Effect of fluid viscosity on elastic wave attenuation in porous rocks

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Summary

Attenuation and dispersion of elastic waves in fluid-saturated rocks due to the pore fluid viscosity is investigated using an idealized exactly solvable example of a system of alternating solid and viscous fluid layers. Waves in periodic layered systems at low frequencies can be studied using an asymptotic analysis of the Rytov's exact dispersion equations. Since the wavelength of the shear wave in the fluid (viscous skin depth) is much smaller than the wavelength of the shear wave in the solid, the presence of viscous fluid layers requires a consideration of higher terms in the asymptotic expansions. For a shear wave with the directions of propagation and of particle motion in the bedding plane, the attenuation (inverse Q) obtained by this procedure is $Q^{-1} = \omega\eta\phi/\mu_s(1-\phi) + \omega\phi\rho_f^2 h_f^2/12\rho\eta + o(\omega)$, where ω is frequency, ρ is weighted average density of the solid/fluid system, h_f is the thickness of fluid layers, ϕ is fluid volume fraction, i.e., the "porosity", μ_s is solid shear modulus, and ρ_f and η are the density and viscosity of the fluid, respectively. The term proportional to η is responsible for the viscous shear relaxation, while the term proportional to η^{-1} accounts for the visco-inertial (poroelastic) attenuation of Biot's type. This result shows that the characteristic frequencies of visco-elastic ω_{VE}, poroelastic ω_B, and scattering ω_R attenuation mechanisms obey the relation $\omega_{VE}\omega_B = \omega_R^2$, which explains why the visco-elastic and poroelastic mechanisms are usually treated separately in the context of macroscopic theories that imply $\omega \ll \omega_R$. The poroelastic mechanism dominates over the visco-elastic one when the frequency-indepenent parameter $B = \omega_B/\omega_{VE} = 12\eta^2/\mu\rho_f h_f^2 \ll 1$, and vice versa.

Introduction

It is generally believed that the phenomena associated with the viscosity of the pore fluid are one of the main causes of the attenuation of elastic waves in reservoir rocks and other fluid-saturated porous materials. However, despite decades of theoretical as well experimental research in this area, there is still some confusion as to the dependency of the attenuation on fluid viscosity at low frequencies. Indeed, according to Biot's theory of poroelasticity, dimensionless attenuation (inverse quality factor) in the low-frequency limit is proportional to frequency and to the inverse of viscosity (Biot, 1956a). On the other hand, the attenuation due to the local flow (squirt) mechanism is proportional to the product of frequency and viscosity (Mavko and Nur, 1975; O'Connell and Budiansky, 1977). Despite the fact that the two mechanisms have the same basic physical cause (viscosity), there is as yet no sound theory that provides a comprehensive model for both mechanisms. In this paper we investigate the effects of pore fluid viscosity on elastic wave propagation using an idealized exactly solvable example.

One example which proved particularly useful in various studies of porous media is a medium consisting of periodically alternating fluid and solid layers (Rytov, 1956; Brekhovskikh, 1981; Schoenberg, 1984; Bedford, 1986). However, most of the research has been focused on ideal and low viscosity fluids and relatively high frequencies when the layered system exhibits behavior typical of fluid-saturated media as described by high-frequency Biot's theory of poroelasticity (Schoenberg, 1984; Schoenberg and Sen, 1986; Molotkov and Bakulin, 1999; Molotkov and Khilo, 1990). In this paper we focus on low frequencies and relatively high viscosity fluids.

The properties of waves in periodic layered systems at low frequencies can be studied using a low-frequency asymptotic analysis of the known exact dispersion equations (Rytov, 1956; Brekhovskikh, 1981). For the asymptotic analysis to be valid, the wavelengths of all the waves involved must be larger than the spatial period of the periodic system. Since the wavelength of the shear wave in the fluid (viscous skin depth) is much smaller than the wavelength of the shear wave in the solid or of the acoustic wave in the fluid, the presence of viscous fluid layers requires a consideration of higher terms in the asymptotic expansions. The procedure is exactly the same for shear waves with the directions of propagation and of particle motion in the bedding plane, and for extensional waves propagating parallel to layering. In this paper we show how this can be done for shear waves.

Low-frequency dispersion equation

Consider a system of periodically alternating solid and fluid layers of period d. The elastic solid is characterized by density ρ_s, bulk modulus K_s and shear modulus μ_s. The viscous fluid is characterized by density ρ_f, bulk modulus (inverse compressibility) K_f, and dynamic viscosity η. The solid and fluid layer thicknesses are h_s and $h_f = d - h_s$, respectively.

We consider propagation of a shear wave in the direction x parallel to layering with the displacement in the direction y normal to x but also parallel to the bedding (SH wave). For a given frequency ω the solution of the mechanical problem can be looked for in the form

$$u_y = u_{y0} e^{i(ax-\omega t)}.$$

We want to obtain low-frequency asymptotic of the wavenumber a or the phase velocity $c = \omega/a$ as a function of ω. To employ the known results for solid layered systems, we can regard the fluid as another solid with a complex shear modulus $\mu_f = -i\omega\eta$. SH wave propagation in a periodic system of solid layers of two types s and f is governed by an exact dispersion equation (Rytov, 1956; Brekhovskikh, 1981)

$$p\left[\left(\tan\frac{\beta_s h_s}{2}\right)^2 + \left(\tan\frac{\beta_f h_f}{2}\right)^2\right] + (1+p^2)\tan\frac{\beta_s h_s}{2}\tan\frac{\beta_f h_f}{2} = 0. \quad (1)$$

Here $\beta_s^2 = \omega^2\left(1/c_s^2 - 1/c^2\right)$, $\beta_f^2 = \omega^2\left(1/c_f^2 - 1/c^2\right)$, where $c_s = (\mu_s/\rho_s)^{1/2}$, and $c_f = (\mu_f/\rho_f)^{1/2}$ are shear velocities in the materials s and f, respectively, and p is given by

$$p = \frac{\mu_f \beta_f}{\mu_s \beta_s}.$$

For sufficiently long waves or low frequencies the arguments of the tangents are small. Thus, the tangents in eq. (1) can be replaced by their respective arguments. The so simplified equation can be solved analytically to give

$$c^2 = \frac{h_s\mu_s + h_f\mu_f}{h_s\rho_s + h_f\rho_f}, \quad (2)$$

or

$$c^2 = \frac{(1-\phi)\mu_s + \phi\mu_f}{\rho},$$

where $\phi = h_f/d$ is the volume fraction of the fluid layers (porosity), and $\rho = (1-\phi)\rho_s + \phi\rho_f$ is the average density of the saturated rock. For the fluid layers the substitution

$$\mu_f = -i\omega\eta \quad (3)$$

yields the following expression for the velocity c^2

$$c^2 = \frac{(1-\phi)\mu_s}{\rho}\left(1 - \frac{\phi}{1-\phi}\frac{i\omega\eta}{\mu_s}\right). \quad (4)$$

Due to the effect of viscosity the velocity is now complex, implying the presence of attenuation.

Eq. (4) is the result given in literature as a low-frequency or long-wavelength approximation (see e.g. Brekhovskikh (1981), Molotkov and Khilo (1990)) with an obvious requirement that $\beta_s h_s$ and $\beta_f h_f$ must be small. However, at low frequencies the wavelength of the viscous wave in the fluid is much smaller than that of the shear wave in the solid. Thus, the decrease of frequency ω also increases the relative magnitude of the terms containing β_f, so that higher terms in the power series expansion of $\tan(\beta_f h_f/2)$ may become significant. To analyze this phenomenon in a greater detail, we must retain the second term in this expansion, i.e.

$$\tan\frac{\beta_f h_f}{2} \simeq \frac{\beta_f h_f}{2}\left(1 + \frac{1}{12}\frac{i\omega h_f^2\rho_f}{\eta}\right). \quad (5)$$

Substituting this approximation for $\tan(\beta_f h_f/2)$ while still replacing $\tan(\beta_s h_s/2)$ by its argument and again solving for c^2 yields

$$c^2 = \frac{(1-\phi)\mu_s}{\rho}\left(1 - \frac{i\omega\eta}{\mu_s}\frac{\phi}{1-\phi} - \frac{1}{12}\frac{i\omega}{\eta}\frac{\phi\rho_f^2 h_f^2}{\rho}\right). \quad (6)$$

The imaginary terms indicate the presence of dissipation. The corresponding dimensionless attenuation (inverse quality factor) can be written

$$Q^{-1} = \frac{\text{Im } c^2}{\text{Re } c^2} = \frac{\omega\eta}{\mu_s}\frac{\phi}{1-\phi} + \frac{1}{12}\frac{\omega}{\eta}\frac{\phi\rho_f^2 h_f^2}{\rho}. \quad (7)$$

Equations (6) and (7) are the central results of this paper. The most interesting feature of these equations is the presence of two dissipative terms with the same frequency dependency but different dependencies on fluid viscosity. In fact, both terms are familiar ones. The first term (proportional to η) is the same as in eq. (4) and accounts for the contribution of the complex shear modulus of the fluid to the overall complex shear modulus of the layered system (viscous shear relaxation). The second term which scales with η^{-1} can be identified with the visco-inertial attenuation in a porous medium as described by Biot's theory of poroelasticity (Biot, 1956a). In Biot's theory the shear wave attenuation in the low frequency limit is given by

$$Q_B^{-1} = \frac{\omega}{\eta}\frac{\rho_f^2\kappa}{\rho}, \quad (8)$$

where κ denotes permeability. The permeability of a system of plain slits is (Bedford, 1986; Biot, 1956b)

$$\kappa = \frac{\phi h_f^2}{12}. \quad (9)$$

Substitution of eq. (9) into eq. (8) yields an expression identical to the second term in the right hand side of eq. (7).

One can see that both terms in eq. (7) are related to the well known mechanisms of wave attenuation in porous media: viscoelastic mechanism (viscous shear relaxation) (Mavko and Nur, 1975; O'Connell and Budiansky, 1977) and visco-inertial Biot's mechanism (Biot, 1956a; Biot, 1956b). In our treatment both terms have been derived, for an idealized porous medium, from the same standpoint.

Figure 1 shows the result expressed by equation (7) against the numerical solution of the exact dispersion equation (1). The parameters of the medium were chosen such that the attenuation factors caused by the two mechanisms are of the same order of magnitude. One can see that the combined effect of the two mechanisms, as expressed by eq. (7), does, indeed, represent the low frequency asymptotic to the exact solution.

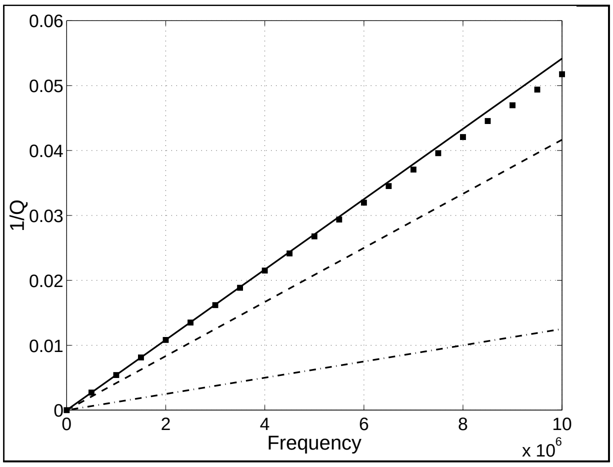

Fig. 1: Viscosity-related attenuation as a function of frequency: Numerical solution of eq. (1) (large dots) versus low-frequency asymptotics. One can observe a perfect agreement between the numerical solution and the compound effect (eq. (7), solid line) of viscoelastic (dash-dotted line) and Biot's visco-inertial (dashed line) mechanisms.

Discussion

To further analyze these results, we rewrite eq. (7) in the form

$$Q^{-1} = \frac{\omega}{\omega_{VE}} \frac{\phi}{1-\phi} + \frac{\omega}{\omega_B} \frac{\phi \rho_f}{\rho}. \qquad (10)$$

Here $\omega_{VE} = \mu_s/\eta$ is the characteristic frequency of the viscoelastic mechanism. At this frequency the absolute value of the complex shear modulus of the viscous fluid equals the solid shear modulus. In turn, $\omega_B = \eta\phi/\kappa\rho_f$ is the Biot's characteristic frequency (Biot, 1956b), at which the wavelength of the shear wave (viscous skin depth in the fluid) equals the thickness of the fluid layers h_f. The expressions for the two characteristic frequencies may be multiplied to give

$$\omega_{VE}\omega_B = \frac{\mu_s \phi}{\kappa \rho_f} = \frac{12\mu_s}{h_f^2 \rho_f} = A\left(\frac{2\pi c_0}{d}\right)^2 = A\omega_R^2, \qquad (11)$$

where

$$c_0 = \lim_{\omega \to 0} c = \sqrt{\frac{(1-\phi)\mu_s}{\rho}}$$

is the static shear velocity in the system,

$$A = \frac{3\rho}{\pi^2 \phi^2 (1-\phi)\rho_f}$$

is a dimensionless parameter of order 1 depending only on the porosity and the ratio of solid-to-fluid densities, and ω_R is approximately the resonant frequency of the layered periodic system, at which the wavelength of the shear wave equals the period of the system. For a disordered (non-periodic) system of layers ω_R corresponds to the characteristic scattering frequency. Equation (11) partly explains why the viscoelastic and visco-inertial mechanisms of attenuation are usually treated separately. Indeed, from eq. (11) it follows that either

$$\omega_{VE} < \sqrt{A}\omega_R < \omega_B$$

or

$$\omega_B < \sqrt{A}\omega_R < \omega_{VE}.$$

In other words, if one of the characteristic frequencies is smaller than the resonant (scattering) frequency ω_R, the other one is bound to be larger than ω_R. But any theory that aims at describing the macroscopic effects can, by definition, describe only the behavior of the media on spatial scales much larger than the grain or pore size (or period for periodic media), i.e. for frequencies much smaller than ω_R. Thus any macroscopic poroelastic theory can describe either the viscoelastic or the visco-inertial (Biot's) mechanism of attenuation in our idealized example of a porous medium. For a general three-dimensional periodic porous medium with a single characteristic pore size, this fact was proved mathematically by Boutin and Auriault (1990) in the context of the theory of asymptotic homogenization of periodic structures, the theory that explicitly employs the ratio ω/ω_R as a small parameter. In our analysis we have managed to obtain both terms together only because our approach is based not on any macroscopic theory, but on the dispersion equation (1) which is exact for all frequencies.

Furthermore, we can define a fundamental parameter of a layered system or a porous medium

$$B = \frac{\omega_B}{\omega_{VE}} = \frac{\eta^2 \phi}{\mu_s \kappa \rho_f}$$

that shows which of the two viscosity-related dissipation mechanisms dominates at frequencies $\omega \ll \omega_R$ when the macroscopic description makes sense. We emphasize that the parameter B does not depend on the frequency, but only on the physical and geometrical properties of the layered system (or a porous rock). If $B < 1$, the Biot's poroelastic mechanism is the primary mechanism, whereas for $B > 1$ the viscoelastic mechanism dominates. For the permeability of 1 Darcy and viscosity of water, the parameter B is about 10^{-4}, but it may be larger for more viscous fluids (heavy oil, bitumen) and/or lower permeabilities. Also observe that the closer the parameter B to 1, the closer both characteristic frequencies to each other and to the resonant (scattering) frequency ω_R, resulting in very small attenuation at frequencies $\omega \ll \omega_R$ where macroscopic description makes sense.

The three characteristic frequencies ω_{VE}, ω_R, and ω_B introduce three dimensionless frequencies: $V = \omega/\omega_{VE} = \omega\eta/\mu_s$, $\Omega = \omega/\omega_R = \omega d/c_0$, and $l = \omega/\omega_B = \omega\kappa\rho_f/\eta\phi$. Equation (11) shows that $V/l = A^{-1}(\Omega/l)^2 = A(V/\Omega)^2 = B$, and $\Omega^2 = AVl$. The parameters $L = \sqrt{l}$

and Ω were introduced by Schoenberg and Sen (1986). Their work focused on the "low frequency" ($\Omega \ll 1$) but "small viscous skin depth" ($l \gg 1$) regime, i.e. on frequencies ω that are in the interval

$$\omega_B \ll \omega \ll \omega_R,$$

which implies the medium with $B < 1$. On the other hand, our asymptotic low frequency relations (6) and (7) are valid when $V \ll 1$, and $l \ll 1$ at the same time, i.e., when frequency is small compared with any of the characteristic frequencies

$$\omega \ll \omega_B, \; \omega \ll \omega_{VE}.$$

As mentioned above, the viscoelastic and visco-inertial mechanisms of attenuation in porous media are usually treated separately. In particular, the viscoelastic phenomenon is ignored in Biot's theory by simply neglecting the shear stress in the fluid in the microscopic (pore-scale) constitutive equations. Pride et al. (1992) analyzed the effect of this approximation and showed that it requires that the parameter $V = \omega\eta/\mu_s$ be small. Indeed, if V is very small, the viscoelastic attenuation is also very small, see eq. (10). However, if at the same time the parameter l is even smaller than V, i.e., $l < V \ll 1$, the poroelastic effects would be even less pronounced than the viscoelastic ones. The condition for neglecting the viscoelastic effects *relative* to the poroelastic ones is $B = V/l \ll 1$. And, most importantly, this condition involves medium parameters only and is independent of frequency. Thus if this condition holds for a particular medium, Biot's theory would apply for all frequencies below the resonant frequency of individual pores. This is consistent with observations of Bedford (1986), who compared numerically the solutions of the exact dispersion equation for a layered solid/fluid system (with very small parameter B) with the prediction of Biot's theory and found an excellent agreement in a wide frequency range. This is not surprising in light of the results of Schoenberg and Sen (1986) and Molotkov and Bakulin (1999) who showed analytically that in the case of low viscosity $B = V/l \ll 1$ the exact constitutive equations for a solid/fluid layered medium represent a partial case of the anisotropic Biot's equations.

Conclusions

For a shear wave with the directions of propagation and of particle motion in the bedding plane, the attenuation (inverse Q) of an elastic wave in a layered viscous fluid/solid medium is a sum of two terms: one proportional to viscosity η and one proportional to η^{-1}. The term proportional to η corresponds to the viscoelastic mechanism (viscous shear relaxation), while the term proportional to η^{-1} accounts for the visco-inertial (poroelastic) attenuation of Biot's type. The characteristic frequencies of viscoelastic ω_{VE}, poroelastic ω_B, and scattering ω_R attenuation mechanisms obey the relation $\omega_{VE}\omega_B = \omega_R^2$, which explains why the visco-elastic and poroelastic mechanisms are usually treated separately in the context of macroscopic theories that imply $\omega \ll \omega_R$. The poroelastic mechanism dominates over the visco-elastic one when the frequency-indepenent parameter $B = \omega_B/\omega_{VE} = 12\eta^2/\mu\rho_f h_f^2 \ll 1$, and vice versa.

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References

Bedford, A., 1986, Application of Biot's equations to a medium of alternating fluid and solid layers: Journal of Wave-Material Interaction, **1**, 34–53.

Biot, M. A., 1956a, Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range: J. Acoust. Soc. Amer., **28**, 168–178.

——— 1956b, Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher frequency range: J. Acoust. Soc. Amer., **28**, 179–191.

Boutin, C., and Auriault, J.-L., 1990, Dynamic behaviour of porous media saturated by a viscoelastic fluid. Application to bituminous concretes: Int. J. Engng. Sci, **28**, 1157–1181.

Brekhovskikh, L. M., 1981, Waves in layered media: Academic Press.

Mavko, G., and Nur, A., 1975, Melt squirt in aesthenosphere: J. Geophys. Res., **80**, 1444–1448.

Molotkov, L. A., and Bakulin, A. V., 1999, Effective model of solid–fluid layers as a partial case of Biot model: J. Math. Sci., in press.

Molotkov, L. A., and Khilo, A. E., 1990, On the equations of the effective model for an elastic medium with fractures filled with a viscous fluid: Voprosy Dynamicheskoj Teorii Rasprostraneniya Seismicheskikh Voln (in Russian), **30**, 88–95.

O'Connell, R. J., and Budiansky, B., 1977, Viscoelastic properties of fluid-saturated cracked solids: J. Geophys. Res., **82**, 5719–5740.

Pride, S. R., Gangi, A. F., and Morgan, F. D., 1992, Deriving the equations of motion for porous isotropic media: J. Acoust. Soc. Amer., **92**, 3278–3290.

Rytov, S. M., 1956, Acoustical properties of a thinly laminated medium: Sov. Phys. Acoust., **2**, 68–80.

Schoenberg, M., and Sen, P. N., 1986, Wave propagation in alternating solid and viscous fluid layers: Size effects in attenuation and dispersion of fast and slow waves: Appl. Phys. Lett., **48**, 1249–1251.

Schoenberg, M., 1984, Wave propagation in alternating fluid/solid layers: Wave Motion, **6**, 303–320.