ELSEVIER

Journal of Hydrology 269 (2002) 122–133

Journal of Hydrology

www.elsevier.com/locate/jhydrol

Improving the analysis of slug tests

C.D. McElwee*

Kansas Geological Survey, University of Kansas, 1930 Constant Avenue, Campus West, Lawrence, KS 66047-3726, USA

Received 12 December 2000; revised 2 July 2002; accepted 12 July 2002

Abstract

This paper examines several techniques that have the potential to improve the quality of slug test analysis. These techniques are applicable in the range from low hydraulic conductivities with overdamped responses to high hydraulic conductivities with nonlinear oscillatory responses. Four techniques for improving slug test analysis will be discussed: use of an extended capability nonlinear model, sensitivity analysis, correction for acceleration and velocity effects, and use of multiple slug tests. The four-parameter nonlinear slug test model used in this work is shown to allow accurate analysis of slug tests with widely differing character. The parameter β represents a correction to the water column length caused primarily by radius variations in the wellbore and is most useful in matching the oscillation frequency and amplitude. The water column velocity at slug initiation (V_o) is an additional model parameter, which would ideally be zero but may not be due to the initiation mechanism. The remaining two model parameters are A (parameter for nonlinear effects) and K (hydraulic conductivity). Sensitivity analysis shows that in general β and V_o have the lowest sensitivity and K usually has the highest. However, for very high K values the sensitivity to A may surpass the sensitivity to K. Oscillatory slug tests involve higher accelerations and velocities of the water column; thus, the pressure transducer responses are affected by these factors and the model response must be corrected to allow maximum accuracy for the analysis. The performance of multiple slug tests will allow some statistical measure of the experimental accuracy and of the reliability of the resulting aquifer parameters.

Keywords: Ground water; Aquifer characterization; Slug tests; Hydraulic tests; Data analysis; Ground water models

1. Introduction

Slug testing has been an important technique for estimating hydraulic conductivity (K) at least since Horslev's work Hvorslev (1951). Slug testing is relatively easy and inexpensive to perform, so it has been widely done. However, the quality control on slug testing has been less than ideal, calling into question the results obtained many times. Properly performed, slug tests can give valuable information about the hydraulic conductivity. Butler et al. (1996) have given some guidelines to improve the quality of slug testing results. This paper examines several additional issues that have the potential to improve the quality of slug test analysis. In particular, this paper discusses techniques which span the range from low K with overdamped responses to high K with nonlinear oscillatory responses.

This paper will deal with four techniques for improving slug test analysis: use of an extended capability model, sensitivity analysis, correction for acceleration and velocity effects, and use of multiple slug tests. The model used for slug test analysis should

* Tel.: +1-785-864-2097; fax: +1-785-864-5317.
E-mail address: cmcelwee@ku.edu (C.D. McElwee).

PII: S0022-1694(02)00214-7

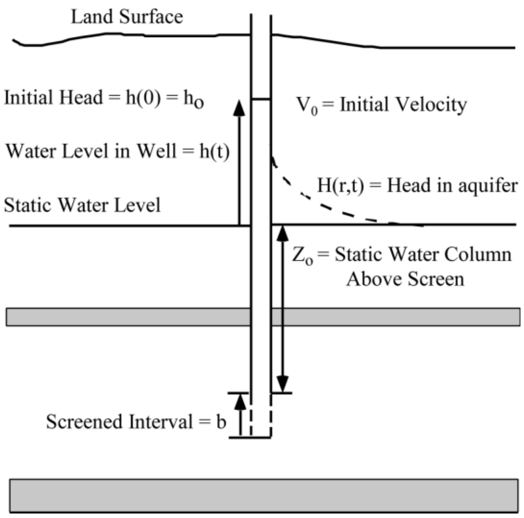

Fig. 1. Typical slug test situation (McElwee, 2001).

be able to simulate the range of responses from overdamped to underdamped seamlessly. In particular, for high conductivity zones the model should be able to simulate nonlinear behavior. The use of sensitivity analysis as an integral part of the fitting procedure will give an estimate of the quality of estimation for each parameter and indicate the degree of interdependence between parameters (correlation). When significant accelerations and velocities are present in the wellbore, the response of the pressure transducer will be affected. For maximum accuracy in the analysis, corrections should be applied for acceleration and velocity to the model response before comparing to the transducer response. Finally, multiple slug tests should be performed at nearly identical initial heights and for differing initial heights. The repeatability of slug tests performed at nearly identical initial heights will give some indication of the magnitude of experimental noise present. Slug tests performed with differing initial heights are very effective indicators of nonlinear behavior. Significant spread in the normalized responses indicates nonlinear effects are important and linear models should not be used. The performance of multiple slug tests will allow some statistical measure of the experimental accuracy and of the resulting aquifer parameter reliability.

2. An extended capability model

Traditionally, slug tests have been analyzed with linear theories as either overdamped (Hvorslev, 1951; Cooper et al., 1967; Bouwer and Rice, 1976) or as underdamped (Krauss, 1974, 1977; van der Kamp, 1976). Some attempts were made to span the complete range of responses (Kipp, 1985; Springer and Gelhar, 1991). In addition, several nonlinear models for slug test responses were developed (Kipp, 1985; Kabala et al., 1985; Stone and Clarke, 1993; Zlotnik and McGuire, 1998). However, most studies concluded that nonlinear models were not needed. McElwee and Zenner (1998) developed a nonlinear model for slug test responses and applied it to data that definitely showed nonlinear effects. Nonlinear effects will be most pronounced when slug tests are performed in aquifers with high K. Therefore, when analyzing slug test data from an aquifer with widely varying K, one should use a model that can span the range from overdamped to underdamped responses and can incorporate nonlinear responses when they are present.

The nonlinear model developed by McElwee and Zenner (1998) is based on the Navier–Stokes equation, nonlinear frictional loss, nonDarcian flow, acceleration effects, radius changes in the wellbore, and a Hvorslev model for the aquifer. This will be referred to as an extended capability model. The basic equation for their model is given by

$$(h + z_o + b + \beta)\frac{d^2h}{dt^2} + A\left|\frac{dh}{dt}\right|\frac{dh}{dt} + \frac{g\pi r_c^2}{FK}\times\left(\frac{dh}{dt}\right) + gh = 0. \quad (1)$$

Some quantities appearing in Eq. (1) are defined in Fig. 1, which shows a typical slug test arrangement. Initial values for slug height or head (h_o) and velocity (V_o) must also be known. Parameter β is primarily related to radius changes in the water column. Radius changes can result from the use of a packer to perform the slug test or by differing radii of the solid casing and screen sections. Parameter A is related to nonlinear head losses that can occur along the casing walls, in the screen, or in the throat of a packer. K is the hydraulic conductivity of the aquifer and is of the most interest. Additional parameters in Eq. (1) are the

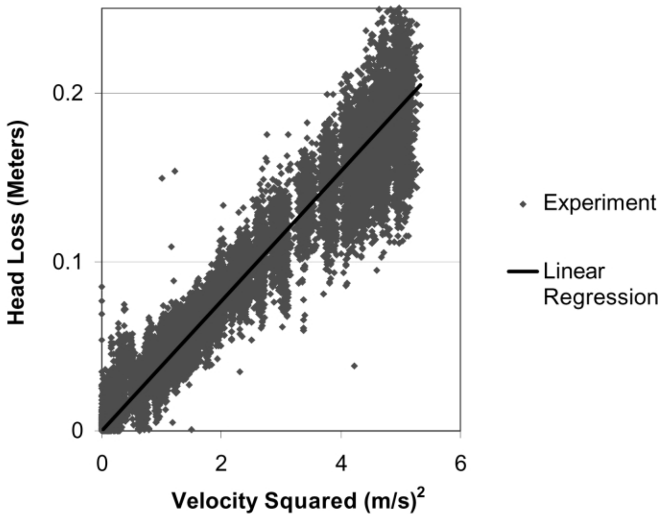

Fig. 2. Plot of head loss versus velocity for clean 5 cm (2 in.) steel pipe.

acceleration of gravity (g), the casing radius (r_c), and the Hvorslev shape factor (F). McElwee and Zenner (1998) have shown typical expressions for β and A based on simple physical considerations. Ignoring β and A makes the analysis for K inaccurate when using traditional theories. Since it may be difficult to calculate β and A from first principles, it is often assumed that they are parameters to be adjusted to best fit the field data.

The parameter A can have a rather complicated representation which changes with water velocity (McElwee and Zenner, 1998). The general form for A involves the product of a wetted length over which the measured head loss occurs and a friction factor. The friction factor is inversely proportional to the Reynolds number in the laminar region and the empirical Colebrook formula (Streeter and Wylie, 1985) can be used for the turbulent region. In the limit of high Reynolds numbers the friction factor goes to a constant value, meaning that A would approach a constant value as long as the wetted length was constant. In order to investigate the experimental dependence of A on velocity two experiments were performed. In both experiments a section of 5 cm (2 in.) pipe was used with two pressure transducers located at known distances along the pipe; and, a digital flow meter was employed to measure the flow rate (velocity). In one experiment the pipe was clean and unobstructed between the two pressure transducers. In the other experiment the packer used to collect the slug test data presented in this paper was located between the two pressure transducers. This arrangement allowed the measurement of head loss over a known distance as a function of water velocity. The results are plotted as head loss versus velocity squared in Figs. 2 and 3 and indicate that, for the range of velocities measured by the flow meter, A can be represented as a constant value (slope of the best fit line). There is considerable scatter in the experimental

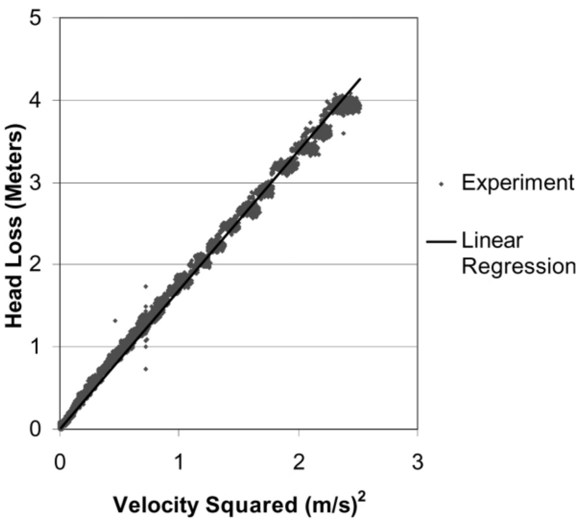

Fig. 3. Plot of head loss versus velocity for 5 cm (2 in.) PVC pipe with packer.

data of Figs. 2 and 3 indicating hysteretic and random behavior. For the analyses presented in this paper it will be assumed that A is a constant parameter above some critical velocity separating the laminar and turbulent regions.

McElwee (2000, 2001) has added an additional parameter representing the initial velocity of the water column at slug initiation, V_o. Normally V_o would be zero, but it appears that sometimes the water column obtains an initial velocity from the initiation mechanism. For example, in this work a piston inside a packer throat was used to initiate the slug tests that can induce a nonzero initial velocity. Some initiation methods, such as the pneumatic system, may be less prone to impart an initial velocity. McElwee (2000) has developed an analysis package that implements the model of Eq. (1) with the additional parameter V_o. As will be shown here, the model seems to be quite robust in estimates of K over varying conditions and allows a wide range of slug test data to be analyzed with greater accuracy than traditional linear methods. The model covers the entire range of responses from overdamped to underdamped and allows a nonlinear response.

3. Correction for transducer response

Normally, in a static water column a pressure transducer reads the pressure caused by the gravity field and the height of water above the measurement point can be simply calculated. However, if the water column is accelerating and has some velocity then the normal equations for calculating the height of water above the measurement point must be modified. Since oscillatory slug test responses in aquifers with high K involve significant accelerations and velocities in the water column, the pressure transducer responses are affected and should be corrected. Theoretically, the correction is given by (McElwee, 2000, 2001)

$$h_e(t) = \left(1 + \frac{a}{g}\right)h_m(t) + \frac{a}{g}Z_s - \frac{v^2}{2g} \qquad (2)$$

where h_e is the experimentally measured height, h_m is the theoretical model height, Z_s is the submergence of the pressure transducer, a is the acceleration of the water column, g is the acceleration of gravity, and v is the water velocity. For maximum accuracy of analysis, the model response must be corrected for acceleration before comparison to the transducer response. This correction may be significant for aquifers with very high K and large oscillatory responses.

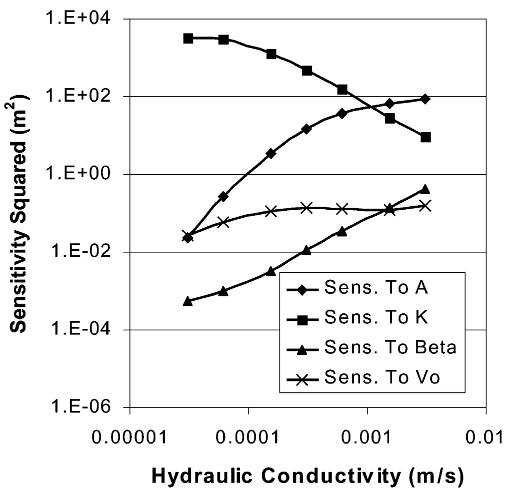

Fig. 4. Sensitivity plot for the four model parameters as a function of hydraulic conductivity ($\beta = 2.1$ m (6.9 ft), $A = 20$, $V_o = -0.15$ m/s (-0.5 ft/s)).

4. Sensitivity analysis

Sensitivity analysis is a formalism that allows the estimation of the change in model response due to changing one parameter (McElwee, 1987). This can be combined with a model fitting routine to iteratively determine the set of parameters that give the best fit to experimental data (McElwee, 2000, 2001). The normalized sensitivity coefficient at space location i and time step n for parameter k is defined as

$$U'^{\,n}_{i;k} = P_k \frac{\partial h_i^n}{\partial P_k} \qquad (3)$$

which is a measure of how much the model output (h_i^n) changes when the parameter P_k is changed by a small amount. If there are N parameters ($k = 1, 2, \ldots, N$), an $N \times N$ sensitivity matrix can be defined and used in the least squares fitting procedure to produce the following equation, which is to be solved for fractional parameter increments $\Delta P_k/P_k$ (McElwee, 2000, 2001)

$$\mathbf{U}'^T\mathbf{U}'\frac{\Delta\mathbf{P}}{\mathbf{P}} = \mathbf{R} \tag{4}$$

$$R_k = \sum_i \sum_n U'^{\,n}_{i;k}[\mathrm{he}_i^n - h_i^n] \tag{5}$$

The right hand side of Eq. (4) involves the difference between experimental slug height (he_i^n) and modeled slug height multiplied by the sensitivity coefficient, as shown in Eq. (5). $\mathbf{U}'^T\mathbf{U}'$ is the $N \times N$ least squares sensitivity matrix.

The diagonal elements of the sensitivity matrix ($\mathbf{U}'^T\mathbf{U}'$) provide a measure of the model sensitivity to a given parameter. The current model has four parameters (β, A, K, and V_o). The diagonal elements of the sensitivity matrix for these four parameters are plotted in Fig. 4 as a function of K (with typical values of β, A and V_o held constant). Sensitivity analysis shows that in general β and V_o have the lowest sensitivity and K usually has the highest sensitivity, however, for very high K the sensitivity to A may surpass the sensitivity to K. Clearly at larger values of K all four parameters must be considered to obtain the most accurate fit of the experimental data to the model. However, for low K, it can be seen that the model is mainly sensitive to K and reduces to the usual Hvorslev (1951) model. For intermediate K values the model has varying sensitivity to the parameters. Sensitivity analysis should be used to decide which parameters need to be fit and which ones can be held at a constant value. The sensitivities for each parameter vary with K but the model has been found to be robust with regard to estimates of K over the whole range.

Sensitivity analysis can be used to obtain a number of other diagnostic results (McElwee, 2000, 2001). The sensitivity matrix ($\mathbf{U}'^T\mathbf{U}'$) can be used to estimate the parameter covariance matrix once the error variance in head has been estimated for the model fit. From the parameter covariance matrix one can estimate the standard error and confidence intervals for each parameter. Any parameter having a large standard error and confidence interval will have low sensitivity and perhaps should be eliminated from the fitting process by being held at a constant value (perhaps near zero). Finally, one can approximate the parameter correlation matrix. If any of the off diagonal elements of the parameter correlation matrix are greater than 0.9, one should have concern about the ability to estimate those two parameters simultaneously. Perhaps one parameter should be held constant at a reasonable value.

5. Performance of multiple slug tests

In order to obtain the maximum amount of information from a slug testing program multiple tests should be executed at a given location. This will allow a definitive test for nonlinear effects and will determine if the tests are repeatable (Butler et al., 1996). Linear theories for slug testing indicate that responses should be independent of initial height if plotted as normalized height versus time. Repeat tests for a given initial height will indicate the magnitude of noise present at that site during the time of testing.

The model presented in Eq. (1) has two nonlinear contributions. One is a weak nonlinearity in the first term, which occurs because the length of the accelerating water column is changing slightly with h. This effect will be small if the total column length is large in comparison to the initial slug height. The second term in Eq. (1) involves the velocity magnitude squared and can be very important when the velocity in the wellbore is high, which will be the case for high values of K. The easiest way to determine if nonlinear effects are important is to perform multiple slug tests having different initial heights (h_o, see Fig. 1) at a given location. If nonlinear effects are significant, the results for differing initial heights will plot separately when plotted on a graph of normalized head versus time, with higher initial heights taking longer to decay. This is in contrast with linear theories, which would predict identical curves for differing initial heights. Examples will be shown and discussed in Section 6.

For a given initial height, multiple slug tests should also be done at least some of the time. If multiple slug tests with the same initial height are not very repeatable that implies either a noisy environment for data collection or that the well is changing characteristics. Noisy environments are common due to conditions (such as pumping or transient stresses on the aquifer) in an area that cannot be controlled by the experimenter. One of the more common situations involves other pumping wells in the area that are being turned on and off. This may give rise to a time trend in the local water levels. In addition to man-made noise there will undoubtedly be other natural sources of noise. Noise data records should be taken periodically while performing slug tests so that the background noise can be quantified. One possibility is to record a channel of background noise with every slug test record by using a pressure transducer in a nearby well. The nearby well should be close enough to have similar noise as the location being slug tested but far enough away so that the slug test does not significantly affect the noise record. Noise having a varying time trend will be particularly troublesome for longer duration slug tests produced by lower values of K. On the other hand, for wells with high K the slug test is over in a matter of seconds and the correction for noise with significantly longer characteristic times will be easier to obtain. Random noise should produce random effects in the slug test data. However, if a systematic behavior is noted that cannot be explained by Eq. (1), that may be an indication that the well is changing its characteristics. Examples of this kind of behavior would be when the measured K depends on the initial slug height, the order in which tests are performed, and the direction of flow (falling head or rising head tests). Wells can change effective hydraulic conductivity from one slug test to another if the well has not been properly developed since it was drilled. Very aggressive development can cause a zone around the screen to be depleted in fine material, giving a K that varies with radius. Aquifers containing fine material that can be mobilized by the slug test can show a directional dependence for the response (falling head versus rising head). Multiple slug tests will allow the experimenter to distinguish between some of these sources of error and to obtain some statistical measure of the experimental accuracy.

6. Example slug tests

A field site for research and teaching has been developed in the Kansas River alluvium in close proximity to the University of Kansas at the Geohydrologic Experiment and Monitoring Site (GEMS). The alluvial aquifer at the site is about 21 m (70 ft) thick and consists of coarse sand and gravel (about 10.5 m (35 ft)) overlain by silt and clay (also about 10.5 m (35 ft) thick). It is known from extensive drilling, sampling, and a tracer test that the hydraulic conductivity varies a great deal spatially (McElwee and Butler, 1995). Over 70 wells have been completed at various depths. Typically, there is a fully screened well and several wells with short screens completed at various depths for each well nest. Generally speaking, it is a fining upward sequence in the sand and gravel region. The site exhibits some very high K values and nonlinear behavior for slug tests in the sand and gravel region. Slug tests have been performed in wells that are completed in the sand and gravel interval using a packer system with a piston for slug test initiation, allowing accurate determination of the initial height and starting time for the slug test. The packer used for all the tests shown in this paper has a 2.5 cm (1 in.) diameter flow through pipe, which is about as large as possible for a packer used in 5 cm (2 in.) wells. It is important to keep the resistance of the packer to a minimum.

The three following examples of slug tests from wells at GEMS span the range from overdamped to underdamped slug test responses. The slug tests were performed by adding a given volume of water to the well. One liter raises the water level in these 5 cm (2 in.) wells by about 0.54 m (1.8 ft). The slug test data have been analyzed with the nonlinear model (McElwee, 2000, 2001). In each case some normalized slug test plots with differing initial heights are shown to investigate nonlinear effects. Examples of repeat slug tests for a given initial height are also shown to look for noise or changing well characteristics. The application of sensitivity analysis yields an estimate of the confidence intervals for the parameters. For the test involving the highest K the effect of making the acceleration correction is illustrated.

6.1. Well 1–2

This well is completed with a total casing length of 11.5 m (37.7 ft) including a screen length of 60 cm (2.0 ft) and is located in fairly fine grained material directly under the semiconfining layer at GEMS. Therefore, one would expect a fairly low K and an overdamped response. Four slug tests were performed by adding 4, 2, 1, and 2 L of water. Two tests of 2 L

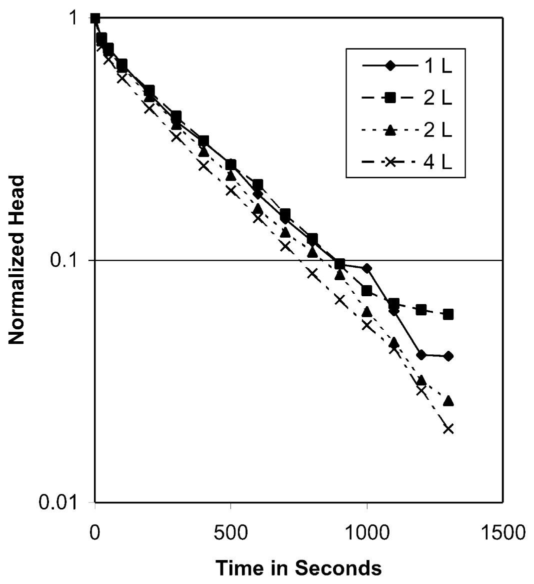

Fig. 5. Comparison of slug test responses at Well 1–2 for various initial heights.

were performed to examine repeatability. Plots of the data in the usual Hvorslev format are shown in Fig. 5. There is some spread in the responses for differing initial heights indicating noise, however, there is no definite trend as there would be for either nonlinear responses or changing well characteristics. There is some upward curvature at early time indicating some

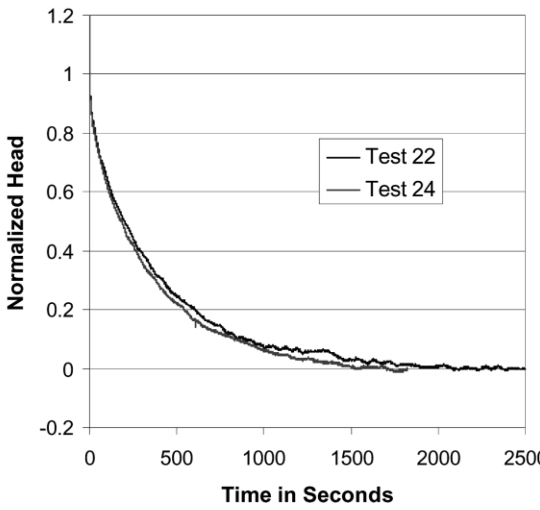

Fig. 6. Comparison of repeat slug tests at Well 1–2 when 2 l are added.

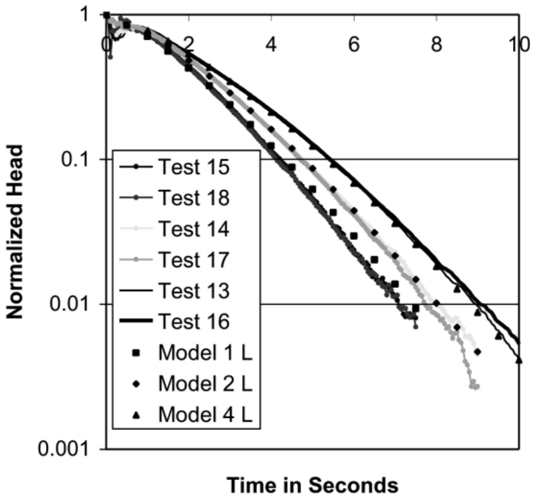

Fig. 7. Comparison of multiple slug test responses and the theoretical results at Well 00–6.

storage effects (Chirlin, 1989). The nonlinear model indicates that there is very little sensitivity to β, A, or V_o and that the best fit for the suite of tests is $K = 4.27 \times 10^{-6}$ m/s (1.40×10^{-5} ft/s). When the four tests are analyzed separately the average K is 3.99×10^{-6} m/s (1.31×10^{-5} ft/s), with a standard deviation of 3.72×10^{-7} m/s (1.22×10^{-6} ft/s), which is a little less than 10% of the average value. In this low K well the Hvorslev (1951) method or Cooper et al. (1967) method gives similar results within a few percent to the model presented in this paper.

Fig. 6 shows two repeat slug test responses for 2 L added to the well. The difference between the two curves represents experimental error during these tests. Part of the error could be due to experimental technique and some will be due to ambient noise at that location. Test 24 in Fig. 6 was truncated at a little over 1800 s, while test 22 was allowed to proceed to about 2500 s. As a consequence, the determination of the static level for the two tests was probably biased by the difference in record length. In retrospect, it appears that the experimental data should have been collected for about 3000 s to allow a better definition of the static level. The spread in the responses shown in Figs. 5 and 6 represent experimental error and probably contains a component due to ambient noise. There are some periodic pumping wells (rural water district) in the

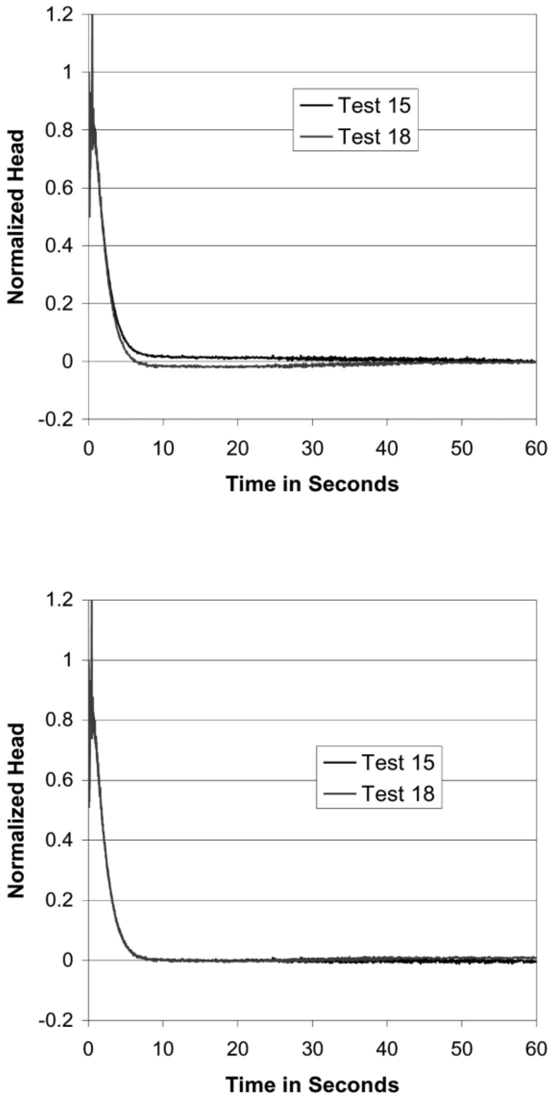

Fig. 8. Slug test responses at Well 00–6 for 1 l added: (A) before water level trend correction; (B) after correction for a linear trend in local water levels.

vicinity of GEMS and these might have caused some varying time trends for the local water levels during the tests shown in Figs. 5 and 6. Since these slug tests were fairly slow to recover, the effect of a time trend is more pronounced and more difficult to correct for than in shorter tests. However, even with noise it appears the K estimate is good within about 10%.

6.2. Well 00–6

This well is completed with a total casing length of 12.9 m (42.4 ft) including a screen length of about 76 cm (2.5 ft) and is located in medium grained material less than 2 m below the semiconfining layer at GEMS. The completion depth results in a moderate value for the hydraulic conductivity; however, the response is still a damped one as shown in Fig. 7. The semilog Hvorslev plot of Fig. 7 shows responses for 1, 2, and 4 L added, with two tests at each volume. There is a definite indication of nonlinear behavior here as the responses for differing initial heights are well separated. There is downward curvature in these responses in distinct contrast to Well 1–2. The early oscillations near zero time are due to elastic effects (water hammer) in the water column and are ignored in this analysis. Sensitivity analysis with the nonlinear model indicates that there is low sensitivity to β and V_o but good sensitivity to K and A. The best fit for the suite of tests is $K = 7.04 \times 10^{-4}$ m/s (2.31×10^{-3} ft/s) and $A = 14.1$. For this data the estimated 95% confidence interval for K is $\pm 4.6 \times 10^{-6}$ m/s ($\pm 0.15 \times 10^{-4}$ ft/s) and for A it is ± 0.33. The calculated responses from the nonlinear model are shown as black symbols on Fig. 7 and match the observed responses very well.

Traditional methods cannot be correctly applied to the data in Fig. 7 because of the nonlinearity exhibited. If one tries to apply a Hvorslev model (straight line fit) to the data, the results for K depend on the time interval used for the fit and the volume of water added. The Hvorslev analysis K values vary from 7.80×10^{-4} m/s (2.56×10^{-3} ft/s) when fitting all the data for 4 L added to 1.14×10^{-3} m/s (3.74×10^{-3} ft/s) when analyzing the data for the 4–10 s time interval for 1 L added. The worst result is about 62% too high. The best result is close to the earlier nonlinear model analysis; however, the data analyst has no objective way to pick the best Hvorslev result.

The tests shown in Fig. 7 are over in about 10 s so they are not very susceptible to long term trends in local water levels. Short-term trends in the water levels that can be approximated by linear trends can be subtracted out fairly easily to improve the data. All the data in Fig. 7 have been examined for trends and corrected accordingly. As an example of this process

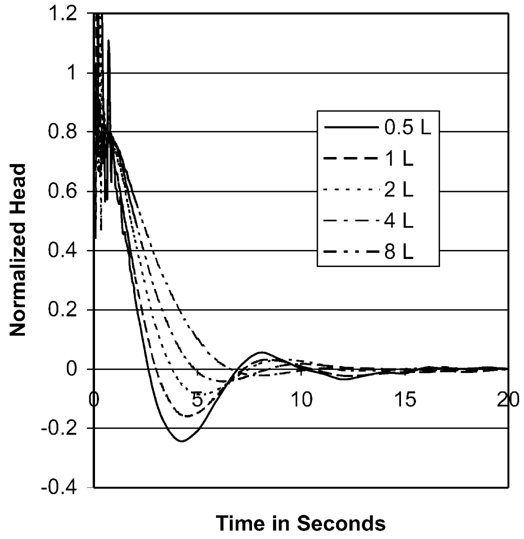

Fig. 9. Comparison of slug test responses at Well 2–7 for various initial volumes of added water.

Fig. 8A shows the two slug test responses for 1 L added to the well, after initial processing. It is clear that there is a problem with repeatability. A linear trend in the local water table with time can be subtracted out (McElwee, 2000) to obtain the result shown in Fig. 8B. Now the two data records agree very well and are the ones shown in Fig. 7.

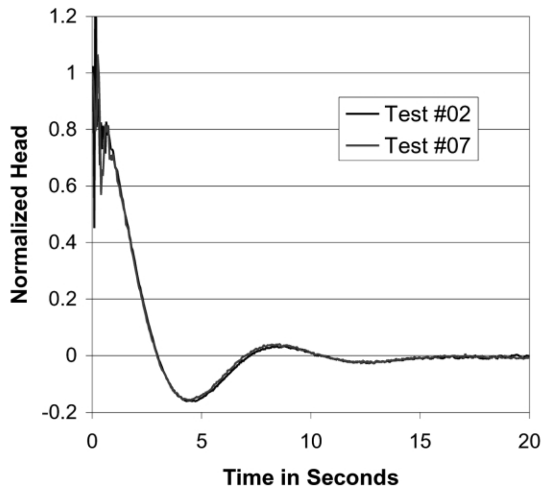

Fig. 10. Repeatability of 1 l slug tests at Well 2–7.

6.3. Well 2–7

This well is completed with a total casing length of 17.2 m (56.4 ft) including a screen length of 79 cm (2.6 ft) and is located in fairly coarse grained material near the center of the sand and gravel aquifer at GEMS. A fairly high K results in an underdamped response with significant oscillation. The plot in Fig. 9 is of normalized height or head versus time, which makes it appear that the oscillation is greater for smaller initial heights. However in fact, the amplitude of the first negative oscillation is only weakly dependent on the initial height, varying from about -0.064 m (-0.2 L ft) for the 0.5 L tests to about -0.094 m (-0.3 L ft) for the 8 L tests. There is good separation of all responses indicating strong nonlinear effects, with higher initial heights taking longer to decay. These data records also show oscillation at early time due to elastic effects in the water column, which are ignored in the present analysis. After basic processing and correction for any local water level trends, the slug test responses for identical volumes have good repeatability and show evidence of only a small amount of other water level noise, as shown for the 1 L tests in Fig. 10.

The parameter β represents a correction to the water column length caused by radius variations in the wellbore and is most useful in matching the oscillation frequency and amplitude in wells with underdamped responses. For simple radius variations in the wellbore, the theoretical value for β can be computed; for the packer used in these tests β is calculated to be 2.1 m (6.9 ft). Holding β at this theoretical value, the best fit for the suite of tests analyzed together is $K = 1.47 \times 10^{-3}$ m/s (4.83×10^{-3} ft/s) and $A = 18.2$. For 10 tests analyzed separately the average K is 1.54×10^{-3} m/s (5.05×10^{-3} ft/s) with a standard deviation of 6.83×10^{-5} m/s (2.24×10^{-4} ft/s) or about 4.4% of the average value. Sensitivity analysis used with the nonlinear model indicates that there is moderate sensitivity to β and V_o and strong sensitivity to A and K.

Traditional methods of analysis for oscillatory slug tests would involve linear models (van der Kamp, 1976; Kipp, 1985; or Springer and Gelhar, 1991). Linear models can be approximated by setting $A = 0$ in the nonlinear model. When the data in Fig. 9 are analyzed in this manner, the results vary from

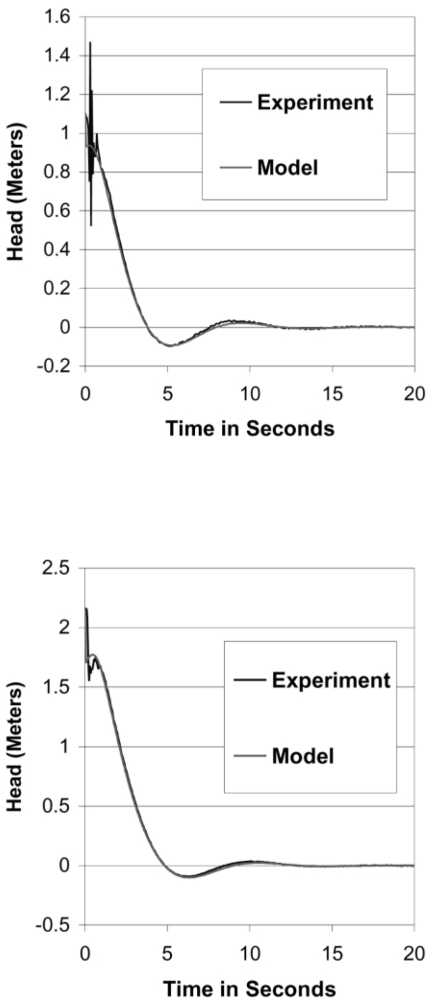

Fig. 11. Comparison of experimental and fitted head at Well 2–7 for two initial added volumes: (A) 2 l; (B) 4 l.

1.52×10^{-3} m/s (4.99×10^{-3} ft/s) for 0.5 L added to 5.61×10^{-4} m/s (1.84×10^{-3} ft/s) for 8 L added. The 8 L added result is only 38% of the nonlinear analysis result, with a fit to the data that is not very good. The 0.5 L added result is close to the nonlinear result and is a good fit to the data. This shows that nonlinear effects decrease as the initial slug height is reduced. However, as the initial height is reduced any ambient noise that is present becomes more of a problem and the volume of the aquifer tested becomes smaller (Guyonnet et al., 1993). Very small initial heights may give results that are unduly influenced by the aquifer conditions very near the screen.

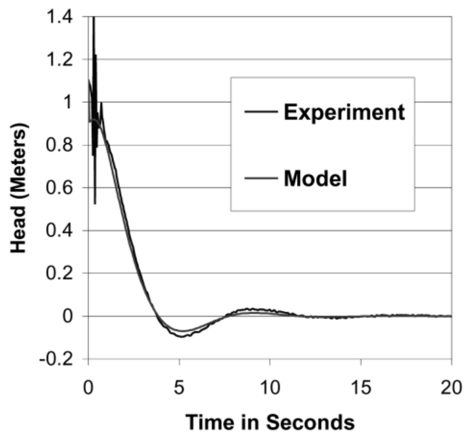

Fig. 12. Comparison of experimental and calculated head at Well 2–7 with $\beta = 0$. Initial added volume is 2 l.

The results of fitting the nonlinear model to the 2 and 4 L slug tests in Well 2–7 are shown in Fig. 11A and B. The response is underdamped and highly oscillatory and shows strong nonlinear effects. These two tests are matched well, indicating the model is describing the system adequately. The sum of squared errors for the fit is 0.61 m^2 (6.6 ft^2). The fit shown in these figures will be compared later to a fit when $\beta = 0$ and the acceleration correction is not applied.

When the slug test responses are recalculated using $\beta = 0$ instead of the theoretical value of 2.1 m (6.9 ft) (all other parameter values are held constant at their fitted values), the sum of squared errors increases to 0.98 m^2 (10.4 ft^2). The resulting response for the 2 L test is shown in Fig. 12 and is seen to deteriorate

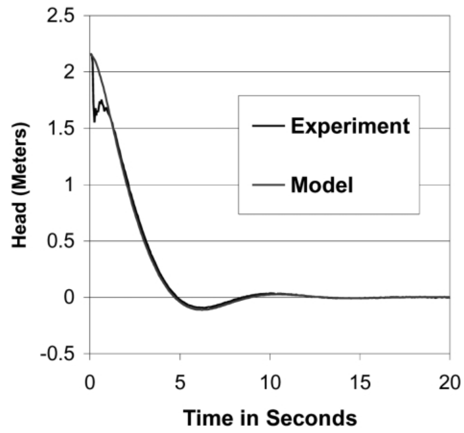

Fig. 13. Comparison of experimental and calculated head at Well 2–7 with no acceleration correction (Initial added volume 4 l).

somewhat. It is clear that the beta parameter is important in matching the amplitude and frequency of the oscillations.

Oscillatory slug tests involve accelerations and higher velocities of the water column and the pressure transducer responses are affected by these factors. If the acceleration and velocity correction is not applied and the slug test responses are recalculated for the previously converged values of the parameters, the sum of squared errors increases to 0.95 m^2 (10.2 ft^2). The model response without acceleration and velocity correction is compared to the 4 L slug test data in Fig. 13. It is clear that the early part of the experimental record is not as well matched as before. Although it is more difficult to see, the fit with the experimental data in other areas is not as good also. The acceleration and velocity correction is largest at initiation and is about 0.45 m (1.5 ft). From this value it decays to zero and goes negative, reaching a maximum negative value of -0.026 m (-0.087 ft) at about 2.8 s. The correction becomes negligible as time continues to increase.

7. Conclusions

To obtain maximum accuracy in analyzing slug test data one should employ a number of techniques: an extended capability model, sensitivity analysis, correction for acceleration and velocity effects, and multiple slug tests. Use a model that will seamlessly simulate responses for the overdamped region through the critically damped region and on into the underdamped region. This is particularly important when taking multilevel data sets at a site where the hydraulic conductivity changes dramatically from location to location. The four-parameter nonlinear slug test model used in this work has been shown to allow accurate analysis of slug tests with widely differing character. As the hydraulic conductivity and the velocities in the wellbore increase, the nonlinear effects represented by the parameter A also increase. At some point the slug test response will become oscillatory as the hydraulic conductivity increases. The parameter β represents a correction to the water column length caused primarily by radius variations in the wellbore and is most useful in matching the oscillation frequency and amplitude. The water column velocity at slug initiation (V_o) would ideally be zero, but may not be due to the initiation mechanism. Therefore, V_o is used as a model parameter. The use of this extended capability model increases the accuracy of the data analysis. Sensitivity analysis shows that in general β and V_o have the lowest sensitivity and hydraulic conductivity (K) usually has the highest, however, for very high K the sensitivity to A may surpass the sensitivity to K. The sensitivities to the four parameters vary with hydraulic conductivity; but we find the model to be robust with regard to estimates of hydraulic conductivity over a wide range of conditions. Since oscillatory slug tests involve accelerations and higher velocities of the water column, the pressure transducer responses will be affected by these factors. For maximum accuracy of analysis, the model response must be corrected for acceleration and velocity effects before comparison to the transducer response. This effect is most noticeable for slug tests in very high conductivity material. Multiple slug tests should be taken at a given location to test for nonlinear effects and to determine repeatability. Nonlinear effects will be revealed as a dependence on the initial height of the slug test. Performing multiple slug tests may allow some insight as to the type of noise present. Random noise, time trends of water levels, and changing well characteristic are possible sources of error and can be identified. The performance and analysis of multiple slug tests will allow some estimation of the experimental accuracy and of the reliability of the estimated aquifer parameters.

References

Bouwer, H., Rice, R.C., 1976. A slug test for determining hydraulic conductivity of unconfined aquifers with completely or partially penetrating wells. Water Resources Research 12 (3), 423–428.

Butler, J.J. Jr., McElwee, C.D., Liu, W., 1996. Improving the quality of parameter estimates from slug tests. Ground Water 34 (3), 480–490.

Chirlin, G.R., 1989. A critique of the Hvorslev method for slug test analysis: the fully penetrating well. Ground Water Monitoring Review 9 (2), 130–138.

Cooper, H.H., Bredehoeft, J.D., Papadopulos, I.S., 1967. Response of a finite-diameter well to an instantaneous charge of water. Water Resources Research 3 (1), 263–269.

Guyonnet, D., Mishra, S., McCord, J., 1993. Evaluating the volume of porous medium investigated during slug tests. Ground Water 31 (5), 627–633.

Hvorslev, M.J., 1951. Time Lag and Soil Permeability in Ground-Water Observations, Waterways Experimental Station, Corps of Engineers, US Army, Bulletin no. 36, 50 pp..

Kabala, Z.J., Pinder, G.F., Milly, P.C.D., 1985. Analysis of well-aquifer response to a slug test. Water Resources Research 21 (9), 1433–1436.

Kipp, K.L. Jr, 1985. Type curve analysis of inertial effects in the response of a well to a slug test. Water Resources Research 21 (9), 1397–1408.

Krauss, I., 1974. Die Bestimmung der Transmissivität von Grundwasserleitern aus dem Einschwingverhalten des Brunnen-Grundwasserleitersystems. Journal of Geophysics 40, 381–400.

Krauss, I., 1977. Determination of the transmissibility from the free water level oscillation in well-aquifer systems. In: Morel-Seytoux, H.J., (Ed.), Surface and Subsurface Hydrology, Proceeding of the Fort Collins Third International Hydrology Symposium, on Theoretical and Applied Hydrology, Water Resources Publications, Fort Collins, pp. 268–279.

McElwee, C.D., 1987. Sensitivity analysis of ground-water models. In: Bear, J., Corapcioglu, M.Y. (Eds.), Proceedings of the 1985 NATO Advanced Study Institute on Fundamentals of Transport Phenomena in Porous Media, Martinus Nijhoff, Dordrecht, pp. 751–817.

McElwee, C.D., 2000. Implementation of a Nonlinear Model for Analysis of Slug Tests, Kansas Geological Survey Computer Program Series 2000–01, 99 pp.

McElwee, C.D., 2001. Application of a nonlinear slug test model. Ground Water 39 (5), 737–744.

McElwee, C.D., Butler, J.J. Jr., 1995. Characterization of heterogeneities controlling transport and fate of pollutants in unconsolidated sand and gravel aquifers: final report. Project Report to Air Force Office of Scientific Research, University Research Initiative Research Initiation Program, US Department of Defense, Kansas Geological Survey Open-File Report no. 95-16, 594 pp.

McElwee, C.D., Zenner, M., 1998. A nonlinear model for analysis of slug-test data. Water Resources Research 34 (1), 55–66.

Springer, R.K., Gelhar, L.W., 1991. Characterization of large-scale aquifer heterogeneity in glacial outwash by analysis of slug tests with oscillatory responses, Cape Cod, Massachusetts. In: Mallard, G., Aronson, D. (Eds.), US Geological Survey, Toxic Substances Hydrology Program Proceeding of the Technical Meeting, Monterey, California, March 11–15, US Geological Survey, WRI Report. 91-4034, pp. 36–40.

Stone, D.B., Clarke, G.K.C., 1993. Estimation of subglacial hydraulic properties from induced changes in basal water pressure: a theoretical framework for wellbore response tests. Journal of Glaciology 39 (132), 327–340.

Streeter, V.L., Wylie, E.B., 1985. Fluid Mechanics, McGraw-Hill, New York.

van der Kamp, G., 1976. Determining aquifer transmissivity by means of well response tests: the underdamped case. Water Resources Research 12 (1), 71–77.

Zlotnik, V.A., McGuire, V.L., 1998. Multi-level slug tests in highly permeable formations: 1 modification of the Springer–Gelhar (SG) model. Journal of Hydrology 204, 271–282.