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Modified Biot-Gassmann Theory for Calculating Elastic Velocities for Unconsolidated and Consolidated Sediments*

Myung W. Lee
U.S. Geological Survey, Denver Federal Center, Box 25046, MS-939 Denver, Colorado 80225, U.S.A. (E-mail: mlee@usgs.gov)

Key words:

Abstract

The classical Biot-Gassmann theory (BGT) generally overestimates shear-wave velocities of water-saturated sediments. To overcome this problem, a new theory is developed based on BGT and on the velocity ratio as a function of $G(1-\phi)^n$, where ϕ is porosity and n and G are constants. Based on laboratory data measured at ultrasonic frequencies, parameters for the new formulation are derived. This new theory is extended to include the effect of differential pressure and consolidation on the velocity ratio by making n a function of differential pressure and the rate of porosity reduction with respect to differential pressure. A scale G is introduced to compensate for discrepancies between measured and predicted velocities, mainly caused by the presence of clay in the matrix. As differential pressure increases and the rate of porosity reduction with respect to differential pressure decreases, the exponent n decreases and elastic velocities increase. Because velocity dispersion is not considered, this new formula is optimum for analyzing velocities measured at ultrasonic frequencies or for sediments having low dispersion characteristics such as clean sandstone with high permeability and lack of grain-scale local flow. The new formula is applied to predict velocities from porosity or from porosity and P-wave velocity and is in good agreement with laboratory and well log data.

Introduction

The velocity ratio (V_p/V_s), where V_p is the *P*-wave velocity and V_s is the *S*-wave velocity, has been used for many purposes, such as identifying lithologies, determining the degree of compaction, identifying pore fluid, and predicting velocities. The velocity ratio generally depends on lithology, porosity, degree of compaction and consolidation, clay content, differential pressure, frequency, pore geometry, and other factors. For dry rock or gas-saturated rock, the velocity ratio is almost a constant irrespective of porosity and differential pressure (Winkler, 1985; Krief et al., 1990), whereas the velocity ratio of wet rock depends largely on porosity and differential pressure. The purpose of this paper is to accurately predict elastic velocities of water-saturated clastic sediments by utilizing the dependence of the velocity ratio on porosity.

Pickett's cross plot (1963) shows that P-wave to S-wave velocity ratio (V_p/V_s) for brine-saturated sandstone is about 1.6 in low porosity rocks, drifting to 1.8 in relatively higher porosity rocks. His observation implies the dependence of V_p/V_s on the porosity for sandstone. Gardner and Harris (1968) showed that V_p/V_s values > 2.0 are characteristic of water-saturated unconsolidated rocks, and values < 2.0 indicate either well-consolidated rock or the presence of gas in unconsolidated sands. Gregory (1976) confirmed this relationship between the velocity ratio and consolidation and suggested the dependence of velocity ratio on porosity.

Hornby and Murphy (1987) and Murphy et al. (1993) showed that (1) the velocity ratio increases as the clay content increases, (2) the Biot-Gassmann theory (BGT) accurately predicts the velocity ratio of unconsolidated water-saturated sand with respect to effective pressure. Castagna et al. (1985) and Han et al. (1986) empirically derived relations between velocity ratio and the porosity and clay content. Han et al. (1986) showed that the velocity ratio increases linearly with clay content and porosity. The equation by Castagna et al. (1985) also implies increase of the velocity ratio with increasing porosity.

The prediction of S-wave velocities for water-saturated rocks based on the velocity ratio with a first-order application of the BGT is given by Greenberg and Castagna (1992). Empirical relations by Castagna et al. (1985) and Han et al. (1986) can be used to predict S-wave velocity either from porosity and clay content or velocity ratio with porosity. Xu and White (1996) investigated the S-wave velocity prediction based on the bulk and shear moduli of the dry rock frame by a combination of the Kuster and Toksöz theory (1974) and the differential effective medium theory, using pore aspect ratio. They listed a number of disadvantages of empirical relations, including lack of physical mechanisms of rock properties.

In this paper, a new method of modeling velocities for consolidated and unconsolidated sediments is presented, based on the assumption that the velocity ratio depends on porosity (Lee, 2002, 2003). Lee (2002) developed this new theory for unconsolidated sediments, particularly for gas-hydrate-bearing sediments. Lee (2003) extended the earlier version to include consolidated sediments and to incorporate the effect of differential pressure on velocity. Pickett (1963), Gregory (1976), Castagna et al. (1985), and Han et al. (1986) each suggested a relation between porosity and the velocity ratio, but their functional relations between porosity and the velocity ratio are different from the one proposed in this paper. Parameters for this investigation were derived from laboratory data by Domenico (1977), Han et al. (1986), Huffman and Castagna (2001), and Prasad (2002) without considering velocity dispersion. A number of measured velocities for consolidated and unconsolidated sediments are in good agreement with the predicted velocities by the new method.

Theory

Elastic velocities (i.e., compressional-velocity (V_p) and shear velocity (V_s)) of water-saturated sediments can be computed from the elastic moduli by the following formulas:

$$V_p = \sqrt{\frac{k + 4\mu/3}{\rho}} \quad \text{and} \quad V_s = \sqrt{\frac{\mu}{\rho}}, \tag{1}$$

where k μ, and ρ are bulk modulus, shear modulus, and density of the formation, respectively. The formation density is given by

$$\rho = (1-\phi)\rho_{ma} + \phi\rho_{fl}, \tag{2}$$

where ϕ, ρ_{ma} and ρ_{fl} are the porosity, matrix (constitutes the skeleton of the formation) density , and pore fluid density, respectively.

The new method proposed here to calculate elastic velocities is based on an assumption that shear moduli of sediments can be estimated from the following relation of the velocity ratio:

$$V_s = V_p G\alpha(1-\phi)^n, \tag{3}$$

where α is the V_s/V_p ratio for the matrix material and G is a scale to incorporate the V_s/V_p discrepancy between measured and calculated.

The bulk and shear moduli are given by the following formulas using the Biot coefficient β (Lee, 2003)

$$k = k_{ma}(1-\beta) + \beta^2 M, \tag{4}$$

$$\mu = \frac{\mu_{ma}k_{ma}(1-\beta)G^2(1-\phi)^{2n} + \mu_{ma}\beta^2 MG^2(1-\phi)^{2n}}{k_{ma} + 5\mu_{ma}[1 - G^2(1-\phi)^{2n}]/3}. \tag{5}$$

where

$$\frac{1}{M} = \frac{(\beta-\phi)}{k_{ma}} + \frac{\phi}{k_{fl}},$$

and k_{ma}, μ_{ma}, and k_{fl} are the bulk modulus of matrix, the shear modulus of the matrix, and the bulk modulus of the fluid, respectively.

For soft formations or unconsolidated sediments, the following Biot coefficient is used (Lee, 2002):

$$\beta = \frac{-184.05}{1 + e^{(\phi+0.56468)/0.09425}} + 0.99494. \tag{6}$$

For hard formations, the equation by Raymer et al. (1980), which is written by the following form by Krief et al. (1990), is used.

$$\beta = 1 - (1-\phi)^{3.8}. \tag{7}$$

Under the low frequency approximation, the Biot (1956) equation yields:

$$\mu = \mu_{ma}(1-\beta). \tag{8}$$

Equation (4) with Equation (8) is the original formulation of Biot-Gassmann theory (BGT) (Biot, 1941, 1956; Gassmann, 1951) under low-frequency approximation. Equation (8) also can be derived under the assumption that the ratio of S-wave velocity to the P-wave velocity is constant irrespective of the porosity and equals to the velocity ratio of matrix for dry rock

(Krief et al., 1990) or the shear modulus of rock is not affected by fluid saturation (e.g., Wang, 2000).

In this paper, only water-saturated sediments are considered. To differentiate this new theory from BGT, the new theory is called the BGT by Lee or BGTL, where Equation (5) is used to calculate the shear modulus. It is emphasized that the difference between BGT and BGTL is the way shear modulus of the formation is derived. As indicated in Lee (2003), there is no difference between BGT and BGTL for gas saturated sediments. BGTL with the Biot coefficient for consolidated sediments works well for porosity less than about the critical porosity (0.4 for sandstone, Nur et al., 1998). Above this porosity range, the Biot coefficient for unconsolidated sediments is preferable.

Parameters *n* and *G*

The velocity ratio depends on many factors such as porosity, differential pressure, consolidation, clay content, and frequencies of measurements (Lee, 2003). It has been known that elastic wave velocities in water-saturated sediments are dispersive (e.g., Winkler, 1983; Murphy, 1985), so the velocity ratio would be dependent on the frequency of measurement. In the BGTL formulation, the exponent n incorporates the effect of differential pressure and the scale G compensates for the effect of clay on the velocity ratio. However, the proposed BGTL does not include the effect of velocity dispersion.

There are two adjustable parameters in BGTL that can be estimated from the physical nature of sediments. Based on the laboratory data compiled by Prasad (2002) with frequencies ranging from 100 kHz to 1 MHz, the following equation for the exponent n is derived.

$$n = [10^{(0.426-0.235 Log_{10} p)}]/m, \quad (9)$$

where p is differential pressure in MPa and m is a constant to be determined.

To a first order approximation, the constant m appears to depend on the rate of porosity change with respect to differential pressure ($\partial\phi/\partial p$) and is given by the following equation .

$$m = 1.0 + 4.95289 e^{5212 \partial\phi/\partial p}. \quad (10)$$

Equation (10) indicates that as $\partial\phi/\partial p$ approaches zero, m approaches about 6, and as it approaches a large number, m approaches one. In deriving Equation (10), only three input points - Domenico (1977) measured at about 0.5 MHz, Han (1986) measured at 0.6–1 MHz, and by Prasad (2002) measured at 0.1 MHz (Table 4 of Prasad) - are used and the least squares fitting to the exponential function is exact. Thus, it is not known whether Equation (10) is appropriate for other values of $\partial\phi/\partial p$. In practice, $\partial\phi/\partial p$ is rarely known, thus a direct application of Equation (10) is limited. Measured data indicate that $m \approx 5$ is appropriate for consolidated sediments and $m \approx 1.5$ is suitable for unconsolidated sediments.

The accuracy of BGTL depends on how accurately the observed V_p/V_s of sediments agrees with the predicted ratio by Equation (3). Many factors such as the aspect ratio of pore space, clay volume content, degree of compaction, frequency, differential pressure, and others affect the velocity ratio and cause discrepancy between predicted and observed velocities. A constant G is introduced to correct the discrepancy mainly caused by clay in the matrix. Laboratory data by Han et al. (1986) indicate that $G = 1$ is good for clean sandstone and as clay volume increases, G appears to decreases according to the following equation:

$$G = 0.9552 + 0.0448 e^{-C_v/0.06714}. \quad (11)$$

Equation (11) is derived for consolidated sediments having an average clay volume content of 15%. For practical applications of BGTL, G may be treated as a free parameter to fit the observation. As n increases, velocities decrease and n has a more pronounced effect on the S-wave velocity than on the P-wave velocity. As G decreases, velocities decrease and G has a more pronounced effect on the S-wave velocity. Velocity errors associated with parameters n and G are given in Lee (2003).

Modeling with BGTL

Modeling velocities with respect to porosity

BGTL is most suitable to model velocities with respect to porosity. Figure 1 shows measured and computed velocities with respect to porosity. Black dots and circles in Figure 1 are measured velocities at $p = 30$ MPa by Han et al. (1986). In order to model Han's data, parameters of n and G are calculated using an average clay content of $C_v = 0.15$ with $m = 5$ and $p = 30$ MPa. Predicted P- and S-wave velocities, shown as solid lines, follow the average trend of measured velocities quite well. The large scattering of measured velocities compared to predicted velocities is due to

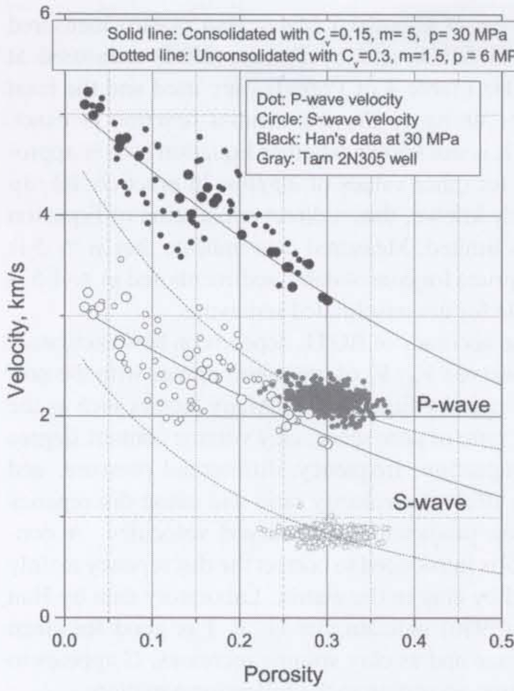

Figure 1. Predicted and measured velocities. To model Han's data (1986), $p = 30$ MPa, $m = 5$ and an average clay content of $C_v = 15\%$ are used. Dots denote P-wave velocity and circles represent S-wave velocity. Hans's data (1986) at 30 MPa are shown as black color and Tarn 2N305 data are shown as gray color. Large dots and circles are velocities of Han's data for sediments having clay volume content between 10 and 20%. For Tarn 2N305 well data (depth range between 500–600 m), $P = 6$ MPa, $m = 1.5$, and $C_v = 30\%$ are used.

the variable clay contents in Han's data. Large dots and circles are velocities of Han's data for sediments having clay volume contents between 0.1 to 0.2, and these values follow the predicted velocities with less scattering. Gray dots and circles are well-log velocities measured by DSI tool for unconsolidated sediment at the Tarn 2N305, northern Alaska. Typical sonic frequencies for a DSI tool are 12 kHz for the monopole P-wave and 2.5 kHz for dipole S-wave (Guerin and Goldberg, 2002). Predicted velocities from BGTL with $C_v = 0.3$, $m = 1.5$, and $p = 6$ MPa are shown as dotted lines and agree well with measured velocities.

Modeling velocity ratio, V_p/V_s

Primarily velocities of sediments depend on differential pressure and porosity. For a given sediment, as differential pressure increases, porosity of sediment decreases. In order to accurately predict elastic velocities with respect to differential pressure, porosity change owing to differential pressure should be incorporated, particularly for unconsolidated sediments. The general behavior of porosity variation with respect to differential pressure is not accurately known. Therefore, in this paper, the data presented by Prasad (2002, Table 4) are used to derive a relation between porosity and differential pressure and its relationship is given as follows.

$$\phi = 0.38452 - 0.00319p. \tag{12}$$

The porosity range for the Prasad (2002) data is between 0.382 at $p = 0.89$ MPa and 0.321 at $p = 19.67$ MPa (actually, Prasad (2002) provided density instead of porosity data and the porosity is calculated assuming a matrix density of 2.65 g/cm^3 and a fluid density of 1.0 g/cm^3).

Figure 2 shows measured velocity ratios with computed ratios. Circles are measured ratios compiled by Prasad (2002). The measured velocity ratio appears to be a linear function of differential pressure in log-log scale and Prasad derived a least squares fitting curve, which is given by $V_p/V_s = 5.6014p^{-0.2742}$. As indicated in Figure 2, predicted ratio by BGTL using n given by Equation (9) with $m = 1$ and $G = 1$ is close to the measured ratio at differential pressure greater than about 0.2 MPa, but the computed ratio is much larger than that predicted by the linear function for differential pressure less than about 0.2 MPa. The dashed line is the least square fitting curve for the data analyzed by Huffman and Castagna (2001) and agrees well with the prediction of BGTL with $m = 1.3$ and $G = 1$ for differential pressure greater than about 0.1 MPa, and as differential pressure decreases, the difference between the two curves increases.

Predicting S-wave velocity

One application of BGTL is in predicting S-wave velocities from P-wave and porosityor from porosity alone. In order to demonstrate the effectiveness of BGTL, data by Han (1986) are used. Porosity of Han's data ranges from 2% to 30% and C_v ranges from 0 to 51%. Both BGT and BGTL can be used to predict velocities from porosity or the S-wave velocity from porosity and P-wave velocity. With BGTL, the S-wave velocity can be predicted from porosity and the P-wave velocity using the following formula or by using Equation (3)

$$\mu = \frac{\mu_{ma} G^2 (1-\phi)^{2n} \rho V_p^2}{k_{ma} + 4\mu_{ma}/3}. \tag{13}$$

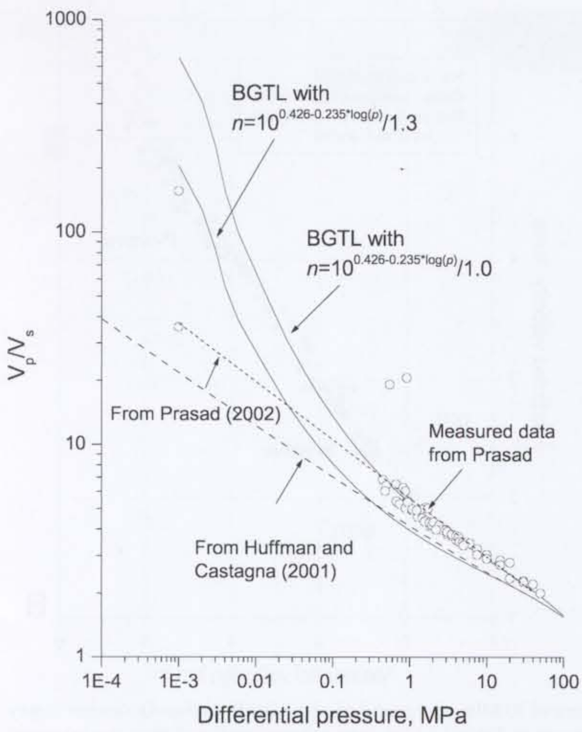

Figure 2. Graph showing computed and measured velocity ratio (V_P/V_S) with respect to differential pressure. Circles are measured V_p/V_s from data compiled by Prasad (2002, Table 3), the dotted line is a least squares fitting curve to the measured data by Prasad, and the dashed line is the least square fitting curve to the measured data by Huffman and Castagna (2001).

Table 1. Elastic constants used for velocity models

	Values used	Sources
Shear modulus of quartz	44 Gpa	Carmichael (1989)
Bulk modulus of quartz	38 Gpa	Carmichael (1989)
Shear modulus of clay	6.85 Gpa	Helgerud et al. (1999)
Bulk modulus of clay	20.9 Gpa	Helgerud et al. (1999)
Bulk modulus of water	2.29 Gpa	
Density of quartz	2.65 g/cm^3	Helgerud et al. (1999)
Density of clay	2.58 g/cm^3	Helgerud et al. (1999)

Figure 3a shows S-wave velocities predicted from P-wave velocities and porosities using Han' s data at 30 MPa based on BGTL and BGT. A variable exponent n from Equation (9) with $m = 5$, $p = 30$ MPa, the scale G given in Equation (11) , and parameters shown in Table 1 are used for BGTL. The fractional S-wave velocity error ($\Delta V_s/V_s$) from BGTL is 0.01 $\pm$ 0.03 and error from BGT is 0.06 $\pm$ 0.03.

Figure 3b shows P- and S-wave velocities predicted using only porosities. Figure 3b indicates that both BGTL and BGT predict reasonable velocities, but velocities predicted from BGTL are more accurate. Overall, BGT slightly overestimates the velocities and BGTL slightly underestimates velocities. The fractional errors for P-wave velocities predicted from BGT and BGTL are 0.03 ± 0.03 and -0.01 ± 0.02, respectively, indicating the same amount of error for both methods. However, the fractional errors for S-wave velocities predicted from BGT and BGTL are 0.09 $\pm$ 0.05 and -0.00 ± 0.05, indicating more accurate S-wave velocities from BGTL.

Discussion

Biot coefficient and parameters

Within the poroelastic framework, skeleton or frame moduli, k and μ, are undetermined and must be specified a priori. Therefore, the Biot coefficient, which relates the frame moduli of the sediments to those of the matrix material for dry rocks, should be known to accurately predict velocities. Because velocities depend on differential pressure, the Biot coefficient depends on differential pressure as well as porosity.

Generally, frame moduli are measured at the laboratory or are predicted by the grain contact theory (Digby, 1981; Murphy, 1984; Murphy et al., 1993), which predicts that k and μ (or the Biot coefficient)are simple functions of porosity. The Biot coefficient shown in Equations (6) and (7) depends only on porosity. As indicated in Lee (2003), the Biot coefficient shown in Equation (6) is adequate for clean sandstone at differential pressure of about 20 MPa, and Equation (7) or similar Biot coefficient by Murphy et al. (1993) for consolidated sediments is adequate for high differential pressure at about 50 MPa.

In general, the Biot coefficient is not available at particular differential pressure. In order to predict pressure dependent velocities for clean/shaly sandstones, BGTL utilizes a pressure-dependent exponent n and a scale G. Therefore, BGTL formulation attempts to implicitly calculate the pressure dependent Biot coefficient for clean/shaly sandstones by using exponent n and scale G from pressure independent Biot coefficient for clean sandstone, such as shown in Equations (6) and (7).

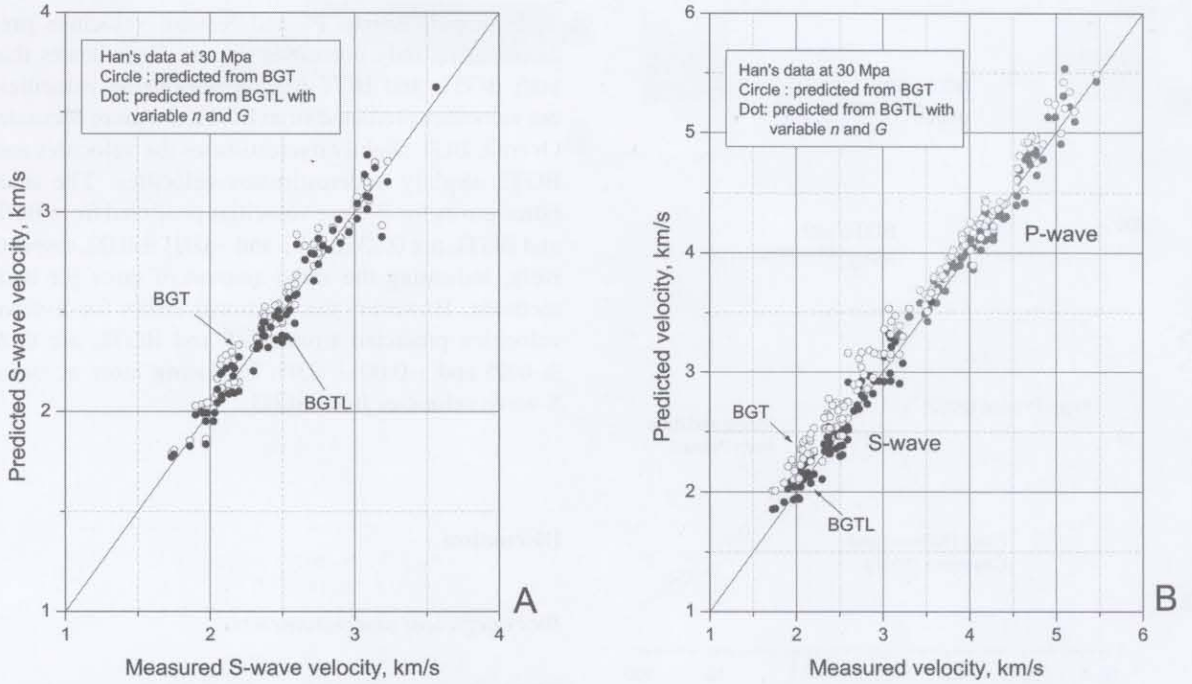

Figure 3. Measured velocities by Han et al. (1986) at the differential pressure of 30 MPa, and predicted velocities from Biot-Gassmann theory (BGT) and Biot-Gassmann theory by Lee (BGTL) using elastic moduli shown in Table 1. A) S-wave velocity predicted from the measured P-wave velocity and porosity. For BGT, the Biot coefficient is calculated from Lee (2003). B) P- and S-wave velocities predicted from porosity. Biot coefficient by Raymer et al. (1980), Equation (7), is used.

Comparison with Kuster and Toksöz

According to Kuster and Toksöz (1974), the aspect ratio of pore space can change the moduli of dry sediments. One implication of Kuster and Toksöz's theory (KT) (1974) is that the Biot coefficient, and the velocity ratio, depends on the aspect ratio of pore space. Toksöz et al. (1976) indicated that for rocks containing only spherical pores (aspect ratio of 1), KT theory is identical to the Gassmann equation. Castagna et al. (1985) speculated that clean sandstones dominated by equant porosity yield lower V_p/V_s than sandstones with elongated pores. As the aspect ratio of pore space decreases, velocities decrease and V_p/V_s increases. Xu and White (1996) used KT to predict S-wave velocity from P-wave or porosity and recommended an aspect ratio of 0.12 for sandstone and 0.03 for shale for Han's data (1978).

The result of using data measured at 40 MPa by Han et al. (1986) indicates that the predicted velocities using an aspect ratio of 0.3 is as close as those predicted using BGTL with $n = 0.2$ and G depending on clay content, whereas BGT overestimates S-wave velocities. In BGTL, lowering velocities are achieved by using larger exponent n or lowering scale G and in KT by using a lower aspect ratio of the pore space. As the differential pressure increases, all cracks with lower aspect ratios will be closed and velocities increase. Therefore, qualitatively, the role of exponent n and G in BGTL is similar to the role of aspect ratio in KT. Figure 4 shows the predicted velocities using various methods, namely BGT, KT, and BGTL for Han's data at 5 MPa. Because velocities decrease as differential pressure decreases, the aspect ratio of 0.25 is used in KT to predict velocities. Predicted S-wave velocities from P-wave velocity and porosity from KT is reasonable, but those velocities predicted from porosity are not accurate. KT with other aspect ratios improves the accuracy of predictions, as shown by Xu and White (1998). However, it is not clear how to choose the aspect ratio with respect to differential pressure. Also the results by Xu and White (1998) are not better than results from BGTL with $n = 0.6$, shown in Figure 4C. The overestimations of both P- and S-wave velocities from BGT comes from the fact that the Biot coefficient shown in Equation (7) is used for 5 MPa data. As indicated previously, the Biot coefficient shown in Equation (7) works better for velocities at high differential pressure.

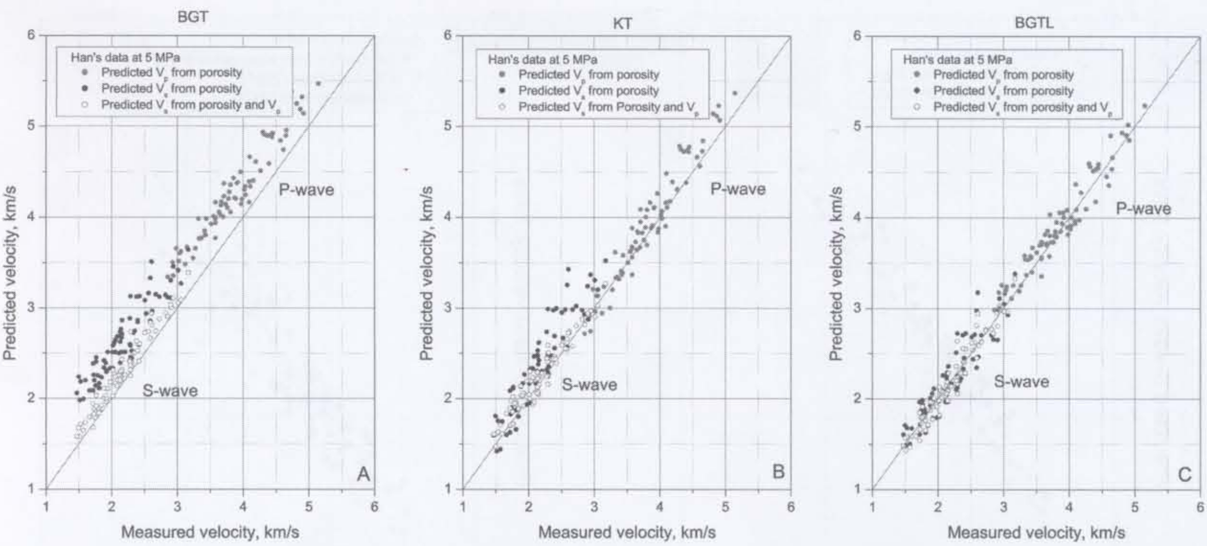

Figure 4. Predicted and measured velocities at 5 MPa by Han et al. (1986). A). Predicted by Biot-Gassmann theory (BGT). B) Predicted by Kuster and Toksöz theory (KT) using the aspect ratio of 0.25 . C) Predicted by Biot-Gassmann theory by Lee (BGTL) with $n = 0.6$.

Comparison with some empirical formulas

Many empirical formulas to predict S-wave velocities are available (e.g., Castagna et al., 1985; Han et al., 1986; Wang, 2000; Koesoemadinata and McMechan, 2001). Wang shows that the S-wave velocity can be predicted from P-wave velocity and density by

$$V_s^2 = (0.4211 + 0.0061k_{fl})V_p^2 - \frac{1.1255k_{fl}}{\rho} \qquad (14)$$

Figure 5A shows predicted S-wave velocity from measured P-wave velocity and porosity for BGTL and P-wave velocity and density for Wang's formula. The average predicted fractional error from BGTL is 0.00 $\pm$ 0.05, while it is 0.06 $\pm$ 0.05 from Wang. Because density is related to porosity, both approaches use the same input data to predict S-wave velocity. For 40 MPa data, the errors are 0.02 $\pm$ 0.03 and 0.03 $\pm$ 0.04 for BGTL and for Wang, respectively. As differential pressure increases, the performance of Wang's empirical formula improves, whereas the performance of BGTL remains steady.

Koesoemadinata and McMechan (2001) introduced the following empirical formula for the prediction of S-wave velocity

$$V_s = 2.664 - 5.039\phi - 1.691C_v + 0.169\ln(p) + 0.368f. \qquad (15)$$

Note that Koesoemadinata and McMechan (2001) included the dispersion relation explicitly in their formula. Figure 5B shows the predicted S-wave velocity from Koesoemadinata and McMechan (2001), with $f = 0.6$ MHz and $p = 5$ MPa. The fractional error of the predicted S-wave velocity is -0.03 ± 0.07 at 5 MPa and -0.01 ± 0.05 at 40 MPa. The accuracy of predicted S-wave velocity from Koesoemadinata and McMechan (2001) is almost identical to that from BGTL for 40 MPa data, but the accuracy for 5 MPa data is inferior to that from BGTL. In all cases, the performance of BGTL is better than or equal to that from Wang (2000) or Koesoemadinata and McMechan (2001).

Velocity ratio and frequencies

As indicated previously, the proposed BGTL does not include the effect of velocity dispersion in its formulation. The exponent n and scale G were based on the data acquired at ultrasonic frequencies (Domenico (1977) measured at about 0.5 MHz (pulse width of 2 μsec), Han et al. (1986) measured at 1 MHz for the P-wave velocity and 0.6 MHz for the S-wave velocity, and compiled by Prasad (2002) measured at 100 kHz $-$ 1 MHz excluding two samples by Ayers and Theilen (1999)). The question is how much error is introduced by ignoring velocity dispersion in BGTL using the exponent n based on measurements at ultrasonic frequencies.

In order to examine the magnitude of error in BGTL by ignoring the velocity dispersion, an empirical formula by Koesoemadinata and McMechan (2001), who used a variety of data sets measured with

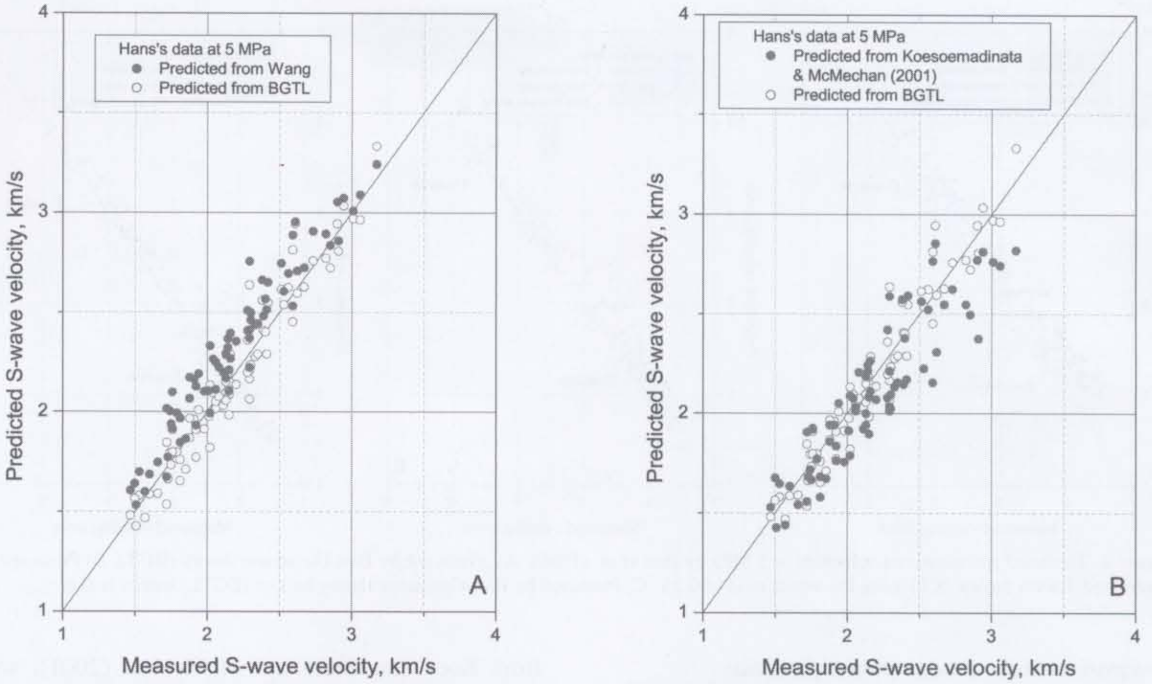

Figure 5. Predicted and measured velocities at 5 MPa by Han et al. (1986) A). Predicted S-wave velocities by the empirical formula by Wang (2002) and by Biot-Gassmann theory by Lee (BGTL) with n computed using $P = 5$ MPa and $m = 5$. B) Predicted S-wave velocities by an empirical formula by Koesoemadinata and McMechan (2001) and by Biot-Gassmann theory by Lee (BGTL) with n computed using $P = 5$ MPa and $m = 5$.

frequency ranges from 380 Hz to 1 MHz, is applied. Their analysis indicates that the dispersive part of P-wave velocity is $0.338f$, where f is the frequency in MHz, and is $0.368f$ for the S-wave velocity. The P-wave velocity of a sandstone having a porosity of 15% is 4.237 km/s, and the S-wave velocity is 2.532 km/s at 0 Hz and $p = 40$ MPa. These velocities are 4.575 km/s and 2.9 km/s for P- and S-wave velocity, respectively, at 1 MHz. Therefore, V_p/V_s at 0 Hz is 1.673 and is 1.578 at 1 MHz, which is about a 6% decrease of V_p/V_s going from 0 Hz to 1 MHz. However, velocity itself has a larger error, 8% for the P-wave velocity and 15% for the S-wave velocity.

Velocity dispersion requires that the exponent n should be dependent on the frequency as well as differential pressure to accurately predict velocities. If the exact amount of velocity dispersion is known, the exponent n can be adjusted to incorporate the frequency dependent velocity. However, quantifying velocity dispersion is difficult because it is not known precisely how much velocity dispersion occurs in fluid-saturated rock from seismic to laboratory ultrasonic frequencies (Wang and Nur, 1990). McDonal et al. (1958) observed that there appears to be no detectable dispersion of velocity with frequency in consolidated sedimentary rocks over a frequency range less than 1 MHz. Blangy et al. (1993) analyzed Troll sands and concluded that ultrasonic measurements can be used directly for detailed seismic work without correction for frequency dispersion, probably because of high permeability of Troll sands and the lack of grain-scale local flow effect (Mavko and Jizba, 1991).

Therefore, the proposed BGTL is optimum for velocities measured at ultrasonic frequencies, and for sandstones with high permeability without grain-scale local flow, or consolidated sediments with no appreciable velocity dispersion. Because the exponent n is based on the ultrasonic frequency, the appropriate n for velocities measured at frequency less than ultrasonic frequency would be a little larger than n calculated from Equation (9). However, sonic data at the Tarn 2N305 well demonstrate that parameters derived from ultrasonic frequencies appear to work well for sonic frequency ranges.

Conclusions

Biot-Gassmann theory by Lee (BGTL), which is formulated under the assumption that the velocity ratio is a function of $G(1-\phi)^n$, accurately predicts the velocity ratio or velocities with respect to porosity. The effect of differential pressure on velocities is incorporated by making the exponent n a function of differential pressure and the rate of porosity change with respect to differential pressure. However, the velocity dispersion is not included in the formulation. This investigation indicates that BGTL is preferable to BGT in most cases. The performance of BGTL is similar to that of Kuster Toksöz theory (1974) and to empirical formulas by Wang (2000) and Koesoemadinata and McMechan (2001) at high differential pressure, but performs better as differential pressure decreases for consolidated sediments. However, because parameters are derived from velocities measured at ultrasonic frequencies, BGTL would be optimum for velocities measured at high frequencies near 1 MHz, or for sediments having small velocity dispersion.

The prediction of S-wave velocity based on BGTL requires properties of matrix material including clay content, porosity (or porosity and P-wave velocity), the Biot coefficient such as proposed by Raymer et al. (1980) or Lee (2002), and differential pressure. This study demonstrates that BGTL works well for both laboratory and well log data.

The application of BGTL is more complex than the application of BGT, because the exponent n depends on many factors. A judicious choice of n and G is essential to accurately predicting velocities, and n and G values can be estimated from the general guideline and examples presented in this paper.

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