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Strange attractors in magmas: evidence from lava flows

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Abstract

Magma mixing structures from three different lava flows (Salina, Vulcano and Lesbos) are studied in order to assess the possible chaotic origin of magma mixing processes. Structures are analysed using a new technique based on image analysis procedures that extract time series that are representative of the relative change in composition through the structures. These time series are then used to reconstruct the attractors underlying the magma mixing process and to calculate the fractal dimension of the attractors. Results show that attractors exist and possess fractional dimensions. This evidence suggests that the mixing of magmas is a chaotic process governed by a low number of degrees of freedom. In addition, fractal dimension analyses allows us to discriminate between different regimes of mixing in the three lava flows. In particular our analyses suggest that the lava flow of Salina underwent more turbulent mixing than the lava flows of Lesbos and Vulcano.

Keywords: Magma mixing; Time series; Strange attractors; Chaotic dynamics

1. Introduction

The past three decades have seen an increasing research on chaotic dynamics and many applications have appeared in recent years (e.g. Turcotte, 1992; Abarbanel, 1996). Chaotic dynamics have been observed and analysed in a wide variety of experimental and natural contexts (e.g. Cvitanovic, 1984; Crilly et al., 1993), but the application of concepts and methods of chaos theory to igneous processes is scarce (e.g. Flinders and Clemens, 1996; Hoskin, 2000; Perugini and Poli, 2000).

In this paper, we use principles of chaos theory to study the processes of mixing in magmas. In particular we show that structures produced by magma mixing in some lava flows outcropping on the islands of Lesbos (Greece), Salina and Vulcano (Italy) can be regarded as produced by chaotic dynamics. The approach that is used to reach this result is based on the analysis of time series extracted from images of magma mixing structures. In particular, time series are used to reconstruct the strange attractors underlying the process of magma mixing and to quantify the attractors by calculating their fractal dimension.

2. General features of magma mixing structures

Magma mixing structures occurring in lava flows are studied because they provide an instantaneous picture of the ongoing magma mixing process thanks to the rapid cooling of magmatic masses. Macroscopic

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observations (Fig. 1) show that in the three lava flows magmas underwent mixing processes that produced intimate dispersion of magmas and generated structures evidencing the pattern of the flow fields inside the magmatic masses. Since the host magma is always more acidic than the "dispersed" magma, we refer to the former as "A" (acid) magma and to the "dispersed" magma as "B" (basic) magma (Table 1). Both magmas have a glassy structure with percentages of crystals less than 4%.

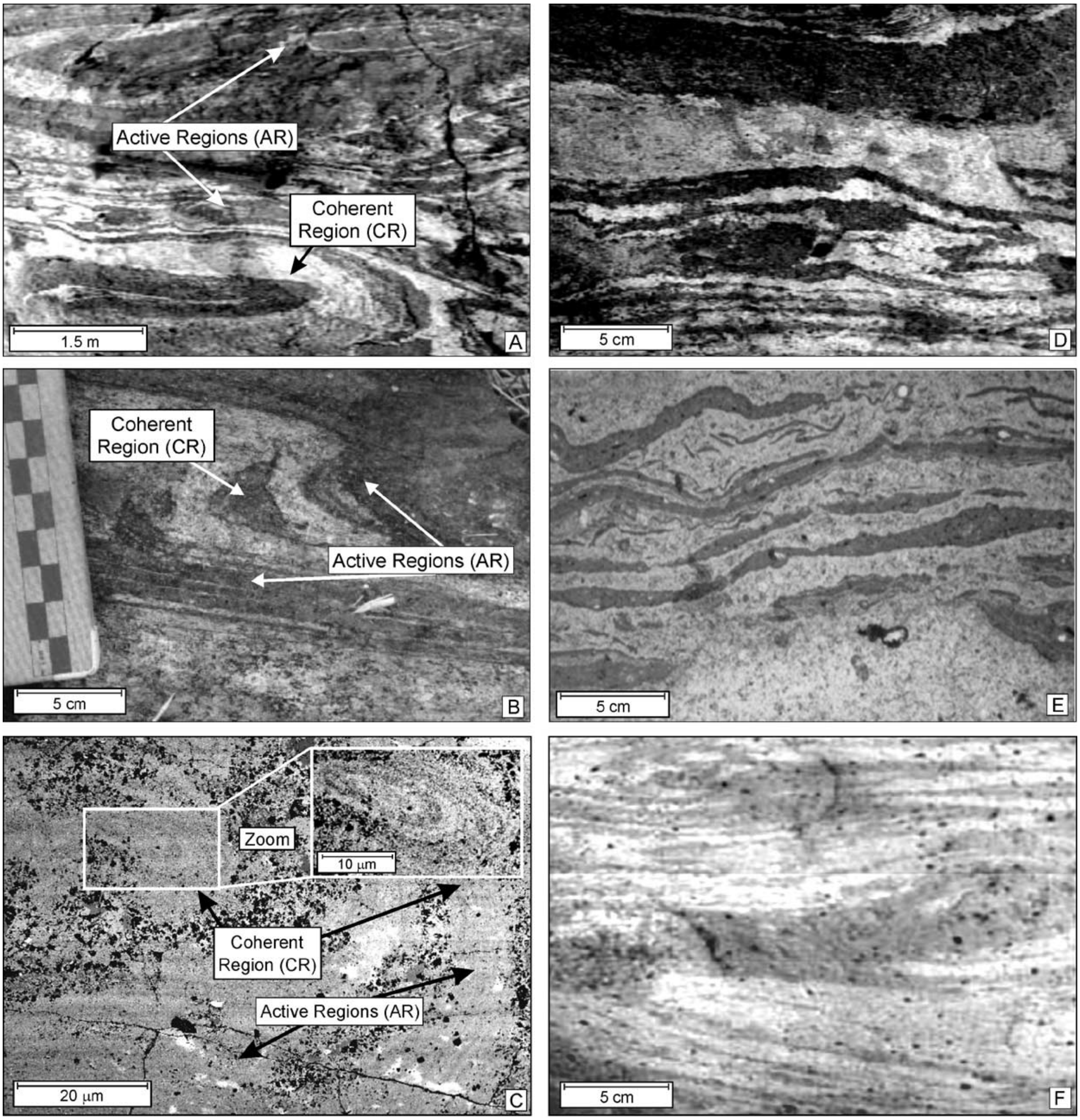

Fig. 1. Magma mixing structures in lavas from the island of Lesbos (A–D), Salina (E) and Vulcano (F). The darker flow structures consist of B magmas dispersed through light coloured A magmas.

Table 1
Rheological properties of the A and B magmas belonging to the three lava flows studied

	Salina		Vulcano		Lesbos	
	Magma A	Magma B	Magma A	Magma B	Magma A	Magma B
Rock	Latite	Andesite	Trachyte	Basalt	Rhyolite	Qz-Trachyte
μ (log Pa s)	5.94	4.42	5.01	4.39	7.19	5.97
ρ (g/cm^3)	2.62	2.85	2.70	2.82	2.36	2.46

Calculation of viscosity and density are based on the geochemical composition of glasses (Shaw, 1972) measured by an electron probe micro-analyser.

Although a wide variety of structures can be recognized in Fig. 1, they can be grouped into two main categories: (i) filament-like regions of B magmas inside A magmas (Active Regions; Fig. 1), and (ii) Coherent Regions of B magmas that did not disperse through A magmas showing a globular shape and occurring between filament-like regions (Fig. 1). It is interesting to note that such structures propagate inside the magmatic masses over a large range of scales showing self-similarity and suggesting a fractal nature of the process (Fig. 1A–C).

The occurrence of coherent and filament-like regions coupled with the occurrence of fractal structures during fluid mixing has been widely documented in the literature in both real and simulated systems and it has been demonstrated that they are produced by chaotic dynamics (e.g. Ottino et al., 1992; Liu et al., 1994; Aref and El Naschie, 1995; Bresler et al., 1997). The similarities between the structures produced by chaotic fluid mixing reported in literature (Fig. 2) and the mixing structures of Fig. 1 suggests a chaotic origin of magma mixing structures in lava flows.

2.1. Extraction of time series from magma mixing structures

Different methods and algorithms can be used to determine quantitatively whether a dynamical system exhibits chaotic behaviour (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997). However, in our case most of the techniques cannot be applied because they require detailed knowledge of initial conditions of the systems that cannot be known from the observations

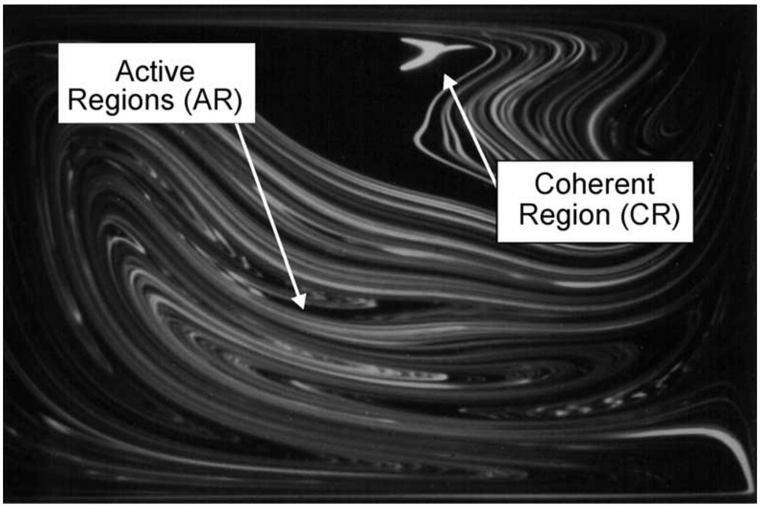

Fig. 2. Example of chaotic fluid mixing experiment from Bresler et al. (1997) evidencing the same structures observed in natural magma mixing structures (Fig. 1).

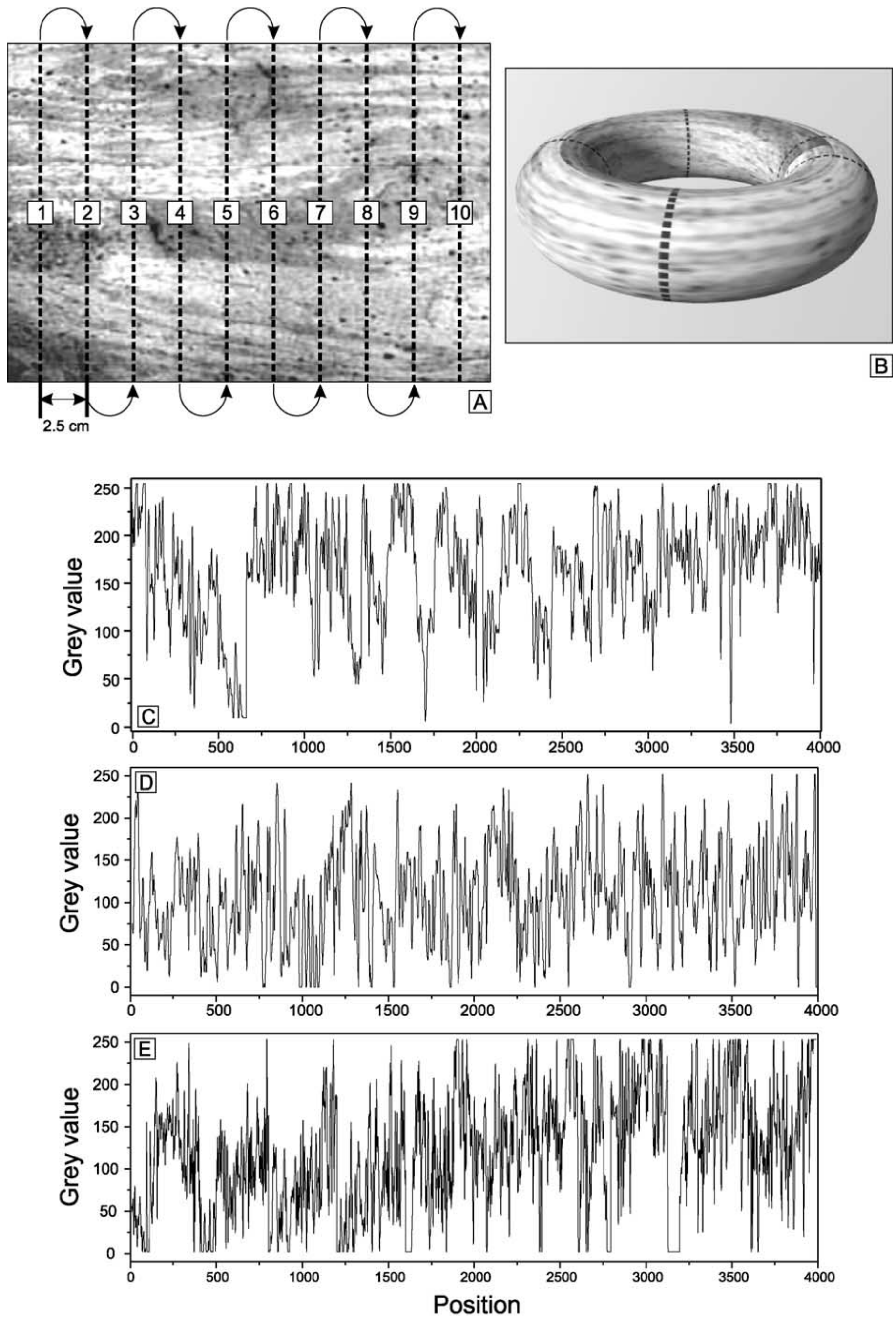

Fig. 3. Method used to extract times series from the magma mixing structures; grey scale image (A) from which the time series have been extracted; (B) schematic illustration of the toroidal approximation used to obtain a single time series from each magma mixing structure; examples of time series extracted from three structures belonging to the lava flow of Vulcano (C), Salina (D) and Lesbos (E).

of magma mixing structures. In fact, such structures represent the final state of dynamical systems and it is not possible to reconstruct their history from the beginning. This limits the number of techniques that can be utilised to determine whether magmatic interaction processes are chaotic or not.

To circumvent such difficulties, the most used and powerful techniques to detect chaotic behaviour within a system are based on the analysis of time series associated with the variations in time and space of a given variable of the system (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997).

In order to extract time series from magma mixing structures the following procedure was adopted. A number of structures from the lava flows (five for Salina, five for Vulcano and six for Lesbos) were selected. Selection has been done considering well exposed two-dimensional sections of fresh unaltered samples. Colour pictures of the magma mixing structures were processed to produce grey scale images in which the lowest colour (black) and the highest (white) are represented by the colour codes 0 and 255, respectively (Fig. 3A). It follows that all the shades of greys constituting the images range between 0 and 255 and can be considered representative of the relative changes of concentrations of A and B magmas in the rocks.

In order to extract time series data from the structures, a number of transverses passing through the grey scale images were traced (Fig. 3A). Considering that the size of images is 25×15 cm (corresponding to 600×400 pixels) and that the distance between transverses has been taken to be equal to 2.5 cm (corresponding to 60 pixels), over each image were traced 10 transverses that produced an equal number of time series whose values range between 0 and 255 in grey intensity values.

Then, assuming spatial continuity of the structures, each time series has been connected to the successive to obtain a single time series for each image. This assumption is supported by the fact that each image is a portion of the entire outcrop where the same structures propagate widely in all directions. Thus the connection of time series can be thought as a toroidal approximation of images (Fig. 3B) to have a representative sampling of each image incorporating information about its spatial variability.

Fig. 3C, D and E gives three representative time series extracted from three magma mixing structures of the lava flows of Lesbos, Salina and Vulcano, respectively.

3. The fingerprint of chaos in magma mixing structures

There are several ways how to detect chaotic patterns in time series. In this study the correlation dimension method (Grassberger and Procaccia, 1983a,b) is used because it has been extensively tested and is computationally efficient and relatively fast when implemented for attractor dimension estimation (e.g. Addison, 1997). The method is based on the reconstruction of the phase space of the dynamical system under consideration. A phase space can be defined as the space in which each direction corresponds to a variable, also called a degree of freedom, of the system (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997). Once the phase space is defined, the evolution of the system is described by a point in this space whose coordinates give the different values of the physical variables at the time of interest. With the passing of time, the point in the phase space moves on and gives a curve which describes the dynamical evolution of the system. In the limit of long time one expects the dynamics to settle to a dynamical stationary state, and the subspace on which the motion of the point remains is called the attractor of the dynamics (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997). For instance, if the motion is periodic, the attractor will be a simple closed curve. It is noteworthy that the number of degrees of freedom needed to characterize a system increases as the system dynamics become more and more random, that is as the behaviour of the system becomes progressively less predictable. In general, the motion of the point may describe very complex curves that mix producing attractors that propagate inside the phase space at many scales generating self-similar domains having a fractional dimension. In this case, the attractor is called "strange" and the process is chaotic. Besides, the fractal dimension of the attractor corresponds to the number of degrees of freedom needed to characterize completely the system dynamics (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997).

Starting from time series it is possible to reconstruct an approximate phase space of a given system using the method developed by Packard et al. (1980; see Appendix A). This methods allows the construction of a phase space of the system possessing the same topological properties of the true phase space. This does not mean that the attractor obtained in the construction is identical to that in the original phase space, but that the new representation of the attractor retains the same topological properties. The mathematical justification of this scheme has been proven by Takens (1981).

Using the time series extracted from the magma mixing structures occurring in the studied lava flows we reconstructed the attractors of the magma mixing system. For comparison, we also reconstructed the Lorenz attractor (Lorenz, 1963) using a time series obtained iterating the Lorenz system and monitoring the displacement of the x variable.

Fig. 4 reports the reconstructed attractors for the Lorenz time series and the three series of Fig. 3 in three dimensions. The coordinates of the three dimensional space are defined by $x=x(t)$, $y=x(t+\tau)$ and $z=x(t+2\tau)$ where τ is the delay (e.g. Abarbanel, 1996; Addison, 1997; see Appendix B). The graph of the Lorenz series shows the classical shape of the Lorenz attractor evidencing the good quality of the techniques in capturing the essential dynamics of the system (Fig. 4A). The graph of time series of magma mixing structures show attractors having structures

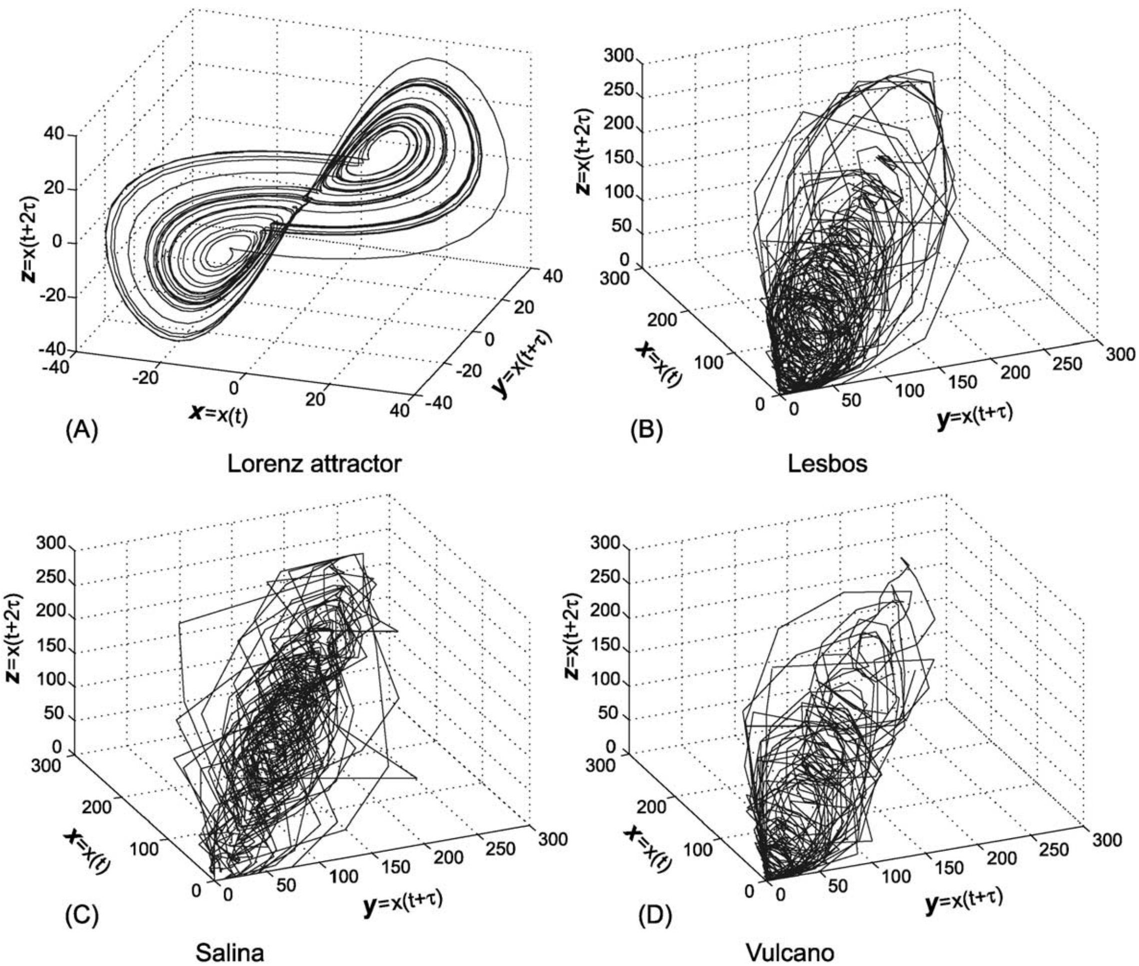

Fig. 4. Three-dimensional graphs of the reconstructed strange attractors for the chaotic time series (A) and the three time series of Lesbos (B), Salina (C) and Vulcano (D) shown in Fig. 3.

resembling the wings of the Lorenz attractor on which a great number of orbits recur. However, the attractors of magma mixing structures display some differences among them that are worth discussing. In particular, the structure of the attractor of the Lesbos series exhibits, at first sight, a pattern in which orbits are more dense in the lower corner of the graph (Fig. 4B) with respect to the attractors of Vulcano and Salina where the highest density of orbits is progressively displaced to the centre of the graph (Fig. 4C and D). The same features are observed in all the attractors reconstructed for all the analysed magma mixing structures. These observations show that different dynamics may have governed the evolution of the three magma mixing systems in Lesbos, Vulcano and Salina lava flows.

4. Quantification of chaos in magma mixing structures

As introduced above, the existence of an attractor does not imply that the system is chaotic. In fact, in order to define a system as chaotic the attractor must be "strange" and this means that it must have a fractal dimension.

We can determine if an attractor is strange or not, determining its so-called "correlation dimension" D as a function of the embedding dimension d (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997; see Appendix B). If D increases with increasing d, this implies that the time series is random, the attractor cannot be reconstructed and more and more degrees of freedom are taken into account by increasing the embedding dimension d. On the contrary, if D saturates to some fractional value D_{max} as d increases, the attractor can be reconstructed in the phase space of the embedding dimension d, and if its dimension (D_{max}) is not an integer, it is "strange" and hence the system is chaotic.

This method has been applied to all time series extracted from the magma mixing structures in order to calculate the dimension (D_{max}) of the reconstructed attractors. For comparison we also calculate the dimension of the Lorenz attractor using the same method.

Fig. 5 shows that for the Lorenz time series D values saturate as d increases according to its chaotic

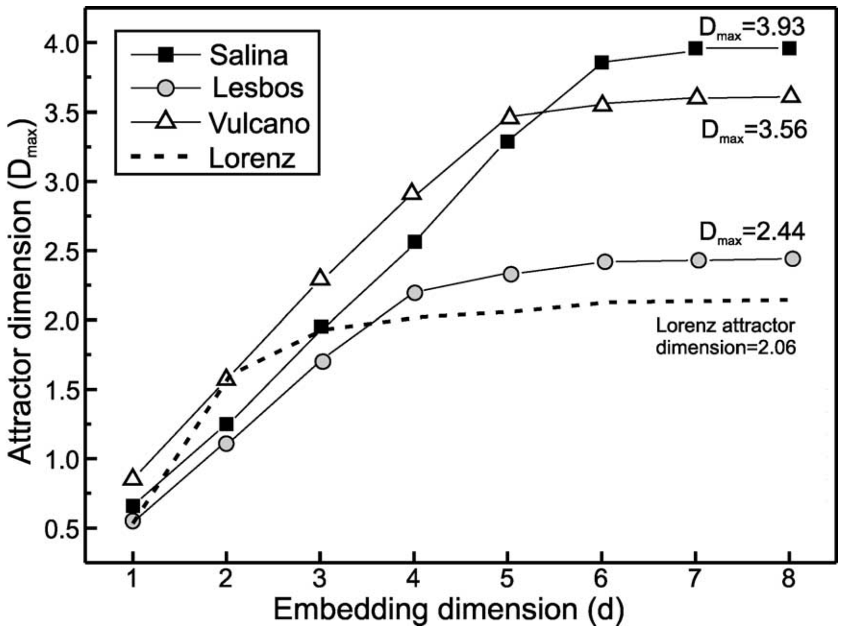

Fig. 5. Variation of fractal dimension of the attractor (D) vs. the embedding dimension (d) for the chaotic time series (dashed line) and for the three time series extracted from the mixing structures belonging to the lava flow of Lesbos, Vulcano and Salina of Fig. 3.

nature. In particular the fractal dimension D_{max} calculated for the attractor approaches a value of 2.06 that is the estimated fractal dimension for the Lorenz attractor (e.g. Addison, 1997).

The graph of Fig. 5 also shows the behaviour of D as d increases for the three representative time series extracted from the magma mixing structures shown in Fig. 3. D approaches constant values as in the case of the Lorenz time series showing that the series of natural structures exhibit typical chaotic behaviour and have few degrees of freedom governing their evolution. It is noteworthy that the saturation of D values has been observed for all sixteen natural time series; D_{max} values are reported in Table 2. The remarkable point here is that all the attractors have a fractional dimension indicating that these dynamical systems can be regarded as chaotic.

Moreover, although D_{max} varies within each lava flow, considering the mean values of D_{max} (Table 2) it is suggested that the three lava flows underwent different mixing dynamics. Remembering that D_{max} corresponds to the number of degrees of freedom, the three dynamical systems need different numbers of degrees of freedom to be completely characterized. In particular the number of degrees of freedom increases from Lesbos to Vulcano and Salina. Considering that the number of degrees of freedom increases as the system become more and more random, this implies that the magma mixing process goes towards dynamical states progressively more "random" passing from Lesbos to Vulcano and Salina.

These results may be related to the degree of turbulence of the magma mixing process. In fact, Gollub and Swinney (1975) show that a strict relationship between the fractal dimension of the attractor and the Reynolds number (*Re*) in fluid dynamic systems exists. In particular, it is demonstrated that the dimension of the attractor increases with the increasing turbulence within the system (Gollub and Swinney, 1975). Turbulence implies a very large number of degrees of freedom whose coupling and superimposition generate progressively more random dynamics. Thus, the increase of D_{max} in natural structures may indicate increasing degrees of turbulence ("randomness") in the different lava flows. In particular, the lava flows of Salina and Vulcano underwent more turbulent mixing than the lava flow of Lesbos.

However, it is worth noting that turbulence does not necessarily imply good mixing as common sense may indicate. In fact, Raynal and Gence (1995) have shown, using numerical calculations based on mixing time and energy dissipation in fluid mixing systems governed by laminar and turbulent chaotic dynamical regimes, that laminar mixing is generally more efficient than turbulent mixing. From this point of view, the degree of magmatic interaction suffered by the magmas constituting the lava flows of Salina and Vulcano has to be lower than that in the lava flow of Lesbos. Recent results reported by Perugini et al. (submitted), where quantitative analyses of the degree of mixing have been reported on the same mixing structures, evidence that the mixing process has been much more efficient in the lava flow of Lesbos in respect to Salina and Vulcano, corroborating the results reported in this study.

Table 2
Values of the fractal dimension of attractors (D_{max}) calculated for the time series extracted from the structures of Lesbos, Salina and Vulcano

Attractors dimension (D_{max})			
Provenance	Lesbos	Salina	Vulcano
Struct. n. 1	2.42	3.93	3.56
Struct. n. 2	2.44	4.36	4.35
Struct. n. 3	3.41	5.04	4.56
Struct. n. 4	4.27	5.28	4.77
Struct. n. 5	4.88	5.63	4.93
Struct. n. 6	5.42		
Mean	3.81	4.85	4.43

Errors in the estimation of D_{max} are better than 0.5%.

5. Conclusions

The mixing of magmas has been studied in three lava flows using a method that allows us to extract time series form magma mixing structures. Time series have been utilised to reconstruct the strange attractors underlying the process of mixing of magmas and to quantify attractor fractal dimension. In all analysed structures, the dimension of the attractors has a non-integer value. This is a remarkable result since it allows us to state that the mixing of magmas is a chaotic process governed by a low number of degrees of freedom and is not a random process as one would expect it to be. This is a crucial point since it gives a theoretical framework for the simulation of the magma mixing processes that, from this point of view, has to be carried out using models based on chaotic dynamical systems in order to take into account the long term unpredictability of natural systems.

In addition, it is shown that different chaotic systems having different numbers of degrees of freedom may have been responsible for the genesis of magma mixing structures occurring within each studied lava flow. In particular, the Salina and Vulcano lava flows exhibit higher numbers of degrees of freedom with respect to Lesbos. Since the number of degrees of freedom increases with the increasing of the degree of turbulence inside the systems, it is suggested that the lava flow of Salina underwent more turbulent mixing than the lava flows of Lesbos and Vulcano. However, turbulence is not a synonym of good mixing because the higher the turbulence, the lower magmas are mixed. It follows that the Lesbo lava flow, being characterized by a less turbulent regime, contains magma mixing structures exhibiting the highest degree of mixing with respect to those occurring in the lava flows of Salina and Vulcano.

Acknowledgements

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Appendix A. Phase space reconstruction

From the variation of the variable $X(t)$ (the time series), it is possible to reconstruct an approximate phase space by the following procedure (Packard et al., 1980; Froehling et al., 1981; Takens, 1981) where a set d of new variables $[X_j(t), j=1-d]$ are defined by

$$X_j = X[t+(j-1)\tau] \qquad j=1-d$$

The phase space, of dimension d, where d is called the "embedding dimension", is thus constructed with these different variables: $X_1(t)=X(t)$, $X_2(t)=X(t+\tau)$, $X_3(t)=X(t+2\tau)$, ..., $X_d(t)=X[t+(d-1)\tau]$. In this scheme, the time series $X(t)$ is considered to be independent of the same time series at a later time $X(t+\tau)$ where τ is an arbitrary constant called the delay. Different methods can be employed to estimate

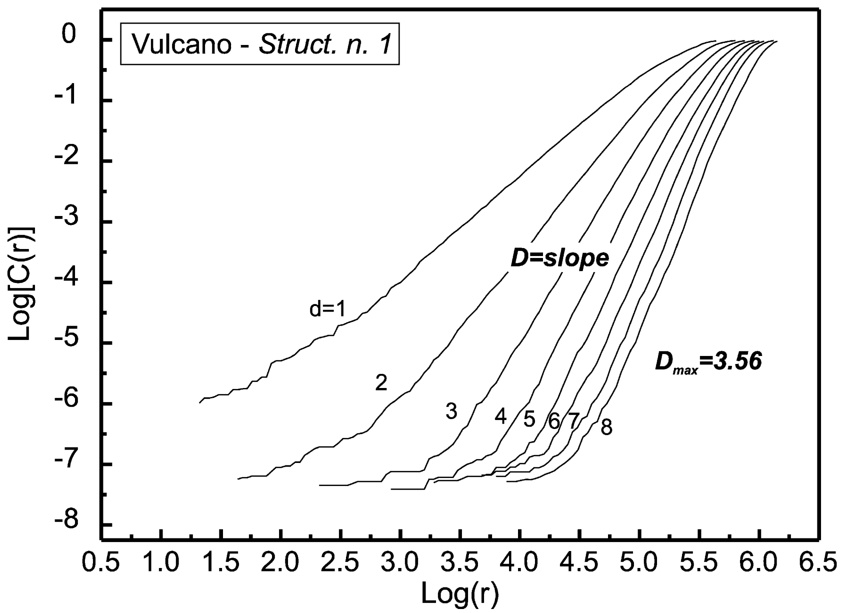

Fig. 6. Variation of $\log[C(r)]$ vs. $\log(r)$ for different values of the embedding dimension (d) for the time series extracted from one sample of the lava flow of Lesbo (Struct. n. 1; Table 2). In the graph is also reported the value of D_{max}.

τ and among them the most used are the autocorrelation and the mutual information function (e.g. Abarbanel, 1996; Addison, 1997). In order to estimate the value of τ used for the reconstruction of attractors of the magma mixing structures, in this paper both methods have been employed. Results show that autocorrelation and the mutual information function give identical estimates of τ for each time series extracted from the analysed magma mixing structures.

Appendix B. Computation of the fractal dimension of attractors

Once the attractor has been reconstructed, we can quantify it determining the so-called "correlation dimension" D as function of the embedding dimension d. D is the "correlation" fractal dimension of the attractor (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997), that is the set defined by all the points of coordinates $\{X_1(t)=X(t),\ X_2(t)=X(t+\tau),\ X_3(t)=X(t+2\tau),\ \ldots,\ X_d(t)=X[t+(d-1)\tau]\}$.

To compute D, the following method has been used (e.g. Strogatz, 1994; Abarbanel, 1996; Addison, 1997), which consists of determining the following correlation function $C(r)$:

$$C(r) = \left(\frac{1}{N^2}\right)\{\text{number of pairs } (a,b) \text{ of points in phase space whose distance } |x_a - x_b| < r\}$$

The correlation dimension D as a function of the embedding space dimension d is then defined by the following power law:

$$C(r) \sim r^D$$

Using logarithms the above expression can be written as:

$$\log[C(r)] = D\log(r) + q$$

D is the slope of the linear regression of the graph $\log[C(r)]$ vs. $\log(r)$, where q is the intercept. As an example the graph of Fig. 6 shows the variation of $\log[C(r)]$ as a function of $\log(r)$ for a time series of the Vulcano lava flow (Struct. n. 1; Table 2) for different values of d. As d increases, the slope of the straight part of the trend (i.e. D) approaches a constant value (D_{max}) that, in the case reported in the figure, is equal to 3.56.

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