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Inversion of surface nuclear magnetic resonance data by an adapted Monte Carlo method applied to water resource characterization

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Abstract

Inversion of surface nuclear magnetic resonance (SNMR) provides important information about aquifers, such as their depths, thickness, pore size and water content. Different methods (inverse quadratic, linear programming) have been applied to the problem of SNMR data inversion, but there has not yet been any attempt to explore model space. We propose that an adaptation of the Monte Carlo method presented here is suitable for exploring the set of solutions consistent with SNMR data. We also demonstrate the capability of this method applied to the interpretation of SNMR data with examples of both synthetic and field data inversions. We also show that the method can be used to obtain various results, such as a posteriori water distribution with depth and a posteriori pore-size distribution with depth.

Keywords: Inversion; Surface nuclear magnetic resonance; Monte Carlo

1. Introduction

Surface nuclear magnetic resonance (SNMR) is a particular application of the nuclear magnetic resonance (NMR) phenomenon (Varian, 1962; Semenov, 1987; Semenov et al., 1987), wherein only the protons in the hydrogen nucleus are investigated. It is the only known surface geophysical method that allows direct detection of groundwater, in addition to providing information about groundwater distribution and the mean size of the pores of water-saturated rocks. The interpretation of SNMR measurements is based on the solution of an inverse problem and involves various relevant hydrogeological parameters, such as bulk volume of the water, aquifer depth and thickness. By adopting certain assumptions, the problem becomes a linear one. Nevertheless, the nonuniqueness of the solution requires the application of additional criteria or knowledge about the solution in order to select just one solution from amongst other equivalent solutions.

Although the Tikhonov regularization method (Tikhonov and Arsenin, 1977) or linear programming techniques can be successfully used for inverting SNMR data (Legchenko and Shushakov, 1998; Guillen and Legchenko, 2002), these methods cannot provide a statistical sampling of the solution space. It is, therefore, necessary to find a method that will make it possible to explore extensively the model space that conform with the known data on:

- the SNMR measurements,
- the probability density function (pdf) of the groundwater content,
- the pdf of the decay time in the subsurface region (as a function of the pore size),
- the pdf of the aquifer thickness.

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Hereafter, we demonstrate how a technique, such as an adapted Monte Carlo method, can be applied to the interpretation of SNMR measurements and why this proposed approach is better than using the classical Monte Carlo method. We first use a synthetic model and then apply the method to a field test case close to the Dead Sea in Israel.

A certain amount of information is required to perform the SNMR data inversion by this method, the three main items being ground type, probable thickness of the geological layers and the pdf of water content and decay time in the subsurface.

Two relevant questions from the point of view of the data interpretation can, for instance, be formulated as follows:

- What is the probability of there being a water content greater than *X*% between 20- and 30-m depth?
- What is the probability of there being a water content greater than *X*% between 0- and 40-m depth if there is a barren zone between depths of 50–100 m?

2. Basic theory of SNMR sounding

A circular or square antenna, between 10 and 100 m diameter/side according to the required depth of investigation is laid out on the ground and used to transmit an alternating signal: $i(t)=I_0\cos(\omega_0 t)$, $0\le t\le\tau$, where I_0 and τ are the amplitude and the duration of the signal excitation pulse.

The current frequency $f_0=\omega_0/2\pi$ is equal to the Larmor frequency of the protons in the geomagnetic field of the site, i.e., $f_0=\lambda H_0/2\pi$ with λ being the gyromagnetic ratio of the protons and H_0 the amplitude of the geomagnetic field.

The precession of protons around the geomagnetic field creates an alternating magnetic field, which is measured with the transmitting antenna at the end of the transmission. The measured NMR signal oscillates at the Larmor frequency and has an exponential envelope

$$e(t) = e_0\exp(-t/T_2^*)\cos(\omega_0 t + \varphi_0),$$

where T_2^* is the spin–spin relaxation or decay time and φ_0 the phase of the NMR signal, T_2^* being a function of the pore size of the rock and increases with pore size. This information is relevant because it gives an idea of the available free groundwater. The amplitude of the NMR signal e_0 is equal to (Trushkin et al., 1995)

$$e_0(q) = \omega_0 M_0 \int_V h_{I\perp}\sin(0.5\lambda h_{I\perp}q)w(\mathbf{r})\mathrm{d}\boldsymbol{r},$$

where $q=I_0\tau$, which represents the excitation parameter (here after called charge); M_0 is the magnetic moment; and $h_{I\perp}=H_{I\perp}/I_0=f(\mathbf{r}, \rho(\mathbf{r}), \alpha)$; $H_{I\perp}$ is the component of the signal created perpendicular to the Earth's magnetic field with an inclination of α, $\rho(\mathbf{r})$ and $w(\mathbf{r})$ represent, respectively, the ground resistivity and the water content at point $\boldsymbol{r}$.

If we assume that we have m measurements of the NMR signal, and k samples of the exponential envelope, and that the model is contained in a volume V of the subsurface, then the data e_{0jl} are connected to the groundwater concentration and to the decay time T_2^* of the model by the equation

$$e_{0jl} = \int_v G_j(\mathbf{r})w(\mathbf{r})\exp(-t_l/T_2^*(\mathbf{r}))\mathrm{d}\boldsymbol{r},$$

$$j = 1,\ 2,\ \ldots,\ m \text{ and } l = 1,\ 2,\ \ldots,\ k,$$

where $G_j(\mathbf{r})$ is the Green's function for the measurement j, with $G_j(\mathbf{r}) = \omega_0 M_0 \int_V h_{I\perp}\sin(0.5\lambda h_{I\perp}q_j)\mathrm{d}\boldsymbol{r}$, and we have $0\le w(\mathbf{r})\le 1$ throughout the investigated space V.

If we now assume a model of horizontally stratified ground with each layer being homogeneous, the above equation becomes

$$e_{0jl} = \sum_i G_{ij}w_i \exp(-t_l/T_{2j}^*),$$

$$j = 1,\ 2,\ \ldots,\ m \text{ and } l = 1,\ 2,\ \ldots,\ k,$$

$$\text{with } G_{ij} = \omega_0 M_0 \int_{x=-\infty,\ y=-\infty,\ z=z_i}^{x=\infty,\ y=\infty,\ z=z_i+1} h_{I\perp}$$

$$\sin(0.5\lambda h_{I\perp}q_j)\mathrm{d}\boldsymbol{r} \tag{1}$$

where z_i and z_i+1 are the depths, respectively, to the top and bottom of layer i, w_i represents the groundwater content in layer i, and T_2^* represents the decay time associated with the same layer.

3. Inverse problem

Taking g_{jl} to be the SNMR effect at time t_l for a charge q_j, and G_{ij} to be the contribution of a layer i with a concentration w_i for a charge q_j, we obtain

$$g_{jl} = \sum_i G_{ij} w_i \exp(-t_l/T_{2i}^*)$$

$$(j = 1, \ldots, m \text{ and } l = 1, \ldots, k) \quad (2)$$

where k represents the number of samples of the exponential envelope, m represents the number of pulse acquisition (charge values), w_i represents the water content in the ith layer, with $0 \leq w_i \leq 100\%$ (i=1, n), and T_{2i}^* represents the decay time in the same layer with $0 \leq T_{2i}^* \leq 1000$ ms (i=1, n).

We define a model m by the vector of water content and decay time. Using the bayesian approach to inverse problems, describing the relation between the a priori *information* on a model m defined by a probability density $P_{\text{prior}}(m)$ and the information provided by the measurements e_{0jl} and by the physical theory $L(m, e_0)$, we define a probability density of the a posteriori *information* $P_{\text{post}}(m)$ equal to (Tarantola and Valette, 1982)

$$P_{\text{post}}(m) = kL(m, e_0)P_{\text{prior}}(m)$$

where k is an appropriate normalization constant.

$L(m, e_0)$ is a function of the fit between the measurements e_{0jl} and the effect of the model g_{jl}. We want maximise the probability of the a posteriori *information* so we must maximise $L(m, e_0)$ defined now as $L(m)$.

If we assume that the measurement error obeys a Gaussian law of distribution and that σ^2, defined by the noise measurement before transmitting the signal, represents the variance of the data which is assumed to be the same for all measurements, then we need to maximize $k\exp(-S(g)/\sigma^2)$. Here, $S(g)$ represents the misfit function between the SNMR measurements and the model effect, i.e., $S(g) = 1/2\sum_{j=1,\ l=1}^{m,\ k}(g_{jl} - e_{0jl})^2$ with g_{jl} being the total effect of the model for the observation pulse j at time t_l, and e_{0jl} being the measurement of the field at the same point.

3.1. Inversion strategy

The proposed inversion strategy is a Monte Carlo type of inversion (Metropolis and Ulam, 1949; Metropolis et al., 1953; Mosegaard and Tarantola, 1995) that makes it possible to randomly search a large number of possible models, for a hypothesis defined by:

- the electrical conductivity distribution of the medium,
- the pdf of the water content,
- the pdf of the decay time,
- the pdf of the aquifer thickness.

Monte Carlo methods are pure random search methods, where each parameter is allowed to vary within an a priori pdf. Starting from an initial model we generate a new model by a random perturbation of the parameters. The model is retained once the misfit between the effect of the new model and the observations is smaller than a given threshold. Clearly, this approach fails when the misfit function has several peaks and troughs. An alternative for finding a global minimum is the simulated annealing methods (Sen and Stoffa, 1995), of which the Metropolis algorithm is a particular case. Mosegaard and Tarantola (1995) propose slightly modifying the current model so as to find a neighboring model and then, sequentially sampling all the possible models in order to solve the inverse problem using an efficient Metropolis method that enables a posteriori sampling of models.

The definition of the neighboring model is important (Mosegaard and Tarantola, 1995) because it influences the success of the method. We define a model as a set of aquifers with a water content and a decay time, and where disturbances involves modification of the following parameters: the water content of an aquifer, the decay time of an aquifer and the geometry of an aquifer.

4. Inversion algorithm

To perform the inversion, we first consider the water content distribution using only the amplitude

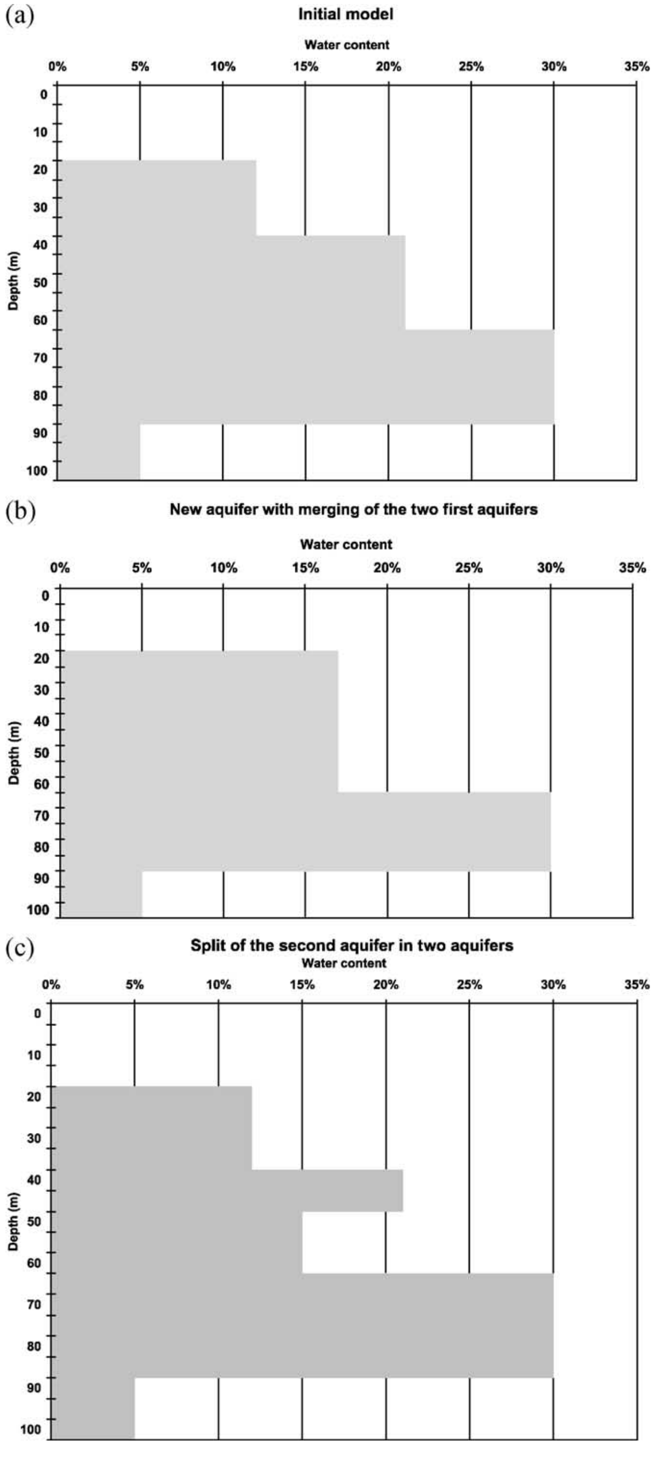

Fig. 1. Modification of the geometry of an aquifer. (a) Initial model. (b) Merging of two aquifers. (c) Splitting of an aquifer into two aquifers.

value of the signal at $t=0$, and then look for a decay time distribution that can explain the exponential envelope.

The initial model for a given hypothesis defined by a pdf for water content and decay time is m_0. The construction of the initial model provides a topology of aquifers, each aquifer A_a consists of an assemblage of unit layers j. For ease and rapidity of calculation, we propose the following method:

- assume a distribution of conductivity in the subsurface;
- calculate the elements G_{ij} (Eq. (1)) for this distribution of conductivity and for layers of unit thickness (one meter);
- sum the G_{ij} of the unit layers j composing the ath aquifer. This will always be possible if we work in whole numbers (rounded off to the next higher or lower integer) to define the thickness and position of the aquifers. The effect of the ath aquifer for the measurement i will then be $EA_{ai} = \sum_j G_{ij}$ for all j, such that the unit layer j belongs to the ath aquifer.

In order to be able to apply the Monte Carlo type method (disturbance of the model) described above in an efficient manner, the disturbances must be small.

The first disturbance will be a modification of the water content of a randomly selected aquifer. The new content must obey the previously given probability density function for the distribution of water in the subsurface region.

The second disturbance will be a modification of the geometry of an aquifer (Fig. 1). In selecting an aquifer (Fig. 1a), we can either destroy the separation between two aquifers and choose a water content and a decay time for the new aquifer obeying the pdf for the distribution of water and the distribution of the decay time in the subsurface (Fig. 1b), or create a new aquifer by inserting an interface into the aquifer, and then allocate a new water and decay time distribution for each of the two new aquifers, obeying the previously given pdf for the water and decay time distribution in the subsurface (Fig. 1c).

To compute the SNMR effect of a model m_0, we calculate the elementary SNMR effect for each aquifer of the model obtained as follows. Let A_a be the aquifer of the a priori geometrical model m_0 and EA_{ai} the effect of this aquifer at the measurement point i for a distribution of water equal to 100%. The SNMR effect of the model m_0 at a measurement point i and at time l will then be: $g_{jl} = \sum_a EA_{ai}(w_a) \exp\left(-t_l/T^*_{2a}\right)$, with w_a representing the distribution of water and T^*_{2a} representing the distribution of decay time in the ath aquifer.

4.1. A posteriori sampling

The different steps for the a posteriori sampling of models using an inversion for the a priori geometrical model m_0, are as follows.

(1) Construct the a priori model m_0 (i.e., define the distribution of conductivities, pdf of water content, pdf of decay time and of aquifer thickness).

(2) Compute the elementary SNMR effect for each elementary layer, with water content of 100%. The

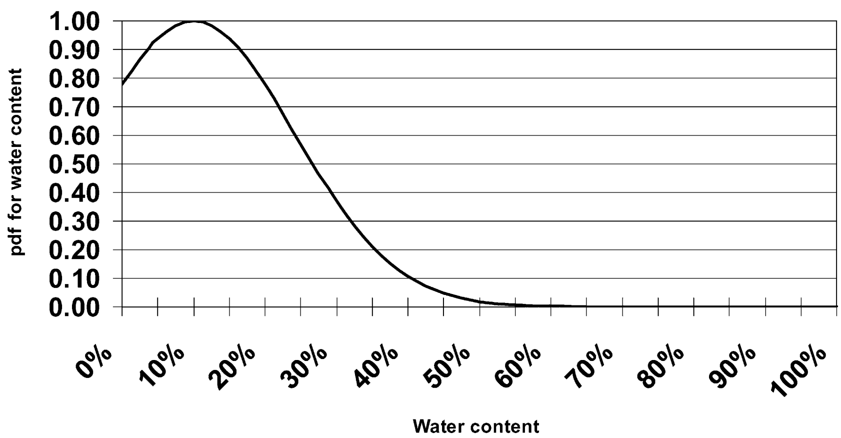

Fig. 2. A priori probability density function for water content.

effect of a unit layer j at a measurement point l will then be G_{jl}.

(3) Determine the laws of a priori pdf of water content and decay time in the subsurface region and of the aquifer thickness.

(3a) For the aquifer thickness, the pdf can be represented by a uniform law

$$f(th) = (1/th_{\max})(H(th) - H(th - th_0)),$$

$$\text{with } H(th) = \begin{cases} 1 & \text{if } th \geq 0 \\ 0 & \text{if } th < 0 \end{cases}$$

and $th_{\max}$ representing the maximum thickness.

(3b) For the water content pdf in the subsurface region (Fig. 2), a law of the following type can be used:

$$fw(w) = \begin{cases} 0 & \text{if } w < 0 \\ C_{st} & \text{if } w = 0 \\ (1/\sigma\sqrt{2\pi})\exp(-(w - \overline{w})^2/2\sigma^2) & \text{if } w > 0 \text{ and } w \leq 1 \\ 0 & \text{if } w > 1 \end{cases}$$

where $\overline{w}$ represents the mean water content and where C_{st} is chosen to obtain $\int_0^1 f_w(w)\mathrm{d}w = 1$.

(3c) For the pdf of decay time in the subsurface region, a law of the following type can be used:

$$f_{T_2}(T_2^*) = (H(T_2^*) - H(T_2^* - 1000)),$$

$$\text{with } H(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}.$$

(4) Sample the thickness of each aquifer (from the top down) and the water content for all the aquifers according to the law determined at step 3.

(5) Calculate the effect of the model and initiate the inversion process.

(6) Initialize the likelihood of the model $L(m_0)=k \exp(-S(m_0)/\sigma^2)$ to zero. This likelihood will be called the current likelihood, $L(m_{\text{cur}})$. With $S(m_0)$ representing the deviation from the geophysical data, for example, $S(m_0) = 1/2\sum_{i=1}^{N}(g_i(m_0) - e_{0i})^2$ or $g_i(m_0)$ represent the total effect of the model at the observation point i and for e_0 the measured field at the same point. Let σ^2 represent the a priori variance of the data, which we assume to be identical for all the data.

(7) Calculate the effect of the model by applying formula (2). The model is called the disturbed model m_{dis}.

(8) Disturb the current model m_{cur}: Choose the type of modification, i.e., modification of water content or the geometry of the aquifers. This choice is made randomly with a probability of 0.5.

(a) If the geometry is modified:

- Select an aquifer by equiprobable choice selection. Let A_a be the aquifer selected.
- Decide by equiprobable choice if the aquifer A_a is to be divided in two, or if it is to be combined with the overlying aquifer.
- Allocate a value of water content and of decay time to the new aquifers using the laws determined above.
- Calculate the geophysical disturbance at time t=0 by which the model is affected; for this, it is simply necessary to recalculate the effect of the disturbed aquifers.

(b) If the water content is modified:

- Select an aquifer by equiprobable choice selection. Let A_a be the selected aquifer.
- Sample the parameter (water content and decay time) using the probability law determined at step 3.
- Calculate the geophysical disturbance at time t=0 by which the model is affected; for this, it is simply necessary to recalculate the effect of the disturbed aquifer.

(9) Calculate the likelihood of the model $L(m_{dis})=k \exp(-S(m_{dis})/\sigma^2)$.

(10) If $L(m_{\text{dis}})>L(m_{\text{cur}})$, then the model is disturbed and we set $m_{\text{cur}}=m_{\text{dis}}$. If not, we choose to retain m_{dis} with a random selection and a probability of $L(m_{\text{dis}})/L(m_{\text{cur}})$. If we keep m_{dis}, then we set $m_{\text{cur}}=m_{\text{dis}}$; if not, m_{cur} is not modified. An example of the evolution of $S(m_{\text{dis}})$ is illustrated in Fig. 3.

(11) Store the model obtained, i.e., the thickness, water content and decay time of each aquifer.

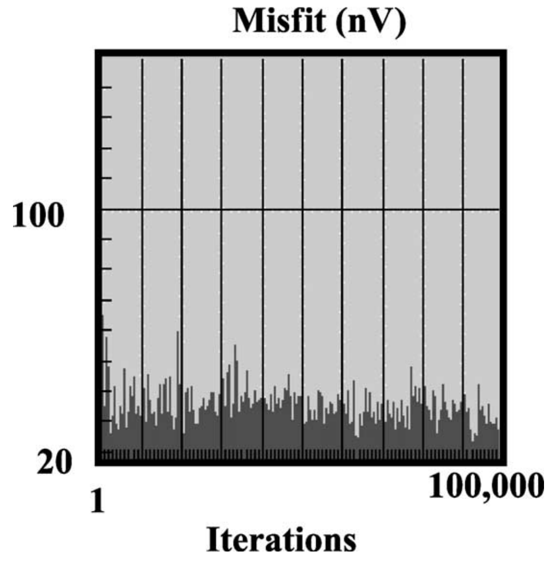

Fig. 3. Evolution of the mismatch between a current model m_{cur} and measurements after 100,000 iterations.

(12) Increment the number of inversions n performed. After 100,000 inversions, return to step 1 to perform an inversion for a new initial model.

(13) If a new disturbed model m_{dis} results, return to step 8 for a new sampling.

5. Applications

5.1. Synthetic data cases

We applied the method to a synthetic model composed of two aquifers:

- 10–20-m depth with 10% water and T_2^* equal to 500 ms,
- 40–50-m depth with 15% water and T_2^* equal to 300 ms.

A Gaussian noise level equal to 5% of the signal was added to the computed response.

The initial model consists of 10 aquifers, each 10 m thick and containing 0% water.

Fig. 3 clearly shows the evolution of the mismatch from the data and the oscillations that enable a wide range of solutions to be investigated. The oscillations are created by the retention of models that increase the misfit to the data, but which are nevertheless retained with a certain probability (see step 10 in a posteriori sampling). This strategy makes it possible to explore the space of the possible models. In the same way, the evolution of the mean of the solutions with the number of iterations can be illustrated along the inversion process. This mean does not in itself represent a model, but gives a good idea of the probable distribution of subsurface water, because the response is linear. The final distribution of the water content as a function of depth is illustrated in Fig. 4, which shows that the distribution of water in this evolution becomes progressively concentrated between 10 and 20 m with a mean value near 10%, and between 35 and 55 m with a mean value of 12.5%. The mean of the water content and of the T_2^* value as a function of depth is illustrated in Fig. 5. Here again, we retrieve the two aquifers; the first with a T_2^* between 600 ms and 700 ms, the second with a T_2^* close to 400 ms. The distribution of T_2^* at a given depth is more spread out than for the water content (Fig. 5) due to the multiple solutions in the inversion of a sum of exponentials.

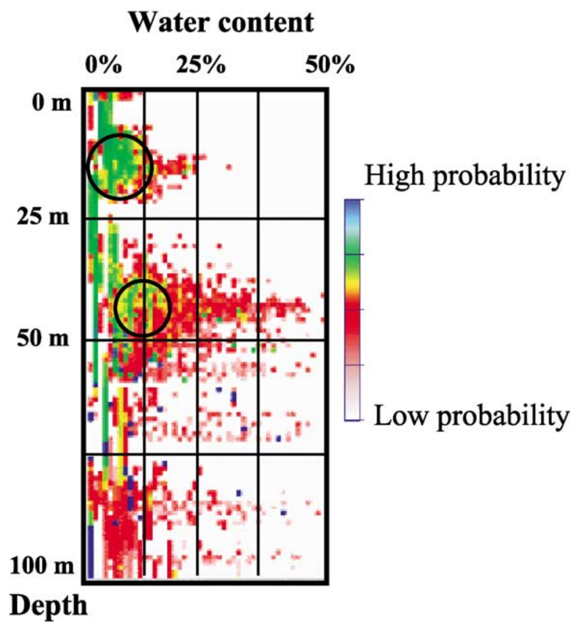

Fig. 4. Probability density function of the water content versus depth.

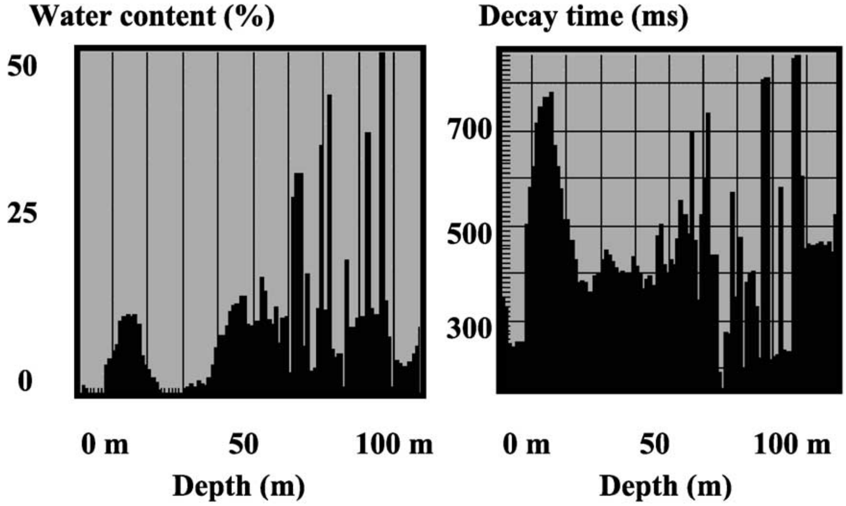

Fig. 5. Mean of water content and T_2^* versus depth.

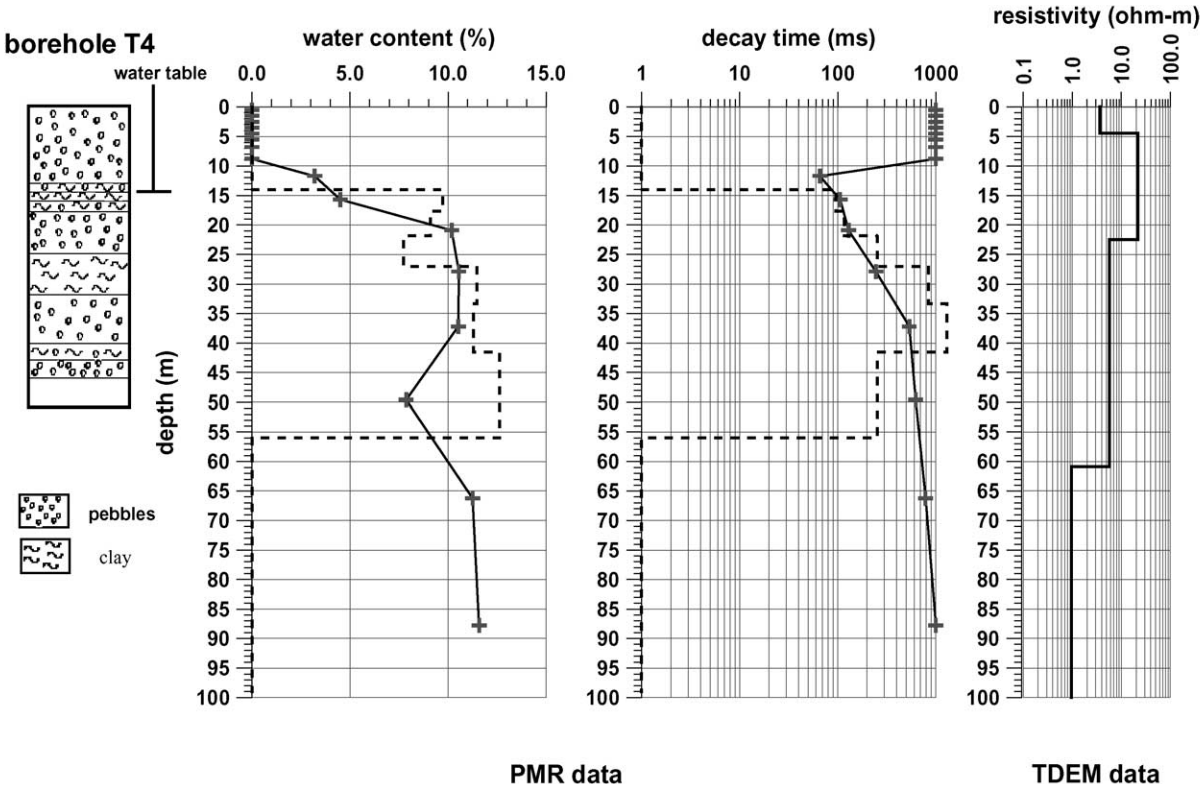

Fig. 6. Field test case in Israel: lithological description of the borehole, water content and decay time obtained by Tikhonov inversion, resistivity obtained by TDEM (Goldman et al., 1994).

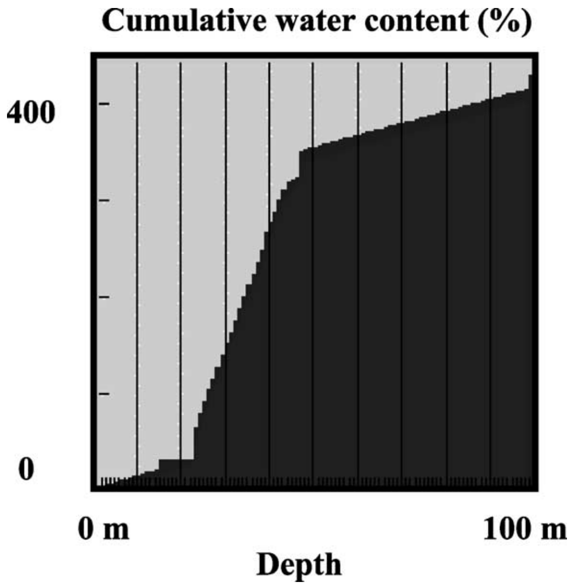

Fig. 7. Cumulative distribution of water content versus depth for the model corresponding to the minimum of total water.

5.2. Field test case

During 1998, 30 surface nuclear magnetic resonance (SNMR) soundings were performed in Israel in two distinct areas, one located near the Mediterranean coast and the other near the Dead Sea (Legchenko and Beauce, 1998). For the area close to the Dead Sea, which had been studied by Goldman et al. (1994), two boreholes T2 and T4 were used to verify the SNMR results.

5.2.1. Results using the Tikhonov regularization method for inversion

A comparison of the borehole data and the SNMR results obtained using the Tikhonov regularization method for inversion are depicted in Fig. 6 for the sounding corresponding to borehole T4. The water table depth deduced from the SNMR sounding is in good agreement with the depth measured in the borehole T4. The interpretation is also in agreement with the available hydrogeological data for this area. Some variations in the decay time were detected by SNMR sounding close to borehole T4, although not in the sounding near borehole T2. The lower decay time in the shallow part of the resolved aquifer could be explained by the presence of a 5-m-thick layer of clay at a depth of 25 m, which was revealed by the borehole and by TDEM (Goldman et al., 1994). This relatively thin layer, which is not resolved by inversion of the water content, causes a decrease in the decay time due to the direct relationship existing between decay time and pore size.

5.2.2. Results using the adapted Monte Carlo method for inversion

We analyzed the results obtained from this SNMR sounding using the proposed Monte Carlo method for inversion. Assuming a Gaussian a priori probability density function, with a mean value of 5% and a standard deviation of 30%, for water content, the results are:

(i) The minimum mass of water is 410, corresponding to an equivalent 41-m-thick aquifer with water content of 10% (Fig. 7).
(ii) Two water-bearing zones are clearly distinguished; the first around 10 m and the second beginning at 20 m with higher water content (Fig. 8).

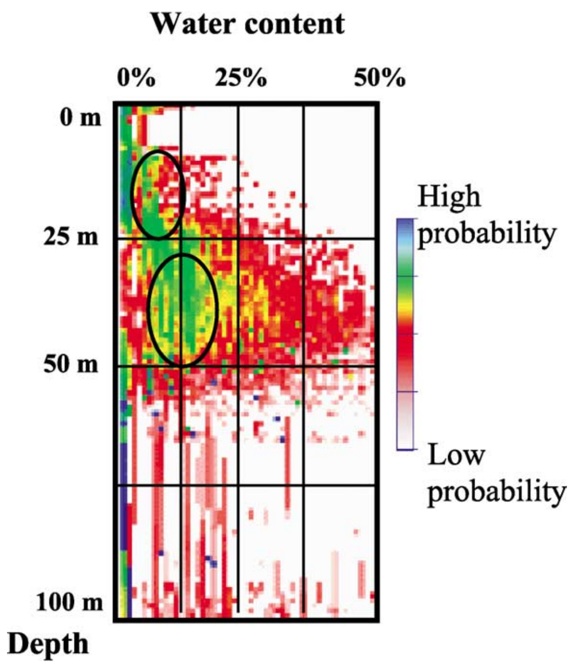

Fig. 8. Probability density function water content versus depth.

(iii) A 5-m-thick aquifer corresponding to a decrease in decay time could be identified around 20 m (Fig. 9).

Not only are these results in good agreement with the results obtained by the more classical Tikhonov regularization method, but also they provide more additional useful information as discussed below.

If we store the different models obtained during the inversion process, we can, for example, seek answers to different questions, such as:

- What is the evolution of the maximum water content with depth?
- What is the probability of a given water content between two depths?
- What is the probability of obtaining more than x% of water between 30 and 50 m if there is less than 5% of water in the first 20 m?

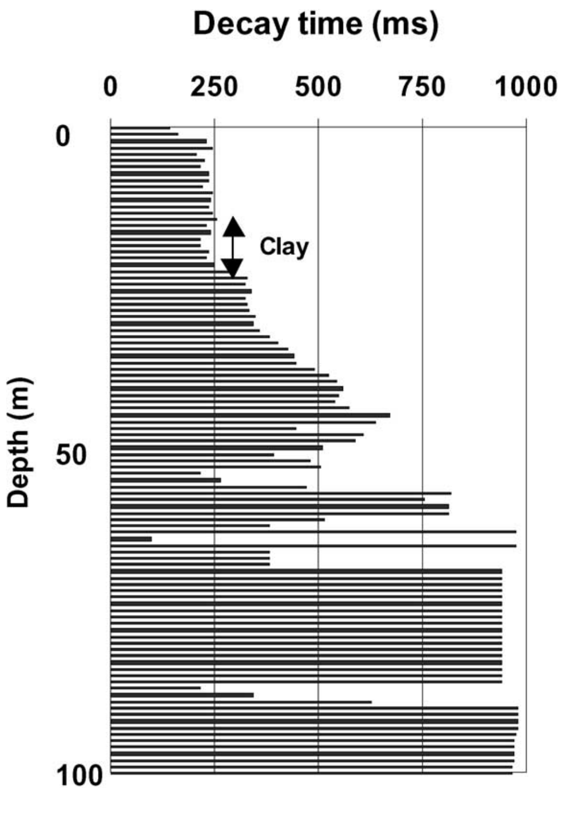

Fig. 9. Mean of T_2^* versus depth.

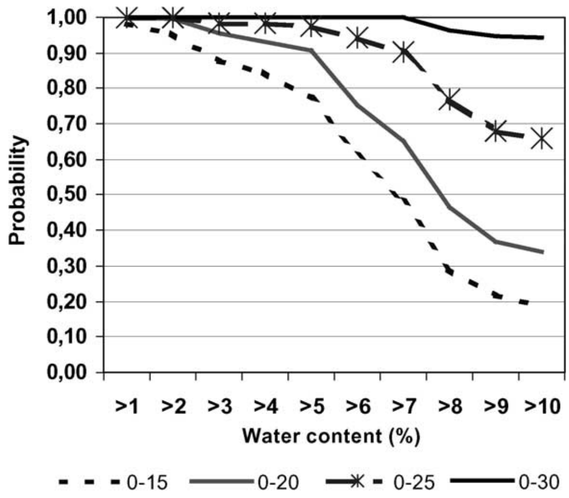

Fig. 10. Evolution of the maximum water content with depth.

- What is the probability of obtaining more water in a given 5-m-thick layer than in the 5 m above this layer?

6. Model interpretation

6.1. What is the evolution of the maximum water content with depth?

The results to this question give us an idea of the maximum water content that we could expect at a given depth. To do this we compute the function:

$$f(a, z) = \text{Prob}(\text{Max}(w(h)) > a),$$

with $a \in \{0, 1, 2, 3, \ldots, 15, \ldots\}$ and $h \in [0, z]$,

where w represents the water content and z represents a depth, looking for all the models that satisfy this condition in the set of stored models. Fig. 10 illustrates this function for $a \in \{1, 2, 3, \ldots, 10\%\}$ and for different $z \in \{15, 20, 25, 30\ m\}$ for the field test case near borehole T4. We see on the curve corresponding to the depth interval 0–20 m that the probability of a water content greater than 5% is 0.9 and on the curve corresponding to the depth interval 0–30 m that the probability of a water content greater than 10% is greater than 0.9.

6.2. What is the probability of given minimum mean water content between two depths?

The answer to this question gives us an idea of the distribution of water in some areas of the sounding. To do this, we compute the function

$$f(a,\ z_0,\ z_1) = \mathrm{Prob}((1/(z_1 - z_0)) \int_{z_0}^{z_1} w(z)\mathrm{d}z) > a),$$

with $a \in \{1, 2, 3, \ldots, 15, \ldots\}$, and where w represents the water content and z_0 and z_1 represent two depths defining an interval, looking for all the models that satisfy this condition in the set of stored models. Fig. 11 illustrates this function for $a \in \{1, 2, 3, \ldots, 20\%\}$ and for different pairs of z_0 and $z_1 \in \{(0, 15), (0, 20), \ldots, (0, 60)\}$ for the SNMR sounding near borehole T4. We see on this plot that the probability of a mean of water content greater than 5% between 0 and 40 m is 0.8 and it is 0.9 to have a mean greater than 8% between 0 and 50 m.

6.3. What is the probability of obtaining more than X% of water between 30 and 50 m if there is less than 5% of water in the first 20?

The answer to this question gives us an estimate of the expected water resources between 30 and 50 m if the first 20-m contain less than 5% of water. To obtain these results we compute the function

$$f(a, z, z_0, z_1) = \mathrm{Prob}(A/B),$$

where A is $((1/(z_1 - z_0))\int_{z_0}^{z_1} w(z)\mathrm{d}z) > a)$ with $a \in \{0, 1, 2, 3, \ldots, 20\%\}$, B is $\max(w(z))<5\%$ for $0<z<20$ m, where w represents the water content, and where z_0=30 m and z_1=50 m representing the interval for which we want known the resources, looking for all the models that satisfy this condition in the set of stored models. Fig. 12 illustrates this function applied to the SNMR sounding near borehole T4. We see on this curve that we are sure (the probability is one) to obtain more than 10%, and we have a probability of 0.8 to obtain more than 14%.

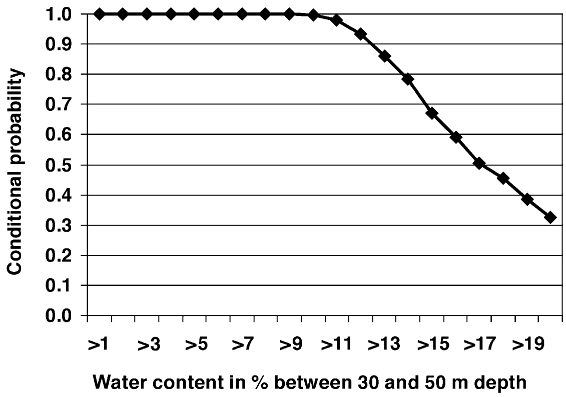

Fig. 12. Function representing the probability of mean water content greater than a given value between 30 and 50 m depth.

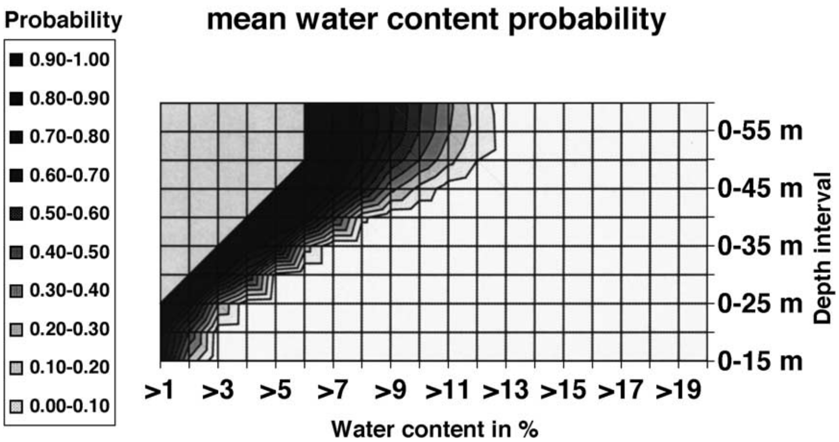

Fig. 11. Plot representing the probability of a mean water content greater than a given value in a depth range (0–X m).

6.4. What is the probability of obtaining more water in a given 5-m-thick layer than in the 5-m above this layer?

The results to this question give us an idea of the evolution of the water content versus depth, pointing out the areas where the water content increases or decreases. To obtain these results we compute the function

$$f(z_0, \Delta z) = Prob(A > B)$$

where $A = 1/(\Delta z)\int_{z_0-\Delta z}^{z_0} w(z)\mathrm{d}z$ and $B = 1/(\Delta z \int_{z_0-2*\Delta z}^{z_0-\Delta z} w(z)\mathrm{d}z)$. Here w represents the water content, z_0 represents the depth and Δz=5 m is the interval in which we want to find the probability, looking for all the models that satisfy this condition in the set of stored models. Fig. 13 illustrates this function applied to the field test case near borehole T4. We see on this curve that the probability of obtaining more water between 15 and 20 m than in the 10–15 m layer is 0.3, whereas between 35 and 40 m, the probability of an increase in the water content is 0.6.

These different results demonstrate that in using the adapted Monte Carlo method for inversion we are able to describe the studied aquifer statistically and obtain a good idea of the distribution of the water content, maximum water content expected, location of the water and mean water content.

The calculation time for 100,000 iterations with graphic representation every 100 iterations is of the order of 4–5 min, using a PC with 128-Mbyte RAM.

7. Conclusion

We have successfully adapted a Monte Carlo technique to the inversion of SNMR data. The examples that we have presented show that the strategy adopted for exploring the space of the models is effective and efficient.

This strategy allows us to describe an aquifer statistically and evaluate, using water content and the decay time, the resources and production risk of the aquifer through exploring a range of models in order to answer questions relevant to groundwater exploration, viz:

- What is the probability of there being a water content greater than *X*% between 20 and 30 m?
- What is the probability of there being a water content greater than *X*% between 0 and 40 m if there is a barren zone between 50 and 100 m?

In the near future, we expect to be able to use inversion results and geostatistics to run 3D simulations of the water content and decay time. In the same way, we should be able to invert and investigate 2D and 3D data.

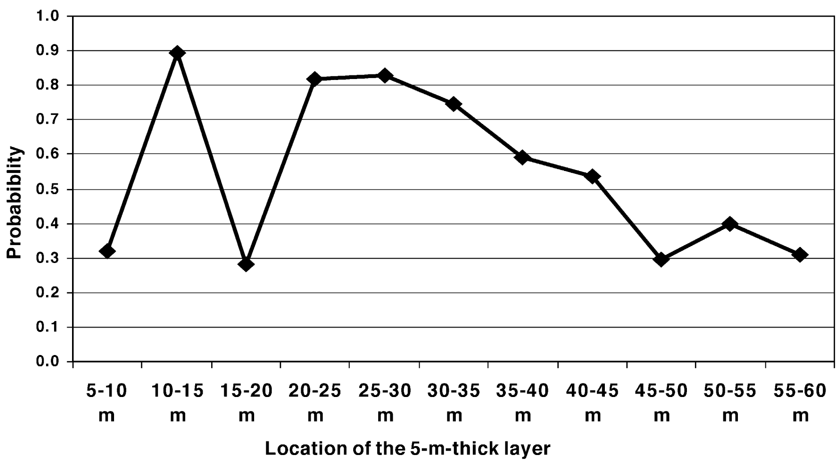

Fig. 13. Probability versus depth of greater water content in a 5-m-thick layer than in the 5-m-thick layer above it.

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