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Application of linear programming techniques to the inversion of proton magnetic resonance measurements for water prospecting from the surface

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Abstract

In addition to direct detection of groundwater from the surface, the Proton Magnetic Resonance (PMR) method can provide important information concerning aquifers, such as their number, depth, thickness and water content. This requires the solution of an inverse problem that like many other geophysical inverse problems is ill posed and often leads to nonuniqueness of the solution. In this paper, we present the results of applying various techniques originally developed for gravity and magnetic inversion, such as linear programming, 'ideal body' and 'compact body', in order to select just one solution out of the solution space. These methods assume minimization of specific functionals that was obtained by using a linear programming technique and comparing with those obtained by the Tikhonov regularization method. Although mathematically equivalent in terms of fit between measured and synthetic data, the selected solutions may differ in their physical significance. Analysis of the obtained solutions provides additional information for the interpretation of field measurements. A few examples of both synthetic and field data inversion are presented to demonstrate that these techniques enable the groundwater distribution to be characterized through the following parameters: top and bottom depths of the acquifer formation, minimum and maximum concentration of water, water quantity, total water thickness, etc. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Inverse problem; Water prospection; Proton magnetic resonance

1. Introduction

Nuclear Magnetic Resonance (NMR) is a technique that was developed in 1945 and is widely used in chemistry and medicine. Although the original idea to use NMR for the surface prospecting of hydrocarbons was proposed by Varian (1962) some 15 years later, the method and the equipment were developed for geophysical application by a Russian team (Semenov, 1987; Semenov et al., 1987).

Proton Magnetic Resonance (PMR) is a particular case of Nuclear Magnetic Resonance (NMR) in which only the protons in the nucleus are investigated; it is probably the only existing surface geophysical method that allows direct detection of groundwater. It provides information about the groundwater distribution and the mean pore size of water-saturated rocks. Interpretation of the PMR measurements is based on the solution of an inverse problem, which in turn involves various relevant hydrogeological parameters such as the bulk volume of water and the depth and thickness of aquifers. Although with some assumptions the problem is a linear one, the nonuniqueness of the solution requires the application of certain criteria, or additional

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knowledge about the solution, in order to select just one solution out of the potential solution space. Using the Tikhonov regularization method (Tikhonov and Arsenin, 1977) to invert the PMR data gives a single solution based on the only knowledge of the measurement. However, a unique solution can also be obtained by restricting the geometry of water-saturated layers, or other parameters that depend on the physical meaning of the resolved problem. An investigation of the results obtained with different assumptions about the solution can be useful for interpreting the measured data.

Here, we will demonstrate how different gravity and magnetic data interpretation techniques such as 'ideal body' and 'compact body' inversion can be adapted for interpreting PMR measurements. We use synthetic models and then apply the method to an actual field test case at the Saint-Cyr-en-Val site in France.

2. Forward problem

We begin by describing the forward problem. A circular loop of wire that acts as both the transmitting and receiving antennas is laid out on the ground, with a diameter varying between 20 and 150 m, which largely determines the depth of investigation (Fig. 1). The loop is energized with a pulse of alternating current.

$$i(t) = I_0\cos(\omega_0 t), \qquad 0 \le t \le \tau,$$

where I_0 and τ are the amplitude and the duration of the pulse, respectively.

The current frequency $\omega_0 = 2\pi f_0$ is set equal to the Larmor frequency of the protons in the geomagnetic field, i.e. $\omega_0 = \lambda H_0$ where λ is the gyromagnetic ratio and H_0 is the magnitude of the geomagnetic field. An alternating magnetic field is created by the precession of the nuclear magnetization M_0 of protons after it has been tilted by the pulse, and it is this secondary field that is measured after the pulse is terminated. The PMR signal oscillates at the Larmor frequency and has an exponential envelope

$$e(t) = E_0(q)\sin(\omega_0 t + \varphi_0)\exp(-t/T_2^*), \tag{1}$$

where T_2^* is the spin–spin relaxation time and φ_0 is the phase. The amplitude of the signal is (Trushkin et al., 1995)

$$E_0(q) = \omega_0 M_0 \int_{x^2+y^2 \le (2D)^2,\, z \le 2D} h_{1\perp}\sin\left(\frac{1}{2}\lambda h_{1\perp} q\right) \times w(\mathbf{r})\mathrm{d}\mathbf{r}, \tag{2}$$

where $q = I_0\tau$ is the pulse parameter, $h_{1\perp}(\mathbf{r},\rho(\mathbf{r}),\alpha)$ is a component of the antenna's magnetic field created by I_0, perpendicular to the geomagnetic field, $\rho(\mathbf{r})$ is the rock resistivity, α is the inclination of the geomagnetic field, $w(\mathbf{r})$ is the water content, $\mathbf{r} = r(x,y,z)$ is the radius vector of the point in the space, and D is the diameter of the antenna.

The depth, thickness and water content of detected aquifers can be derived from the amplitude of the signal. The relaxation time, T_2^*, correlates with the mean size of the pores of water-saturated rocks (Shirov et al., 1991) and the phase depends on the electrical conductivity of rocks (Legchenko et al., 1990). Here, we shall focus our attention on the amplitude of the signal.

If we have M measurements of the signal, then Eq. (2) can be written as

$$E_0(q_j) = E_{0j} = \int_{x,y,z} G_j(\mathbf{r})w(\mathbf{r})\mathrm{d}(\mathbf{r}),$$
$$j = 1,2,\ldots,M, \tag{3}$$

where $G_j(\mathbf{r})$ is Green's function for a pulse q_j and $G_j(\mathbf{r}) = \omega_0 M_0 h_{1\perp}\sin((1/2)\lambda h_{1\perp} q_j)$.

Eq. (3) needs to be solved for the distribution of groundwater. To simplify the problem, we assume homogeneity of the geomagnetic field and horizontal stratification, where upon Eq. (3) can be written as

$$E_{0j} = \sum_i G_{i,j} w_i, \qquad i = 1,2,\ldots,N, \tag{4}$$

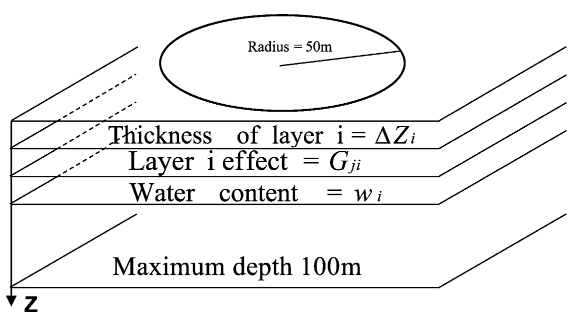


Fig. 1. Model geometry and discretization.

where N is a number of layers in the model and

$$G_{i,j} = \omega_0 M_0 \int_{x=-\infty, y=-\infty, z=z_i}^{x=+\infty, y=+\infty, z=z_{i+1}} \times h_{1\perp} \sin\left(\frac{1}{2}\lambda h_{1\perp} q_j\right) \mathrm{d}\mathbf{r}, \quad (5)$$

where z_i and z_{i+1} are, respectively, the depth of the top and bottom of layer i, and $0 \leq w_i \leq 100\%$ is the water content in layer i.

The amplitude of the signal E_{0j} can be measured and so, using these data, the water content w_i can be derived from Eq. (4).

3. Inverse problem

Taking E_{0j} to be the voltage induced in the loop by the free precession of protons initiated by the pulse q_j, and $G_{i,j}w_i$ to be the contribution of layer i with water content w_i to the total signal produced by the pulse q_j, and ε_j to be the error for the measurement j, one obtains the following linear system

$$\begin{cases} E_{0j} - \varepsilon_j \leq G_{i,j} w_i \leq E_{0j} + \varepsilon_j \\ 0 \leq w_i \leq 100\% \end{cases} . \quad (6)$$

System (6) is satisfied by more than one solution, which create a solution space. It is then necessary to choose one of the solutions out of the solution space, based on additional criteria.

The data obtained by the PMR can be inverted by the Tikhonov regularization method, which gives a unique, and in some senses optimal, solution with noisy data using knowledge of measurement accuracy. This method assumes a minimization of the Tikhonov functional (Tikhonov and Arsenin, 1977)

$$\| GW_\eta - \mathbf{e}_\varepsilon \|_{L_2} + \eta \| W_\eta \|_{L_2} \rightarrow \min_\eta \ , \quad (7)$$

where $\mathbf{G}=[G_{i,j}]$, $W_\eta=(w_1,w_2,\ldots,w_M)$ is a vector of the solution, $\mathbf{e}_\varepsilon=(e_{\varepsilon_1},e_{\varepsilon_2},\ldots,e_{\varepsilon_M})$ is a vector of the noisy data, and $\eta=\eta(\varepsilon)$ is the regularization parameter.

System (6) can also be solved using other information. This is done by looking for parameters that characterize the groundwater distribution, such as depth and thickness of the aquifers and the water content of each aquifer, and applying a linear programming technique that is based on ideas proposed by Parker (1974) for gravity data inversion and further developed by Safon et al. (1977) and Cuer and Bayer (1980).

First, however, it is necessary to verify whether system (6) has a solution with a given level of error ε. If such is the case, it will be called a feasible solution. If such is not the case, it is necessary to increase ε and try again since the solution space is a function of experimental errors. In practice, the value ε can be well estimated by studying the noise level during the data acquisition process, and so, for simplicity, we assume that $\varepsilon_j=\varepsilon$ for all j.

Once the system is known to have a solution, it is then necessary to investigate the solution space. For this purpose, we introduce specific parameters that may characterize the water-saturated intervals.

The *maximum depth of the top of the water-saturated zone* (z_{top}) is the top of the first water-saturated layer k such that $w_i=0$ when $i<k$ and $w_i>0$ when $i=k$. In order to find z_{top}, we need to solve the system

$$\begin{cases} w_i = 0, \text{ for } i = 1,2,\ldots,k-1 \\ w_i > 0, \text{ for } 1 = k \\ E_{0j} - \varepsilon \leq G_{i,j} w_i \leq E_{0j} + \varepsilon \\ 0 \leq w_i \leq 100\% \end{cases} . \quad (8)$$

This evaluation will give us the maximum drilling depth required to find an aquifer, which is an important economic consideration in exploration.

The *minimum depth of the bottom of the water-saturated zone* (z_{bot}) is the bottom of the last water-saturated layer l such that $w_i>0$ when $i \leq l$ and $w_i=0$ when $i>l$. This parameter can be deduced from the system

$$\begin{cases} w_i > 0, \text{ for } i = l \\ w_i = 0, \text{ for } i = l+1, l+2, \ldots, M \\ E_{0j} - \varepsilon \leq G_{i,j} w_i \leq E_{0j} + \varepsilon \\ 0 \leq w_i \leq 100\% \end{cases} . \quad (9)$$

This evaluation will give us the minimum drilling depth required to intersect all the aquifer formations.

Both these parameters are retrieved using extremal points in the solution space of system (6). Although, ideally, the water-saturated zone must be between the calculated top and bottom, in practice, it may extend further in both directions. In some cases, if the resolution is poor, the calculated minimum depth of the bottom may be shallower than the calculated maximum depth of the top.

The *maximum* ($V_{\max}$) *and minimum* ($V_{\min}$) *volume of water* in the investigated area may be found by an optimization of the linear form $V = \sum \Delta z_i s_i w_i = \max(\min)$, where Δz_i is the thickness of layer i and $\Delta z_i s_i$ is the total volume of the layer. In order to avoid any problems related to antenna geometry, we can prescribe $s_i = 1$. Thus, we can evaluate an average volume of water in a vertical column of total volume $1 \times 1 \times \Delta z_i$. The linear system will now take the form

$$\begin{cases} \sum_i \Delta z_i w_i \rightarrow \max(\min) \\ E_{0j} - \varepsilon \leq G_{1,j} w_i \leq E_{0j} + \varepsilon \, . \\ 0 \leq w_i \leq 100\% \end{cases} \tag{10}$$

This evaluation gives us the maximum and minimum volume of water that should be intersected by a borehole drilled down to the maximum depth of PMR investigation. In practice, the minimum volume of water is a matter of interest if the objective is to locate a source of water supply, while the maximum volume of water should be estimated if one is looking to find a relatively dry area for building construction or other reasons.

The 'ideal body' solution, as developed by Parker (1974, 1975), is a uniform water-saturated layer of least possible water content associated with the measured data. The corresponding linear system is (Cuer and Bayer, 1980)

$$\begin{cases} \max_i (w_i) \rightarrow \min \\ E_{0j} - \varepsilon \leq G_{i,j} w_i \leq E_{0j} + \varepsilon \, . \\ 0 \leq w_i \leq 100\% \end{cases} \tag{11}$$

This solution allows us to determine the *minimum value of water content* and, consequently, the maximum thickness of aquifers. Groundwater must be located within the detected aquifers (layers with $w_i > 0$), but the exact position of the center of the layers remains unknown and unsaturated intervals ($w_i = 0$) in the 'ideal body' solution should be considered as reliably dry.

In order to find the location of water with greater certainty, we can apply the 'compact body' solution (Last and Kubik, 1983; Guillen and Menichetti, 1984), which represents the most compact distribution of water. It can be found by minimizing the total thickness of all detected water-saturated layers and applying the linear system.

$$\begin{cases} \sum_i v_i \rightarrow \min \\ E_{0j} - \varepsilon \leq G_{i,j} w_i \leq E_{0j} + \varepsilon \, , \\ 0 \leq w_i \leq 100\% \end{cases} \tag{12}$$

where

$$v_i = \begin{vmatrix} 1 \text{ for } w_i > 0 \\ 0 \text{ for } w_i = 0 \end{vmatrix}$$

This type of solution gives us the *maximum value of water content* and the minimum aquifer thickness. We consider the detected intervals with nonzero water

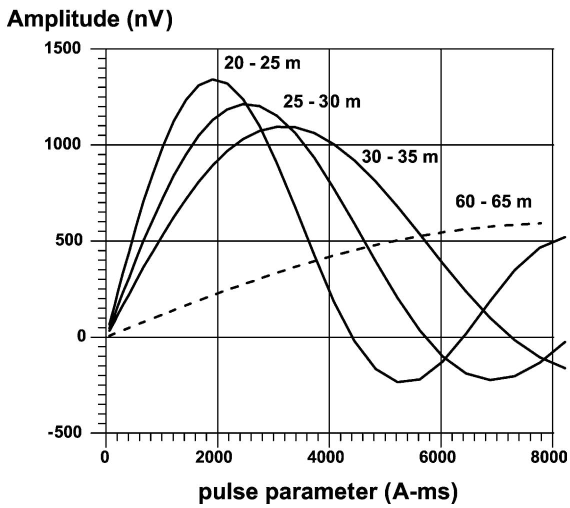


Fig. 2. Examples of the calculated PMR signal.

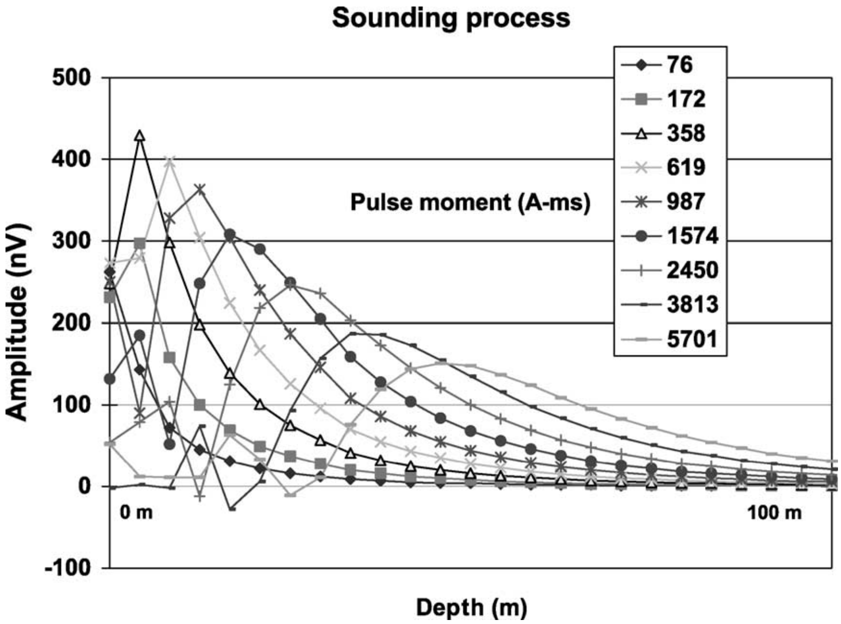


Fig. 3. The contribution of a 1-m-thick layer of water to the total signal as a function of layer depth.

content as reliably water-saturated, but the thickness of detected water-saturated layers is uncertain. We are also not sure whether or not the derived dry interval ($w_i = 0$) contains water. In the gravity interpretation, this solution gives the center of gravity of the investigated body.

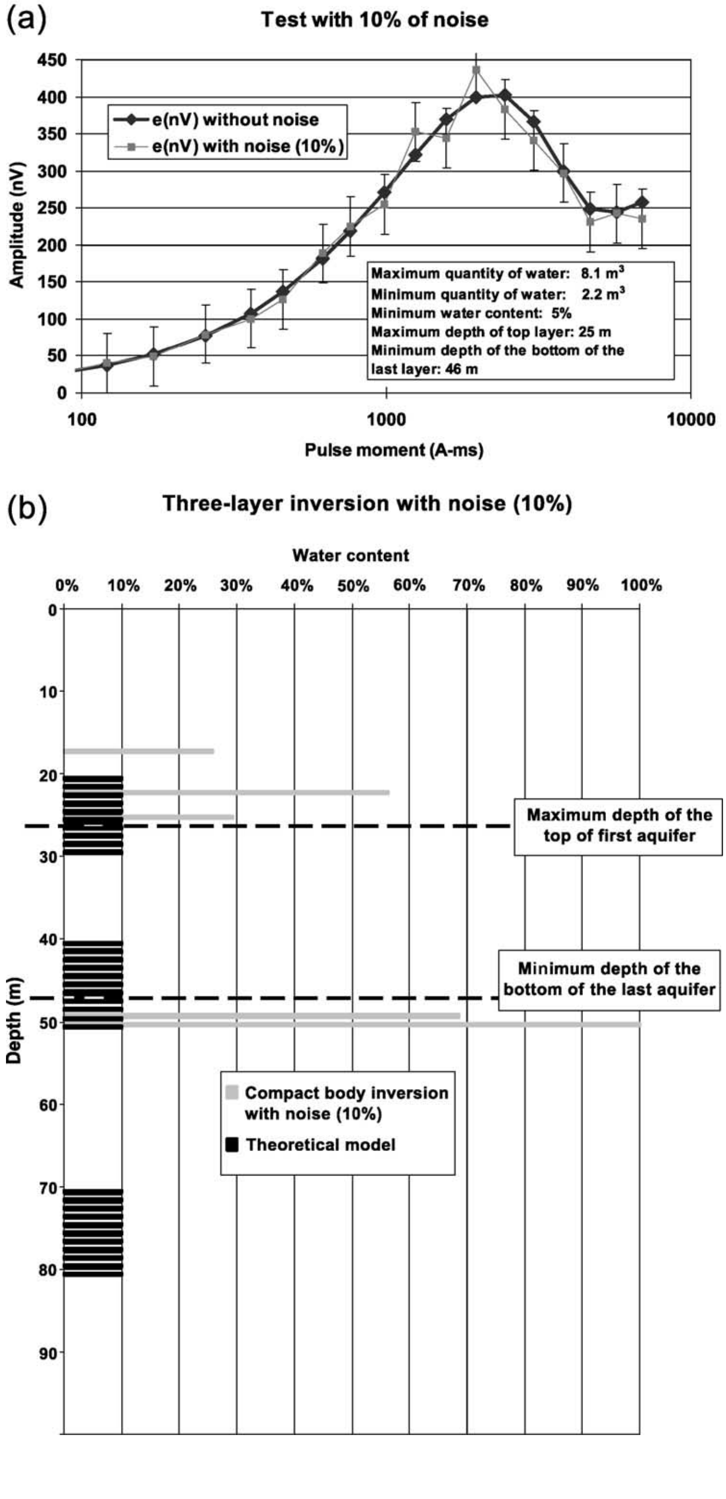
The 'ideal body' and 'compact body' solutions are two extreme cases for the possible location of the detected aquifer. In practice, it means that the aquifers are located around the position obtained by the 'compact body' solution and do not spread outside the region given by the 'ideal body' solution.

We are now in a position to write down the strategy for PMR data interpretation:

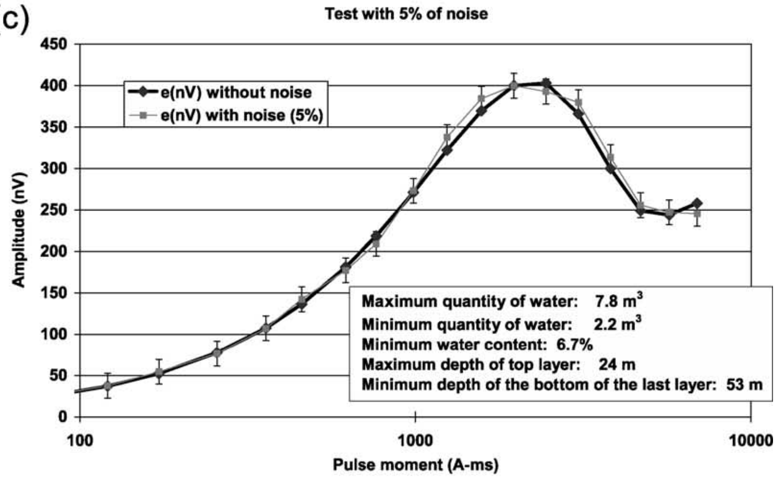
- find a feasible solution using the knowledge about experimental errors;
- compute the top and bottom of the water-saturated zone;
- estimate the maximum and minimum volume of water;

Table 1
Estimation of the water-saturated zone and volume of water for a one-layer model

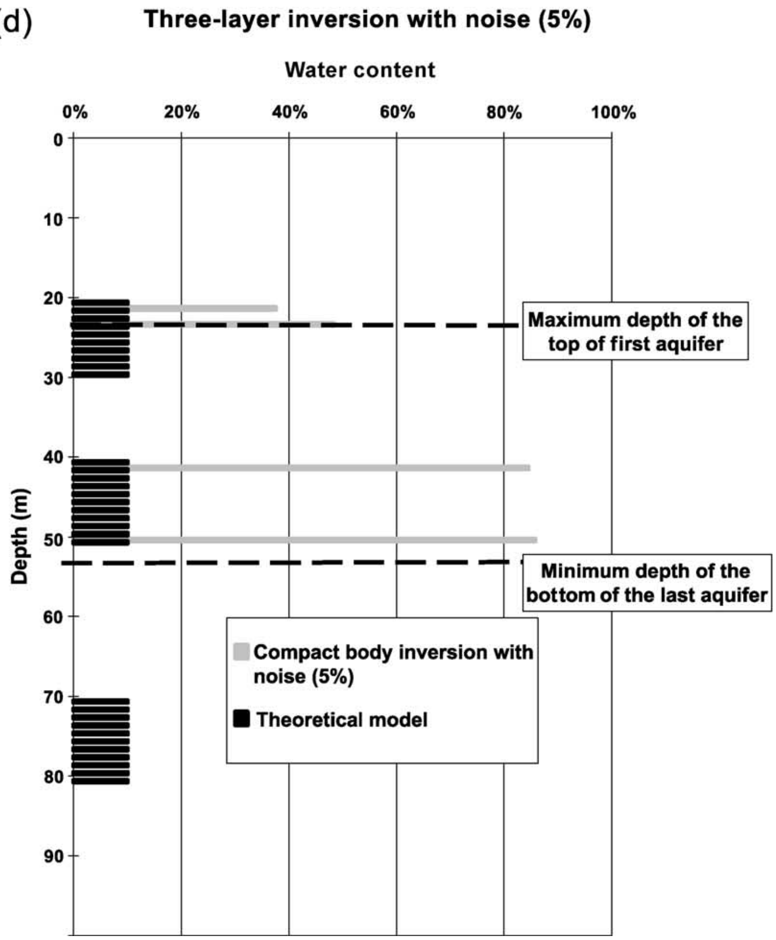
Parameter	Model (5–15 m)	Solution (5–15 m)	Model (20–30 m)	Solution (20–30 m)	Model (60–70 m)	Solution (60–70 m)
z_{top} (m)	5	4	20	21	60	96
z_{bot} (m)	15	13	30	25	70	54
V_{min} (m^3)	1.0	0.987	1.0	0.964	1.0	1.42
V_{max} (m^3)	1.0	1.36	1.0	1.38	1.0	3.2
'Ideal body' minimum water content (%)		10		7.6		1.7
'Ideal body' equivalent maximum thickness (m)		10		12.7		83
'Compact body' center of layer (m)		9–10		26–27		65


(a)
Test with 10% of noise
e(nV) without noise
e(nV) with noise (10%)
Amplitude (nV)
Maximum quantity of water: 8.1 m³
Minimum quantity of water: 2.2 m³
Minimum water content: 5%
Maximum depth of top layer: 25 m
Minimum depth of the bottom of the last layer: 46 m
Pulse moment (A-ms)
(b)
Three-layer inversion with noise (10%)
Water content
Maximum depth of the top of first aquifer
Minimum depth of the bottom of the last aquifer
Depth (m)
Compact body inversion with noise (10%)
Theoretical model

(c)


Test with 5% of noise
e(nV) without noise
e(nV) with noise (5%)
Amplitude (nV)
Pulse moment (A-ms)
Maximum quantity of water: 7.8 m³
Minimum quantity of water: 2.2 m³
Minimum water content: 6.7%
Maximum depth of top layer: 24 m
Minimum depth of the bottom of the last layer: 53 m

(d)


Three-layer inversion with noise (5%)
Water content
Depth (m)
Maximum depth of the top of first aquifer
Minimum depth of the bottom of the last aquifer
Compact body inversion with noise (5%)
Theoretical model

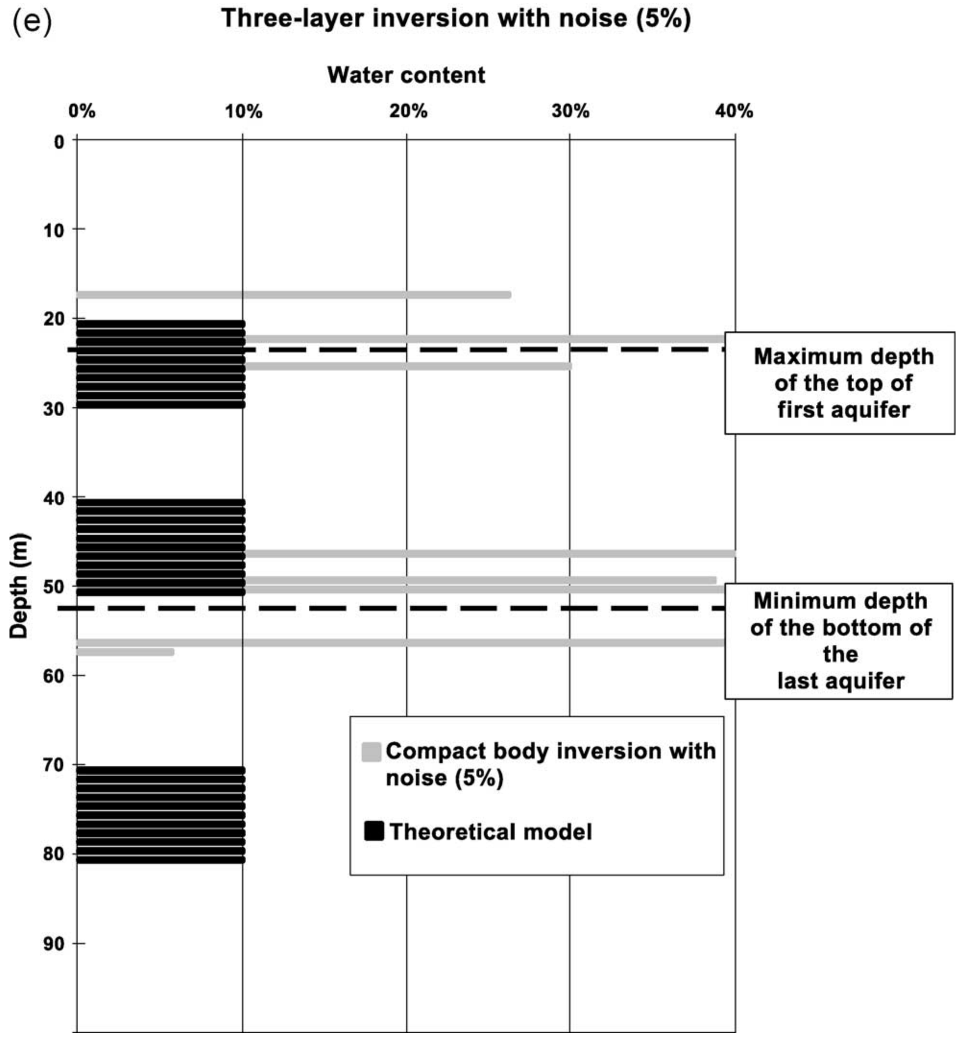


Fig. 4. Inversion of data for a three-layer model using both noisy and noise-free data. (a) PMR signal for the three-layer model using 0 and 10% noise added to the synthetic data. (b) Inversion of the PMR signal for the three-layer model using 0 and 10% noise added to the synthetic data. (c) PMR signal of the three-layer model using 0 and 5% noise added to the synthetic data. (d) Inversion of the PMR signal for the three-layer model using 5% noise added to the synthetic data. (e) Inversion of the PMR signal for the three-layer model using 5% noise and a maximum water content of 40%.

- find the minimum value of water content and guaranteed waterless (completely dry) intervals using the 'ideal body' solution;
- find the number and the position of the different aquifers using the 'compact body' solution.

Before illustrating the method, we need to discuss some physical features of the PMR inverse problem. The PMR signal, calculated for a 5-m-thick layer of water ($w_i = 100\%$) situated at different depths of 20, 25, 30 and 60 m, is presented in Fig. 2. It is seen that both the amplitude and the shape of the signal vary according to the depth of the layer; with a given level of noise ε, the signal to noise ratio for deeper layers is less favorable than for shallow ones. The contribution to the signal energy from a 1-m-thick layer of water, located at different depths for different values of the pulse parameter q_j, demonstrates the attenuation of the signal caused by depth (Fig. 3).

4. Results

For a demonstration of the inversion of PMR measurements for water prospecting from the surface, we first used two classes of synthetic data sets and then applied the inversion to field measurements at the Saint Cyr-en-Val site in France. In each case, the subsurface is assumed to be homogeneous and non-conductive.

4.1. One-layer synthetic data inversion

The first model demonstrates the PMR method resolution as a function of depth. We used a model of one 10-m-thick water-saturated layer with a water content equal to 10%. The simulation was calculated using both the ‘ideal body’ and ‘compact body’ inversions for depths of 5–15, 20–30 and 60–70 m. No noise was added to the theoretical signal. For each case, we have performed both the “ideal body” and “compact body” inversions. Nevertheless, despite the noiseless data, we used an assumed error $\varepsilon = 10$ nV, which enlarges the solution space, and also makes our results more realistic.

If the resolution is poor, we should expect a difference, in terms of thickness, position and number of the aquifers, between the ‘ideal body’ and ‘compact body’ solutions.

The results presented in Table 1 indicate that the difference between the extremal estimations of the layer thickness and water content is relatively small when the layer close to the surface, but increases with increasing depth of the layer. This can be explained principally by the decrease in the resolution of the PMR method with increasing depth, a result that agrees very well with the results of Legchenko and Shushakov (1998) using an eigenvalue analysis.

On comparing the results of the inversion with the model data (Table 1), we can see that:

- as expected, the water-saturated zone is well detected for the shallow (5–15 m) and intermediate (20–30 m) layers;
- the minimum volume of water was found to be very close to the model for the two shallow depths;
- the maximum volume of water is much greater than the true value in the model;
- the ‘ideal body’ gives good results of the minimum water content for the shallow and for the intermediate layers;
- the ‘ideal body’ gives good results of the equivalent maximum thickness for the shallow and for the intermediate layers;
- the ‘compact body’ for the three cases gives good results regarding the position of the center of the layer.

Nevertheless, the accuracy of detection of the water-saturated zone decreases with increasing layer depth. For the deepest model (60–70 m), the inversion gives a maximum depth of the top of the water-saturated zone that is deeper than the minimum depth of the bottom. This result demonstrates that the inversion method has no resolution at this depth and that the parameters of an aquifer cannot be computed

Table 2
Estimation of the water-saturated zone and volume of water for a three-layer model

Parameter	Model (3 layers)	Solution (3 layers, 5% noise)	Solution (3 layers, 10% noise)
z_{top} (m)	20	24	25
z_{bot} (m)	80	53	46
V_{min} (m^3)	3.0	2.2	2.2
V_{max} (m^3)	3.0	7.8	8.1
‘Ideal body’ minimum water content (%)		6.7	5
‘Ideal body’ equivalent maximum thickness (m)		32.8	44
‘Compact body’ center of layer (m)		23–25 and 43–50	19–26 and 50–51

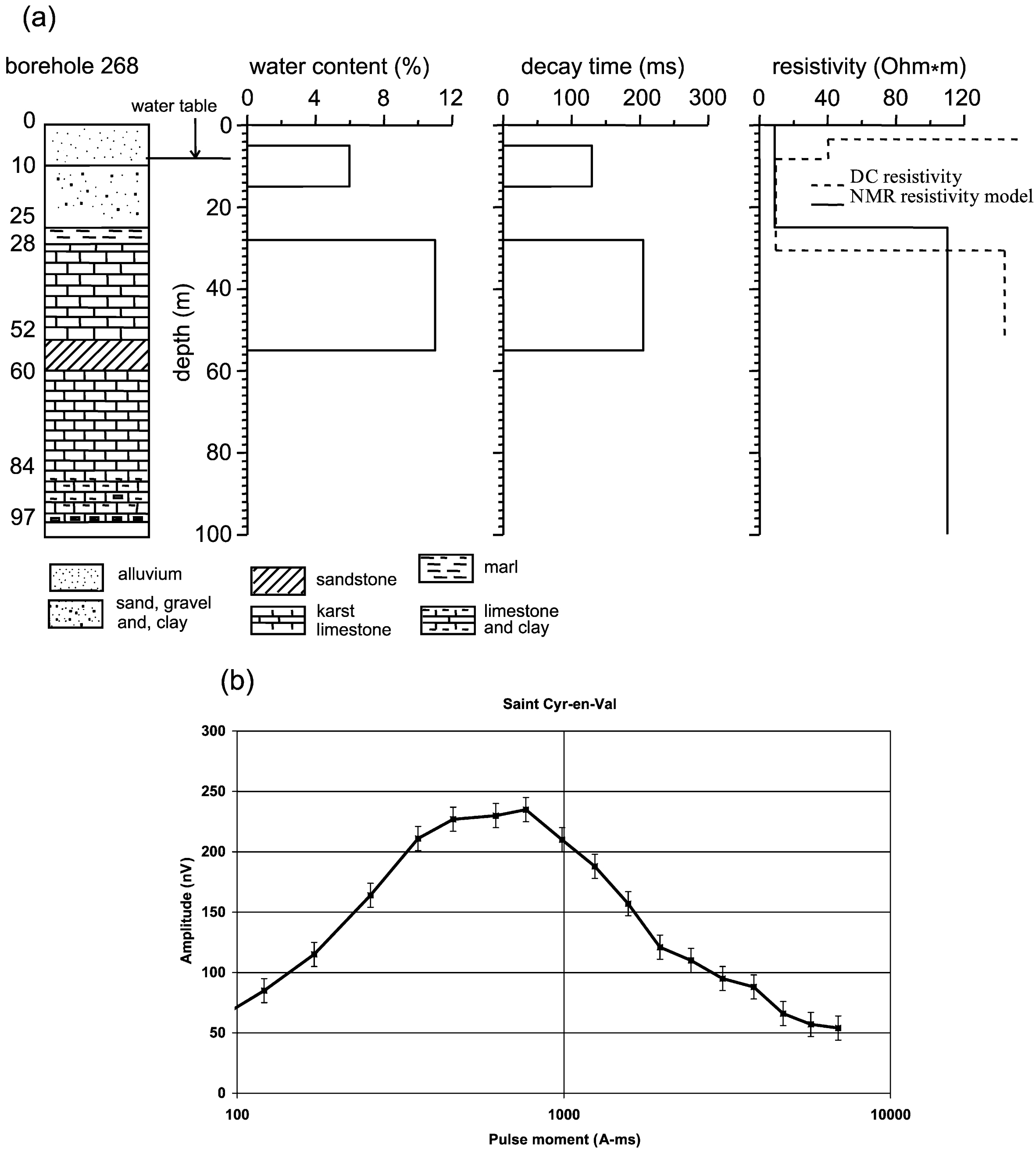


Fig. 5. Field case of Saint-Cyr-en-Val. (a) Lithological description of borehole 268, water content inversion using Tikhonov regularization, decay time inversion and resistivity. (b) Measured PMR signal. (c) Inversion of the PMR signal using 'ideal body' solution. (d) Inversion of the PMR signal using 'compact body' solution.

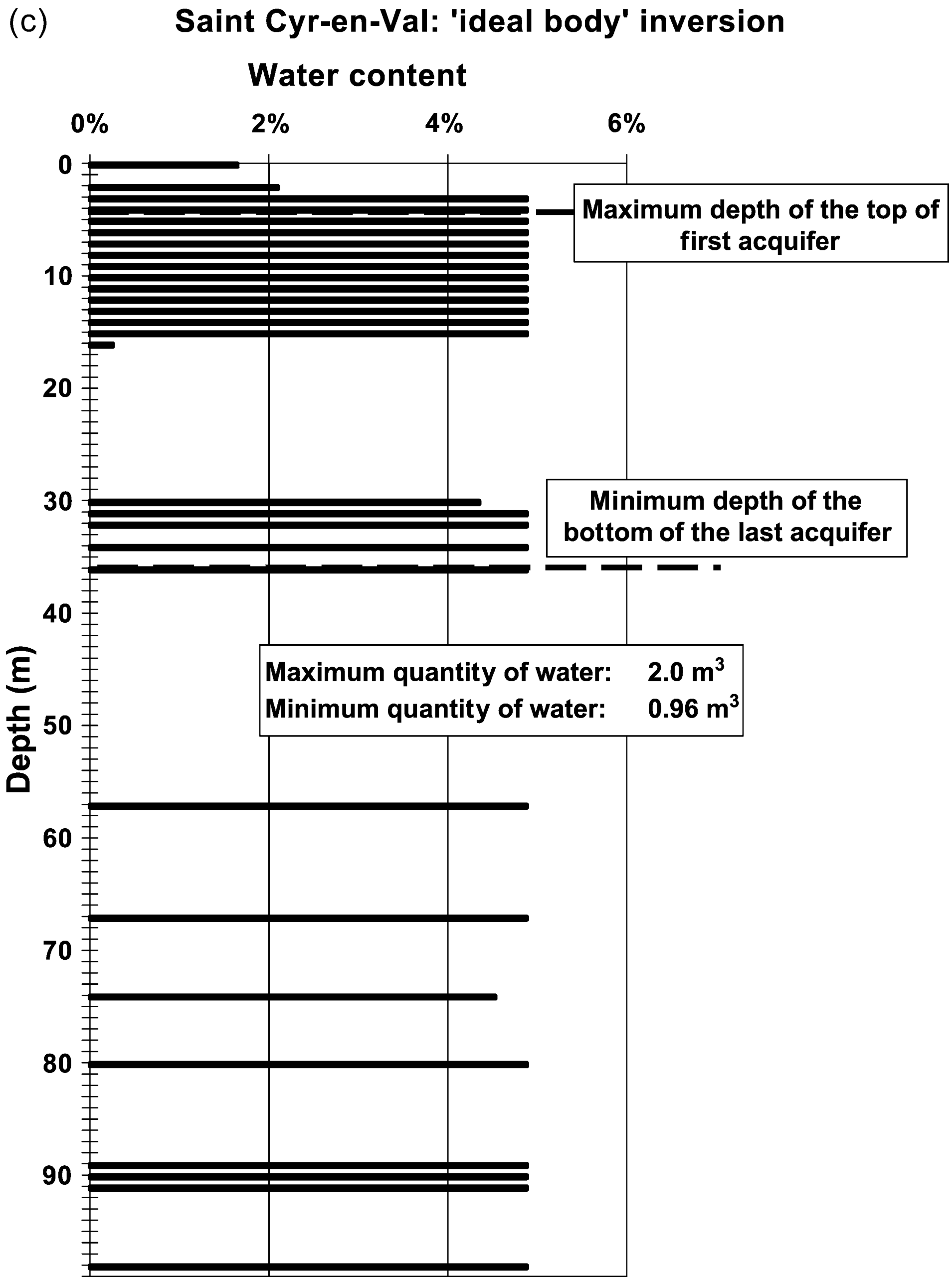


Fig. 5 (*continued*).

satisfactorily from the measurements. However, the evaluation of the minimum volume of water is quite close to the model, and although the evaluation of the maximum volume of water is too optimistic, it does indicate the existence of water.

4.2. Three-layer synthetic data inversion

A more complicated model consists of three 10-m-thick layers lying at 20–30, 40–50 and 70–80 m, with the water content in each layer being equal to

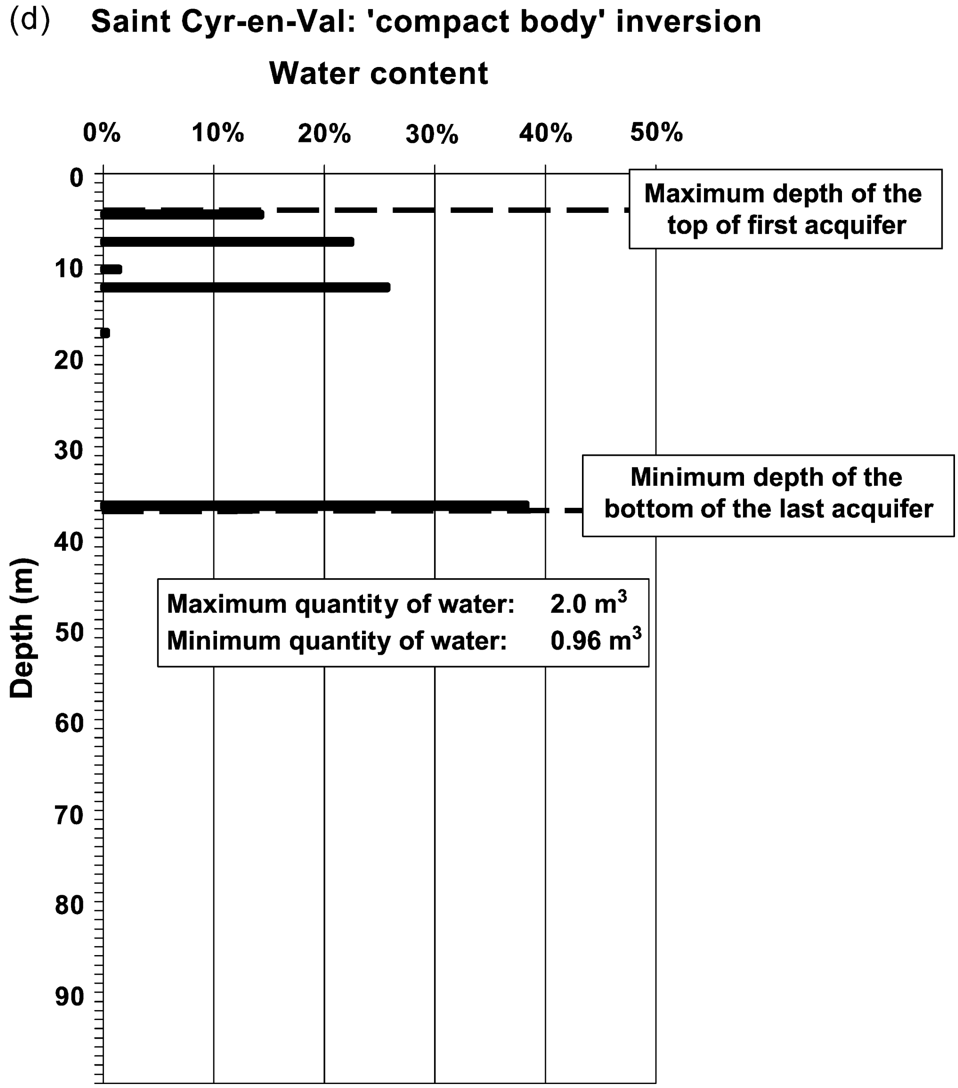


Fig. 5 (*continued*).

10%. Inversion was performed using the linear programming method, with an error of $\varepsilon = 10$ nV being used for noiseless data as in the single-layer case. When adding random noise to the theoretical data, using a noise amplitude of 10 and 5% of the signal amplitude (Fig. 4a and c), it was necessary to increase the error to $\varepsilon = 10$ nV, in order to solve system (6) with noisy data.

Results of the inversion are presented in Table 2 and depicted in Fig. 4b, d and e. Once again, we can see (Fig. 4d and e) that the solution space for the shallow layer is smaller than for the deeper layers, and that the deepest layer (70–80 m) cannot be retrieved even using the less noisy (5%) data. The results also show (Fig. 4b and d) that no water is expected below 50 m.

Here, we used the most common physical limitation $0 \leq w_i \leq 100\%$ for the inversion. However, it makes sense to use a more realistic evaluation of the maximum possible water content based on hydrogeological knowledge about the aquifers: this would significantly improve results obtained by linear pro-

Table 3
Estimation of the water-saturated zone and volume of water for the Saint Cyr-en-Val field data

Parameter	Field data: linear programming, 'ideal body' and 'compact body'	Field data: Tikhonov regularization
z_{top} (m)	4	3
z_{bot} (m)	37	52
V_{min} (m^3)	0.96	1.78
V_{max} (m^3)	2.0	1.78
'Ideal body' minimum water content (%)	4.8	
'Ideal body' equivalent maximum thickness (m)	20	
'Compact body' center of layer (m)	5–13 and 37	

gramming, for example, one could use the maximum possible porosity of the water-saturated rock when the rock type is known.

Results of the inversion of the three-layer model with the limitation $0 \le w_i \le 40\%$ are presented in Fig. 4e. We used the same noise level in the data that was used for inversion shown in Fig. 4c and it can be seen that the result is improved.

4.3. Field data inversion

For the field test of the various inversion schemes, data were acquired with the newly developed NUMIS system (Legchenko et al., 1995) in the vicinity of Orleans city (Central France); the corresponding hydrogeological data were obtained from a nearby water-supply well for the village of Saint Cyr-en-Val (France) during a BRGM geological study. A lithologic log of borehole 268 is given in Fig. 5a.

Three aquifers were intersected by this borehole: an upper aquifer of mixed gravel, sand and clay, approximately 20–25 m thick, and two aquifers of water-saturated karst limestone separated by a sandstone layer at a depth of 50–60 m. The total thickness of the water satured zone is about 70–80 m and the average porosity is approximately 10%. Water transmissivity within the aquifer is around 0.28 m^2/s. The measured signal is presented in Fig. 5b.

Inversion of these field data was performed by both the linear programming (Fig. 5c and d) and regularization (Fig. 5a) methods. According to the linear programming algorithm, we first concluded that a solution exists with a noise $\varepsilon = 10$ nV, and then, we evaluated the presence and character of a water-saturated zone. The results are presented in Table 3. The 'ideal body' inversion (Fig. 5c) showed a water-saturated zone between 3 and 37 m; the water predicted below 37 m is discussed later. Using 'compact body' inversion (Fig. 5d), we can distinguish two aquifers: a shallow aquifer between 5 and 13 m and a deeper aquifer at 37 m. The waterless interval between 13 and 30 m was confirmed by both the 'ideal body' and 'compact body' solutions. The water content cannot be more than 52%, but this high value is hardly new information.

Using the Tikhonov regularization method, we obtained a unique solution (Fig. 5a). Evaluating the water-saturated zone in the same manner as with the linear programming method, two aquifers are detected at about the same depths and the volume of water is close to the maximum that was found by linear programming (Table 3). Both the linear programming and the Thikonov regularization indicate a small amount of water below 40 m, thus we can expect that, according to the inversion, there is a third aquifer situated deeper than 50 m. This is borne out by the lithologic log of borehole 268, which indicates a third aquifer between 60 and 80 m. However, this deeper aquifer cannot be computed reliably from the measured data because of lack of resolution of the PMR data set at this depth.

5. Conclusion

We have presented examples of solutions for the inversion of surface PMR data applied to groundwater investigation. Inversion was performed using a linear programming technique that permits not only the finding of a solution but also a study of the solution space, which gives us a better idea of possible parameter variations in view of experimental errors.

The method enables the groundwater distribution to be characterized through parameters that constrain the depths of the top and bottom of the aquifer formations; it also indicates other properties of groundwater distribution such as minimum concentration, total water thickness, water quantity, etc.

In order to compare the performance of different methods for inverting PMR data, we used both numerical simulations and field data. The results obtained using the linear programming compared favorably with those obtained using the Tikhonov regularization method. Moreover, these algorithms can be used with a portable computer, which means that interpretations can be made rapidly in the field. Generalization to the nonuniformly stratified 2D case and even to the general 3D case can be envisioned.

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