Combining stochastic facies and fractal models for representing natural heterogeneity

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Abstract Sedimentary deposits are often characterized by various distinct facies, with facies structure relating to the depositional and post-depositional environments. Permeability (k) varies within each facies, and mean values in one facies may be several orders of magnitude larger or smaller than those in another facies. Empirical probability density functions (PDFs) of log(k) increments from multi-facies structures often exhibit properties well modeled by the Levy PDF, which appears unrealistic physically. It is probable that the statistical properties of log(k) variations within a facies are very different from those between facies. Thus, it may not make sense to perform a single statistical analysis on permeability values taken from a mix of distinct facies. As an alternative, we employed an indicator simulation approach to generate large-scale facies distributions, and a mono-fractal model, fractional Brownian motion (fBm), to generate the log(k) increments within facies. Analyses show that the simulated log(k) distributions for the entire multi-facies domain produce apparent non-Gaussian log(k) increment distributions similar to those observed in field measurements. An important implication is that Levy-like behavior is not real in a statistical sense and that rigorous statistical measures of the log(k) increments will have to be extracted from within each individual facies.

Résumé Les dépôts sédimentaires sont souvent caractérisés par des faciès variés, avec une structure de faciès associée aux environnements de sédimentation et post-sédimentaires. La perméabilité (k) varie dans chaque faciès et la valeur moyenne d'un faciès peut être de plusieurs ordres de grandeur supérieure ou inférieure à celle d'un autre faciès. Des fonctions empiriques de densité de probabilité (FDP) des incréments log(k) de structures multi-faciès présentent souvent des propriétés bien modélisées par la FDP de Lévy, qui apparaît physiquement non réaliste. Il est probable que les propriétés des variations de log(k) dans un même faciès sont très différentes de celles entre les faciès. Ainsi, cela n'a pas de sens de réaliser une analyse statistique des valeurs de perméabilité provenant d'un mélange de faciès distincts. Nous avons utilisé, comme alternative, une approche d'indicateur de simulation pour générer des distributions de faciès à grande échelle, et un modèle mono-fractal, le mouvement brownien fractionnaire (mBf), pour générer des incréments log(k) à l'intérieur des faciès. Les analyses montrent que les distributions simulées de log(k) pour l'ensemble du domaine multi-faciès produit des distributions apparentes non gaussiennes des incréments log(k) semblables à celles observées dans les mesures de terrain. Une implication importante est que ce comportement semblable à celui de Lévy n'est pas réel au sens statistique et que des mesures statistiques rigoureuses des incréments log(k) devront être extraites de chacun des faciès individuels.

Resumen Los depósitos sedimentarios se caracterizan a menudo por varias facies distintas, cuya estructura está relacionada con ambientes deposicionales y post-deposicionales. La permeabilidad (k) varía dentro de cada facies, y los valores medios en una de ellas pueden ser órdenes de magnitud superiores o inferiores que los correspondientes a otra. En estructuras multi-facies, las funciones de densidad de probabilidad (FDP) empíricas de los incrementos del logaritmo de la permeabilidad, log(k), exhiben a menudo propiedades que pueden ser modeladas mediante la FDP de Levy, la cual carece de un significado físico evidente. Es probable que las propiedades estadísticas de las variaciones del log(k) dentro de una misma facies sean muy diferentes a las existentes entre facies. Así, puede no tener sentido efectuar un análisis estadístico sencillo de los valores de permeabilidad obtenidos a partir de una mezcla de facies diferentes. Como alternativa, se ha utilizado un enfoque de simulación in-

Received: 9 May 2001 / Accepted: 2 May 2002
Published online: 19 June 2002

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 DOI 10.1007/s10040-002-0212-x

dicadora para generar distribuciones de facies a gran escala, y el modelo mono-fractal del movimiento fraccional Browniano (mfB) para generar los incrementos de log(k) dentro de las facies. Los análisis indican que las distribuciones simuladas de log(k) para todo el dominio multi-facies producen distribuciones de los incrementos de log(k) aparentemente no Gaussianas, de forma análoga a lo que se observa en medidas de campo. Una implicación importante es que el comportamiento tipo-Levy no es real en el sentido estadístico y que hay que efectuar medidas estadísticas rigurosas de los incrementos de log(k) dentro de cada facies.

Keywords Facies model · Fractal model · Heterogeneity · Hydraulic conductivity · Sedimentary rock

Introduction

Modern statistically-based studies of heterogeneity in hydraulic conductivity (K) or intrinsic permeability (k) recognize that K or (natural) log(K) distributions are typically non-stationary, which means that statistical parameters such as the mean and variance depend on position. This has led to the study of non-stationary processes with stationary increments (Feller 1968), wherein the increments of log(k) (i.e., differences between log(K) values measured with a separation, Δx, are assumed to be stationary (i.e., for each Δx chosen, the statistical properties of the log(K) increments (increments $\equiv\Delta\log(K)=\log[K(x+\Delta x)]-\log[K(x)]$) are independent of position in the medium). If the probability density functions (PDFs) of the increments for any given lag are Gaussian, then the resulting distribution of the log(K) increments over all lags is given by the Gaussian stochastic fractal known as fractional Brownian motion (fBm; Mandelbrot and Van Ness 1968; Hewett 1986; Molz and Boman 1993, 1995; Liu and Molz 1996, 1997a). The so-called scaling properties of fBm result from the implications of the classical central limit theorem (Feller 1968). Scaling refers to how the variance of the increments [variance of $\Delta\log(K)$] varies with Δx, with fBm yielding a power-law variogram of $\Delta\log(K)$ (Molz et al. 1997).

Numerous studies of log(K) field data have supported stationarity of the increments (Painter and Paterson 1994; Painter 1995, 1996a,1996b, 2001; Liu and Molz 1997b). However, the calculated PDFs of the $\Delta\log(K)$ have typically shown non-Gaussian behavior as characterized by increased peaking around the mean of zero and more heavy (slowly-decaying) non-Gaussian PDF tails (Painter and Paterson 1994; Painter 1995, 1996a, 1996b; Liu and Molz 1997b). This and other lines of evidence led Painter and Paterson (1994) to propose the Levy PDF as a candidate distribution for log(K) increments. The Levy PDF is a generalization of the Gaussian distribution, but includes the Gaussian PDF as a special case (Fig. 1). Moreover, a generalized central limit theorem applies to the Levy family of PDFs, which results in a class of non-Gaussian stochastic fractals known as fractional Levy motions (fLm) (Feller 1968; Painter and Paterson 1994; Samorodnitsky and Taqqu 1994). However, the Levy PDF has an infinite variance, which leads to the divergence of all statistical moments of the resulting fLm when it is exponentiated to get K itself (efLm). Many researchers view this behavior as unrealistic physically (Liu and Molz 1997b). The problem was noted by Painter and solved in a practical sense by truncating the Levy PDF being used to generate the fLm. Further recent study by Lu and Molz (2001) confirms that log(K) increment PDFs resemble Levy distributions, but after a certain distance above the mean, have tails that decay too quickly to maintain Levy behavior. Herein we refer to such distributions as Levy-like.

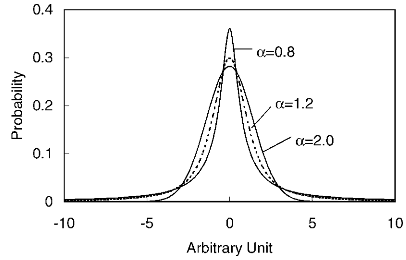

Fig. 1 An example of Levy probability density distributions, including the Gaussian (α=2) limiting case

Geologists have observed that sedimentary deposits are often characterized by various distinct facies that may change abruptly in both the vertical and horizontal directions, with facies structure relating to the depositional and post-depositional environments (Davis et al. 1993; Allen-King et al. 1998). Hydraulic conductivity varies within each facies, and mean values in one facies may be several orders of magnitude larger or smaller than those in another facies. In many situations it is possible that, and perhaps even probable, the statistical properties of permeability variations within one facies are very different from those in another facies. In such a situation it will not make sense to perform a single statistical analysis on permeability values taken from a mix of distinct facies because the statistical parameters are likely tied to the underlying depositional processes that may vary greatly between facies. Similar concerns have been expressed recently by other researchers (Davis et al. 1993, 1997; Allen-King et al. 1998). As an alternative, we employed the transition probability, Markov approach with indicator Kriging (Carle and Fogg 1996, 1997; Carle et al. 1998), to simulate large-scale facies distributions in which a single mean permeability value was assigned to each facies. To further represent the natural heterogeneity of sedimentary deposits on an intrafacies scale, we use a mono-fractal model, fractional Brownian motion (fBm) to represent the log(K) variations inside each facies with a single Hurst coefficient, H=0.3, and different means and variances estimated from the measured log(K) data. Analyses show that the simulated log(K) distributions for the entire multi-facies domain produce non-Gaussian log(K) increment distri-

 DOI 10.1007/s10040-002-0212-x

butions similar to those measured in the field, i.e., they are Levy-like. Thus, the main objectives of the present paper are to (1) present new data and data analyses supporting the Gaussian behavior of log(K) increments in single facies, and (2) show how the superposition of facies exhibiting Gaussian behavior leads to overall Levy-like behavior, thus suggesting that the underlying statistical behavior is not Levy-based after all. Overall, the study may be viewed as a synergistic union of facies geology with stochastic hydrology, possibly leading to improved concepts and methodology for simulating transport processes in natural porous media.

As implied by the extensive review of Koltermann and Gorelick (1996), classifying heterogeneity in sedimentary deposits is not a simple task. In their terminology, we are combining “spatial statistical methods” to generate fractal structure within facies and “sedimentation pattern imitation methods” to generate facies geometry itself. All of this fits under their category of “structure imitating methods” for creating maps of heterogeneity. However, if the facies geometry were created deterministically based directly on field data and observations, then we would be combining their categories of structure–imitating and descriptive methods.

Motivation for Combining Stochastic Facies and Fractal Models

We conceive of facies in a manner similar to that used by Allen-King et al. (1998), i.e., “facies commonly are defined on the basis of distinct textural, structural, and/or lithologic features that reflect changes in transport or depositional mechanisms, including changes in flow competence, capacity, and/or variability.” The key property from our perspective is that a facies reflect the unique combination of processes through which it was created. Thus, permeability may be conceived as exhibiting two gross components of variation: variation between facies and variation within facies (Goggin 1988).

Previous studies have presented evidence that log(k) increments (k = intrinsic permeability) within multifacies structure often follow non-Gaussian distributions that are Levy-like (Painter and Paterson 1994; Painter 1995, 1996a, 1996b; Liu and Molz 1997b). For example, a k data set from a vertical core of the Page Sandstone was obtained in the laboratory using a surface gas mini-permeameter (Goggin 1988). It consists of 2,884 consecutive k measurements with half-inch (1.27-cm) spacings. Recently, Lu and Molz (2001) have concluded that both the k increments and log(k) increments have non-Gaussian distributions with heavy, non-Gaussian tails when the entire data set is used for the analysis. (By “heavy tails” we mean anomalously slow PDF decay towards zero as one moves away from the mean.) However, under the assumption that permeability is related to at least three of the primary facies types found in eolian sequences (that is, grain-flow, wind-ripple, and inter-dune), Goggin (1988) provided evidence that k in each facies is log-normally distributed. Therefore, the log(k) increments for each facies should be approximately normally distributed. We checked for this behavior using Goggin’s (1988) data. Shown in Fig. 2 are results for the inter-dune data, indicating that the cumulative distribution of log(k) increments are well approximated by a Gaussian cumulative distribution function. Rescaled range (R/S) analysis of log(k) (Fig. 3) also shows that long-range correlated log(k) structure is found with a Hurst coefficient H=0.39 (Liu and Molz 1996). (“Long-range” means correlation of Δlog(k) over the entire length of the domain of measurements, which was a 1.4-m length of sandstone core.)

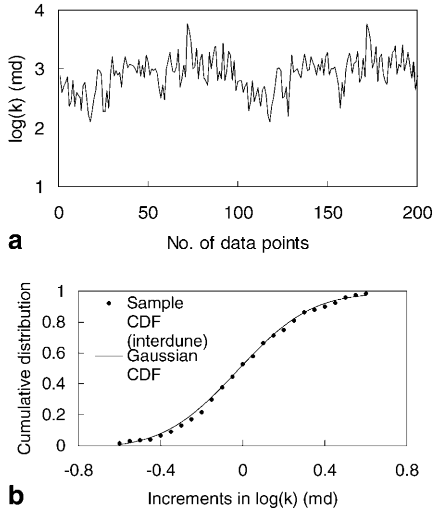

Fig. 2 **a** Permeability data collected from the inter-dune facies in eolian sandstone. **b** The cumulative distribution of the increments of log(k) indicates the distribution is well fit by a Gaussian distribution

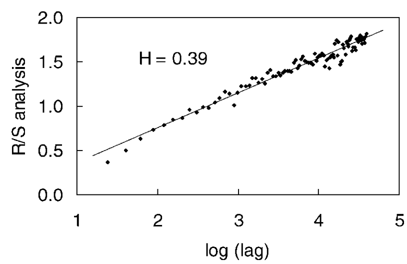

Fig. 3 Rescaled range analysis of log(k) collected from the inter-dune facies in eolian sandstone. The Hurst coefficient (H) is given by the slope of the best-fitting straight line

Recently, field k measurements have been made by three of the present authors (Castle, Lu, and Molz) and others using a newly designed drill-hole, gas, mini-permeameter, with a 15-cm measurement spacing on a 6×21-m sandstone outcrop near Escalante, Utah (Dinwiddie et al. 2000; Lorinovich et al. 2000; Lu et al. 2000). About 500 k measurements were collected along three horizontal transects and four vertical profiles. Among them, two horizontal transects with 269 measurements are located in a bioturbated, shallow-marine, sandstone that is considered as a single facies. Analysis

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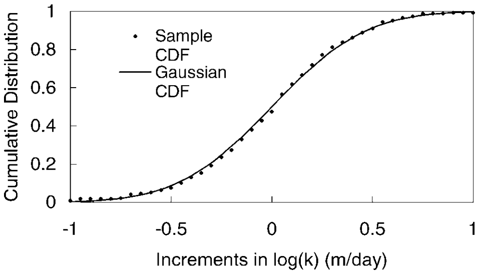


Fig. 4 The cumulative distribution of the increments of log(k) collected from a shallow marine sandstone in Utah. Once again the CD is well fit by a Gaussian distribution

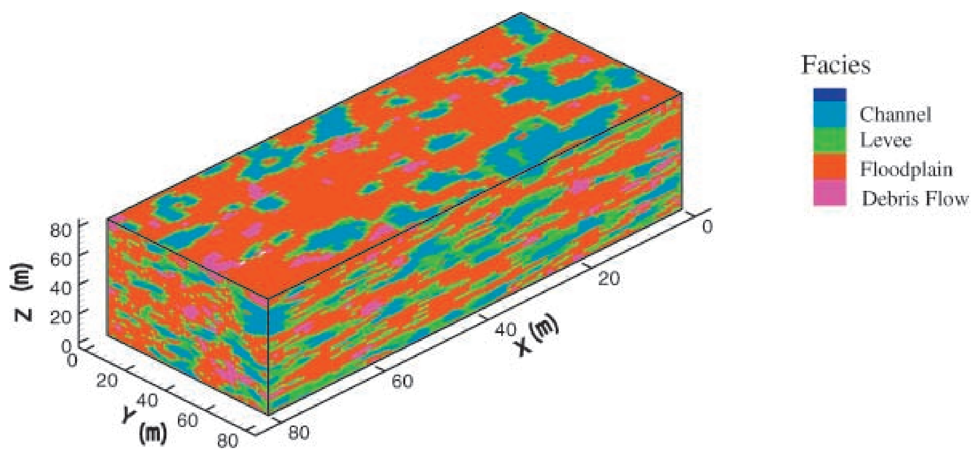
shows that the log(k) increments have a Hurst coefficient of 0.34 and are also well fit by a Gaussian cumulative distribution function as shown in Fig. 4. Therefore, we conclude that there is field evidence for Gaussian behavior of log(k) or log(K) increments, at least within some individual facies. This is one of the main motivations for proposing the fractal/facies model. However, more field data are needed in order to better define the positive aspects as well as the probable limitations of this concept. For example, we expect that post-depositional fracturing of a particular facies would destroy potential Gaussian behavior of a property such as the log(k) increments in the original parent material.

Simulation of Log(K) Increments Using the Fractal/Facies Model

In order to illustrate the fractal/facies construction procedure, and to show that an underlying Gaussian log(K) increment structure can give rise to an overall Levy-like increment structure, the alluvial fan deposits at the Lawrence Livermore National Laboratory (LLNL) in the Livermore Valley of California was selected for study. This aquifer–aquitard system has been sampled (cored) and characterized extensively (Thorpe et al. 1990; Carle 1996; Carle et al. 1998; Fogg et al. 1998, 2000). The system is composed primarily of four different facies: channel, levee (proximal floodplain), floodplain, and debris flow deposits, with volume fractions of 0.18, 0.19, 0.56, and 0.07, respectively. A facies realization developed previously and based on the transition probability/Markov chain indicator approach (Carle and Fogg 1996, 1997) is presented in Fig. 5. This realization, which was generated with the program TPROGS, is comprised of four indicators that represent the four different facies present. Mean lengths of the facies are less than 0.1 times the model dimensions. Based on field and laboratory K data, as well as analysis of multiple-well interference testing, it was shown that the multi-facies domain has an overall multi-modal frequency distribution of log(K) as shown in Fig. 6 (Carle 1996; Fogg et al. 2000). The estimated mean values of K are 0.432 m/day for debris flow deposits, 4.32×10^{-5} m/day for floodplain deposits, 0.173 m/day for levee deposits, and 5.184 m/day for channel deposits. The channel facies represents the aquifer units, whereas the other facies are primarily aquitards. Because the variances of log(K) increments for each facies are needed as input in a fractal generation model, they are roughly estimated from Fig. 6 first, that is, 0.50, 0.25, 0.1, 0.06 for floodplain, levee, channel, and debris flow, respectively. In the following fractal log(K) realization procedure, the variance for each facies is adjusted so that the variance of the resulting log(K) simulation approximately matches that of the log(K) frequency for each facies in Fig. 6.

Given a mean and a roughly estimated variance for the log(K) increments for each facies as input, and after several trials, a 3-D fBm log(K) realization was generated for each facies by using the successive random additions algorithm found in Lu and Molz (in revision), and available from the authors upon request. Each of these realizations has a Hurst coefficient H=0.3 that is near the center of reported H values for log(K) (Neuman 1990), and a mean and variance of log(K) corresponding to each individual facies. Thus, a total of four 3-D fBm realizations, one for each facies, were generated. Each fractal realization has the same resolution (nodes and node spacing) as the facies realization. According to the

Fig. 5 Example of a facies distribution generated with the transition probability, Markov approach (from Carle et al. 1998)



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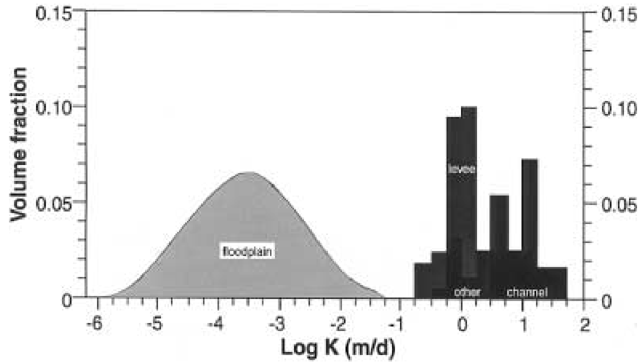


Fig. 6 Frequency distributions derived from sample data [log(K)] for four different facies of an alluvial fan: floodplain, levee, channel, and debris flow (from Fogg et al. 1998)

spatial locations and facies types, K values selected from the corresponding fractal realization are assigned within each facies of the previously developed facies realization. The resulting new fractal/facies K realization, which preserves the facies structure, now has irregularity and scaling within each facies that is characteristic of exponentiated fBm, as illustrated in Fig. 7.

Analysis of the Simulated Multi-Facies Log(K) Data

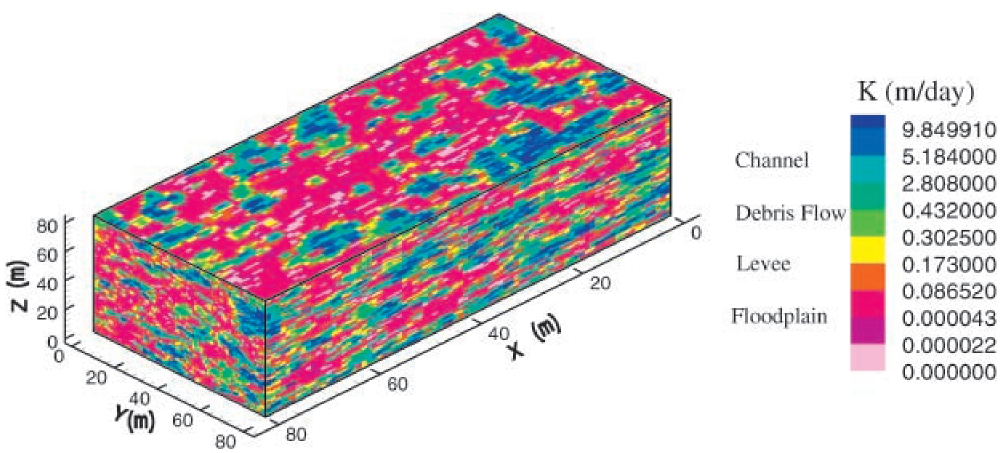
Statistical Analysis

For the purpose of comparing properties of the simulated K values with the measured values, a total of 40 sets of log(K) values along the vertical direction, i.e., 40×81=4,050 simulated data points, are extracted from the fractal/facies log(K) realization. The result of a frequency analysis presented in Fig. 8 shows that the 4,050 simulated log(K) values have an overall multi-modal frequency distribution similar to the measured values shown in Fig. 6. Thus, the global properties of the measured data are reproduced quite easily by the simulation.

Further analysis shows that the log(K) increment PDF for the entire simulated data set, as shown in Fig. 9, takes on a distinctly non-Gaussian appearance with peaking around the mean value of zero and heavy tails, even though the underlying statistics are Gaussian. Qualitatively similar to the observations of Lu and Molz (2001), the tails decay relatively quickly to small values at a sufficient distance above and below the mean. Both of these attributes are observed commonly in multi-facies experimental data sets (Painter 2001). The best fitting Gaussian PDF with a mean of 0 and a variance of 3.16 does not match the central part of the sample distribution well, and the tails are too thin (Fig. 9). The overall distribution definitely takes on a Levy-like appearance, with a Levy PDF having a scale parameter of 0.786 and a stable index of 1.35 (based on the Fama and Roll (1971) estimator) fitting the entire central part of the sample PDF rather well. However, examination of the tail behavior of its cumulative distribution function (CDF), as illustrated in Fig. 10, shows that the sample CDF falls between the Gaussian and Levy cases, as one would expect given the method of construction.

To relate the Levy-like appearance more precisely to the construction procedure, it is important to note that most of the increments of the resulting log(K) simulation are those random numbers that are generated from one of the Gaussian generators representing a single facies in the fractal generation model. Only a few of the increments resulted from the large log(K) variations associated with the facies interfaces. To understand why the combination of these increments results in the Levy-like PDF shown in Fig. 9, let us superimpose and re-normalize four Gaussian distributions with four different variances (σ_1=0.44, σ_2=0.70, σ_3=0.89, σ_4=2.53), which results in the Levy-like distribution shown in Fig. 11. The tails of the Levy-like distribution are dominated by the Gaussian distribution with the largest variance, σ_4=2.53 whereas the center part is mainly controlled by the Gaussian distribution with the smallest variance. Mathematically, the resulting PDF f=1/4 (f1+f2+f3+f4), where f1, f2, f3, and f4 represent four different Gaussian distributions. If the large jumps over the different facies interfaces are considered, the tails will be thicker. Therefore, the magnitude of the difference of variances of the Gaussian PDFs and the degree of the mixture of different

Fig. 7 Example of a log(K) realization showing the facies and hydraulic conductivity distribution generated with the combined fractal/facies approach. Mean K values for the four facies types are indicated on the diagram



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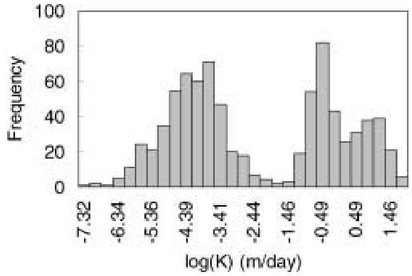

Fig. 8 Synthetic frequency distribution of log(K) derived from the hydraulic conductivity realization shown in Fig. 7. The results compare well with those in Fig. 6

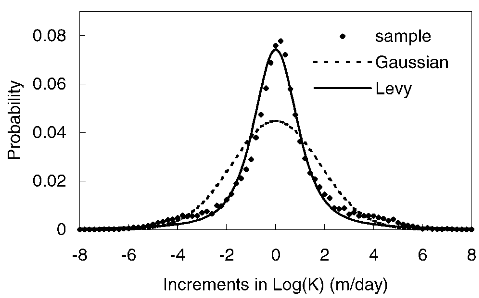

Fig. 9 Probability density distributions of the increments of log(K) derived from the hydraulic conductivity realization shown in Fig. 7. The best-fitting Levy probability density function and the best-fitting Gaussian distribution are shown also

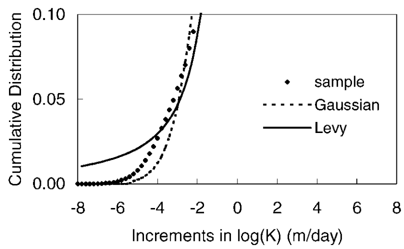

Fig. 10 Diagram showing the tail thickness of the simulated hydraulic conductivity increments and the best-fitting Gaussian and Levy distributions

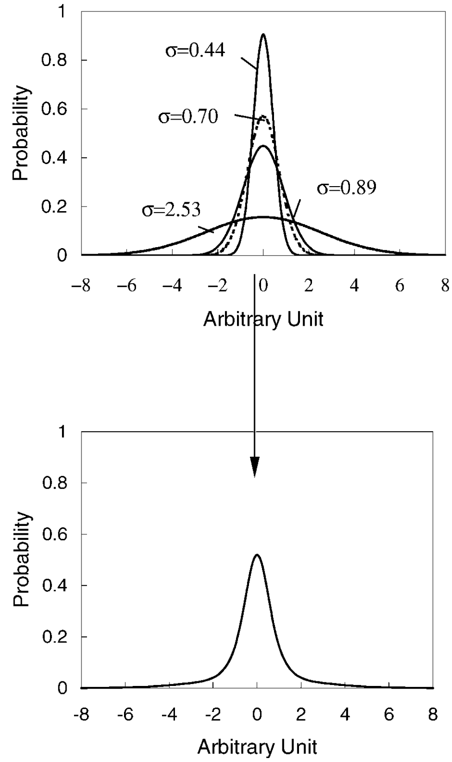

Fig. 11 Illustration of the superposition and re-normalization of four different Gaussian distributions resulting in a Levy-like distribution

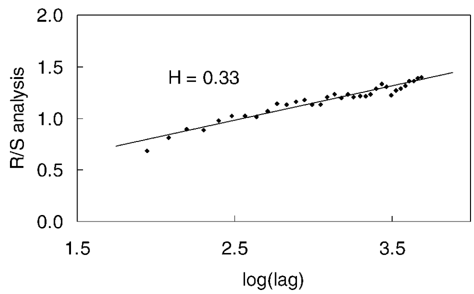

Fig. 12 Rescaled range analysis of data derived from the log(K) realization shown in Fig. 7. The Hurst coefficient (H) is given by the slope of the best-fitting straight line

facies determine how long and thick the tails are. This implies that the tail behavior may vary from site to site, which is supported by two data sets studied in detail by Lu and Molz (2001).

Scaling/Correlation Analysis

Previously published data analyses (Molz and Boman 1993, 1995; Liu and Molz 1997a, 1997b; Boufadel et al. 2000; Lu and Molz 2001; Painter 2001), and the present analysis of Goggin's (1988) data, show that log(K) or log(k) measurements collected from a mix of various types of facies often display a fractal-like structure. It is interesting to see whether a mix of fBm, such as our fractal/facies simulations, still preserves fractal-like long-range correlation in the resulting log(K). Accordingly, ten sets of data, each having 81 data points, were extracted along the vertical direction from the log(K) simulation shown in Fig. 7. Rescaled range (R/S) analysis (Liu and Molz 1996) was then applied to each set of data. The average of ten R/S analyses shown in Fig. 12 indicates approximate long-range correlation, with a Hurst coefficient H=0.33 (based on least-squares fitting; Bhattacharyya and Johnson 1977) that is slightly larger than the value of 0.3 selected as input for the permeability realizations of each facies. The R/S analysis trends well along a straight line, with some of the smaller scale noise probably caused by the mix of facies.

 DOI 10.1007/s10040-002-0212-x

Discussion and Conclusions

A model for natural heterogeneity, called herein the fractal/facies model, is proposed. This model may be viewed as a union of the geologic concept of facies with stochastic hydrology based on the mathematics of non-stationary stochastic processes that have stationary increments. In order to apply the model, one should deal with a heterogeneous geologic property whose distribution has a structure that can be subdivided into one or more facies that each reflect a unique combination of formation (genetic) processes. Furthermore, the property increment PDF's within each facies should approximate a Gaussian distribution. When such conditions are met, we have shown herein that simulated property distributions, such as log(K) increment distributions, will produce non-Gaussian, multi-peaked frequency distributions that are similar to those observed in multi-facies data sets. In addition, increment distributions will tend to be Levy-like, with peaking around the mean and tails more heavy than Gaussian. Careful examination of the tail behavior, however, will show that it lies between the Gaussian and Levy cases, similar to the results of the more abstract analysis presented recently by Painter (2001). At the same time, fractal-like structure, as reflected by variance scaling and long-range correlation, is maintained in the simulated data sets. This type of behavior was shown previously to hold for the MADE data set (Lu and Molz 2001), which is a multi-facies type, although the facies structure is not known in detail. Both the present analysis and that of Painter (2001) are consistent with empirical Δlog(K) PDFs falling between the Gaussian and Levy cases. A potentially important difference, however, is the implication of our study, or at least the strong suggestion, that Levy-like behavior is an artifact in a statistical sense and that rigorous statistical parameters will have to be extracted from within each facies separately in a multi-facies domain.

Although a stochastic facies model is used in this study, a stochastic approach may not be necessary in some cases if sufficient information about facies distribution is available. In such a case, a deterministic facies model may be appropriate. Both stochastic and deterministic approaches to facies architecture provide a way to reproduce large-scale structure distributions in the heterogeneous property. Thus, the facies structure serves as a way to condition the fractal so that large-scale property variations are included in the overall model.

Such conditioning may be very important from the transport point of view, because multi-facies structures, which contain various distinct facies commonly having mean permeability values that differ by several orders of magnitude, will play a primary role in solute migration (LaBolle and Fogg 2001). However, it is important also to have permeability variations within facies that exhibit realistic variation and scaling because such variations may play an important role in forming potential preferential flow pathways through facies.

The proposed fractal/facies model for simulation of K or k distributions appears sufficiently realistic to justify further study and testing. More field data sets are needed to see if increment distributions within pure facies are commonly Gaussian, and to further study the implications if they are not Gaussian. The advantage of using the fBm model to generate log(k) or log(K) distributions within each facies is that the fBm model is simple and well understood, and is supported by the data studied herein. A possible alternative would be to use a multi-fractal model for intra-facies permeability distributions if permeability data collected within facies were found to exhibit multi-fractal properties (Boufadel et al. 2000).

Acknowledgements This research has been supported by Grant # R82-6171 from the US Environmental Protection Agency's Science to Achieve Results (STAR) program. It has not, however, been subjected to any EPA review and, therefore, does not necessarily reflect the views of the agency, and no official endorsement should be inferred. Support for collecting the data shown in Fig. 4 was provided by Grant DE-AC26-98BC15119 from the US Department of Energy.

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