity for a nonlinear mantle will be laterally heterogeneous and time dependent. In our treatment of nonlinear rheology, we shall neglect the interaction between load-induced stress and the ambient tectonic stress since the strain magnitude due to postglacial rebound is orders of magnitude smaller than that due to tectonics [27].

All the earth models consist of a uniform 100 km thick elastic lithosphere overlying a stratified viscoelastic mantle and an inviscid fluid core. The elastic structure of all our models is given by Model SG5 (Table 1).

Two FE models are used in this study. The first is a 3D spherical model, which is composed of 20 layers. Each layer consists of two rows of 72 six-node elements around the poles (one row at the north pole and one at the south pole) and 34×72

Table 1
Elastic structure of Model SG5

Layer	Radius of the top (km)	Density (kg m^{-3})	Rigidity (GPa)	Gravity (m s^{-2})
Lithosphere	6371	4120	73	9.71
Upper mantle	6271	4120	95	9.66
Transition zone	5950	4220	110	9.57
Lower mantle	5700	4508	200	9.51
Core	3480	10925	0	10.62

eight-node elements in between the polar elements. This model is used to verify that the excitation of the toroidal or non-zonal poloidal mode is negligible for linear and nonlinear rheology. However, even for this coarse spatial resolution (5°), the model is computationally very intensive. Thus, after the verification that the motions are axially symmetric with no toroidal and non-zonal poloidal modes, the second FE model, which is computationally more efficient, is used for the rest of the paper. This second model is an axisymmetric spherical model, which gives a spatial resolution of 0.5°. The details of the models can be found in [25].

Except for Fig. 7, where an axisymmetric uniform disc load is used, the surface loads used for the rest of the paper are all harmonic loads with order $m=0$ (axisymmetric load). These loads are applied on the earth's surface and left there, so the responses computed are Heaviside responses. To implement these loads on a FE grid, the average load over the surface of the elements is applied (i.e. the load is integrated over the surface of the FEs and divided by the surface area).

The validity of the FE method is demonstrated in Fig. 1, where the computed Heaviside responses for a degree $l=2$ harmonic load over a uniform linear mantle and inviscid fluid core are compared to the analytical solutions. The agreement between the numerical and the analytical methods is well within 1%. This is adequate for our present purpose. If better accuracy is required, finer elements with better spatial resolutions are needed, but that would put more demand on computer resources.

In the following, lateral viscosity variations in both linear and nonlinear mantles will be considered. For nonlinear rheology, the mantle is assumed to be uniform and lateral viscosity variations are completely induced by the load. For linear rheology, two axisymmetric models with sharp viscosity contrasts will be used. The first model, used with harmonic loads, is shown in Fig. 2a. Note that there is no radial variation in mantle viscosity below the lithosphere and mantle viscosity from the north pole to co-latitude ψ has $\eta_1=10^{21}$ Pa s, while mantle viscosity from ψ to 180° is higher and has value η_2. The second model, to be used with a uniform disc load of 15° angular radius is shown in Fig. 2b. Below the lithosphere, this earth model has a high viscosity ($\eta_2=10^{22}$ Pa s) root which extends to depth D underneath the load. Outside the high viscosity root, the viscosity of the mantle is $\eta_1=10^{21}$ Pa s. For both earth models, mode coupling induced by lateral viscosity variations has large effects on the radial and tangential displacements and the potential perturbations. However, for the purpose of this paper, only the radial displacements will be considered.

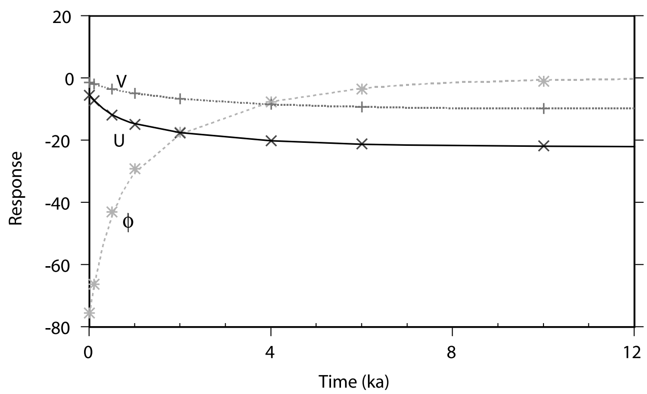

Fig. 1. Comparing theoretical (lines) to numerical results (symbols) for a Heaviside $l=2$ harmonic load over a self-gravitating two-layer earth (with uniform mantle over uniform inviscid fluid core). U, V, ϕ are the harmonic coefficients for radial displacement, tangential displacement (units in m) and gravitational potential perturbations (units in $(m/s)^2$) at the earth's surface, respectively.

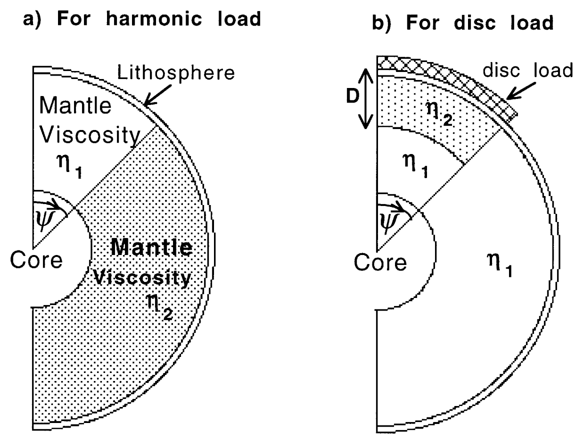

Fig. 2. (a) An axisymmetric linear rheology model where mantle viscosity from the north pole to co-latitude ψ has value $\eta_1 = 10^{21}$ Pa s, while mantle viscosity from ψ to 180° is higher and has value η_2. There is no radial variation of mantle viscosity and the model is used for harmonic loads. (b) An axisymmetric linear rheology model with high viscosity ($\eta_2 = 10^{22}$ Pa s) root underneath the load. The root extends to depth D. Outside the high viscosity root, $\eta_1 = 10^{21}$ Pa s. Elastic structures of both models are given by Model SG5 in Table 1.

3. Results

When the 3D spherical FE model is used to compute the deformation in the linear model (Fig. 2a) with axisymmetric lateral viscosity variation ($\psi = 45°$ and $\eta_2 = 10^{23}$ Pa s), it is found that the toroidal and non-zonal poloidal components of displacements are so small that they are within numerical uncertainties. The same is true when nonlinear rheology is employed. Thus, the axisymmetric earth model is used for the rest of the calculations.

Next, the radial displacements of the linear earth model (Fig. 2a) due to $l = 2$ and 5 harmonic loads are calculated with the axisymmetric FE earth model with and without the inclusion of SG for $\psi = 45°$ and 90° and $\eta_2 = 10^{23}$ Pa s. The amplitudes of these harmonic loads are obtained from the decomposition of a point load with a mass of 2.9×10^{19} kg. The radial displacements are then decomposed into their harmonic components and their spectral peaks are plotted in Fig. 3. An inspection of this figure shows that the elastic deformations ($t = 0$) due to these loads produce no mode coupling because the elastic structures of these earth models are laterally homogeneous. However, as time t increases and the viscoelastic mantle starts to relax, the radial response is able to detect the lateral viscosity variation, thus mode coupling becomes more and more important. If we normalize the radial displacement spectrum of Fig. 3 with the amplitude at l, the harmonic degree of the load, then we will find that the relative amplitudes at neighboring degrees l' will grow rapidly in time during the first 1 ka. However, this growth in relative amplitudes stopped soon after 1 ka, so that the normalized amplitudes for both the SG and NSG curves at, say, 4 and 15 ka are about the same. Comparing the curves with and without the inclusion of SG in Fig. 3, one sees that the effect of SG is significant only at l, the harmonic degree of the load.

In Figs. 4 and 5, the origin of mode coupling and the effects of ψ on mode coupling are investigated for $\eta_2 = 10^{23}$ Pa s (see Fig. 2a) and $\psi = 45°$, 90°, 135° and 180° (the last one corresponds to a laterally homogeneous mantle). In Fig. 4, a degree $l = 2$ and $m = 0$ harmonic load is applied at the surface while in Fig. 5 the harmonic load has $l = 5$ and $m = 0$. In Figs. 4a and 5a, the normalized radial displacements at $t = 15$ ka are plotted as a function of co-latitude θ. The radial displacement curves for the laterally homogeneous model are purely harmonic, with the same degree as the surface load. With lateral heterogeneity introduced, Figs. 4a and 5a show that the deformation over the high viscosity part of the mantle is delayed and thus has its amplitude reduced at $t = 15$ ka. For example, in Fig. 4a the model with $\psi = 135°$ (dashed line) gives almost the same displacement as the laterally homogeneous earth for $\theta < 135°$. But outside, the displacements are reduced by almost 50%. Clearly, to represent the radial deformation of the laterally heterogeneous earth, harmonic degrees other than that for the load are required. This gives us a physical understanding of the origin of mode coupling in laterally heterogeneous earth models. If this is the physical cause of mode coupling, then one can understand why an axisymmetric load will not induce the toroidal and non-zonal poloidal components. This is because a poloidal load only induces poloidal dis-

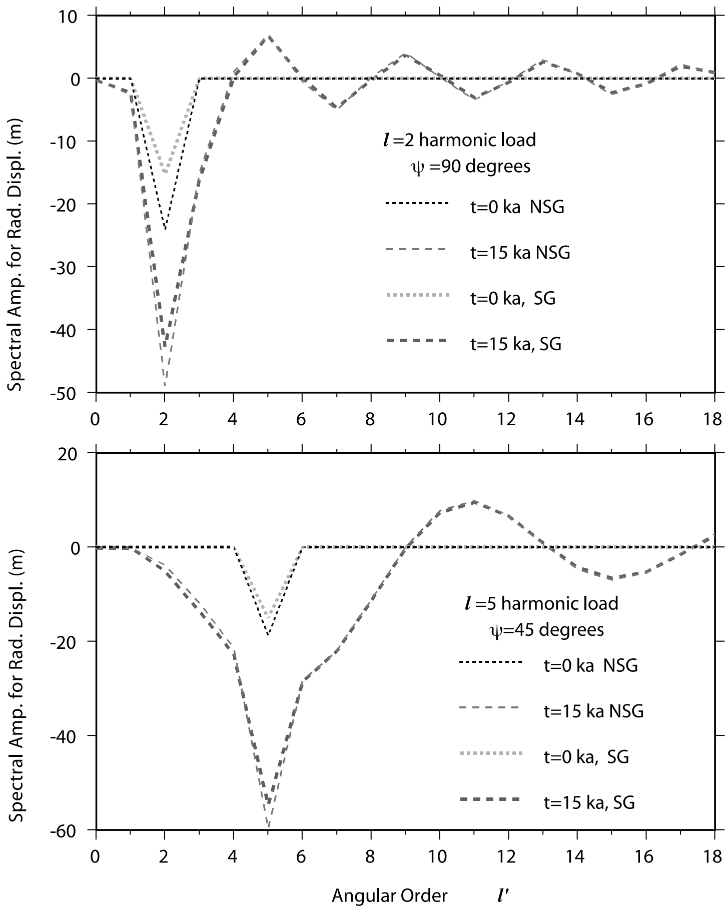

Fig. 3. Spectral amplitude of the radial displacement at $t=0$ and 15 ka after the application of a harmonic load with degree $l=2$ (top) and $l=5$ (bottom). Curves labeled SG have self-gravitation included but those labeled NSG do not include self-gravitation. Two earth models with different ψ (see Fig. 2a) are used.

placements and an axisymmetric load only induces axisymmetric deformations if the lateral viscosity variation is also axisymmetric [24]. However, if lateral viscosity variation is not axisymmetric, then one can expect that the coupling involves both harmonic degree l and order m because the deformation requires all these components to describe it.

In Figs. 4b and 5b, the amplitude spectrum is normalized by the amplitude at l, the harmonic degree of the load. These figures show that mode coupling at $l=5$ is stronger than that at $l=2$. For example, in Fig. 4b with $\psi=135°$, the largest coupling amplitude is about 13% at $l'=3$, but for Fig. 5b and $\psi=135°$, the largest coupling amplitude is 25% at $l'=6$.

Figs. 4b and 5b also show that mode coupling is dependent on the angle ψ. For large ψ, mode coupling is strongest near the degree of the load l but it decreases rapidly away from l. For example, in Fig. 5b where $\psi=135°$, the amplitudes at $l'=4$ and 6 are 21% and 25%, respectively, but for $l'>11$, mode coupling decreases to less than 5%. For smaller ψ, mode coupling becomes even more important. For example, with $\psi=45°$, the largest coupling amplitude in Fig. 5b is about 52% at $l'=6$ and at $l'=15$, the amplitude is still about 12%.

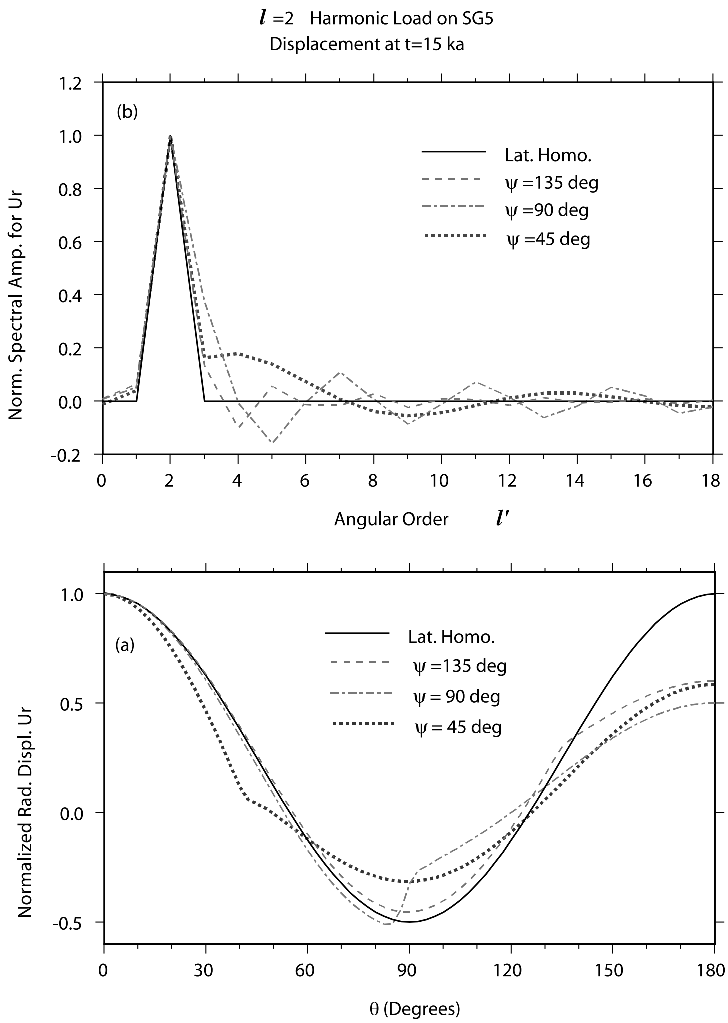

Fig. 4. (a) Normalized radial displacements of the four earth models with different ψ (see Fig. 2a; note that the laterally homogeneous earth corresponds to $\psi = 180°$) are plotted as a function of co-latitude θ. (b) Normalized spectral amplitude of the radial displacement at $t = 15$ ka after the application of a degree $l = 2$ harmonic load.

As pointed out in Section 1, D'Agostino et al. [2] assumed that mode coupling exists only up to degree 10. In Fig. 6, the effect of neglecting the coupling of the higher degree modes on the accuracy of the solution is investigated. Here the earth model of Fig. 2a with $\psi = 45°$ is loaded by a degree five harmonic load. The solid line is the actual solution at $t = 15$ ka (same as the one in Fig. 5), while the dashed and the dotted lines are computed by coupling the first 10 and 16 modes, respectively. An inspection of Fig. 6 shows that the neglect of the higher degree modes results in a loss of accuracy of the computed solution. Even with 16 modes, the computed solution still differs from the actual solution by more than 50% within the range $30° < \theta < 60°$.

The effect of viscosity contrast on mode coupling is shown in Fig. 7. There, the load has harmonic degree $l = 5$, the angle ψ in the earth model (see Fig. 2a) is fixed at 45°, and η_2 is taken to be 10^{21}, 10^{22}, 10^{23} and 10^{24} Pa s. As the viscosity contrast increases, the spectral amplitudes for the $l = 5$ component increases relative to the other harmonics. Thus, as demonstrated in Kaufmann and Wolf [11], mode coupling becomes more important as the viscosity contrast increases. But beyond a factor of 100 increase in viscosity, the changes are not as significant at this time ($t = 15$ ka) so that the curve for 1000 stays close to the curve for 100. Of course, given enough time for the mantle to relax, all the models give the same amplitude spectrum.

Up to now, only harmonic loads have been considered because they show quite plainly the effects of mode coupling. However, harmonic loads are not realistic and it is not clear whether the presence of other harmonics in a real load will affect mode coupling. Furthermore, a viscosity contrast that extends throughout the mantle (Fig. 2a) is also not realistic. Thus, we conclude our discussion on mode coupling with linear rheology by considering the more realistic case shown in Fig. 2b. Here the angular radius of the load and of the high viscosity root ($\eta_2 = 10^{22}$ Pa s) is $\psi = 15°$. The values of D considered are 0 km (i.e. laterally homogeneous case), 420 km and 2891 km (in the latter case, the root extends to the core–mantle boundary). Outside the high vis-

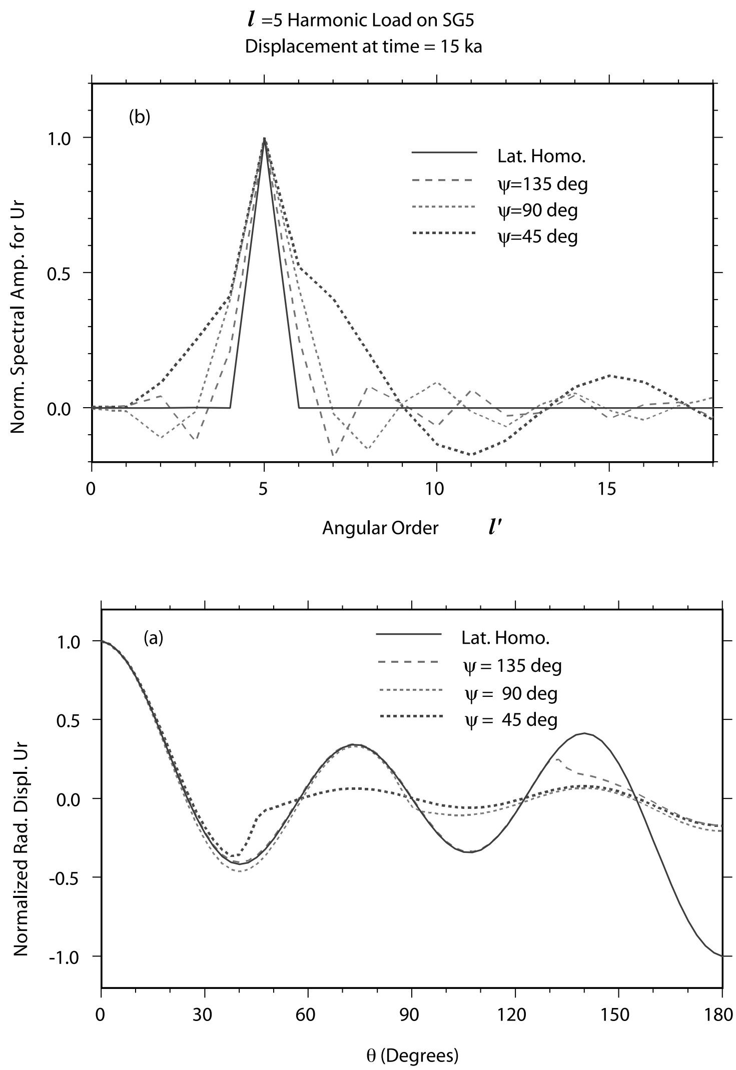

Fig. 5. Same as Fig. 4 except that the load has harmonic degree $l = 5$.

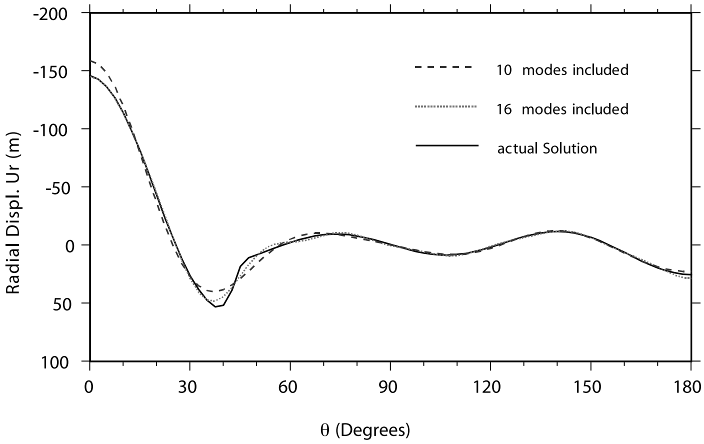

Fig. 6. The effect of mode truncation on the accuracy of the solution is studied for the earth model in Fig. 2a that has $\psi = 45°$ which is forced by a degree $l = 5$ harmonic load. The solid line is the actual solution, while the dashed line is computed with coupling of 10 modes and the dotted line includes 16 modes.

cosity root, the viscosity of the mantle is $\eta_1 = 10^{21}$ Pa s. In Fig. 8, the amplitude spectrum of the load and the surface displacements at 15 ka after Heaviside loading are plotted. Now a positive load induces a negative radial displacement, thus, for comparison purposes, the spectral amplitude of the load is multiplied by negative 10^{-4} before being plotted in Fig. 8. A comparison of this modified load spectrum (chain dash in Fig. 8) with the radial displacement on a laterally homogeneous mantle (solid line) shows that they are in phase with each other (the peaks, troughs and zeros match each other), indicating that there is no mode coupling. The spectral amplitudes of the radial displacement are dependent on the viscosity structure of the mantle and the reduction in amplitude at large angular degree l is due to the presence of the lithosphere. In the presence of lateral viscosity variations, mode coupling causes the spectral amplitude of the displacements to get

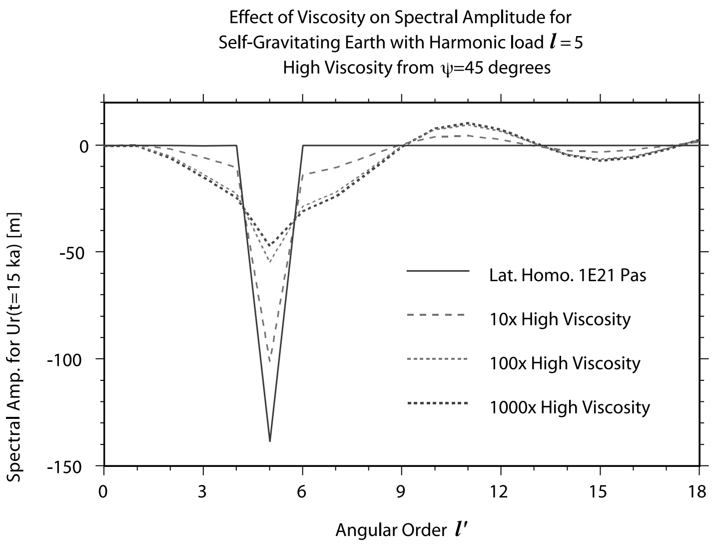

Fig. 7. Effect of viscosity contrast on the normalized spectral amplitude of the radial displacements at $t = 15$ ka when the load has harmonic degree $l = 5$. The earth model used is the same as that in Fig. 2a and has $\psi = 45°$.

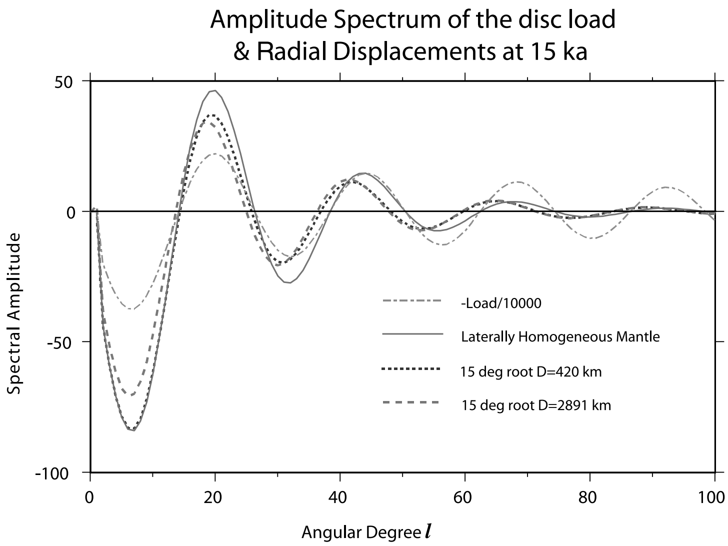

Fig. 8. Amplitude spectrum of the load and the radial displacements at $t = 15$ ka for the models in Fig. 2b. The angular radius of the disc load and the high viscosity root is $\psi = 15°$. The high viscosity root has thickness D and the values of D considered are $D = 0$ (laterally homogeneous case), 420 and 2891 km.

out of phase with the modified load spectrum. With the high viscosity root extending down to 420 km (dotted line), Fig. 8 shows that mode coupling (phase shift) becomes important for angular degree greater than about $l = 30$. Below this angular degree, the long wavelength deformations 'see' deeply into the mantle and thus become less affected by the shallow root. For $D = 2891$ km, even the long wavelength deformations become affected – their amplitudes decrease due to the high viscosity root and the phase shift at low angular degree l increases. Thus, we see that mode coupling remains important even for a disc load over a shallow high viscosity root.

Finally, we investigate the effects of nonlinear rheology on mode coupling. The creep law in the uniform mantle is taken to be $A^* = 3 \times 10^{-35}$ Pa^{-3} s^{-1} and $n = 3$ (to be consistent with the sea-level

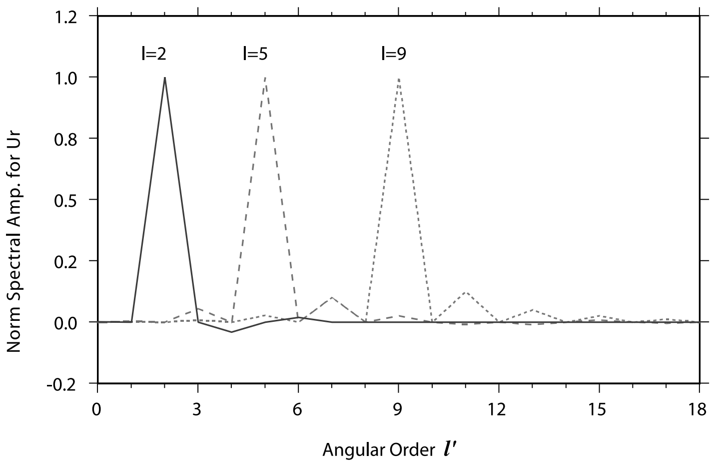

Fig. 9. Normalized spectral amplitude of the radial displacement at $t = 15$ ka for a nonlinear mantle with creep parameter $A^* = 3 \times 10^{-35}$ and $n = 3$. Three harmonic loads with degree $l = 2$, 5 and 9 are used.

curves found around Laurentia [19]). Lateral variations in effective viscosity are induced by the load. The normalized spectral amplitudes are plotted in Fig. 9. Again, the deformation only contains components with $m = 0$. For harmonic load $l = 2$, only components with even degrees (l') are nonzero. Mode coupling for degree 2 is not very strong – the amplitude at degree $l' = 4$ is only 4% of the amplitude at $l = 2$, and the amplitude of the higher modes decreases so rapidly that they are totally negligible for $l' > 10$. For odd harmonic loads $l = 5$ or 9, only components with odd degrees are nonzero. The even harmonics are zero because the lateral viscosity is induced by the load, which has odd harmonics. For $l = 5$, the amplitude of degree $l' = 7$ is 10% while that for degree $l' = 3$ is 5%, thus there is asymmetry of amplitude at about $l = 5$. For $l = 9$, the coupling increases to 12% for $l' = 11$ and 10% for $l' = 7$. Thus we see that with nonlinear rheology, mode coupling becomes more important as l increases. Comparing the curves with and without the inclusion of SG for the nonlinear case, it is found that just like Fig. 3, the effect of SG is significant only at l, the harmonic degree of the load.

4. Conclusion

In this study, an axisymmetric harmonic load with degree l is applied to a spherical, self-gravitating viscoelastic earth whose lateral viscosity variation is also axisymmetric. It is found that:

1. The excitations of the toroidal and non-zonal poloidal modes are completely negligible. This is because the lateral viscosity variation is axially symmetric, so that only modes with $m = 0$ are excited. However, if the lateral viscosity varies in the azimuth direction, it is predicted that modes with $m > 0$ will be excited.
2. SG is found to have a strong effect on the amplitude of the harmonic degree l deformation, but the effects of SG on the other coupled modes are found to be small.
3. For the linear rheology model of Fig. 2a, mode coupling is dependent on the harmonic degree of the load l, the location of the viscosity contrast (angle ψ in Fig. 2a) and the magnitude of the viscosity contrast. For models with small ψ angles, it is important to allow for mode coupling up to a larger number of harmonics than just 10 (D'Agostino et al. [2]). This also means a large coupling matrix in Normal Mode theory [14]. Mode coupling remains important for a uniform disc load over a high viscosity root (Fig. 2b).
4. For uniform mantle with nonlinear rheology, mode coupling is not very important for $l = 2$ but its effect becomes important as l (the angular degree of the load) increases. Furthermore, a load with even harmonics will couple with other even harmonics and a load with odd harmonics will couple with other odd harmonics.

Acknowledgements

I would like to thank an anonymous reviewer of [22] for raising the issue of mode coupling and thus initiating this study. Dr. Bingzhu Wang helped me set up the 3D FE model for this study. I am also indebted to Drs. Bert Vermeersen, Zdenek Martinec, Detlef Wolf and Georg Kaufmann for their constructive criticism on the manuscript. The FE calculation was performed with the ABAQUS package from Hibbitt, Karlsson and Sorensen Inc. This research is supported by an Operating Grant from NSERC of Canada.***[RV]***

References

[1] S. Zhang, U. Christensen, Some effects of lateral viscosity variations on geoid and surface velocities induced by density anomalies in the mantle, Geophys. J. Int. 114 (1993) 531–547.

[2] G. D'Agostino, G. Spada, R. Sabadini, Postglacial rebound and lateral viscosity variations: a semi-analytical approach based on a spherical model with Maxwell rheology, Geophys. J. Int. 129 (1997) F9–F13.

[3] W.-J. Su, A.M. Dziewonski, Simultaneous inversion for 3-D variations in shear and bulk velocity in the mantle, Phys. Earth Planet. Inter. 100 (1997) 135–156.

[4] M. Ishii, J. Tromp, Normal-mode and free-air gravity constraints on lateral variations in velocity and density of Earth's mantle, Science 285 (1999) 1231–1236.

[5] F.A. Dahlen, J. Tromp, Theoretical Global Seismology, Princeton University Press, Princeton, NJ, 1998.
[6] P. Gasperini, R. Sabadini, Lateral heterogeneities in mantle viscosity and postglacial rebound, Geophys. J. 98 (1989) 413–428.
[7] P. Wu, Z. Ni, G. Kaufmann, Postglacial rebound with lateral heterogeneities: from 2D to 3D modeling, in: P. Wu (Ed.), Dynamics of the Ice Age Earth: A Modern Perspective, Trans Tech. Publ., Switzerland, 1998, pp. 557–582.
[8] P. Gasperini, R. Sabadini, Finite element modeling of lateral viscosity heterogeneities and post-glacial rebound, Tectonophysics 179 (1990) 141–149.
[9] C. Giunchi, G. Spada, R. Sabadini, Lateral viscosity variations and postglacial rebound: effects on present-day VLBI baseline deformation, Geophys. Res. Lett. 24 (1997) 13–16.
[10] G. Kaufmann, P. Wu, Lateral asthenospheric viscosity variations and postglacial rebound: a case study for the Barents Sea, Geophys. Res. Lett. 25 (1998) 1963–1966.
[11] G. Kaufmann, D. Wolf, Effects of lateral viscosity variations on postglacial rebound: an analytical approach, Geophys. J. Int. 137 (1999) 489–500.
[12] G. Kaufmann, P. Wu, G. Li, Glacial isostatic adjustment in Fennoscandia for a laterally heterogeneous earth, Geophys. J. Int. 143 (2000) 262–273.
[13] G. Kaufmann, P. Wu, D. Wolf, Some effects of lateral heterogeneities in the upper mantle on postglacial land uplift close to continental margins, Geophys. J. Int. 128 (1997) 175–187.
[14] J. Tromp, J.X. Mitrovica, Surface loading of a viscoelastic planet – III. Aspherical models, Geophys. J. Int. 140 (2000) 425–441.
[15] Z. Martinec, Spectral-finite element approach to three-dimensional viscoelastic relaxation in a spherical earth, Geophys. J. Int. 142 (2000) 117–141.
[16] S. Karato, Defects and plastic deformation in olivine, in: S. Karato, M. Toriumi (Eds.), Rheology of Solids and of the Earth, Oxford University Press, Oxford, 1989, pp. 176–208.
[17] G. Ranalli, Inferences on mantle rheology from creep laws, in: P. Wu (Ed.), Dynamics of the Ice Age Earth: A Modern Perspective, Trans Tech. Publ., Switzerland, 1998, pp. 323–340.
[18] S. Karato, P. Wu, Rheology of the upper mantle: a synthesis, Science 260 (1993) 771–778.
[19] P. Wu, Modeling postglacial sea-levels with power law rheology and realistic ice model in the absence of ambient tectonic stress, Geophys. J. Int. 139 (1999) 691–702.
[20] P. Wu, Postglacial induced surface motion and gravity in Laurentia for uniform mantle with power-law rheology and ambient tectonic stress, Earth Planet. Sci. Lett. 186 (2001) 427–435.
[21] P. Wu, Effects of stress exponent in mantle flow law on postglacial induced surface motion and gravity in Laurentia, Geophys. J. Int. 148 (2002) 676–686.
[22] P. Wu, Effects of nonlinear rheology on degree 2 harmonic deformation in a spherical self-gravitating earth, Geophys. Res. Lett. 29 (2002), 10.1029/2001GL014109.
[23] M.A. Richards, B.H. Hager, Effects of lateral viscosity variations on long wavelength geoid anomalies and topography, J. Geophys. Res. 94 (1989) 10229–10313.
[24] Z. Martinec, D. Wolf, Gravitational viscoelastic relaxation of concentrically nested spheres, Geophys. J. Int. 138 (1999) 45–66.
[25] P. Wu, Using commercial finite element codes for the computation of Glacial Isostatic Adjustment for an incompressible, spherical, self-gravitating viscoelastic earth, in preparation.
[26] P. Wu, Deformation of an incompressible viscoelastic flat earth with power-law creep: a finite element approach, Geophys. J. Int. 108 (1992) 35–51.
[27] S. Karato, Micro-physics of post glacial rebound, in: P. Wu (Ed.), Dynamics of the Ice Age Earth: A Modern Perspective, Trans Tech. Publ., Switzerland, 1998, pp. 351–364.