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Scattering coefficient for *S* wave incident in a random medium characterized by exponential correlation function

Jayant N. Tripathi

Department of Earth and Planetary Sciences, University of Allahabad, Allahabad 211 002, *India. E-mail: jntripa@yahoo.com*

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SUMMARY

Analytical expressions for scattering coefficients for *S*-wave incidence have been derived for the medium defined by the exponential correlation function. Low- and high-frequency asymptotic characteristics of the total scattering coefficients have also been investigated. The scattered energy of the converted waves reaches a maximum value in the high-frequency range for the case of an exponential correlation function. The meridional component of the scattered energy for common-mode scattering has a second-power frequency dependence for the high-frequency range, while the latitude component of the scattered energy for common-mode scattering has a fourth-power frequency dependence for the high-frequency range. A comparison of the scattering coefficient for *P*- and *S*-wave incidence is also discussed. Mode-conversion for *P*-wave incidence is greater than that for *S*-wave incidence. On the other hand, for the common-mode case the scattered energy for *S*-wave incidence is greater than that for *P*-wave incidence.

Key words: exponential correlation function, inhomogeneity, random medium, scattering.

INTRODUCTION

The problem of the seismic wave scattering through the Earth can be considered to be random and some statistical properties of the medium and the parameters describing its heterogeneities can be deduced. Previously, the scalar wave scattering theory for random media (Chernov 1960; Tatarskii 1961) has been applied to estimate heterogeneities in the lithosphere. The single-wave-scattering theory of scalar waves has been successfully applied to deal with the modelling of coda generation of local earthquakes (Aki 1969; Aki & Chouet 1975; Sato 1977) the strength of coda waves (Aki 1981; Sato 1982a,b; Wu & Aki 1985a,b) and attenuation by scattering (Aki & Chouet 1975; Aki 1980a,b; Wu 1982a,b; Sato 1982a,b). The frequency dependence of scattering attenuation is also discussed by other workers (Sato 1982a; Korvin 1983). Using the Born approximation, expressions for the mean-square amplitudes of the scattered elastic waves by a random medium have been obtained for low and high frequencies (Knopoff & Hudson 1964, 1967). Haddon (1978) derived the explicit expressions for the incident *P* waves (see, also Aki & Richards 1980).

Wu & Aki (1985b) have derived the mean square amplitudes of the scattered field and directional coefficients of a random medium for the *P–P*, *P–S*, *S–P* and *S–S* cases using the Born approximation for the exponential correlation function.

The scattering characteristics of the heterogeneous earth medium can be very well observed in the envelopes of high-frequency seismograms, especially *S*-coda waves, of earthquakes. *S*-coda is interpreted as being composed of scattered *S* waves. So, *S–S* scattering has been the basis of its study and analysis (Hoshiba 1991; Mayeda *et al.* 1991; Fehler *et al.* 1992). Regional differences of randomness have also been observed on the basis of *S–S* scattering or strong diffraction of *S* waves (Gusev & Abubabirov 1987; Sato 1989; Scherbaum & Sato 1991; Obara & Sato 1995; Gusev & Abubabirov 1996). Multiple isotropic scattering process has been synthesized including *P–S* conversion for the seismogram envelope on the basis of energy transport theory (Sato 1984) and the numerical integration method (Zeng 1993). Sato & Fehler (1998) have presented a very good summary of observations and models used in this field.

In this paper, analytical expressions for the total scattered power for *S*-wave incidence have been derived. The expressions have been obtained for scattered energy in low- and high-frequency ranges. A comparison has been made for the scattered energy in the case of *P*- and *S*-wave incidences. The total scattered energy for the *P*-wave incidence is more than that for the *S*-wave incidence in the case of mode-conversion, while for the common-mode case the total scattered energy for the *S*-wave incidence is more than that for the *P*-wave incidence.

PROBLEM FORMULATION AND DIRECTIONAL SCATTERING COEFFICIENTS FOR *S*-WAVE INCIDENCE

It is assumed that there is a homogeneous elastic random medium with density ρ and elastic parameters λ and μ. A finite volume is considered in this medium. This arbitrary heterogeneity is characterized by the random perturbation parameters:

$$\rho(\xi) = \rho_0 + \delta\rho(\xi)$$
$$\lambda(\xi) = \lambda_0 + \delta\lambda(\xi) \quad (1)$$
$$\mu(\xi) = \mu_0 + \delta\mu(\xi),$$

where ξ is the position vector within the volume, $\rho(\xi)$ is the density and $\lambda(\xi)$ and $\mu(\xi)$ are the corresponding Lamé parameters of the random inhomogeneities at ξ, and $\delta\rho(\xi)$, $\delta\lambda(\xi)$ and $\delta\mu(\xi)$ are their perturbations. Here it is assumed that

$$\rho_0 = \langle\rho(\xi)\rangle$$
$$\lambda_0 = \langle\lambda(\xi)\rangle \quad (2)$$
$$\mu_0 = \langle\mu(\xi)\rangle$$

and $\delta\rho \ll \rho_0$, $\delta\lambda \ll \lambda_0$, $\delta\mu \ll \mu_0$, i.e. the inclusion is such that the random medium is weakly heterogeneous, where $\langle\ \rangle$ represents an average over the statistical ensemble of the random variable.

It is assumed that the random medium is statistical, homogeneous and isotropic. The perturbations of density ($\delta\rho(\xi)$) and the Lamé parameters ($\delta\lambda(\xi)$ and $\delta\mu(\xi)$) are defined by the same type of correlation functions. Let

$$\left\langle\left(\frac{\delta\rho}{\rho_0}\right)^2\right\rangle = \langle\varepsilon^2\rangle,$$
$$\left\langle\left(\frac{\delta\lambda}{\lambda_0}\right)^2\right\rangle = m^2\langle\varepsilon^2\rangle, \quad (3)$$
$$\left\langle\left(\frac{\delta\mu}{\mu_0}\right)^2\right\rangle = n^2\langle\varepsilon^2\rangle,$$

$$\langle\delta\rho(\xi)\delta\rho(\eta)\rangle = \langle\delta\rho^2\rangle N(|\xi-\eta|)$$
$$\langle\delta\lambda(\xi)\delta\lambda(\eta)\rangle = \langle\delta\lambda^2\rangle N(|\xi-\eta|) \quad (4)$$
$$\langle\delta\mu(\xi)\delta\mu(\eta)\rangle = \langle\delta\mu^2\rangle N(|\xi-\eta|)$$

etc. Where the normalized correlation function of the random media is given by $N(|\xi-\eta|)$. The correlation coefficients between ρ,

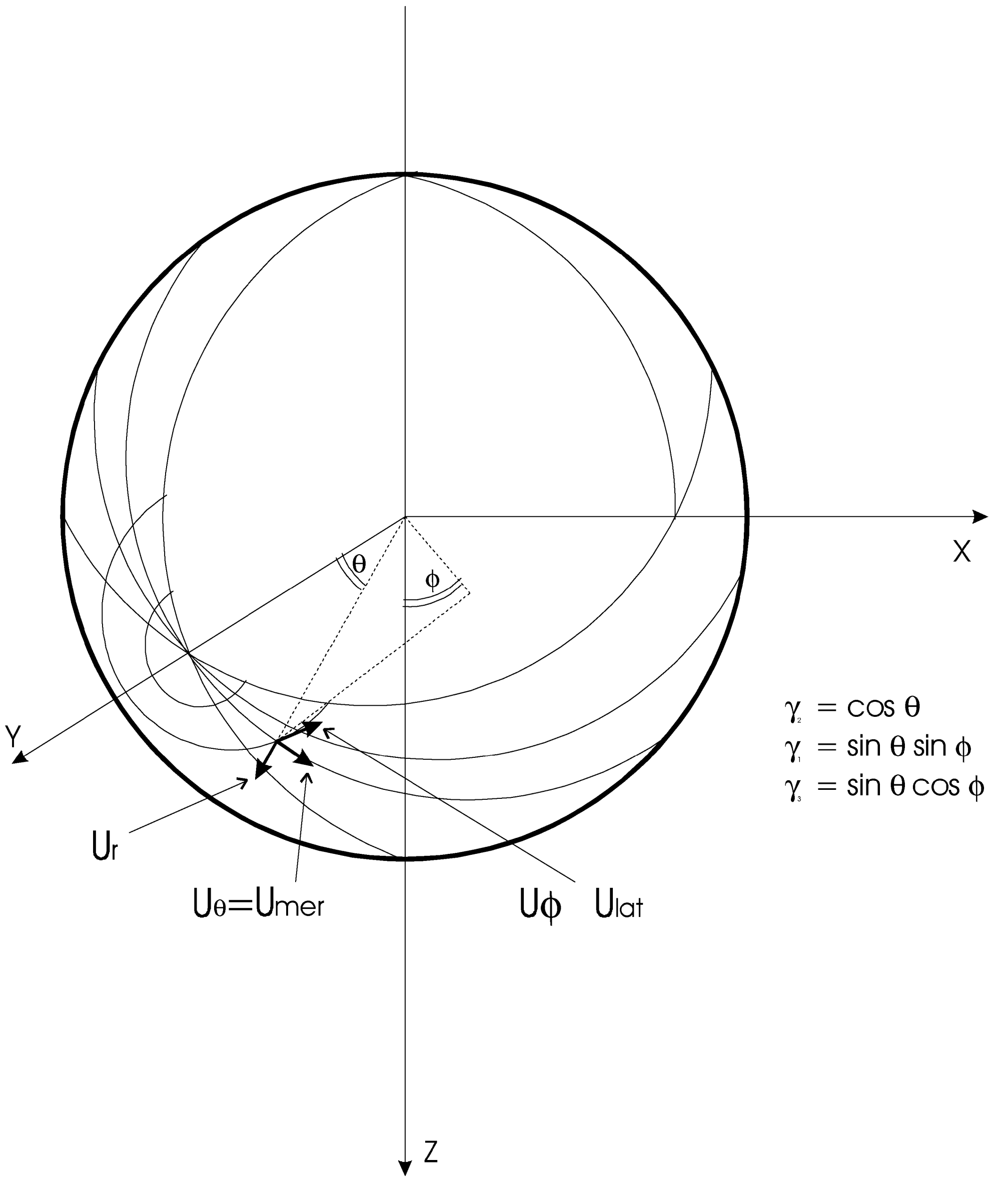

Figure 1. The spherical coordinate system for *S*-wave incidence (along the *x* direction). The polar axis is in the direction of particle motion (*y*-axis). γ_1, γ_2 and γ_3 are the direction cosines of the scattering direction (Wu & Aki 1985b).

λ and μ are represented as $\phi_{\rho\lambda}$, $\phi_{\rho\mu}$ and $\phi_{\lambda\mu}$ and are defined as follows:

$$\phi_{\rho\lambda} = \frac{\langle\delta\rho\delta\lambda\rangle}{\langle\delta\rho^2\rangle^{1/2}\langle\delta\lambda^2\rangle^{1/2}} \tag{5a}$$

$$\phi_{\lambda\mu} = \frac{\langle\delta\lambda\delta\mu\rangle}{\langle\delta\lambda^2\rangle^{1/2}\langle\delta\mu^2\rangle^{1/2}} \tag{5b}$$

$$\phi_{\rho\mu} = \frac{\langle\delta\rho\delta\mu\rangle}{\langle\delta\rho^2\rangle^{1/2}\langle\delta\mu^2\rangle^{1/2}}. \tag{5c}$$

In order to find the directional scattering coefficients for *S*-wave incidence, a spherical coordinate system is considered in which the polar axis is along the direction of particle motion (Fig. 1). The directional scattering coefficients for *S–S* scattering $g^{ss}(\hat{x})$ is defined as 4π times the average scattered power in the x direction per unit solid angle by a unit volume of random medium for a unit incident field (Wu & Aki 1985b). Thus, for the case when $\delta\rho(\xi)$, $\delta\lambda(\xi)$ and $\delta\mu(\xi)$ are totally correlated ($\phi_{\rho\lambda} = \phi_{\rho\mu} = \phi_{\lambda\mu} = 1$), the directional scattering coefficients can be obtained as (Wu & Aki 1985b)

$$g^{sp}(\theta,\phi) = \frac{1}{4\pi}\frac{\omega^4}{\alpha_0^4}\langle\varepsilon^2\rangle\left[\cos\theta - \left(\frac{\beta_0}{\alpha_0}\right) n\sin 2\theta\sin\phi\right]^2 P^{c}(\hat{x}) \tag{6}$$

$$g^{ss}_{\mathrm{mer}}(\theta,\phi) = \frac{1}{4\pi}\frac{\omega^4}{\beta_0^4}\langle\varepsilon^2\rangle(\sin\theta + n\cos 2\theta\sin\phi)^2 P^{s}(\hat{x}) \tag{7}$$

$$g^{ss}_{\mathrm{lat}}(\theta,\phi) = \frac{1}{4\pi}\frac{\omega^4}{\beta_0^4}\langle\varepsilon^2\rangle n^2\cos^2\theta\cos^2\phi P^{s}(\hat{x}) \tag{8}$$

where $g^{sp}(\theta,\phi)$ is the *S–P* scattering coefficient, $g^{ss}_{\mathrm{mer}}(\theta,\phi)$ is the scattering coefficient for the meridian component of the *S–S* scattering and $g^{ss}_{\mathrm{lat}}(\theta,\phi)$ is the scattering coefficient for the latitude component of the *S–S* scattering and

$$P^{c}(\hat{\mathbf{x}}) = \iiint_{-\infty}^{\infty} N(|\zeta|)\exp\left[i\omega\left(\frac{\hat{\mathbf{x}}_1}{\beta_0} - \frac{\hat{\mathbf{x}}}{\alpha_0}\right)\zeta\right] dV(\zeta) \tag{9}$$

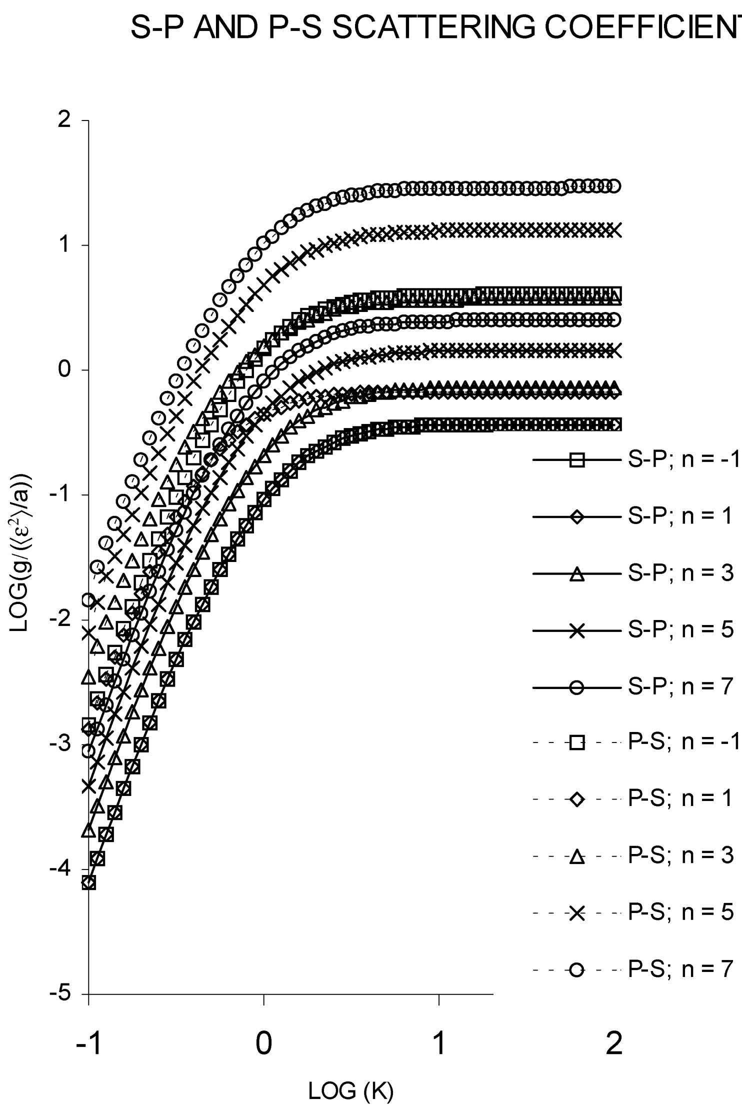

Figure 2. The frequency dependence of the *S–P* scattering coefficient for different values of *n*. The frequency dependence of the *P–S* scattering coefficient for the values of *n* is also given for comparison.

$$P^s(\hat{\mathbf{x}}) = \iiint_{-\infty}^{\infty} N(|\zeta|) \exp\left[i\omega\left(\frac{\hat{\mathbf{x}}_1}{\beta_0} - \frac{\hat{\mathbf{x}}}{\beta_0} \right)\zeta \right] dV(\zeta), \tag{10}$$

where $\hat{\mathbf{x}}_1$ is the unit vector in the incident direction.

The normalized correlation function, $N(|\zeta|)$, is taken as an exponential correlation function that is defined as

$$N(|\zeta|) = \exp(-|\zeta|/a) \tag{11}$$

where a is the correlation length.

Thus, for the exponential correlation function, one can obtain

$$P^c(\theta_x) = \frac{8\pi a^3}{\left[1 + (\omega S_2 a)^2\right]^2} \tag{12}$$

where

$$S_2 = \left\{(1/\alpha_0)^2 + (1/\beta_0)^2 - [2/(\alpha_0\beta_0)]\cos\theta_x\right\}^{1/2} \tag{13}$$

and

$$P^s(\theta_x) = \frac{8\pi a^3}{\left\{1 + [2(\omega/\beta_0)a\sin\theta_x/2]^2\right\}^2} \tag{14}$$

where θ_x is the scattering angle, i.e. the angle between the scattering direction and the incident ($\mathbf{x}_1$) direction.

TOTAL SCATTERED POWER AND SCATTERING COEFFICIENT OF THE MEDIUM FOR *S*-WAVE INCIDENCE

The scattering coefficient g can be defined as the total scattered power by a unit volume of the random medium for a unit incident field (i.e. unit power flux density)

$$g = \frac{1}{4\pi} \iint_{4\pi} g(\theta, \phi)\, d\Omega \tag{15}$$

where $d\Omega$ is the differential solid angle over which $g(\theta)$ is defined. The dimension of g is 1/length or area/volume.

For *S*-wave incidence,

$$\begin{aligned} g^s &= \frac{1}{4\pi} \iint_{4\pi} \left[g^{sp}(\theta, \phi) + g^{ss}_{\mathrm{mer}}(\theta, \phi) + g^{ss}_{\mathrm{lat}}(\theta, \phi)\right] d\Omega \\ &= g^{sp} + g^{ss}_{\mathrm{mer}} + g^{ss}_{\mathrm{lat}}. \end{aligned} \tag{16}$$

For the case when $\delta\rho$, $\delta\lambda$ and $\delta\mu$ are totally correlated, g^{sp}, g^{ss}_{mer} and g^{ss}_{lat}, which are scattering coefficients for *S–P*, *S–S* (meridian and latitude components), can be calculated. Thus, we can express the frequency dependence of *S–P* scattering coefficient as

$$\begin{aligned} g^{sp} = \frac{1}{4\pi} \iint_{4\pi} [g^{sp}(\theta, \phi)]\, d\Omega = \frac{2\langle\varepsilon^2\rangle}{a} \frac{K^4}{b^3} \Bigg\{ b\frac{(2d^2 - b^2)}{(d^2 - b^2)} \\ - d\ln\left(\frac{d+b}{d-b}\right) + \frac{2n^2}{\gamma^2 b^2}\left[\frac{2b^3}{3} - 4d^2 b + d(2d^2 - b^2) \\ \times \ln\left(\frac{d+b}{d-b}\right)\right]\Bigg\} \end{aligned} \tag{17}$$

where $K = \omega a/\alpha_0$, $\gamma = \alpha_0/\beta_0$, $b = 2\gamma K^2$ and $d = 1 + (1 + \gamma^2)K^2$.

It is noticeable that the 'n' has the power of 2 in the above expression. So, the contribution of this component will be same for $n = 1$ (impedance perturbation) and $n = -1$ (velocity perturbation).

When $K \ll 1$, i.e. at low frequencies eq. (17) reduces to

$$g^{sp} = \frac{2\langle\varepsilon^2\rangle}{3a}\left(1 + \frac{4n^2}{5\gamma^2}\right)K^4, \tag{18}$$

which has Rayleigh scattering with the usual fourth-power frequency dependence. The ratio of g^{sp}/g^{ps} (from above equation and eq. 58 of Wu & Aki 1985b) can be obtained at low frequencies as

$$g^{ps}/g^{sp} = 2\gamma^4 \tag{19}$$

which is the same as that obtained by Papanicolaou *et al.* (1996) (the factor 2 is omitted in Aki 1992).

Now, at high frequencies i.e. when $K \gg 1$,

$$g^{sp} = \frac{\langle\varepsilon^2\rangle}{a} \frac{D}{2\gamma^2} \tag{20}$$

where

$$D = \frac{2\xi^2 - 1}{\xi^2 - 1} - \xi\eta + 2\frac{\eta^2}{\gamma^2}\left(\frac{2}{3} - \xi\eta - 4\xi^2 + 2\xi^3\eta\right), \tag{21}$$

where

$$\xi = \frac{1}{2}(\gamma + \gamma^{-1}) \tag{22}$$

and

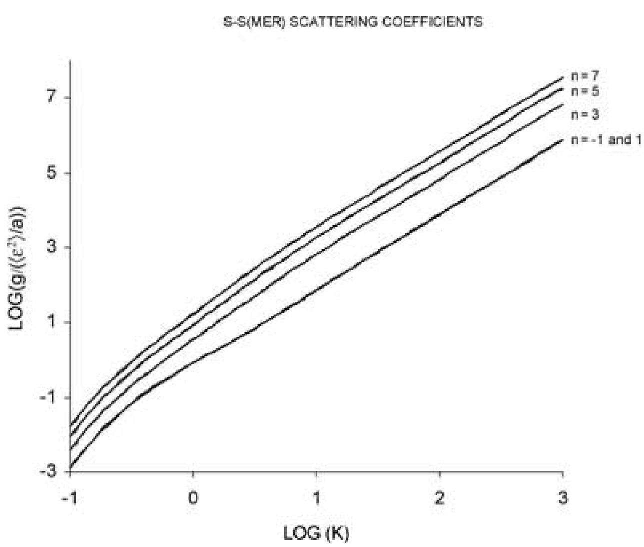

Figure 3. The frequency dependence of the meridian component of the *S–S* scattering coefficient for different values of *n*.

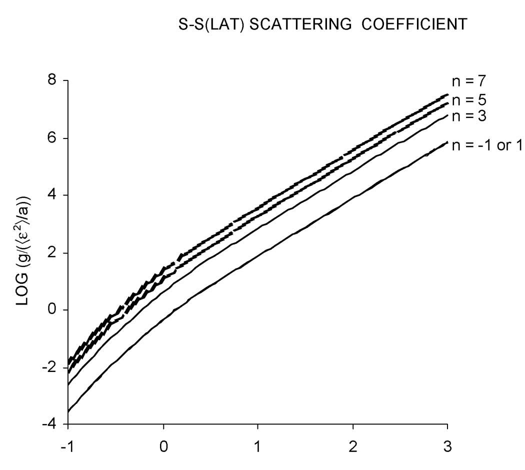

Figure 4. The frequency dependence of the latitude component of the *S–S* scattering coefficient for different values of *n*.

$$\eta = 2\ln\left(\frac{\gamma+1}{\gamma-1}\right). \tag{23}$$

It is clear from eq. (20) that the conversion loss reduces to a constant value at higher frequencies for the exponential correlation function.

Similarly, the value of $g^{ss}_{\text{mer}}(\theta,\phi)$ is substituted and the total scattered *S*-wave (meridian) energy can be computed as

$$g^{ss}_{\text{mer}} = \frac{1}{4\pi}\iint_{4\pi}\left[g^{ss}_{\text{mer}}(\theta,\phi)\right]d\Omega. \tag{24}$$

So,

$$g^{ss}_{\text{mer}} = \frac{\langle\varepsilon^2\rangle}{a}K^4\gamma^4\left\{\frac{2+n^2}{1+2E} - \frac{2(1+2n^2)}{E^3}\left[\frac{EF^4}{1+2E} - F\ln(1+2E) + E\right] + \frac{4n^2}{E^5}\left[\frac{EF^4}{1+2E} - 2F^3\ln(1+2E) + 3EF^2 + \frac{E^3}{3}\right]\right\} \tag{25}$$

where $K = \omega\, a/\alpha_0$, $\gamma = \alpha_0/\beta_0$, $E = 2\gamma^2K^2$ and $F = 1 + E$.

In the above expression, '*n*' also has the power of 2. So, the contribution of this component will also be the same for impedance perturbation and velocity perturbation, in this case.

When $K \ll 1$, i.e. for low frequencies,

$$g^{ss}_{\text{mer}} \approx \frac{\langle\varepsilon^2\rangle}{a}\left(\frac{4}{3} + \frac{7n^2}{15}\right)\gamma^2K^2. \tag{26}$$

So, it has the usual second-power frequency dependence.

When $K \gg 1$, i.e. for high frequencies,

$$g^{ss}_{\text{mer}} = \frac{\langle\varepsilon^2\rangle}{4a}\left(\gamma^2n^2K^2 + \frac{16n^2}{3} - 4\right). \tag{27}$$

So, it has the usual second-power frequency dependence but with the addition of some other factor.

Similarly, g^{ss}_{lat} can be derived as follows:

$$g^{ss}_{\text{lat}} = \frac{\langle\varepsilon^2\rangle}{a}\frac{\gamma^4K^4n^2}{E^3}\left[\frac{E(2+E^2+4E)}{1+2E} - (1+E)\ln(1+2E)\right]. \tag{28}$$

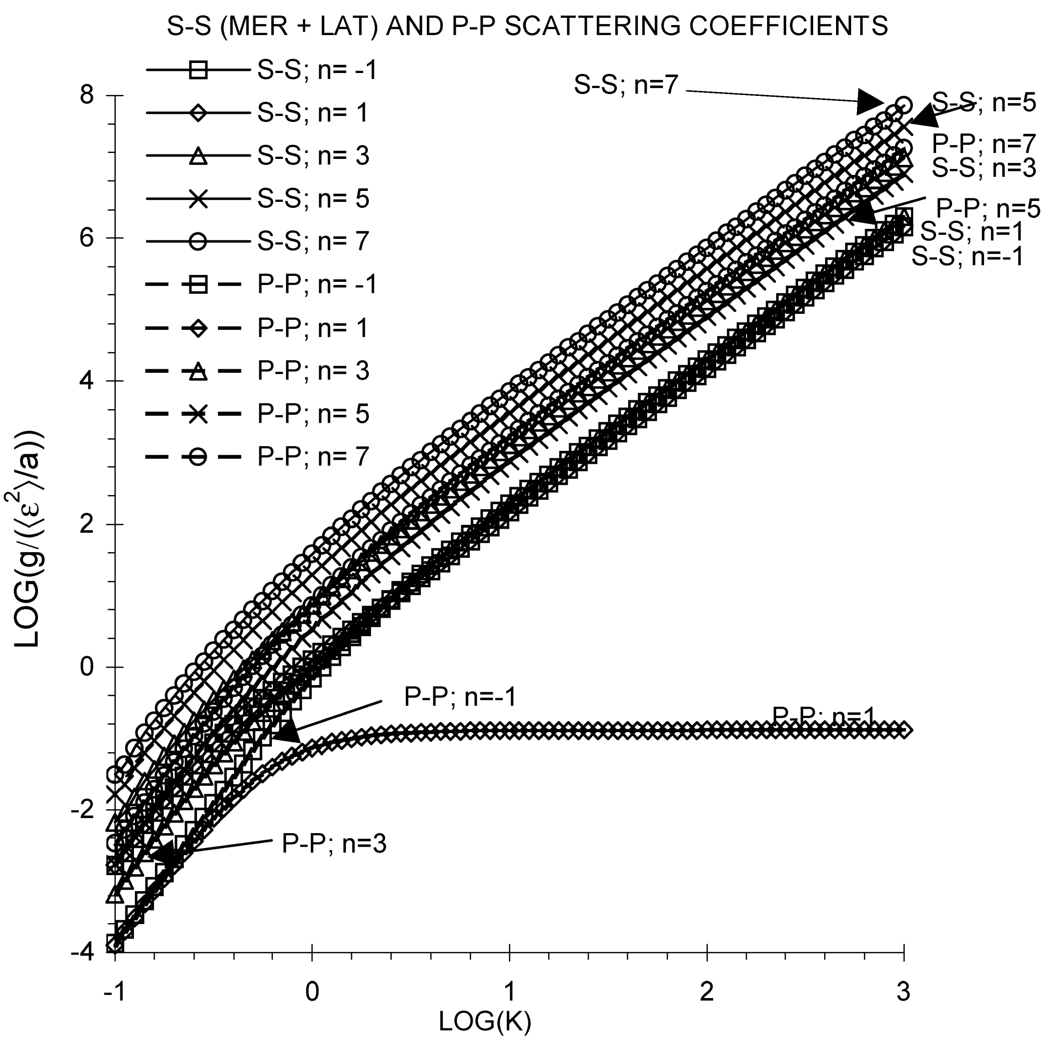

Figure 5. The frequency dependence of the total *S–S* scattering coefficient for different values of *n*. The scattering coefficient for *P–P* scattering is also given in the figure for comparison.

When $K \ll 1$, i.e. at low frequencies

$$g_{\text{lat}}^{ss} = \frac{\langle \varepsilon^2 \rangle}{3a} n^2 \gamma^4 K^4. \tag{29}$$

So, it has a fourth-power frequency dependence. When $K \gg 1$, i.e. at high frequencies,

$$g_{\text{lat}}^{ss} = \frac{E}{2}\left(1 + \frac{7}{2E}\right) + \ln(1 + 2E). \tag{30}$$

These equations should follow the Born approximations (the scattered energy must be very small compared with the energy of the primary field) such that these can be applied to find the parameters of the medium. The frequency ω and propagation distance L for the validity condition for the converted waves (eq. 20) should be such that the following condition must be satisfied (Aki & Richards 1980; Wu & Aki 1985b; Tripathi & Ram 1997)

$$\frac{\langle \varepsilon^2 \rangle}{a} \frac{D}{2\gamma^2} L \ll 1. \tag{31}$$

The validity condition for the common-mode scattered waves can also be written as follows. For eq. (27) it is

$$\frac{\langle \varepsilon^2 \rangle}{4a}\left(\gamma^2 n^2 K^2 + \frac{16n^2}{3} - 4\right) L \ll 1 \tag{32}$$

and similarly, for eq. (30) it will be as follows:

$$\left[\frac{E}{2}\left(1 + \frac{7}{2E}\right) + \ln(1 + 2E)\right] L \ll 1. \tag{33}$$

DISCUSSION AND CONCLUSIONS

A comparison of the scattering coefficient for mode-conversion is given in Fig. 2. The analytical expressions deduced by Wu & Aki (1985b) for the scattering coefficient for *P–S* conversion is used for the calculation. The scattered energy attains a constant value at high frequencies in the case of *S–P* for the exponential correlation function. It is clear that the mode-conversion is more prominent in the case of *P*-wave incidence (i.e. for the *P–S* case) as compared with *S*-wave incidence (i.e. for the *S–P* case) for low- as well as high-frequency range. This conclusion was also inferred by Knopoff & Hudson (1967). From eq. (19) it is obtained that ratio $g^{ps}/g^{sp} = 2\gamma^4$ at low frequencies. If it is assumed that $\gamma = 3^{1/2}$ then $g^{ps}/g^{sp} \sim 18$ at low frequencies. This implies that the *S* wave will dominate after scattering. It has also been observed that the *S* wave dominates the seismological data (Aki 1992; Su *et al.* 1991). Zeng (1993) also observed the dominance of *S* waves in numerical solutions. It is interesting to notice that the curves for $n = 1$ and -1 are the same, because the '*n*' has a power of 2 in the analytical expression for the case of *S*-wave incidence. So, the amount of scattered energy for the mode-conversion caused by impedance and velocity perturbation are the same. This is not the case for *P*-wave incidence, where the scattered energy for the velocity perturbation is greater than that for impedance perturbation (see Fig. 11, Wu & Aki 1985b), particularly at $K \approx 1$ and $K \gg 1$.

The *S–S* scattering coefficient is given in the Figs 3 and 4 for its meridian and latitude components, respectively. It is also clear from these figures that the amounts of scattered energy for the common-mode for impedance and velocity perturbation are the same. The scattering coefficient has a second-power dependence at high frequencies. The total scattered energy for the common-mode is more prominent for *S*-wave incidence compared with *P*-wave incidence (Fig. 5). It is clear from Fig. 5 that the scattering coefficient for the common-mode for *S*-wave incidence is always higher than that for *P*-wave incidence. The ratio g^{ss}/g^{pp} is almost constant for velocity perturbation ($n \neq 1$). Since the ratios g^{ss}/g^{pp} and g^{ps}/g^{sp} are greater than unity, the dominance of *S* waves is quite obvious in the seismological data as observed by other workers (Su *et al.* 1991; Aki 1992; Zeng 1993).

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