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# The Maximum Likelihood Estimator of *b*-Value for Mainshocks

by Anna Maria Lombardi

Abstract Considering the magnitudes of events in a catalog as independent and exponentially distributed random variables, in agreement with the Gutenberg–Richter law, the statistical significance of the difference between the maximum likelihood estimators of the *b*-value for mainshocks ($b_{ms}$) and for all events ($b_{all}$) is discussed. It is shown that, as a consequence of their definition, the mainshocks do not entirely satisfy the Gutenberg–Richter law and that $b_{ms}$ has been frequently estimated as lower than $b_{all}$ because of an incorrect use of the maximum likelihood method.

## Introduction

One of the most analyzed and discussed topics in statistical seismology concerns variations of *b*-value of the Gutenberg–Richter law. In this context, there are different research matters: (a) the spatiotemporal variation of *b*; (b) the variation of *b* for events in different kinds of clusters (foreshock-mainshock-aftershock clusters, swarms, etc.); and (c) the variation of *b* inside a cluster (difference between two values of *b* estimated for aftershocks and for foreshocks or for mainshocks and for secondary events) (see Suyehiro, 1966; Utsu, 1966; Page, 1968; Suyehiro and Sekiya, 1972; Gibowicz, 1973; Papazachos, 1974; Guha, 1979; Smith, 1981; Knopoff *et al.*, 1982; Pacheco *et al.*, 1992; Wyss and Wiemer, 2000). The discussion of these problems cannot leave out of consideration a careful choice of the statistical method used to estimate *b* and a careful evaluation of statistical significance of variations pointed out.

Topic c is particularly significant because of its connection with earthquake prediction. In fact, it is very important in forecasting to judge, for example, if foreshocks or mainshocks can be distinguished from ordinary seismic activity.

Many studies concerned, in the more or less recent past, the statistical significance of the difference between *b*-values estimated for foreshocks and for aftershocks inside the same cluster. They defended very different and sometimes also contradictory views: in fact, in the opinion of some, the difference of the *b*-value for foreshocks and aftershocks is evident (Suyehiro, 1966; Suyehiro and Sekiya, 1972); others assert, in contrast, that the hypothesis of different physical environments in which foreshocks and aftershocks occur does not have statistical significance (Knopoff *et al.*, 1982; Shi and Bolt, 1982). The co-existence of such different opinions is ascribed, by supporters of each of the two theses, to a wrong selection of data or to an incorrect use of statistical tools by researchers defending the opposite point of view.

In my opinion, a subject not yet rightly discussed and certainly not yet fully explained is the magnitude distribution of mainshocks in a catalog and, in particular, the significance of the difference between the *b*-value estimated for mainshocks and for all events.

In connection with these topics, Utsu (1971), in his analysis of the Japanese catalog during 1926–1968, observed that mainshocks and aftershocks had a distribution according with the Gutenberg–Richter law but with different *b*-values; in particular, as it had already been observed, the *b*-value of mainshocks ($b_0$) was lower than that of aftershocks ($b_a$). Consequently the *b*-value of all events depended on $b_0$ and $b_a$ and on the degree of aftershocks activity. In contrast, Purcaru (1974), in his statistical analysis of Japanese and Greek catalogs, inferred that the mainshocks follow the Gutenberg–Richter law with the same parameter *b* of general magnitude–frequency relation of earthquakes.

More recently, Frohlich and Davis (1993) analyzed the difference between the *b*-values estimated for mainshocks or for secondary events (aftershocks and foreshocks) and the one estimated for all events, in four teleseismic catalogs of earthquakes. They concluded that the selection of mainshocks and of secondary events itself creates a bias, causing a considerable difference between the values of *b* mentioned earlier.

In spite of this, in some recent papers, the difference of estimated *b*-values for mainshocks and for all events has been used as evidence for important physical features of the seismogenic process. In fact, Öncel and Alpetekin (1999), estimating by the maximum likelihood method the *b*-value for the mainshock catalog and for the raw catalog of the North Anatolian fault zone, inferred that inclusion of aftershocks changed the value of *b*. By this result they concluded that, for a more realistic hazard estimation, aftershocks should be removed from the earthquake catalog.

Likewise, Knopoff (2000) used the same distribution magnitude for the complete and the declustered catalogs of earthquakes in the southern California region. He used the difference of estimated *b*-values for the two catalogs to contest the assumption of the self-organized criticality of earthquakes. Finally, in some papers, values of $b$ for mainshocks are obtained to support theories about different topics (Molchan *et al.*, 1999; Kagan, 1999; Öncel and Wyss, 2000).

In this article, by statistical means and by the analysis of events that occurred in southern California, I prove that the difference of observed values of $b$ for declustered and complete catalogs can be ascribed to a misuse of statistical estimators of this parameter, explaining why the various authors have obtained such different results.

## Statistical Tools and Data Analysis

Let us suppose that the magnitudes of seismic events that occurred in a region and in a certain time period are independent and identically distributed random variables in agreement with the Gutenberg–Richter law, that is,

$$\log_{10}N(M) = a - bM, \tag{1}$$

where $N(M)$ is the number of events with magnitude larger than or equal to $M$. This hypothesis is equivalent to supposing that the magnitudes of events are exponential variables with parameter $\beta = b \cdot \ln(10)$ and then that their density function is

$$f(m) = \beta e^{-\beta(m-M_c)}, \quad m \geq M_c, \tag{2}$$

where $M_c$ is the cutoff magnitude (Ranalli, 1969).

In this hypothesis, the best statistical estimator of $b$ is the maximum likelihood estimator (MLE). It is obtained by the formula

$$\hat{b} = \frac{1}{\ln(10)(\bar{M} - M_c)}, \tag{3}$$

where $\bar{M}$ is the sample mean of events considered (Aki, 1965; Utsu, 1966). It is obvious that equation (3) can be used only if observed values have distribution (2). It is known, by the order statistics theory, that the largest member ($\mathcal{M}_0$) of a sample of $N$ events with distribution (2) does not have an exponential distribution; in fact, if we impose that $\mathcal{M}_0$ be larger than or equal to a constant $M_c^* \geq M_c$, its density function is

$$f^N_{\mathcal{M}_0}(m) = \frac{N\beta e^{-\beta(m-M_c)}(1 - e^{-\beta(m-M_c)})^{N-1}}{1 - (1 - e^{-\beta(M_c^*-M_c)})^N}, \quad m \geq M_c^* \tag{4}$$

(Feller, 1966; Casella and Berger, 1990; Lombardi, 2002). Then the distribution of $\mathcal{M}_0$ depends not only on $b$, but also on $N$. Figure 1 shows the plot of density function of $\mathcal{M}_0$ for $M_c^* = M_c$. The distribution of $\mathcal{M}_0$ is very different from the exponential one.

To verify the reliability of the model, the catalog compiled by the Southern California Earthquake Data Center has been considered. It includes 62,394 events with magnitude equal to or larger than $M_c$ 1.95, occurring in the time period 1990–2001. This catalog has been declustered using the Reasenberg algorithm with the standard parameter setting: $R_{fact} = 10$, $P = 0.95$, $\tau_{min} = 1$, $\tau_{max} = 10$ (see Reasenberg [1985] for details). Moreover $M_c^*$ has been chosen equal to $M_c$. There are 2443 clusters identified and 41,022 clustered events (6907 foreshocks, 2443 mainshocks, and 31,672 aftershocks).

Figures 2 shows the cumulative magnitude/log-frequency relation for all events and for mainshocks of the California catalog. The Gutenberg–Richter law seems to be reliable for the whole catalog (Fig. 2a), but not for mainshocks (Fig. 2b); in fact, for lower values of $M$, these data are not perfectly fitted by a straight line. These issues are confirmed by the histograms of Figure 3; the first (Fig. 3a) shows that the distribution of all events is very close to an exponential one, while Figure 3b shows that mainshocks have a different distribution.

In the following, the $b$-value for all events and for mainshocks will be denoted with $b_{all}$ and $b_{ms}$, respectively; moreover the estimators of $b_{all}$ and $b_{ms}$ obtained by equation (3) will be denoted with $\hat{b}_{all}$ and $\hat{b}^1_{ms}$. For the Southern California catalog, the results are

$$\hat{b}_{all} = 0.9593 \pm 0.004 \quad \text{(all events)},$$

$$\hat{b}^1_{ms} = 0.5114 \pm 0.010 \quad \text{(mainshocks)}$$

(the formula of Shi and Bolt [1982] has been used for standard errors).

The significant difference between $\hat{b}^1_{ms}$ and $\hat{b}^1_{all}$ can be ascribed to a misuse of equation (3): in fact, it does not result from the vague hypothesis that data satisfy an empirical relation, as the Gutenberg–Richter law, but from the definite hypothesis that data have an exponential distribution. In re-

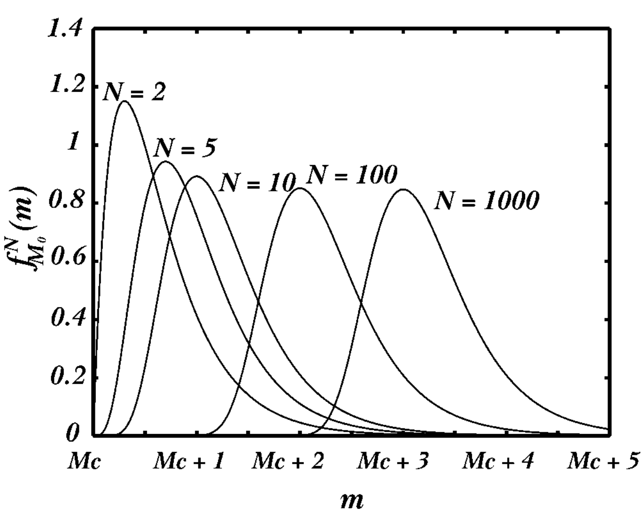


Figure 1. Density functions of mainshocks ($\mathcal{M}_0$) for five different values of $N$, for $M_c^* = M_c$ and for $b = 1$ (equation 4).

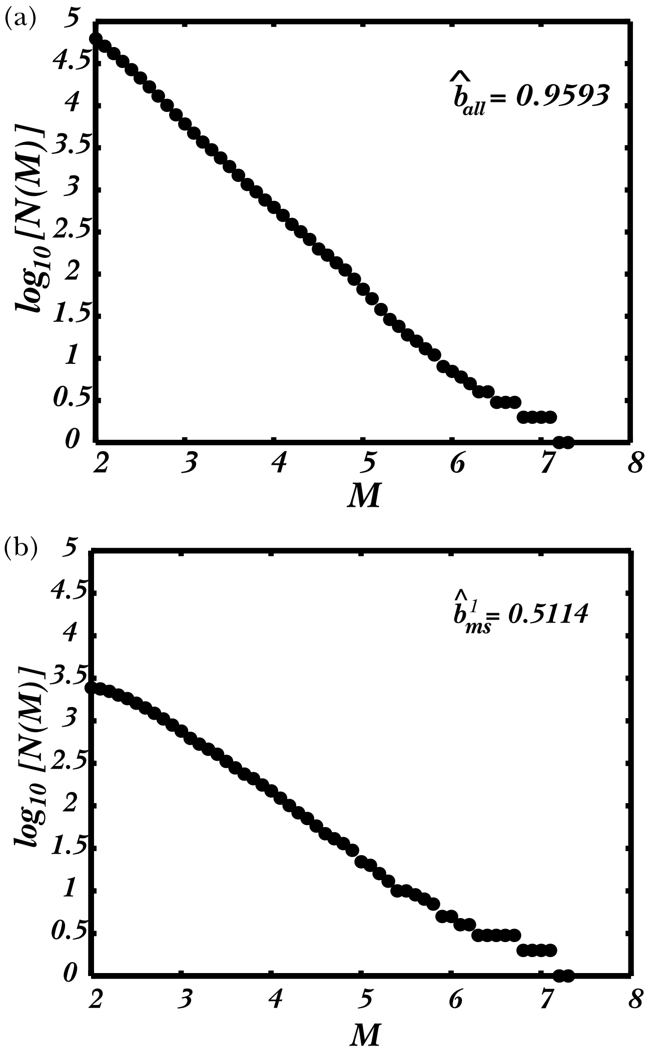


Figure 2. (a) Gutenberg–Richter law for all events (62,394) of the southern California catalog. $\hat{b}_{\mathrm{all}}$ is the maximum likelihood estimator of $b$ (equation 3). (b) The same as (a) for southern California mainshocks; for lower values of $M$, data are not fitted by a line.

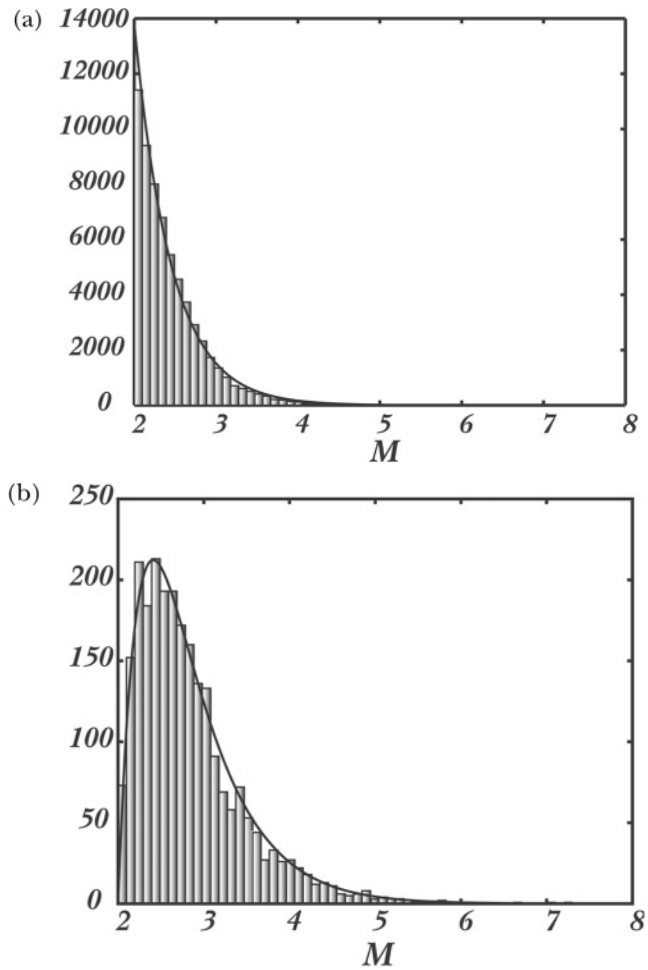


Figure 3. (a) Histogram of southern California events compared with the theoretical density function (2) (solid line), suitably normalized, so that its integral is equal to the one of the histogram. (b) The same as (a) for mainshocks; the theoretical density function is obtained by equation (5). In both cases it has been used that $b = \hat{b}_{\mathrm{all}}$. The catalog has been declustered by the Reasenberg method with the standard parameter setting.

ality, as has been already shown, the mainshocks do not have an exponential distribution, and then it is incorrect to make use of equation (3) to estimate $b_{\mathrm{ms}}$.

Considering that the cluster size $N$ is not constant in a catalog, it becomes necessary to consider a density function of $\mathcal{M}_0, f_{\mathcal{M}_0}(m)$, independent of $N$. Because a theoretical distribution of $N$ is not known, let us use the sample distribution; the result is

$$f_{\mathcal{M}_0}(m) = \sum_{N=2}^{+\infty} f^{\mathrm{N}}_{\mathcal{M}_0}(m) p_N, \qquad (5)$$

where $p_N =$ number of observed clusters with $N$ events/total number of observed clusters.

Figure 3 shows, for complete and declustered southern California catalogs, the histograms of magnitude compared with the theoretical density functions (2) and (5), respectively (solid lines). In both cases the $b$-value used is $\hat{b}_{\mathrm{all}}$. Figure 3b shows an impressive agreement of the data with the model, which is fully confirmed by the $\chi^2$ test with significance level $\alpha = 0.1$: in fact, the hypothesis that mainshocks have the distribution given in equation (5) cannot be rejected.

Then, if we denote with $n_{\mathrm{cl}}$ the number of clusters identified, the log-likelihood function of the $n_{\mathrm{cl}}$ observed values of $\mathcal{M}_0$, $m_0^1, \ldots, m_0^{n_{\mathrm{cl}}}$, is

$$\log L(m_0^1, \ldots, m_0^{n_{\mathrm{cl}}}/b) = \log \left[ \prod_{i=1}^{n_{\mathrm{cl}}} f(m_0^i) \right] = \log \left[ \prod_{i=1}^{n_{\mathrm{cl}}} \sum_{N=2}^{+\infty} f^N(m_0^i) p_N \right]. \qquad (6)$$

Figure 4 shows the plot of log-likelihood function versus $b$ for California mainshocks: the point of maximum is about 0.97 (by numerical approximation, the value 0.9696 is obtained, and then its difference with $\hat{b}_{\mathrm{all}}$ is about 0.0103). The estimator computed as point of maximum of log-likelihood function (6) will be denoted with $\hat{b}^2_{\mathrm{ms}}$.

To judge the agreement with the model of the value $\hat{b}^2_{\mathrm{ms}}$ obtained, 1000 groups of 2443 clusters of independent and exponential random variables, with the same size of California clusters, have been simulated; the $b$-value used is $\hat{b}_{\mathrm{all}} = 0.9593$. Figure 5 shows the histogram of values of $\hat{b}^2_{\mathrm{ms}}$ computed in simulation; more than 38% of values of $|\hat{b}^2_{\mathrm{ms}} - \hat{b}_{\mathrm{all}}|$ are larger than the one of the real catalog ($=0.0103$). Then the estimator of $b_{\mathrm{ms}}$ obtained by equation (3) is incorrect, and a better estimate shows that there is not sufficient statistical evidence to say that $b_{\mathrm{all}}$ and $b_{\mathrm{ms}}$ are different.

It is important, in my opinion, to point out that, since the Gutenberg–Richter relation is not completely applicable to mainshocks, $\hat{b}_{\mathrm{ms}}$ does not represent for them an estimate of proportion of large and small magnitudes.

To show that the results are not dependent on details of the declustering algorithm, the analysis of data has been repeated by varying some basic parameters ($\tau_{\mathrm{min}}$, $\tau_{\mathrm{max}}$, $P$, $R_{\mathrm{fact}}$) used by Reseanberg's method (see Reasenberg [1985] for details). Moreover, data have been selected by different values for the two threshold magnitudes $M_{\mathrm{c}}$ and $M^*_{\mathrm{c}}$. Table 1 lists results of this analysis. With increasing $M^*_{\mathrm{c}} - M_{\mathrm{c}}$, the value of $\hat{b}^1_{\mathrm{ms}}$ approaches $\hat{b}_{\mathrm{all}}$; in fact, as Figure 3b shows, the distribution of $\mathcal{M}_0$ converges to the exponential one with increasing $M^*_{\mathrm{c}}$ (see also equation A5 and figure A1 in Lombardi [2002]). In all cases the results do not seem to depend on the values of parameters, and they do not change the conclusions explained earlier.

## Discussion

The main questions that I try to answer in this work are, (1) Do the magnitudes of mainshocks have the same distribution as the magnitudes of the whole catalog? and (2) Can mainshocks be considered as the largest members of groups of events identified in a set of independent and exponentially distributed random variables? I think my opinion is explained by the arguments of the previous section: the mag-

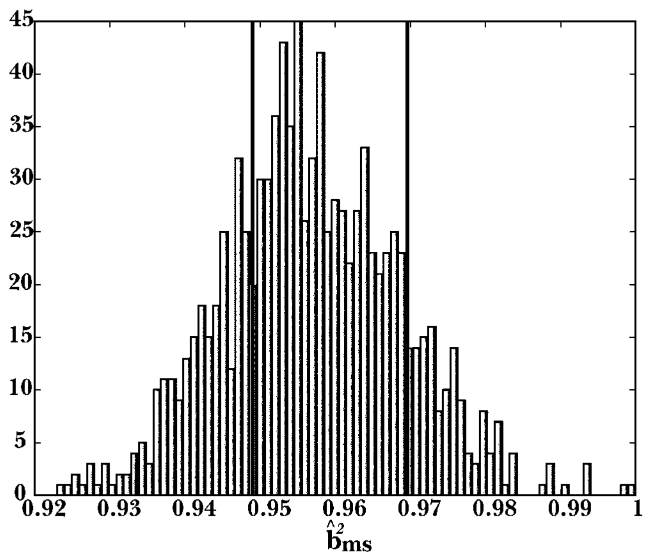


Figure 5. Histogram of values of $\hat{b}^2_{\mathrm{ms}}$ obtained by simulation of 1000 groups of 2443 clusters of independent and exponential random variables with the same size of California clusters and with $b = \hat{b}_{\mathrm{all}} = 0.9593$. Solid lines delimit values of $\hat{b}^2_{\mathrm{ms}}$ for which $|\hat{b}^2_{\mathrm{ms}} - \hat{b}_{\mathrm{all}}| \leq 0.0103$.

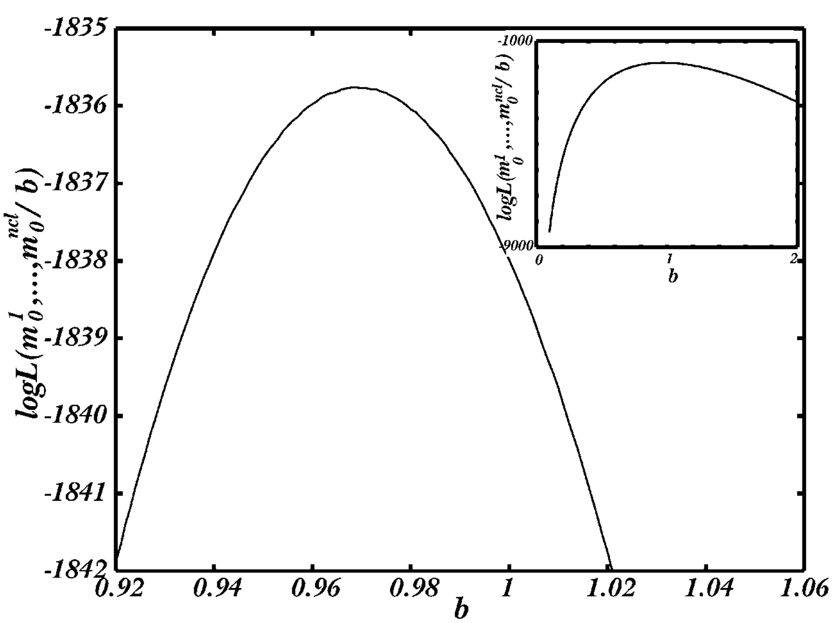


Figure 4. Plot of log-likelihood function (equation 6) for mainshocks of the southern California catalog versus $b$. The smaller panel on the right shows the same plot for a larger range of $b$ ([0.1, 2]). The point of maximum is about 0.97.

Table 1
Results of the Analysis of the Southern California Catalog for Different Values of Parameters used to Select Data

| Parameters | $M_c$ | $M_c^*$ | $N_{ev}$ | $N_{cl}$ | $\hat{b}^1_{ms}$ (equation 3) | $\hat{b}^2_{ms}$ (MLE) | % |
|---|---|---|---|---|---|---|---|
| $\tau_{min} = 1$ (days) | 2.0 | 2.0 | 62394 | 2443 | 0.5114 ± 0.010 | 0.9696 | 38 |
| $\tau_{max} = 10$ (days) | 2.0 | 2.5 | 62394 | 1615 | 0.6757 ± 0.017 | 0.9597 | 100 |
| $P = 0.95$ | 2.0 | 3.0 | 62394 | 760 | 0.7199 ± 0.026 | 0.9419 | 96 |
| $R_{fact} = 10$ | 2.0 | 4.0 | 62394 | 153 | 0.7925 ± 0.064 | 0.9592 | 100 |
| | 3.0 | 3.5 | 6069 | 195 | 0.5863 ± 0.042 | 0.8784 | 18 |
| | 3.0 | 4.0 | 6069 | 109 | 0.6758 ± 0.065 | 0.9099 | 55 |
| $\tau_{min} = 1$ (days) | 2.0 | 2.0 | 62394 | 2169 | 0.4985 ± 0.011 | 0.9613 | 88 |
| $\tau_{max} = 15$ (days) | 2.0 | 2.5 | 62394 | 1452 | 0.6599 ± 0.017 | 0.9522 | 99 |
| $P = 0.99$ | 2.0 | 3.0 | 62394 | 699 | 0.7085 ± 0.027 | 0.9413 | 96 |
| $R_{fact} = 15$ | 2.0 | 4.0 | 62394 | 141 | 0.7659 ± 0.065 | 0.9479 | 91 |
| | 3.0 | 3.5 | 6069 | 192 | 0.5811 ± 0.042 | 0.8838 | 23 |
| | 3.0 | 4.0 | 6069 | 109 | 0.6767 ± 0.065 | 0.9182 | 65 |
| $\tau_{min} = 1$ (days) | 2.0 | 2.0 | 62394 | 2402 | 0.5129 ± 0.010 | 0.9726 | 28 |
| $\tau_{max} = 10$ (days) | 2.0 | 2.5 | 62394 | 1582 | 0.6823 ± 0.017 | 0.9630 | 99 |
| $P = 0.90$ | 2.0 | 3.0 | 62394 | 738 | 0.7269 ± 0.027 | 0.9473 | 97 |
| $R_{fact} = 5$ | 2.0 | 4.0 | 62394 | 146 | 0.7877 ± 0.065 | 0.9563 | 98 |
| | 3.0 | 3.5 | 6069 | 190 | 0.5886 ± 0.043 | 0.8938 | 33 |
| | 3.0 | 4.0 | 6069 | 104 | 0.6642 ± 0.065 | 0.9146 | 61 |

$\tau_{min}$, $\tau_{max}$, $P$, and $R_{fact}$ are the parameters used by the Reasenberg algorithm (see Reasenberg [1985] for details); $M_c$ is the cutoff magnitude; $M_c^*$ is the threshold magnitude for mainshocks; $N_{ev}$ is the number of events with magnitude larger than or equal to $M_c$; $N_{cl}$ is the number of mainshocks with magnitude larger than or equal to $M_c^*$; $\hat{b}^1_{ms}$ is the estimator of $b_{ms}$ obtained by equation (3); $\hat{b}^2_{ms}$ is the maximum likelihood estimator of $b$ for mainshocks; and the last column shows the proportion of simulations with $|\hat{b}_{all} - \hat{b}^2_{ms}|$ larger than that of the real catalog (see text for details).

nitudes of mainshocks do not have the same distribution as the ones of all events. This result does not have to be ascribed to different physical conditions in which they occur, compared with those of the other events, but to simple statistical causes: the largest variable in a sample of independent and identically distributed random variables does not have the same distribution as the sample.

In the past, some authors have already dealt with this topic; their results are very different, but, in my opinion, explainable. Purcaru (1974), in his statistical analysis of aftershocks sequences, inferred that the mainshocks had an exponential distribution with the same parameter $b$ of all events. This result can be ascribed to data selection:

- Aftershocks considered by him had a magnitude larger than or equal to a threshold.
- Mainshocks included in the data had a magnitude larger than or equal to a second threshold.
- The difference between these two thresholds was larger than 2.0.

But, as stated in the previous section, in this case the distribution of $\mathcal{M}_0$ is exponential with a parameter that approaches $b_{all}$.

The first authors that ascribed the large observed values of $|b_{ms} - b_{all}|$ to the act of selecting mainshocks itself were Frohlich and Davis (1993). To prove it, they simulated 5000 clusters of two events according to the Gutenberg–Richter law and with $b = 1.0$ and $M_c$ 4.75. Estimating the $b$-values for the 5000 mainshocks ($\hat{b}_{ms}$) and for the 5000 secondary events ($\hat{b}_{sec}$) by the same statistical estimator, they obtained $\hat{b}_{ms} = 0.91$ and $\hat{b}_{sec} = 2.12$, in agreement with the hypothesis of the bias introduced by selecting data. Their result is absolutely provable by what has been said in the previous section. In fact, from equation (4), for $M_c^* = M_c$ and $N = 2$, it follows that the density function of simulated mainshocks ($\mathcal{M}_0$) is

$$f_{\mathcal{M}_0}(m) = 2\beta e^{-\beta(m-M_c)}(1 - e^{-\beta(m-M_c)}). \quad (7)$$

The density function for secondary events ($\mathcal{M}_1$), if $M_c^*$ is equal to $M_c$, is instead

$$f_{\mathcal{M}_1}(m) = N(N-1)\beta e^{-2\beta(m-M_c)} (1 - e^{-\beta(m-M_c)})^{N-2}, \quad m \geq M_c \quad (8)$$

(see Lombardi [2002] for details).

For $N = 2$, the result is

$$f_{\mathcal{M}_1}(m) = 2\beta e^{-2\beta(m-M_c)}, \quad m \geq M_c. \quad (9)$$

Then $\mathcal{M}_1$ is an exponential random variable with parameter $\gamma = 2\beta = c\ln(10)$, where $c = 2b$; so equation (3) provides the MLE of $c(\hat{c})$, and then the MLE of $b$ from the observed values of $\mathcal{M}_1$ ($\hat{b}_{sec}$) is

$$\hat{b}_{sec} = \frac{\hat{c}}{2}. \quad (10)$$

Then the correct value of $\hat{b}_{sec}$ for simulated data by Frohlich and Davis is 1.06: it is very near the value of $b$ (=1) used by the two authors to simulate magnitudes.

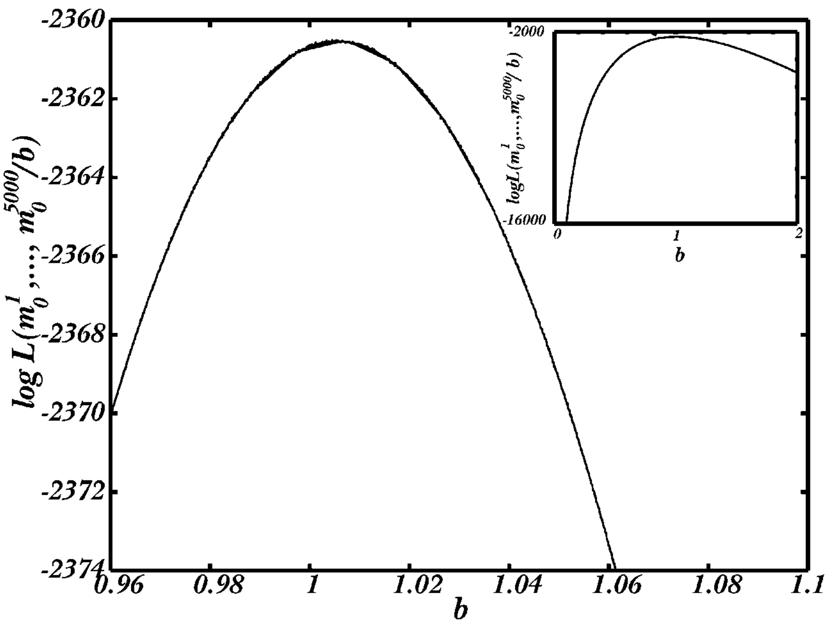


Figure 6. Plot of log-likelihood function (equation 11) versus $b$ for 5000 largest values of simulated exponential samples with two events and with $b = 1$. The smaller panel on the right shows the same plot for a larger range of $b$ ([0.1, 2]). The point of maximum is about 1.0.

Repeating the same simulation of Frohlich and Davis (5000 clusters of two exponential independent random variables with parameter $b = 1$), by equation (3), the result is

$$\hat{b}_{\text{all}} = 0.9979 \pm 0.0141, \quad \hat{c} = 1.9540 \pm 0.0276,$$

in agreement with their result.

Considering the previous discussion, from equation (10) the results is

$$\hat{b}_{\text{sec}} = \frac{\hat{c}}{2} = 0.9770.$$

As far as $\hat{b}_{\text{ms}}$ is concerned, from equation (6) it follows that the log-likelihood function of 5000 simulated values of $\mathcal{M}_0$ $(m_0^1, \ldots, m_0^{5000})$ is

$$\log L(m_0^1, \ldots, m_0^{5000}/b) = 5000 \log(2) + 5000 \log(\beta) - \beta \sum_{i=1}^{5000} (m_0^i - M_c) + \sum_{i=1}^{5000} \log(1 - e^{-\beta(m_0^i - M_c)}). \tag{11}$$

Figure 6 shows the plot of $\log L(m_0^1, \ldots, m_0^{5000}/b)$ versus $b$: it is evident that the point of maximum is very near to 1 (a numerical approximation provides the value 1.0107).

## Conclusions

Considering the magnitudes in a catalog as a set of independent and exponentially distributed random variables and the mainshock as the largest variable of samples identified in this set, the following has been proven:

- The mainshocks have a distribution different from that of all events and not completely described by the Gutenberg–Richter law.
- Consequently, it is incorrect to use the same formula to estimate $b_{\text{all}}$ and $b_{\text{ms}}$; in fact, equation (3) is valid for exponential data only.
- This error is the cause of the low values obtained in the past by estimating $b_{\text{ms}}$.
- Considering a more correct log-likelihood function for mainshocks, the relative maximum likelihood estimator $\hat{b}^2_{\text{ms}}$ is found to be very close to $\hat{b}_{\text{all}}$.

Then the difference between $b_{\text{ms}}$ and $b_{\text{all}}$ does not support the hypothesis that the mainshocks occur in physical conditions different than those that exist for the other events. Moreover their different magnitude distribution can be fully explained by probabilistic analysis.

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Istituto Nazionale di Geofisica e Vulcanologia, Roma
via di Vigna Murata, 605
00143 Roma, Italy
lombardi@ingv.it

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