

## Seismic inversion for the parameters of two orthogonal fracture sets in a VTI background medium

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### ABSTRACT

Characterization of naturally fractured reservoirs often requires estimating parameters of multiple fracture sets that develop in an anisotropic background. Here, we discuss modeling and inversion of the effective parameters of orthorhombic models formed by two orthogonal vertical fracture sets embedded in a VTI (transversely isotropic with a vertical symmetry axis) background matrix.

Although the number of the microstructural (physical) medium parameters is equal to the number of effective stiffness elements (nine), we show that for this model there is an additional relation (constraint) between the stiffnesses or Tsvankin's anisotropic coefficients. As a result, the same effective orthorhombic medium can be produced by a wide range of equivalent models with vastly different fracture weaknesses and background VTI parameters, and the inversion of seismic data for the microstructural parameters is nonunique without additional information. Reflection moveout of

*PP*- and *PS*-waves can still be used to find the fracture orientation and estimate (in combination with the vertical velocities) the differences between the normal and shear weaknesses of the fracture sets, as well as the background anellipticity parameter  $\eta_b$ . Since for penny-shaped cracks the shear weakness is close to twice the crack density, seismic data can help to identify the dominant fracture set, although the crack densities cannot be resolved individually.

If the VTI symmetry of the background is caused by intrinsic anisotropy (as is usually the case for shales), it may be possible to determine at least one background anisotropic coefficient from borehole or core measurements. Then seismic data can be inverted for the fracture weaknesses and the rest of the background parameters. Therefore, seismic characterization of reservoirs with multiple fracture sets and anisotropic background is expected to give ambiguous results, unless the input data include measurements made on different scales (surface seismic, borehole, cores).

### INTRODUCTION

In our previous papers on seismic fracture characterization (Bakulin et al., 2000a,b,c), we discussed a series of typical reservoir models with one or two vertical fracture sets. If two fracture sets are orthogonal to each other and the background matrix is azimuthally isotropic, the effective medium is orthorhombic with a horizontal symmetry plane. [For seismic signatures of models with nonorthogonal fracture sets, see papers by Liu et al. (1993), Sayers (1998), and Bakulin et al. (2000c).]

Inversion of seismic data for the parameters of two orthogonal fracture sets embedded in a purely isotropic background is described by Bakulin et al. (2000b; hereafter referred to as Paper I). Although orthorhombic symmetry is generally defined by nine stiffnesses or Tsvankin's (1997) parameters, for

the model from Paper I only six of them are independent; they are expressed through the four compliances (or weaknesses) of the two fracture sets (Bakulin et al., 2000a,b) and the two Lamé parameters of the background. Paper I introduces practical methodologies for estimating the fracture weaknesses using reflection moveout and/or amplitudes. Note that reflection seismic data have to be supplemented with the vertical velocities (or reflector depth) in order to resolve the anisotropic coefficients and invert them for the fracture weaknesses.

The rock matrix, however, is often intrinsically anisotropic, which makes fracture characterization considerably more complicated. Another orthorhombic model treated in Paper I did have a VTI background, but it contained only one vertical fracture set. Here, we extend the previous results to a more complicated orthorhombic medium formed by two orthogonal

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fracture sets embedded in a VTI background matrix. Since vertical transverse isotropy is defined by five independent stiffnesses or Thomsen's (1986) parameters, the total number of microstructural (physical) parameters increases to nine. We explore the possibility of estimating both the VTI background parameters and fracture weaknesses from seismic data and show that the inversion is nonunique without additional information.

### ANALYTIC FORMULATION

#### Effective stiffness matrix

We consider a model containing two orthogonal sets of parallel vertical fractures embedded in a VTI background medium. In the long-wavelength limit, such a model is orthorhombic with two vertical symmetry planes parallel to the fractures (Appendix A).

Since we intend to relate fracture parameters to dimensionless anisotropic coefficients, it is convenient to replace the fracture compliances  $K_N$ ,  $K_V$ , and  $K_H$  (e.g., Schoenberg and Sayers, 1995) by dimensionless weaknesses  $\Delta_N$ ,  $\Delta_V$ , and  $\Delta_H$  (Bakulin et al., 2000a,b):

$$\Delta_N = \frac{K_N c_{11b}}{1 + K_N c_{11b}}, \quad \Delta_V = \frac{K_V c_{44b}}{1 + K_V c_{44b}}, \quad (1)$$

$$\Delta_H = \frac{K_H c_{66b}}{1 + K_H c_{66b}}.$$

For simplicity, we assume that the shear weaknesses  $\Delta_V$  and  $\Delta_H$  for each fracture set are identical,

$$\Delta_V = \Delta_H = \Delta_T, \quad (2)$$

which reduces the number of model parameters for a given fracture orientation from 11 (five background and six fracture parameters) to nine. If the background were isotropic, assumption (2) would make the fractures rotationally invariant with equal shear compliances ( $K_V = K_H$ ). However, since in VTI media  $c_{44} \neq c_{66}$ ,  $K_V$  differs from  $K_H$  even if  $\Delta_V = \Delta_H$ . Only for weak anisotropy (small weaknesses and small anisotropy parameters of the host rock) can the difference between  $c_{44}$  and  $c_{66}$  be ignored, and setting  $\Delta_V = \Delta_H$  is equivalent to  $K_V = K_H$ .

The compliance matrix of the effective medium is obtained in Appendix A [equation (A-2)] using the linear-slip theory (e.g., Schoenberg and Sayers, 1995). Inverting matrix (A-2) and using equations (1) and (2) yields the following stiffness matrix of the effective medium:

$$\mathbf{c} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{c}}_1 & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{c}}_2 \end{pmatrix}. \quad (3)$$

Here  $\mathbf{0}$  is the  $3 \times 3$  zero matrix, and the matrices  $\tilde{\mathbf{c}}_1$  and  $\tilde{\mathbf{c}}_2$  can be written as

$$\tilde{\mathbf{c}}_1 = \frac{1}{d} \begin{pmatrix} c_{11b} l_1 m_3 & c_{12b} l_1 m_1 & c_{13b} l_1 m_2 \\ c_{12b} l_1 m_1 & c_{11b} l_3 m_1 & c_{13b} l_2 m_1 \\ c_{13b} l_1 m_2 & c_{13b} l_2 m_1 & c_{33b} l_4 \end{pmatrix}, \quad (4)$$

$$\tilde{\mathbf{c}}_2 = \begin{pmatrix} c_{44b}(1 - \Delta_{T2}) & 0 & 0 \\ 0 & c_{44b}(1 - \Delta_{T1}) & 0 \\ 0 & 0 & c_{66b} \frac{(1 - \Delta_{T1})(1 - \Delta_{T2})}{(1 - \Delta_{T1} \Delta_{T2})} \end{pmatrix}, \quad (5)$$

where

$$l_1 = 1 - \Delta_{N1}, \quad l_2 = 1 - r \Delta_{N1}, \quad l_3 = 1 - r^2 \Delta_{N1},$$

$$l_4 = 1 - r'(\Delta_{N1} + \Delta_{N2}) + r(2r' - r)\Delta_{N1} \Delta_{N2}, \quad (6)$$

$$m_1 = 1 - \Delta_{N2}, \quad m_2 = 1 - r \Delta_{N2}, \quad m_3 = 1 - r^2 \Delta_{N2},$$

$$r = \frac{c_{12b}}{c_{11b}}, \quad r' = \frac{c_{13b}^2}{c_{11b}c_{33b}}, \quad d = 1 - r^2 \Delta_{N1} \Delta_{N2}.$$

Note that in VTI media the coefficient  $c_{12b}$  can be expressed through  $c_{11b}$  and  $c_{66b}$ , so the effective stiffness matrix is determined by five background parameters and four fracture weaknesses. Only eight stiffnesses, however, are independent because  $c_{11}$ ,  $c_{22}$ ,  $c_{12}$ ,  $c_{13}$ , and  $c_{23}$  are related by the constraint

$$c_{13}(c_{22} + c_{12}) = c_{23}(c_{11} + c_{12}), \quad (7)$$

which follows directly from equation (A-3).

Since equation (7) involves only elements of the submatrix  $\tilde{\mathbf{c}}_1$ , this equation remains valid even if we introduce different shear weaknesses in the vertical and horizontal directions for each fracture set [this is also clear from equation (A-2)]. The constraint (7) was previously given by Schoenberg and Helbig (1997) for orthorhombic media containing a single fracture set in a VTI background.

#### Approximate anisotropic parameters

Equations (3)–(6) allow us to express Tsvankin's (1997) anisotropic parameters in terms of the fracture weaknesses  $\Delta_{T1,2}$  and  $\Delta_{N1,2}$ . Tsvankin (1997) used the similarity between the Christoffel equation in TI media and symmetry planes of orthorhombic media to extend Thomsen (1986) notation to orthorhombic anisotropy. The parameters  $\epsilon^{(1,2)}$ ,  $\delta^{(1,2)}$ , and  $\gamma^{(1,2)}$  are defined in the two vertical symmetry planes of orthorhombic media by analogy with vertical transverse isotropy. Another coefficient,  $\delta^{(3)}$ , is defined in the horizontal symmetry plane. Combined with the vertical velocities of the  $P$ -wave and one of the split  $S$ -waves, these seven anisotropic parameters fully describe an orthorhombic medium with a known orientation of the symmetry planes. Tsvankin (1997) and Grechka et al. (1999) show that this Thomsen-style notation is much more convenient in seismic velocity analysis and inversion than the stiffness coefficients.

For weakly anisotropic models in which the weaknesses and the background anisotropic coefficients are much smaller than unity, the effective parameters of orthorhombic media can be simplified by linearizing them in  $\Delta_{T1,2}$ ,  $\Delta_{N1,2}$ ,  $\epsilon_b$ ,  $\delta_b$ , and  $\gamma_b$ . The structure of the weak-anisotropy approximation can be understood from the "addition rule" discussed in Paper I. For a model with two fracture sets, any linearized effective anisotropic coefficient  $\epsilon$  is given by

$$\epsilon = \epsilon_b + \epsilon_{f1}^{ISO} + \epsilon_{f2}^{ISO}, \quad (8)$$

where  $\epsilon_b$  is the corresponding anisotropic coefficient of the unfractured background and  $\epsilon_{f1}^{ISO}$  and  $\epsilon_{f2}^{ISO}$  are the coefficients due to each fracture set embedded separately in a reference isotropic medium. In our case, the reference model should have the ratio  $V_S/V_P$  of the  $S$ - and  $P$ -wave velocities close to that of the actual VTI background.

**Symmetry plane  $[x_2, x_3]$ .**—The anisotropic parameters with the superscript “(1)” are defined in the symmetry plane  $[x_2, x_3]$ , which is parallel to the first set of fractures and orthogonal to the second set. In the absence of the second set and background anisotropy, the  $[x_2, x_3]$ -plane would coincide with the isotropy plane of the transversely isotropic medium with a horizontal symmetry axis (HTI) associated with the first set of fractures. Therefore, we can expect those parameters to be largely independent of the properties of the first fracture set. On the other hand, removing both fracture sets would make the parameters in the  $[x_2, x_3]$ -plane (and, for that matter, in any other vertical plane) equal to the anisotropic parameters of the background VTI medium.

Indeed, the linearized anisotropic coefficients in the  $[x_2, x_3]$ -plane depend only on the weaknesses of the second fracture set and the corresponding background parameters:

$$\epsilon^{(1)} = \epsilon_b - 2g(1-g)\Delta_{N2}, \quad (9)$$

$$\delta^{(1)} = \delta_b - 2g[(1-2g)\Delta_{N2} + \Delta_{T2}], \quad (10)$$

$$\gamma^{(1)} = \gamma_b - \frac{\Delta_{T2}}{2}, \quad (11)$$

$$\eta^{(1)} \equiv \frac{\epsilon^{(1)} - \delta^{(1)}}{1 + 2\delta^{(1)}} = \eta_b + 2g[\Delta_{T2} - g\Delta_{N2}], \quad (12)$$

where  $g \equiv V_{S0}^2/V_{P0}^2$  is the squared ratio of the vertical  $S$ - and  $P$ -wave velocities in the background, and  $\eta_b \equiv (\epsilon_b - \delta_b)/(1 + 2\delta_b)$ . The coefficients  $\epsilon^{(1)}$ ,  $\delta^{(1)}$ , and  $\gamma^{(1)}$  are a subset of the core parameters defined by Tsvankin (1997), while  $\eta^{(1)}$  is a measure of anellipticity in the  $[x_2, x_3]$ -plane needed (along with the parameters  $\eta^{(2)}$  and  $\eta^{(3)}$  defined below) for time processing of  $P$ -wave data in orthorhombic media (Grechka and Tsvankin, 1999a).

As expected from the above discussion, equations (9)–(12) are identical to the corresponding expressions given in Paper I for the linearized anisotropic coefficients of orthorhombic media due to a single fracture set (orthogonal to the  $x_2$ -axis) embedded in a VTI background. Also, the fracture-related terms in equations (9)–(12) coincide with those for a fracture set in a purely isotropic background with the squared vertical-velocity ratio  $g$  (Bakulin et al., 2000a).

**Symmetry plane  $[x_1, x_3]$ .**—Similarly, the linearized anisotropic coefficients in the symmetry plane  $[x_1, x_3]$  are controlled by the weaknesses of the first fracture set and the background anisotropy. In agreement with the symmetry of the model, the expressions for the parameters  $\epsilon^{(2)}$ ,  $\delta^{(2)}$ ,  $\gamma^{(2)}$ , and  $\eta^{(2)}$  are analogous to equations (9)–(12):

$$\epsilon^{(2)} = \epsilon_b - 2g(1-g)\Delta_{N1}, \quad (13)$$

$$\delta^{(2)} = \delta_b - 2g[(1-2g)\Delta_{N1} + \Delta_{T1}], \quad (14)$$

$$\gamma^{(2)} = \gamma_b - \frac{\Delta_{T1}}{2}, \quad (15)$$

$$\eta^{(2)} \equiv \frac{\epsilon^{(2)} - \delta^{(2)}}{1 + 2\delta^{(2)}} = \eta_b + 2g[\Delta_{T1} - g\Delta_{N1}]. \quad (16)$$

**Symmetry plane  $[x_1, x_2]$ .**—The remaining anisotropic coefficient  $\delta^{(3)}$  defined in the horizontal symmetry plane  $[x_1, x_2]$  is given by

$$\delta^{(3)} = 2g[\Delta_{N1} - \Delta_{T1}] - 2g[(1-2g)\Delta_{N2} + \Delta_{T2}]. \quad (17)$$

The linearized  $\delta^{(3)}$  is exactly the same as for the model with two fracture sets in a purely isotropic background (Paper I) because the horizontal plane in VTI media is the plane of isotropy.

The anellipticity coefficient  $\eta^{(3)}$  in the horizontal plane is given by

$$\eta^{(3)} = 2g[\Delta_{T1} - g\Delta_{N1}] + 2g[\Delta_{T2} - g\Delta_{N2}]. \quad (18)$$

**Relationship between the anisotropic coefficients.**—If the background were isotropic, the anisotropic parameters in each vertical symmetry plane would satisfy the following constraints (Paper I):

$$\gamma^{(i)} = \frac{1}{4g} \left[ \delta^{(i)} - \epsilon^{(i)} \frac{1-2g}{1-g} \right], \quad (i=1, 2). \quad (19)$$

Those constraints do not hold individually in the presence of anisotropy in the background. However, as follows from equations (9)–(11) and (13)–(15), a valid constraint can be obtained by subtracting equations (19) for  $i=1$  and  $i=2$ , which cancels the background coefficients:

$$\gamma^{(2)} - \gamma^{(1)} = \frac{1}{4g} \left[ \delta^{(2)} - \delta^{(1)} - (\epsilon^{(2)} - \epsilon^{(1)}) \frac{1-2g}{1-g} \right]. \quad (20)$$

Note that equation (20) can be also derived by linearizing the exact constraint (7) in the anisotropic coefficients.

## ESTIMATION OF FRACTURE PARAMETERS

### General analysis

The two fracture sets are aligned with the vertical symmetry planes of the effective orthorhombic medium. The symmetry-plane azimuths can be found from seismic data using the polarization vectors of the split  $S$ -waves (or  $PS$ -waves) at vertical incidence or the orientations of the NMO ellipses of pure or converted reflections from horizontal interfaces (Paper I).

The parameters  $\epsilon^{(1,2)}$ ,  $\delta^{(1,2)}$ ,  $\gamma^{(1,2)}$  govern the azimuthally dependent NMO velocities of  $P$ -waves and split  $S$ -waves from horizontal reflectors. As discussed in detail by Grechka et al. (1999) and in Paper I, those coefficients can be obtained by combining the NMO ellipses of horizontal  $P$  and  $S$  (or converted  $PS$ ) events with the vertical  $P$ - and  $S$ -wave velocities. An efficient algorithm for reconstructing NMO ellipses from 3-D wide-azimuth data was presented by Grechka and Tsvankin (1999b), who also developed a correction of normal moveout for lateral velocity variation. The remaining anisotropic parameter,  $\delta^{(3)}$ , can be estimated from the NMO ellipse of dipping  $P$  events or (with less accuracy) from  $P$ -wave nonhyperbolic moveout (Grechka and Tsvankin, 1999a). Finally, the squared vertical-velocity ratio  $g$  is close to the ratio of the vertical traveltimes of the  $P$ -wave and fast  $S$ -wave (or it is possible to use the fast  $PS$ -wave).

Therefore, seismic data may constrain seven anisotropic coefficients ( $\epsilon^{(1,2)}$ ,  $\delta^{(1,2,3)}$ ,  $\gamma^{(1,2)}$ ) which depend on seven microstructural model parameters ( $\Delta_{N1,2}$ ,  $\Delta_{T1,2}$ ,  $\epsilon_b$ ,  $\delta_b$ , and  $\gamma_b$ ). If all anisotropic coefficients were independent, it might be possible to invert them for the fracture weaknesses and the VTI background coefficients. However, the exact constraint (7) and its weak-anisotropy counterpart (20) show that only six out of seven anisotropic coefficients are independent. Hence, we can expect that there is a family (set) of equivalent models which produce the same anisotropic coefficients (or the same stiffnesses) of the effective orthorhombic model.

### Equivalent models

To verify the existence of equivalent models, we conducted a numerical test based on the exact equations for the effective anisotropic coefficients. The input effective parameters were as follows:  $\epsilon^{(1)} = 0.06$ ,  $\epsilon^{(2)} = 0.01$ ,  $\delta^{(1)} = 0.02$ ,  $\delta^{(2)} = -0.07$ ,  $\gamma^{(1)} = 0.05$ ,  $\gamma^{(2)} = -0.01$ , and  $\delta^{(3)} = -0.07$ . The underlying fracture model was described by the background parameters  $V_{S0}/V_{P0} = 0.5$ ,  $\epsilon_b = \delta_b = \gamma_b = 0.1$ , and the fracture weaknesses  $\Delta_{N1} = \Delta_{T1} = 0.2$  and  $\Delta_{N2} = \Delta_{T2} = 0.1$ .

The value of  $\gamma_b$  was varied from 0.09 to 0.18, and for each  $\gamma_b$  within this interval the exact anisotropic coefficients were inverted for the weaknesses  $\Delta_{N1,2}$ ,  $\Delta_{T1,2}$ , and the background parameters  $\epsilon_b$  and  $\delta_b$ . Smaller values of  $\gamma_b < 0.09$  were not considered because they produce negative weaknesses which are unphysical. Although values of  $\gamma_b > 0.18$  yield plausible weaknesses, they were also excluded because the corresponding background parameters become unrealistically large ( $\epsilon_b > 0.5$  and  $\delta_b > 0.5$ ; see Thomsen, 1986). The inversion results are marked by small dots in Figure 1. Increasing  $\gamma_b$  leads to an in-

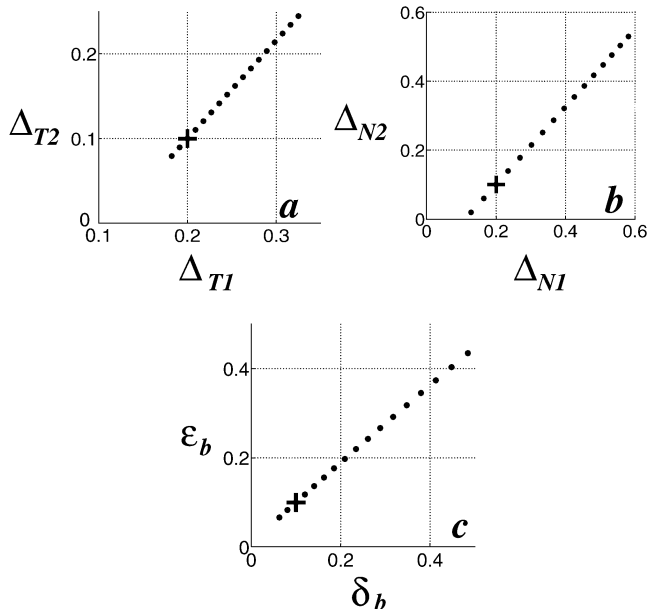


FIG. 1. Equivalent fracture models which produce the effective orthorhombic parameters given in the text. The background coefficient  $\gamma_b$  is varied from 0.09 to 0.18, which leads to a monotonic increase in all other parameters. The crosses mark the actual model parameters. (a)  $\Delta_{T1}$  versus  $\Delta_{T2}$ ; (b)  $\Delta_{N1}$  versus  $\Delta_{N2}$ ; (c)  $\delta_b$  versus  $\epsilon_b$ .

crease in all inverted parameters, which helps us to understand the correspondence between the three plots. The maximum error in the effective anisotropic coefficients of all models displayed in Figure 1 does not exceed 0.003. Clearly, the family of equivalent models includes a wide range of weaknesses and background VTI parameters.

### Constrained parameter combinations

Although the ambiguity in the inversion discussed above prevents us from estimating the individual values of the microstructural parameters, it is still possible to infer some useful information about the fractures and the VTI background. According to the weak-anisotropy approximations (9)–(16), the effective parameters of the orthorhombic model constrain the following quantities:

$$\Delta_{T1} - \Delta_{T2} = 2(\gamma^{(1)} - \gamma^{(2)}), \quad (21)$$

$$\Delta_{N1} - \Delta_{N2} = \frac{\epsilon^{(1)} - \epsilon^{(2)}}{2g(1-g)}, \quad (22)$$

$$\eta_b = \frac{\eta^{(1)} + \eta^{(2)} - \eta^{(3)}}{2}. \quad (23)$$

Since the shear weakness is close to twice the crack density (for penny-shaped cracks; see Paper I), equation (21) implies that seismic data can detect the dominant fracture set and estimate the difference in the crack densities. Note that the term  $[\gamma^{(1)} - \gamma^{(2)}]$  is close to the fractional difference between the vertical velocities of split shear waves (Tsvankin, 1997). Therefore, the difference in the shear weaknesses can be found directly from the time delay between the fast and slow  $S$ -waves at vertical incidence.

According to equation (22), the coefficients  $\epsilon^{(1)}$  and  $\epsilon^{(2)}$  can be used to evaluate the difference between the saturations of the two fracture sets because the normal weakness  $\Delta_N$  is usually large for dry cracks and vanishes for isolated fluid-filled ones (Bakulin et al., 2000a). However, if fluid-filled cracks are connected to pore space, interpretation of  $\Delta_N$  becomes ambiguous. Finally, the  $\eta$  coefficients of orthorhombic media, which can be determined solely from  $P$ -wave moveout data (Grechka and Tsvankin, 1999a), constrain the anellipticity coefficient  $\eta_b$  of the VTI background.

The above conclusions are supported by Figure 1, where all inverted parameters closely follow linear trends. However, to test the accuracy of the weak-anisotropy equations (21)–(23), Figure 2 reproduces the differences  $\Delta_{T1} - \Delta_{T2}$  and  $\Delta_{N1} - \Delta_{N2}$  and the parameter  $\eta_b$  from Figure 1. The variations of all three parameter combinations within the family of equivalent models do not exceed  $\pm 0.035$  (for  $\Delta_{T1} - \Delta_{T2}$  and  $\eta_b$ ) and  $\pm 0.06$  (for  $\Delta_{N1} - \Delta_{N2}$ ), which confirms the weak-anisotropy result.

Figure 3 shows the results of inverting the exact equations for  $\Delta_{T1} - \Delta_{T2}$ ,  $\Delta_{N1} - \Delta_{N2}$ , and  $\eta_b$ . The errors in the estimated parameter combinations in Figure 3 are close to those introduced in the input anisotropic coefficients.

### Additional information for the inversion

Estimating only the differences in the weaknesses of the two fracture sets may be insufficient for reservoir characterization. Since we are missing just one equation, a priori knowledge of

one of the model parameters can help to resolve the rest of them using seismic data.

For example, one or more background VTI coefficients may be known from laboratory measurements on rock samples or from borehole data. Indeed, the average fracture spacing in situ often is much larger than the dimensions of rock samples (e.g., Schoenberg and Sayers, 1995) or the scale of borehole surveys. In contrast, seismic waves probe the reservoir at a much larger scale and provide an estimate of effective long-wavelength anisotropy due to both the fractures and the background medium.

Even if fracturing occurs at a fine scale comparable to the rock-sample dimensions, the background parameters can still be evaluated by making velocity versus pressure laboratory measurements (Vernik, 1993; Bakulin, 1995). With increasing confining pressure, cracks or fractures (especially dry) tend to close, and the elastic parameters of the sample become close to those of the unfractured rock. Extrapolating the obtained background parameters back to the low-pressure range using

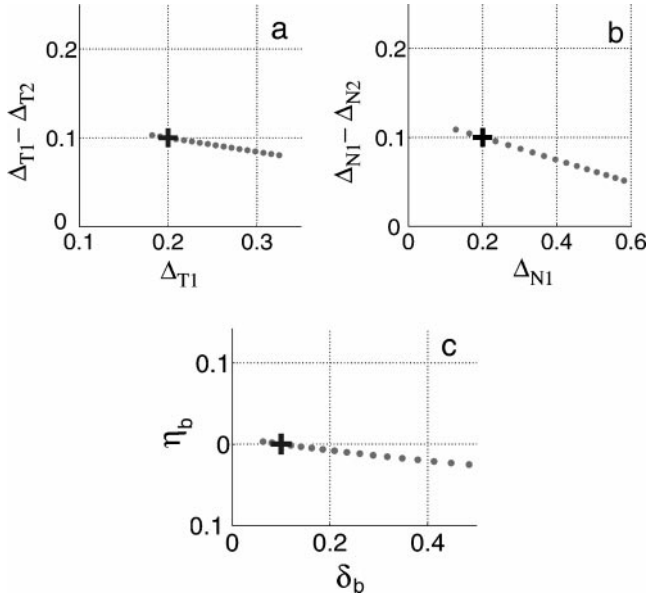


FIG. 2. The differences  $\Delta_{T1} - \Delta_{T2}$  (a),  $\Delta_{N1} - \Delta_{N2}$  (b), and the parameter  $\eta_b$  (c) computed from the inversion results in Figure 1.

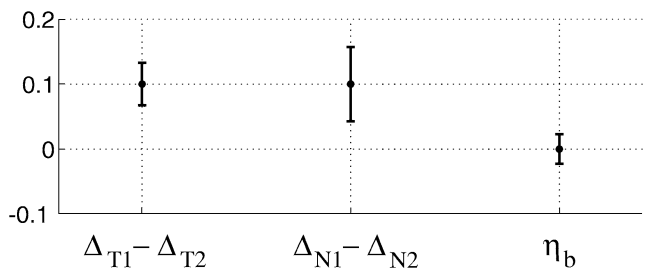


FIG. 3. Inversion of the exact anisotropic coefficients  $\epsilon^{(1,2)}$ ,  $\delta^{(1,2)}$ ,  $\gamma^{(1,2)}$ , and  $\eta^{(3)}$  for the differences in the weaknesses  $\Delta_{T1} - \Delta_{T2}$ ,  $\Delta_{N1} - \Delta_{N2}$  and the background anellipticity coefficient  $\eta_b$ . The input parameters were distorted by Gaussian noise with the standard deviation 0.03. The dots mark the actual parameter values; the bars correspond to the  $\pm$  standard deviation in each parameter.

Murnaghan's theory (Bakulin, 1995) can give a reasonably accurate estimate of the in-situ background properties (e.g., for shales). This methodology, however, is sensitive mostly to the intrinsic component of the matrix anisotropy and does not account for the contribution of thin layering to the background VTI model. Alternatively, in some cases the background parameters may be determined at adjacent unfractured reservoirs with a similar lithologic composition.

### Special cases

Here we consider several special cases in which certain relationships between the model parameters reduce the number of unknowns to be determined from the data. In principle, such a simplification of the model may help to overcome the nonuniqueness in the inversion. However, as shown below, by introducing relations between the fracture parameters, we create additional constraints [the constraint (7) remains valid for all our models] on the effective anisotropic coefficients, and the number of unknowns still exceeds the number of measurements. Some practically important special cases, along with the corresponding constraints, are listed below. For completeness, the same special cases are analyzed in terms of fracture compliances in Appendix A.

#### 1. Identical shear or normal weaknesses.

If the shear weaknesses  $\Delta_{T1}$  and  $\Delta_{T2}$  are equal to each other, the anisotropic parameters satisfy an additional constraint,  $\gamma^{(1)} = \gamma^{(2)}$ , or [using equation (20)]

$$\frac{\delta^{(1)} - \delta^{(2)}}{1 - 2g} = \frac{\epsilon^{(1)} - \epsilon^{(2)}}{1 - g}.$$

For  $\Delta_{N1} = \Delta_{N2}$ , we also lose an independent equation because  $\epsilon^{(1)} = \epsilon^{(2)}$ .

#### 2. Identical fracture sets.

In this case,  $\Delta_{T1} = \Delta_{T2}$  and  $\Delta_{N1} = \Delta_{N2}$ . However, we acquire two additional constraints,  $\delta^{(1)} = \delta^{(2)}$  and  $\epsilon^{(1)} = \epsilon^{(2)}$ . Such a medium is close to VTI, but the horizontal plane is not a plane of isotropy because  $\delta^{(3)} \neq 0$ .

#### 3. One fracture set is dry.

Following Schoenberg and Sayers (1995), we assume that if the first fracture set is dry, in the limit of weak anisotropy  $\Delta_{N1} = \Delta_{T1}/g$  (see also Bakulin et al., 2000a). Then the  $\delta$  coefficients satisfy an additional constraint:  $\delta^{(3)} = \delta^{(1)} - \delta^{(2)}$ .

#### 4. Both fracture sets are dry.

Then  $\Delta_{N1} = \Delta_{T1}/g$  and  $\Delta_{N2} = \Delta_{T2}/g$ , which leads to two additional constraints,  $\delta^{(1)} - \delta^{(2)} = \epsilon^{(1)} - \epsilon^{(2)}$  and  $\delta^{(3)} = \delta^{(1)} - \delta^{(2)}$ .

The above constraints can be used to identify the special features of the model (e.g., whether the sets are identical, have the same shear weakness, etc.) from seismic data. However, for all four cases discussed here even a priori knowledge of those features is not sufficient to overcome the ambiguity in the inversion and resolve the microstructural parameters.

There is at least one model, however, for which the effective anisotropic coefficients can be inverted for all fracture and background parameters. Suppose one of the sets contains isolated fluid-filled cracks, so that  $\Delta_{N1} = 0$  (see Bakulin et al.,

2000a). Then, as follows from the weak-anisotropy approximations (9)–(17), the model parameters can be found as

$$\epsilon_b = \epsilon^{(2)}, \quad (24)$$

$$\Delta_{N2} = \frac{\epsilon_b - \epsilon^{(1)}}{2g(1-g)}, \quad (25)$$

$$\Delta_{T1} = \frac{\delta^{(1)} - \delta^{(2)} - \delta^{(3)}}{4g}, \quad (26)$$

$$\gamma_b = \gamma^{(2)} + \frac{\Delta_{T1}}{2}, \quad (27)$$

$$\Delta_{T2} = 2(\gamma_b - \gamma^{(1)}), \quad (28)$$

$$\delta_b = \delta^{(2)} + 2g\Delta_{T1}. \quad (29)$$

Note, however, that estimation of all parameters except for  $\epsilon_b$  and  $\Delta_{N2}$  requires a measurement of the coefficient  $\delta^{(3)}$  (or  $\eta^{(3)}$ ) that has no influence on the NMO ellipses of horizontal events. As discussed above,  $\delta^{(3)}$  can be obtained from dip or nonhyperbolic moveout of  $P$ -waves (Grechka and Tsvankin, 1999a).

### CONCLUSIONS

Two orthogonal sets of vertical fractures embedded in a VTI background produce an effective orthorhombic model described by nine stiffnesses or Tsvankin's (1997) parameters. Using the linear slip theory, we expressed the effective anisotropic coefficients through the fracture weaknesses and VTI background parameters. Under the assumption of weak background and fracture-induced anisotropy, each anisotropic coefficient is obtained by simply summing up the contributions of the VTI background and both fracture sets. The fracture-related terms are computed for each fracture set embedded separately into a purely isotropic host rock with the velocities close to those of the true VTI background. Those concise approximations help to elucidate the relationship between the effective orthorhombic model and the physical properties of the fractures.

In principle, it is possible to find the azimuths of the vertical symmetry planes and estimate all nine parameters of orthorhombic media from multicomponent, wide-azimuth seismic data (Paper I). Eight parameters (all except for  $\delta^{(3)}$ ) can be found by combining the NMO ellipses of  $P$ - and split  $S$ -waves (or converted  $PS$ -waves) with the vertical velocities (or reflector depth). The coefficient  $\delta^{(3)}$  can be determined from  $P$ -wave reflection traveltimes using dipping events or nonhyperbolic (long-spread) moveout.

Next, the effective orthorhombic model has to be inverted for the fracture and background parameters. The azimuths of both fracture sets coincide with those of the vertical symmetry planes and can be found from either shear-wave polarizations at vertical incidence or the orientation of the NMO ellipses of pure or converted waves. Since the underlying physical model is described by five VTI background parameters and four fracture weaknesses, the number of unknown microstructural parameters is equal to the number of the effective stiffnesses. However, for this model only eight effective stiffnesses turn out to be independent, and seismic data alone do not provide enough information for unambiguous fracture characterization. We show that there exists a broad range of fracture models

producing exactly the same anisotropic coefficients and, therefore, the same seismic signatures.

The only well-constrained parameter combinations are the differences between the normal ( $\Delta_{N1} - \Delta_{N2}$ ) and shear ( $\Delta_{T1} - \Delta_{T2}$ ) fracture weaknesses and the anellipticity coefficient  $\eta_b$  of the background. Therefore, seismic data allow us to identify the predominant fracture set that has the larger shear weakness (i.e., the higher crack density) and estimate the difference between the crack densities of the two sets. It is quite difficult, however, to make any conclusions about fracture saturation just from the difference between the relevant weaknesses  $\Delta_{N1}$  and  $\Delta_{N2}$ .

To resolve the fracture parameters individually, seismic data can be supplemented by additional information from borehole or laboratory measurements. Since seismic anisotropy is a scale-dependent phenomenon, in some cases fine-scale laboratory experiments can give accurate estimates of the anisotropic background parameters, whereas reflection seismic data are influenced by both fracture-induced and background anisotropy. With at least one background parameter known a priori, the inversion of seismic data for the fracture weaknesses and the remaining background parameters becomes unambiguous. Therefore, characterization of complex fractured reservoirs with multiple fracture sets and an anisotropic background matrix requires integrating reflection seismic data with measurements on other scales.

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## APPENDIX A

### EFFECTIVE COMPLIANCE OF A MEDIUM WITH TWO FRACTURE SETS

#### General form of the compliance matrix

Here we obtain the compliance matrix of two orthogonal sets of parallel vertical fractures in a VTI background matrix. The compliance matrix  $\mathbf{s}_b$  of the background is given by

$$\mathbf{s}_b \equiv \mathbf{c}_b^{-1} = \begin{pmatrix} s_{11b} & s_{12b} & s_{13b} & 0 & 0 & 0 \\ s_{12b} & s_{11b} & s_{13b} & 0 & 0 & 0 \\ s_{13b} & s_{13b} & s_{33b} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44b} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44b} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66b} \end{pmatrix}, \quad (\text{A-1})$$

where  $\mathbf{c}_b$  is the stiffness matrix and  $s_{12b} = s_{11b} - s_{66b}/2$  (Hood, 1991).

As in our previous papers (Bakulin et al., 2000a,b,c), the influence of fractures on the effective stiffnesses is incorporated using the linear slip theory (Schoenberg and Sayers, 1995). One of the main advantages of this theory is the simplicity of the expression for the effective compliance matrix that can be obtained by adding the excess fracture compliances to the compliance  $\mathbf{s}_b$  of the background. If the fracture planes are perpendicular to the axes  $x_1$  (first set) and  $x_2$  (second set), and the fractures are described by the normal compliance  $K_N$  and two shear compliances  $K_V$  (in the vertical direction) and  $K_H$  (in the horizontal direction; see Bakulin et al., 2000a), the effective compliance takes the form

$$\mathbf{s} \equiv \mathbf{c}^{-1} = \begin{pmatrix} s_{11b} + K_{N1} & s_{12b} & s_{13b} & 0 & 0 & 0 \\ s_{12b} & s_{11b} + K_{N2} & s_{13b} & 0 & 0 & 0 \\ s_{13b} & s_{13b} & s_{33b} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44b} + K_{V2} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44b} + K_{V1} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66b} + K_{H1} + K_{H2} \end{pmatrix}. \quad (\text{A-2})$$

Note that the first fracture set contributes only to  $s_{11}$ ,  $s_{55}$ , and  $s_{66}$ , whereas the second set contributes to  $s_{22}$ ,  $s_{44}$ , and  $s_{66}$ . Matrix (A-2) corresponds to an orthorhombic medium with an additional constraint

$$s_{13} = s_{23}. \quad (\text{A-3})$$

#### Deconstruction of the compliance matrix

By “deconstruction” (or decomposition) we mean the process of obtaining the background and fracture parameters from measured effective stiffnesses or compliances (Hood

and Schoenberg, 1989). In the compliance formulation, the deconstruction process reduces to solving a system of linear equations. Here, we discuss the deconstruction of matrix (A-2) under the assumption that the background medium has VTI symmetry.

Evidently, it is impossible to estimate 11 unknowns (five background and six fracture parameters) from nine linear equations corresponding to the nine stiffnesses of the orthorhombic model. For example, the parameters  $s_{66b}$ ,  $K_{H1}$ , and  $K_{H2}$  cannot be resolved individually because they contribute to a single stiffness element  $s_{66}$ . To overcome this ambiguity, we introduce a simplifying assumption about the rotational invariance of both fracture sets ( $K_{V1} = K_{H1} = K_{T1}$ ,  $K_{V2} = K_{H2} = K_{T2}$ ), which reduces the number of unknowns to nine. However, due to the additional constraint (A-3), the effective orthorhombic medium is described by only eight independent compliances, and the deconstruction requires additional information.

Since the effective compliances  $s_{12}$ ,  $s_{13}$ , and  $s_{33}$  are equal to the corresponding background compliances, the remaining six unknowns ( $s_{66b}$ ,  $s_{44b}$ ,  $K_{N1}$ ,  $K_{T1}$ ,  $K_{N2}$ , and  $K_{T2}$ ) have to be determined from the following five equations:

$$\frac{s_{66b}}{2} + K_{N1} = s_{11} - s_{12}, \quad (\text{A-4})$$

$$\frac{s_{66b}}{2} + K_{N2} = s_{22} - s_{12}, \quad (\text{A-5})$$

$$s_{44b} + K_{T2} = s_{44}, \quad (\text{A-6})$$

$$s_{44b} + K_{T1} = s_{55}, \quad (\text{A-7})$$

$$s_{66b} + K_{T1} + K_{T2} = s_{66}. \quad (\text{A-8})$$

Let us examine several special cases, which were discussed in the main text in terms of the stiffnesses and anisotropic coefficients.

#### 1. Identical shear or normal compliances.

For equal shear compliances ( $K_{T1} = K_{T2}$ ),  $s_{44} = s_{55}$  and equations (A-6) and (A-7) coincide with each other. If the normal

compliances of the two sets are equal ( $K_{N1} = K_{N2}$ ), equations (A-4) and (A-5) become identical because  $s_{11} = s_{22}$ . Therefore, in either case there is not enough information for the deconstruction because four equations have to be solved for five unknowns.

#### 2. Identical fracture sets.

Combining the two previous assumptions ( $K_{N1} = K_{N2}$  and  $K_{T1} = K_{T2}$ ) makes  $s_{11}$  equal to  $s_{22}$  and  $s_{44}$  equal to  $s_{55}$ . The remaining three equations are still insufficient to constrain the four unknowns ( $K_{N1}$ ,  $K_{T1}$ ,  $s_{44b}$ , and  $s_{66b}$ ).

#### 3. One fracture set is dry.

According to Schoenberg and Sayers (1995), for dry fractures  $K_N = K_T$  (in our case,  $K_{N1} = K_{T1}$ ). Then one of the equations is no longer independent due to the constraint that follows from

equations (A-4)–(A-8):

$$s_{55} + s_{66} - s_{44} = 2(s_{11} - s_{12}). \quad (\text{A-9})$$

Thus, there are only four equations for the five unknowns ( $K_{N1}$ ,  $K_{N2}$ ,  $K_{T2}$ ,  $s_{44b}$ , and  $s_{66b}$ ).

#### 4. Both fracture sets are dry.

In this case,  $K_{N1} = K_{T1}$  and  $K_{N2} = K_{T2}$ , which reduces the number of unknown parameters to four. However, then the effective compliances satisfy two additional constraints,

$$s_{22} - s_{11} = s_{44} - s_{55}, \quad s_{11} + s_{22} - 2s_{12} = s_{66}, \quad (\text{A-10})$$

which preclude us from inverting equations (A-4)–(A-8) for the fracture and background parameters.

As discussed in the main text, the model parameters can be resolved uniquely if one of the fracture sets is known to be formed by isolated fluid-filled cracks ( $K_{N1} = 0$ ).