The development and fracturing of plutonic apexes: implications for porphyry ore deposits

Laurent Guillou-Frottier [1] and Evgenii Burov [2]

1: BRGM, Ressources Minérales, Orléans, France
2: Université Pierre et Marie Curie, Lab. Tectonique UMR 7072, Paris, France

Accepted manuscript EPSL/VCSL/2003-13– June 20th, 2003

Corresponding author: L. Guillou-Frottier, BRGM, Service des Ressources Minérales, 3 av. C. Guillemin - BP6009, 45060 Orléans Cedex 2, France. Phone: +33 (0)2 38 64 47 91; Fax: +33 (0)2 38 64 36 52; e-mail: l.guillou-frottier@brgm.fr

E. Burov, Laboratoire de Tectonique UMR 7072, Case 129 - T26-16 1er étage, Université Pierre et Marie Curie, 4 Place Jussieu, 75252 Paris Cedex 05, France. Phone: +33 (0)1 44 27 38 59; Fax: +33 (0)1 44 27 50 85; e-mail: evgenii.burov@lgs.jussieu.fr

5996 words in the main text *estimated printed length: 14.5 pages*

Abstract

Porphyry ore deposits are generally located above plutonic apexes, described as finger-like extrusions from a large underlying silicic-magma chamber. Fractures and faults that concentrate around these shallow structures allow mineral-enriched hydrothermal and magmatic fluids to circulate and exchange heat and mass with the host rock. Plutonic apexes, however, are not necessarily mineralized. The physical mechanisms invoked for their development and fracturing are focused on the role of volatile pressure, and we have no clear explanation on the associated thermo-mechanical processes. Here we present (a) a semi-quantitative scenario to explain how significant relief could form at the magma chamber roof to give apexes frozen within the shallow crust, and (b) the results of our numerical modeling of fracturing at plutonic apexes. We suggest that morphologic instabilities, expressed by two-directional corrugations (crests and troughs) at the crystallizing roof of the magma chamber, could arise at the top of large silicic batholiths as a result of thermo-mechanical interactions between the reservoir and its surroundings. The corrugated roofs could form with local apexes several kilometers high. Given that a local extensional tectonic regime would surround such systems, crystallization of the apexes would promote a concentration of fractures and faults in their vicinity. In modeling the thermo-mechanical regime around a plutonic apex to show how fractures and faults could develop, we tested different values for temperature contrasts, extension rates and magma viscosity. Two main regimes can be identified, depending on the rheological contrast between the magma and its host rock: the one, a single thick fault connecting the apex to the surface (analogous to a breccia pipe), and the other a network of fractures surrounding the apex (analogous to a stockwork). Where two apexes are close together, one will cluster the shear stresses, regardless of its vertical extension, and thus only a single fracture will develop. We thus infer that barren apexes can be located near mineralized apexes if the distance between them is no greater than the thickness of the brittle layer, which in turn is highly dependent on local thermal and mechanical conditions.

(340 words)

1 - Introduction

A number of porphyry ore deposits have been extensively described and studied over the last decades [1-4]. The major economic products of these deposits, i.e. copper, molybdenum, gold, tungsten, tin and silver, are commonly located above and around small shallow granite intrusions commonly with a porphyritic texture. The edges of these intrusions are intensely fractured, enabling hydrothermal fluids to exchange heat and mass in the permeable zones. However, ore districts containing several shallow mineralized granite extrusions also possess barren intrusions. Regardless of the hydrological and chemical conditions, a channeling of mineralizing fluids requires the presence of faults, vein sets or fracture networks. Consequently, we need to understand the physical aspects related to the thermal and mechanical conditions favoring fracturing above and around plutonic apexes.

Most reviews on the geology of porphyry-type deposits refer to cylindrical intrusions, small-diameter stocks of plutonic rocks, or small “cupolas” emplaced at a few kilometers depth and connected to a large underlying batholith [3, 5-7]. These structures, which we term “apexes”, can cluster fractures and faults above and around their summits.

Numerous types of porphyry ore deposit can be illustrated by the oversimplified sketch of a shallow granite cupola surrounded by a dense fracture network and topped with vein sets and breccia pipes, each allowing hydrothermal fluids to deposit their metals (Figure 1). If we classify these ore deposits according to the associated fracture typology, then two main classes appear. The one comprises a network of fractures, faults and cracks located all around the edges of the plutonic apex and containing complex mineral assemblages with a spatial and temporal zonation [8]; this is the pattern of most porphyry ore deposits. The other comprises veins and breccia pipes connecting the apex summit to the surface and clustering the same type of mineralization as found in the bulk porphyry system (Figure 1). Both classes are often depicted in classic descriptions of porphyry ore deposits, but without any sound explanation as to the thermal and mechanical processes that could account for their coexistence. According to Titley and Beane [3], the degree of fracturing is of major importance in the mineralization process.

Field accounts often refer to both radial and concentric fractures surrounding plutonic apexes (e.g. San Juan deposit, Arizona, [9]). Analytical solutions as well as numerical simulations [10-11] show that stresses concentrating at the top of the buried magma reservoir enable the development of such radial and concentric fractures above the intrusion, but they do not take into account the effect of the temperature-dependent rheology of magma-related rocks. Even after crystallization has begun, the temperature is sufficiently high to strongly modify the location and depth of the transition between the brittle and ductile mechanical regimes [12]. This transition can even occur in quartz-rich rocks at temperatures as low as 250-350 °C.

The main objective of our study is to understand how favorable conditions for fluid circulation become established —the fluids themselves are not incorporated in the models because we consider the formation of faults and fractures above a hot magmatic source to be a prerequisite for mineral deposition. We first present a possible mechanism for the development of plutonic apexes, such as those drawn in numerous conceptual sketches, and then concentrate on analyzing the thermo-mechanical behavior of the host-rock/apex system in order to enlighten the processes that could lead to a given degree of fracturing.

2 - Field observations

Porphyry ore deposits are related to igneous intrusions emplaced at shallow levels of 1 to ~4 km depth with shapes ranging from elongated to cylindrical and diameters of 1 or 2 km [2-3,13]. The porphyry copper deposits of southwest America, for example, are associated with large (2 km) emplacement centers of multiple plutons [9] considered to be genetically related to a large underlying batholith. Despite the many conceptual sketches on the formation of such plutonic apexes, the only suggestions for their genesis invoke a concentration of volatiles from the magma into cupolas at the top of the magma chambers [5,14-15] but without explaining the physical processes giving rise to these cupolas.

The case of the Yerington batholith (western Nevada) and its associated porphyry copper deposits has been described in many papers [e.g. 16-17]. Dilles and Profett [18] were able to reconstruct the batholith geometry together with the buried granite and its associated apexes (see Figure 2). They show that the youngest major intrusion (Luhr Hill Granite) formed at least three cupolas in the center of the batholith, and that the MacArthur and Bear prospects could be located immediately over two other granite apexes, since the underlying porphyry dikes should encounter the Luhr Hill Granite. It can also be supposed that a cupola is present between Bear Prospect and Yerington mine since another dyke swarm has been mapped, although with no indication of mineralization. Although the existence of barren apexes at Yerington have not been proven (J. Dilles, personal communication, 2002), several other field examples can be found in the literature, some of which are given below.

Heithersay and Walshe [19] describe plutonic apexes inferred from geophysical anomalies within the Goonumbla Volcanic Complex, Australia — the largest porphyry-Cu-Au deposit is located on the flanks of one of these apexes. The case of the Henderson porphyry-Mo deposit, Colorado, must also be noted because, according to Carten et al. [20], it consists of at least 11 separate shallow-level intrusions, of which eight are cylindrical and similar in composition.

Field examples show that there is no general rule relating the number of plutonic apexes to the number of mineralized bodies. Before attempting any correlation, one has first to understand the apex-forming mechanisms and the related type of fracturing at their margins. Many studies have provided examples where the roofs of large batholiths form peaks and troughs, which we term here as two-directional corrugations. For example, the first part of the Titley and Beane [3] review paper shows several maps where the mineralization is concentrated above intrusion centers ranging in diameter from 1 to 5 km. Norton [21] gives examples of typical “finger-like” geometries exhibited by plutonic apexes. In Romania, the southern Apuseni Mountains are considered to be underlain by a large (~150 km^2) batholith in which porphyry mineralization (e.g. Rosia Poieni) is located above some of the many plutonic apexes [22]. To the north, in the Baia Mare district, the underlying batholith geometry has been imaged from geophysical surveys [22-23]; each of the six cross-sections of the shallow batholith shows peaks and troughs at the roof (two shown in Figure 3). Thus evidence from a number of field studies indicates that batholith roofs are undulated, and we hope to show that such morphologic instabilities, in the sense of fluid mechanics, may result from thermal and mechanical interactions between the magma and the country rocks.

The few published field studies on fracturing above and around apexes indicate that, once apexes are formed, the surrounding rocks may be too hot to behave mechanically in the brittle regime [12]. However, shortly after the apex formation, lateral cooling from the host rocks should enable brittle failure to occur around and above the extremities of the apexes because they are small in diameter. Previous theoretical work on stress distribution around a prolate magma reservoir was done in the context of a pure elastic regime for the host rocks [e.g. 11, 24], whereas it is now recognized that thermal effects have to be taken into account when considering rock rheology. Heidrick and Titley [25] state that successive heating and cooling stages in magmatic systems may result in multiple fracturing events in porphyry systems, and Titley et al. [26] show that fracturing around porphyry stocks in Arizona was episodic. Homogenization temperatures derived from fluid-inclusion data on primary inclusions in each major vein type are greater than 300 °C; at these temperatures, wall rocks could behave in either the ductile or the brittle regime. It is thus not surprising that field studies provide no clear explanations on fracturing evolution for these “brittle-ductile” systems.

3 – Development of corrugated roofs

3.1. – Thermal convection and silicic magmas

Thermal convection in a silicic magma reservoir provides an efficient means of enabling excess heat to escape. It occurs as soon as the Rayleigh number (*Ra*) of the chamber exceeds a critical value above which buoyancy effects dominate. The Rayleigh number, which compares heat transport resulting from gravity-driven motions to diffusive mechanisms, is expressed by:

$$Ra = \frac{g\ \alpha\ \Delta T\ H^3}{\kappa\ \nu} \tag{1}$$

g is the acceleration due to gravity, α the coefficient of thermal expansion of the magma, ΔT the temperature contrast between the horizontal boundaries, H the thickness of the reservoir, κ the thermal diffusivity of the magma, and ν its kinematic viscosity.

The computed *Ra* for a 3-km-thick batholith having a 500 K temperature contrast with the embedding, a thermal expansion coefficient of 10^{-5} K^{-1}, a thermal diffusivity of 10^{-6} $m^2.s^{-1}$, and a large kinematic viscosity of 10^4 $m^2.s^{-1}$, equals 10^{11}, which is clearly greater than the critical value below which no convection occurs ($\approx 10^3$). For thinner and colder batholiths, the computed *Ra* still exceeds the critical value by several orders of magnitude. A recent study dealing with convective mechanisms below the Campi Flegrei caldera [27] used realistic physical parameters giving Rayleigh numbers between 3.2 10^{10} and 5.1 10^{12}.

When silicic magmas are emplaced in the shallow crust, cooling from the edges leads to crystallization. This then provokes other interactions with the thermal convection, such as increase of the crystal and volatile content and thinning of the convective layer (and thus decrease in the Rayleigh number). The result is a complex behavior due to the evolution of the thermal and mechanical properties of the magma-host rock system with time.

Among the many geological systems in which thermal convection plays a significant role, silicic magma reservoirs are particular in that rheological variations are strongly involved during the lifetime of the system. The thermal and mechanical contrasts in high-viscosity silicic magmas are much lower than in hot, low-viscosity magmas. Thermal processes during the lifetime of a silicic magma chamber result in a dramatic decrease in rheological contrasts between the magma and its host rocks. Coupling of thermal and mechanical processes during magma ascent and emplacement within the brittle-ductile upper crust may thus induce geometric changes to the reservoir. Before tackling the apex fracturing issue, therefore, we shall describe how plutonic apexes can be explained once a realistic rock rheology is taken into account.

3.2. – Modeling of the ascent of magma reservoirs and morphologic instabilities

Some recent studies of large intrusive bodies of igneous rock emplaced at shallow depths [28-29] suggest that the large magma reservoirs may have risen at a relatively high velocity (several cm/yr in the upper brittle crust), especially when realistic rock rheologies are considered. In terms of mechanics, reservoir roofs can be defined by the brittle-ductile transition (BDT), whose depth and precise geometry will depend on local thermal and mechanical conditions [30].

We therefore performed preliminary modeling tests for a diapiric-like context in order to shed light on the BDT evolution when a rising magma chamber encounters the upper brittle crust (see Figure 4 and Table 1 for initial and boundary conditions). A large buoyant magma reservoir, with a temperature sufficiently high to reproduce realistic rheological contrasts between the magma and its embeddings, starts to rise through the overlying crust, whose rheology is also temperature and strain-rate dependent. Details on the numerical algorithm and procedure are given in earlier papers [29, 30] and in Appendix A. Figure 5 shows the result of a numerical experiment where a large volume of light silicic magma rises through the brittle-elasto-ductile crust. During its ascent, the reservoir creates an extensional stress field and thinning in the overlying crustal layers, thus increasing the ascent velocity. The isotherms in Figure 5 reveal the time evolution of the geometry of the BDT, here supposed to be located between the 250 and 450 °C isotherms. When the reservoir reaches shallow depths, it may provoke the formation of multiple localized zones of crustal extension, leading to local rises of magma and thus local uplifts of isotherms — i.e. apex formation. After approximately 550 k.y., one of the apexes starts to grow faster than the others, and deformation in the upper crust brittle layer becomes almost entirely localized at its extremity. After 1.0 m.y., one uplift is well developed: average heights of 4 and 5 km are measured for topographic differences in the 250 and 450 °C isotherms, respectively. In this experiment, an extensional field of 15 mm/yr was imposed, but several tests with no extension confirmed the development of an irregular (undulated) upper boundary. In all cases, roof topography (presence of apexes) was obtained, regardless of the imposed regional tectonic regime. Since our calculations are two-dimensional, it is expected that several such apexes would be triggered with real (three-dimensional) silicic reservoirs, thus leading to unevenly distributed peaks arising from a two-directional corrugation of the upper surface.

3.3. – Convection and crystallization below a corrugated surface

From a dynamic standpoint, the BDT corresponds to the transition from the solid conductive part of the upper boundary layer to the “mushy” zone, where fluids and crystals are intermixed while remaining dynamically stable [e.g. 31]. Laboratory and numerical experiments generally deal with double-diffusive convection processes that occur within the “mushy” zone, and a planar geometry is often assumed for the chamber roof.

Theoretical analyses of crystallization processes in magma chambers have been tackled by a few authors [32, 33], and interactions between convection and solidification are generally studied by laboratory experiments on binary melts [34-38]. In most of these studies, the geometry of the upper cold boundary is horizontal. As far as we know, no theoretical or laboratory studies have been made on crystallization processes below a two-directional corrugated surface. In fact only a very few authors have incorporated such corrugated surfaces within their theoretical or experimental studies on thermal convection, but none of these included crystallization processes [39, 40]. As pointed out by Davis [33], "*the mathematical description of such coupled systems typically involves fifteen or twenty parameters...*" and *"even through the description of the simplest system, only a partial picture exists on how hydrodynamics and morphological changes couple*." Consequently, even if theoretical studies bring some fundamental knowledge on such systems, we only can suggest – at present – a few plausible mechanisms that could explain the behavior and the evolution of corrugated roofs of silicic magma chambers.

3.4. – Scenario for the formation of plutonic apexes

Since most recent work on these particular thermal interactions only deals with simplified cases in terms of geological systems, it is difficult to apply experimental laws or theoretical studies to the thermal and mechanical interactions that take place at silicic-magma chamber roofs. In low viscosity magma chambers, the relief developed at the bottom of the mushy layer [36] is of small amplitude because viscosity contrasts are much larger than in silicic-magma reservoirs. Figure 5 shows that during reservoir ascent, incipient undulations on the reservoir roof appear unsteady. When the ascent velocity of the reservoir decreases, a significant relief develops at around 5 km below the surface, and an extensional regime establishes itself around the "thermal crests". Once the reservoir has reached its final level of emplacement, cooling becomes more efficient. The corrugated upper surface, assumed to resemble the BDT, separates brittle country rocks from the silicic magma that is beginning to crystallize. The apexes have risen to a few kilometers above the “horizontal” part of the chamber roof and, because of their cusp-like geometry, cooling from above and from the sides will quickly solidify them. Meanwhile, the mushy layer develops and follows the corrugated topography. As suggested in the initial experiment, the presence of morphologic instabilities at the top of the convecting magma layer might focus convective motions towards specific locations; for example, upwellings below “crests” and downwellings below “troughs”. Cooling imposed by a thick mushy zone would promote downwellings and crystallization, whereas upwellings would melt part of the thinner mushy zones, enhancing the local Rayleigh number and thus yielding a more pronounced relief (Figure 6).

4. Numerical experiments

In order to focus our study on porphyry deposits related to plutonic apexes, we performed numerical tests on the conditions for fracturing around small stocks of silicic magma (apexes). Contrary to the experiment shown in Figure 5, the apex geometry is now predefined, allowing several other parameters to be tested.

4.1 Algorithm and problem setup

4.1.1 - Algorithm. We modified the fully explicit time-marching large-strain finite-element algorithm initially developed by Podladchidkov and Poliakov [41], which is based on the well-known and heavily documented FLAC® algorithm [42]. The use of this algorithm on physically similar problems (salt diapirism, magma chamber stability, rifting models) is described in a number of articles [30, 41, 43-44]. The code computes stress, velocity and thermal fields versus time around the model apex and its surroundings, reproduces the location and the geometry of the faults in the brittle layers, and predicts principal stress directions and potential brittle failure zones.

4.1.2. - Model geometry and structure. The model presents one or two small magmatic apexes connected to an underlying magma chamber (Figure 7). The model size (12.5 km × 20 km) is limited by the high numerical resolution (100 - 200 m per grid element) needed to resolve fault structures above the magma. Since the lowest parts of the magma body should preserve a sufficient amount of thermal energy during the characteristic time spans of the numerical experiments, and in order to obtain significant overpressures, it is necessary to consider a large reservoir with high thermal inertia. For this reason the model box also includes the topmost 10 km of the crust. The crust has a brittle-elasto-ductile rheology with the physical parameters of a weak quartzite, which represent those of upper crust granites [45]. The lower crust has the same rheology but a higher density (Table 1). The numerical grid consists of 62 × 101 elements. The initial element size is 200 m × 200 m (100 elements per apex); 100 m × 100 m elements were also used in high-resolution experiments.

4.1.3. – Boundary and initial conditions. The upper surface is free with no limitations on the stress and velocity components. At the bottom, we impose a lithostatic pressure with free slip in the horizontal direction, Winkler (i.e. hydrostatic) restoring forces are set in the vertical direction, assuming slightly denser (+ 10 $kg.m^{-3}$) material below the bottom. The lateral boundary conditions are constant horizontal velocity, free vertical slip, lithostatic pressure. When it is not explicitly stated, the horizontal velocities are set to zero (zero displacement).

The background geotherms for the initial crustal thermal state were calculated using a conventional half-space cooling model for continental crust [46, 47]. The initial apex is 350 to 420 °C hotter than the background, and is connected to the magma reservoir at 3.5 km depth. Note that absolute temperature and depth values for the apexes are not crucial as long as the resulting rheological contrasts between the apex and its surroundings are high enough to reproduce physical conditions of thermo-mechanical interactions (see below). In other words, it is not necessary to consider higher ("real") magma temperatures or deeper apexes for the thermo-mechanical problem. "Real" temperatures are needed only for geochemical implications, which are beyond the scope of this study.

4.2 Scaling of rheological contrasts

The experiments were parameterized with minimum rheological contrast, hereafter called R_μ, defined by the ratio of the minimum viscosity of the country rock (for the initial background geotherms) and the minimum allowed viscosity value of the magma (viscosity cut-off). The viscosity is not limited from above, and so local viscosity contrasts may be several orders of magnitude higher than R_μ. The development of mechanical instabilities requires $R_\mu \geq 10^2$ and generally does not change after $R_\mu > 10^5$ [48]. For this reason, the value of R_μ was varied from 10^2 to 10^5, whereas local viscosity contrasts reached $10^5 – 10^9$. These values are still smaller than those in nature, but a higher R_μ would only help to resolve short-time and -length scale motions at the expense of very time consuming computations; decreasing viscosity requires shorter numerical steps. Conductive thermal transfer near the surface is so high that the local effective viscosity is controlled more by cooling than by the global R_μ value. We shall later show that the major impact of R_μ on the system behaviour occurs in the range $10^2 < R_\mu < 10^3$.

Because the viscosity of a particular magma depends on its exact composition, it is rather difficult to define it "in general"; but it can be very small compared to that of the host rock. The time step of the numerical computations is controlled by smallest allowed viscosity, but using "real" magma viscosities for geological-time scale computations would make these impossibly long. A common way to circumvent the problem is to derive and use a minimum physically significant viscosity value because the viscosity contrasts between host rock and magma are very high ($>10^2$). As soon as the flow stress in magma is 100-1000 smaller than that in the host rock, it is useless to account for smaller viscosity values since long time scale behavior of the system will not change. To determine the optimal minimum viscosity value for the experiments, we gradually decreased the minimum value of magma viscosity from 10^{19} Pa.s to 10^{16} Pa.s, which implied a variation of R_μ from 10^2 to 10^5. The maximal viscosity contrast values reached near the brittle-ductile transition zones were considerably higher (10^5 to 10^9).

4.3 A single apex

Having demonstrated the possible formation of localized magma apexes as a result of thermo-mechanical instabilities at the interface between a competent dense upper crust overlying a low-resistant positively-buoyant magma, we concentrated on the interaction between these apexes and the brittle-ductile crust with varying R_μ.

For the apex experiments, R_μ was first limited to 10^2 (Figure 8). The results show that extensional stresses created by a positively-buoyant magma apex are sufficient to provoke the formation of single or multiple cracks or faults at the top of the apex, which then propagate directly to the surface. For this low rheological contrast, there is apparently no fracture network around the apex, but only a single fracturing event.

In the next experiment (Figure 9) the R_μ value was twice as large (200). As shown by the inset of the first time step (300 k.y.), which corresponds to the numerical result of potential failure directions, a broad zone of potential fractures develops all around the edges of the apex, unlike in the previous experiment. These potential fractures are located mainly in the vicinity of the apex but not at the surface. With time, the high strain-rate values are distributed above and around the apex, promoting the development of a surrounding fracture network. Later stages of the system development, as shown at 1.4 m.y., suggest the possibility of distributed faulting at the surface, located several kilometers away from the apex borders.

In the next experiment set (Figure 10) we decreased the lower viscosity limit to a R_μ value of 10^3. The major difference with the previous experiment can be attributed to the appearance of a surface depression due to the low competence of the ascending magma. This phenomenon is also expressed by topographic shoulders above the margins of the apex. A very low viscosity limit for the magma (R_μ value of 10^5) was also tested; here the highest strain-rate values were located within the magma reservoir (low viscosity), and fracturing of the upper crust may not occur.

The variations in fault occurrence and spacing observed in the experiments with increasing R_μ may appear contradictory if we refer to the simple boudinage theory, where viscosity of the ductile lower layer controls fault spacing. In our study, R_μ refers to a global value and not to a ductile layer, so that the lowest viscosities are typically observed far from the brittle layer, near the bottom of the model. Increase in R_μ results in improved heat exchange and in smaller rheological contrasts near the surface. The other explanation to this phenomenon is that the effect of gravity on fault spacing increases with decreasing resistance of the supporting layer and with increasing relative importance of the buoyancy forces.

4.4 Two apexes

The interaction between two spatially distributed apexes was studied in the next experiments (Figure 11) which, to test the evolution of brittle deformation and to resolve second order fractures, involved a two-times finer numerical resolution. These experiments considered a larger numerical box (20 km in depth) and used a higher temperature contrast for pluton and magma (600 °C). Although this choice allowed us to account for uncertainties in the parameters of experimental rheology laws, it should not be considered as a more "realistic" magma temperature, because thermo-mechanical interaction is controlled by the local rheological contrast and not by absolute temperature values. In order to test the ability of this geometry to break the upper crust, we chose a R_μ value of 2 10^4, for which a single apex is not necessarily able to create potential fractures.

As can be seen, two apexes first provoke a stress concentration above each summit, following which the deformation starts to localize around a single apex. Despite the apparent incipient fracturing above the apex on the right (Figure 11a), a single fault is sufficient to accommodate the strain. We also tested the case of tall apexes (6 km high) and found that the above results were not changed: faulting of the upper crust is focused over only one of the two apexes (Figure 11b). This result is not surprising since thermo-mechanical interactions are strongest near the surface.

In nature, the fact that the systems choose one apex to localize deformation may result from natural heterogeneities: the host rock over one of the apexes may be either slightly weaker or the magma below it slightly faster, so that one fault first localizes over this apex. As soon as this happens, the stress field becomes redistributed to the detriment of the possible simultaneous formation of a second fault, which may form later. In the code, heterogeneity in the material properties is modeled via the introduction of a slightly arbitrary (2% Gaussian noise) disturbed grid [41]. Such arbitrary disturbance prevents neither the development of symmetrical structures if the symmetry is favored by the underlying physical processes, nor the development of asymmetric structures in the opposite case [30].

5. Discussion

5.1 Development of plutonic corrugations

Previous attempts to explain the development of plutonic apexes have invoked degassing of the reservoir and a clustering of volatiles at the top of cupolas (small stocks) that are considered to be pre-existing structures. Our preliminary calculations (Section 3) have shown that a significant relief can be triggered at the roof of the reservoir as soon as the reservoir aspect ratio exceeds five [see also 29]. Local extension above the incipient apexes promotes development of these topographic highs, which can extend several kilometers into the upper crust. Since the mechanisms occur before final emplacement of the reservoir, thermal convection within the reservoir adapts to the corrugated upper boundary, focusing upwellings below the “thermal crests” and downwellings below the “thermal troughs”. Crystallization and cooling from the sides of the apexes will then contribute to a rapid solidification (Figure 6).

In addition to the condition of a large aspect ratio for the reservoir geometry, the reservoir ascent must be not too rapid — if the density and temperature contrasts are too high, then thermo-mechanical interactions will not result in a corrugated brittle-ductile transition because the entire reservoir will ascend rapidly towards the surface [29].

5.2 Apex fracturing and porphyry ore deposits

Porphyry ore deposits related to plutonic apexes fall into two main groups. With the first group, the mineralization can be distributed throughout a fracture network around and near the flanks of the apex. Such stockworks (high fracture density) were obtained in the numerical experiments (Figure 9) when the rheological contrast between the magma and its host rock was not too large. Stresses due to the light apex are thus exerted all around the apex, resulting in an even distribution of potential fractures. With the second group, the mineralization is found in “breccia pipes”over the apex. In the experiments (Figure 8) this occurs with smaller rheological contrasts, whereupon the stresses are focused towards the center of the apex. This results in a narrow zone of active faulting above the plutonic apexes. In the event of a high rheological contrast (Figure 10), the stresses will be distributed over a larger distance than in the case of a stockwork formation; this can give rise to distal conjugate veins and could account for other types of ore deposits located close to a plutonic apex [49].

In the model where two apexes are close together (Figure 11), we find that only one is faulted, regardless of vertical size. The experiments show that a minimum distance must exist between two apexes for fracturing to develop above both of them. This result may account for some field data where barren apexes are surrounded by mineralized ones. Because the tops of the apexes (1.5 km in the models) correspond to the level of the brittle-ductile transition, it seems that a distance of 2 km is not sufficient for fracturing to occur above both apexes. This is not surprising since the mechanics of brittle layers predicts that, in the gravity field, faults appear at a spacing about 1.25-1.75 greater than the layer thickness, h. This spacing depends on the friction angle and ratio of bulk modulus to lithostatic pressure at the bottom of the layer, ρgh [42]. The depth of the BDT, however, depends on local thermal and mechanical conditions, and thus apex summits can be located a few kilometers deeper in the crust. In such cases, the distance between two mineralized apexes would be expected to increase.

Our study being focused on the prediction and analysis of fracture patterns formed above plutonic apexes as a consequence of long-term thermo-mechanical processes, post-plutonic effects such as fluid circulation were not incorporated in the model. Obviously, for a complete study of the mineralization processes, hydrological and chemical mechanisms would be required in addition to the thermo-mechanical mechanisms. But, whereas faulting is an irreversible phenomenon, fluid circulation can vary strongly with time. In other words, apex-related fractures that are not initially mineralized by fluid circulation, could become so following dramatic changes in the magma dynamics (e.g. replenishment) and consequent changes in the hydrological regime. Hydraulic fracturing, however, is one of the short-term processes that is not necessarily triggered by long-term thermo-mechanical interactions. This is why we consider it essential that the long time-scale fracturing conditions be first understood through a rigorous analysis with appropriate thermal and rheological parameters.

5.3 Time-dependent processes and regional effects

At the scale of a magmatic system that develops corrugations before completing its ascent, we must consider a number of mechanisms such as thermal convection, location of the BDT, solidification of apexes, and magma dynamics in general, as time-dependent. Any of these processes can strongly modify some of the results presented above. However, once the reservoir is emplaced and the apexes have begun to solidify through crystallization, the fracturing processes will be controlled mainly by local conditions and the geometric features of the pluton. Consequently, we believe that our results would be modified only by large-scale phenomena such as changes in the regional tectonic regime or changes in the regional erosion rates –large-scale phenomena that may indeed play a role in the development of corrugations, but not necessarily in the fracturing phase where local extension dominates. Questions concerning the preferential tectonic regime for porphyry deposits might not be so important because it is local extension (in addition to rheological contrast) that governs the development and fracturing of plutonic apexes. We must emphasize that the fracturing phase is “instantaneous” compared to the magmatic or regional processes, and our results must be considered as "snapshots" of a long-lived system in which apexes have formed.

5.4 Further studies

Although morphologic instabilities were obtained in our numerical experiments, the magmatic processes (crystallization, focusing of convecting currents) were not included in the calculations. Few laboratory experiments have been dedicated to observing corrugations and their effects on local heat transfer, and we suggest that laboratory studies on magma dynamics with a non-predefined reservoir geometry could help in understanding the interactions between associated thermal and mechanical processes. The most difficult part of such studies will be finding suitable analog material to represent the brittle-elasto-ductile crust; a real challenge for future laboratory experiments dedicated to magma dynamics.

Field studies clearly show that even though two classes of fracturing can effectively be defined, i.e. the “breccia” type and the “stockwork” type, the timing of these events is not clear since the dynamic history of a magmatic system can lead to both types of fracturing. A better knowledge of the magmatic-system history may thus help to understand the timing of the fracturing events above plutonic corrugations. However, despite the fact that two regimes have been modeled and compared with field observations, numerical experiments cannot provide a detailed picture of the fracture distribution and connectivity because, at present, higher numerical resolution would require time-consuming calculations that we cannot afford at the scale of the model.

Appendix A. Numerical model.

The 2D numerical scheme was developed from the Paravoz code [41] based on the FLAC® algorithm [42]. This code is a fully explicit time-marching large-strain Lagrangian algorithm that solves the full Newtonian equations of motion

$$\rho \frac{\partial}{\partial t}(\frac{\partial \mathbf{u}}{\partial t}) - div\sigma - \rho \mathbf{g} = 0 \tag{A1}$$

coupled with constitutive equations such as:

$$\frac{D\sigma}{Dt} = F(\sigma, \mathbf{u}, \nabla \frac{\partial \mathbf{u}}{\partial t}, ...T...) \tag{A2}$$

and with equations of heat transfer (advection is computed together with the displacement field):

$$\rho C_p \partial T/\partial t + \mathbf{u}\nabla T - div(\mathbf{k}\nabla T) - H_r = 0 \tag{A3}$$

and surface erosion

$$\partial h_s/\partial t - \nabla(k_e \nabla h_s) = 0 \tag{A4}$$

where $\mathbf{u}$, σ, $\mathbf{g}$, $\mathbf{k}$ are the respective vector-matrix terms for the displacement, stress, acceleration due to body forces, and thermal conductivity. The scalar terms t, ρ, C_p, T, H_r, h_s , k_e respectively designate the time, density, specific heat, temperature, internal heat production, surface elevation, and coefficient of erosion. The terms $\partial/\partial t$, D/Dt, F denote a time derivative, an objective time derivative and a functional, respectively.

In the numerical scheme, the solution of the equations of motion provides velocities at mesh points from which it is possible to calculate element strains. These strains are then used in the constitutive relations to calculate element stresses and equivalent forces, which form the basic input for the next calculation cycle. The Lagrangian coordinate mesh moves with the material; and at each time-step the new positions of the mesh grid nodes are calculated and updated in large strain mode from the current velocity field using an explicit procedure (two-stage Runge-Kutta). To explicitly solve the governing equations, the Paravoz (FLAC®) method uses a dynamic relaxation technique by introducing artificial masses in the inertial system. The adaptive remeshing technique developed by Poliakov and Podladchikov [41] enables large grid distortions to be handled.

The FLAC® method does not imply any inherent rheology assumptions. The main interest in this method refers to its ability to physically model highly unstable processes and handle strongly non-linear rock rheologies. For each time-step in the explicit numerical scheme, the elastic, brittle (plastic) and ductile rheology terms are evaluated using explicit forms of the constitutive laws. The total strain increment is calculated as a sum of elastic, plastic and ductile strain increments. The constitutive parameters we use for the elastic rheology are *E* (Young's modulus) = 0.8 GPa and *v* (Poisson's ratio) = 0.25. The brittle behavior is modeled by Mohr-Coulomb plasticity with a 30° friction angle and 20 MPa cohesion [50]. At high temperatures, the diffusion or dislocation creeping flow is the dominant mechanism of deformation. This behavior is different from that of a Newtonian fluid because the effective viscosity of creeping fluid may vary within 10 orders of magnitude even at adiabatic temperature conditions. The constitutive relations take form [48]:

$$\dot{\varepsilon}_{ij} = A\sigma^{n-1}\sigma_{ij}\,\exp\left(-H/RT\right) \tag{A5}$$

where $\dot{\varepsilon}_{ij}$ is the strain rate, $\sigma = (\frac{1}{2}\sigma_{ij}\sigma_{ij})^{1/2}$ is the effective stress (second invariant of the deviatoric stress), *A* is material parameter, *n* is the effective stress exponent, *H* is the creep activation energy, *R* is the universal gas constant. The variables *A, n, H* describe the properties of a specific material (Table 1).

Acknowledgments. We thank R. Cattin, J. Dilles, L. Lavier, and J.P. Richards for useful comments on different versions of this manuscript. The version of the code Paravoz used is this study was derived from original source code developed during 1991-1997 by A. Poliakov in association with Y. Podladchikov, who are deeply thanked for their generous help and availability. We would like to thank Patrick Skipwith for proofreading the final text and English editing. This is BRGM publication BRGM-CORP-02310.

Table 1. Rheological and physical parameters used in the modeling experiments. The flow law parameters for soft quartzite, which closely represent those of upper-crust granite [45], give a rheology closely matching that of Westerly granite [51]). Other parameters come from [47].

Parameter	**upper crust**	**magma**	**lower crust**	**residual surface material**
ρ ($kg\ m^{-3}$)	2700	2400-2600	2800	2400
n	3	3	3	3
A ($Pa^{-n}\ s^{-1}$)	$5.\times10^{-12}$	$5.\times10^{-12}$	$5.\times10^{-12}$	$5.\times10^{-12}$
H ($J\ mol^{-1}$)	$2.\times10^{5}$	$2.\times10^{5}$	$2.\times10^{5}$	$2.\times10^{5}$
λ (Pa)	$3.\times10^{10}$	$3.\times10^{10}$	$3.\times10^{10}$	$3.\times10^{10}$
G (Pa)	$3.\times10^{10}$	$3.\times10^{10}$	$3.\times10^{10}$	$3.\times10^{10}$
σ (cohesion, Pa)	$1.\times10^{7}$	$1.\times10^{7}$	$1.\times10^{7}$	0
ϕ (friction angle, °)	30	30	30	15
k (thermal conductivity, $W\ m^{-1}K^{-1}$)	2.5	2.5	2.5	1.5
κ (thermal diffusivity, $m^2.s^{-1}$)	10^{-6}	10^{-6}	10^{-6}	10^{-6}
α (thermal expansion, K^{-1})	10^{-5}	10^{-5}	10^{-5}	10^{-5}

Figure captions

Figure 1: Schematic diagram of a porphyry copper system associated with a plutonic apex, after Kirkham and Sinclair [13]. Porphyry mineralization can be located within the fracture network (crosses) or within the neighboring veins and breccia pipes (triangles).

Figure 2: Sketch showing the reconstructed Yerington batholith (left) after removing Cenozoic tilting and faulting episodes. The reconstructed cross-section (A-A') is taken from Dilles and Proffett [18], and (B-B') is inferred from their study (see text for details).

Figure 3: The mineralized Baia Mare district, Romania, related to a large underlying batholith. Two cross-sections, inferred from gravimetric and aeromagnetic surveys, show undulations of the batholith roof. After Crahmaliuc and Crahmaliuc (written communication, 1995, [23]) and Borcos and Vlad [22].

Figure 4: Model set-up for preliminary experiments on the ascent and emplacement of large reservoirs within the shallow crust. The numerical grid consists of 200 × 80 elements. The upper surface condition is free, and isostatic restoring forces are allowed at the bottom boundary. Constant velocities and lithostatic pressures are assumed at the lateral boundaries. Other details are given in [29].

Figure 5: Results from the first set of numerical experiments. The temperature field is shown for different time steps. Arrows indicate the velocity field (at time 200 k.y., the reservoir rises at a mean velocity of 6 $cm.yr^{-1}$ and at time 1 m.y., the maximum of apex velocity reaches 20 $cm.yr^{-1}$). Black and white lines show the geometric evolution of the 450 and 250 °C isotherms during magma ascent and emplacement.

Figure 6: Scenario for successive processes promoting the genesis of plutonic apexes. Top: Time evolution of magma reservoir emplacement within the shallow crust, with incipient morphologic instabilities and apex development. Middle: Role of the local extension field above the corrugated brittle-ductile transition. Bottom: Thermal processes controlling apex formation, apex cooling, crystallization, and decreasing convection.

Figure 7: Numerical model set-up for predefined apex experiments. See text for details.

Figure 8: Effective shear stress (left) and strain-rate (right) for the experiment with a single predefined apex, and with a minimum rheological contrast of 10^2. Vertical and horizontal scales are identical. Dashed line shows the location of the apex margins. A single fracturing event is obtained. The faulted zone connects the center of the apex summit to the surface.

Figure 9: Experiment with a minimum rheological contrast of $2\ 10^2$. A network of potential fractures is obtained (see the inset at time 300 k.y.) around the edges of the apex. Distal faults are obtained in the two first kilometers of the crust, more than 1.0 m.y. after the first fracturing event.

Figure 10: Experiment with a minimum rheological contrast of 10^3. The lower magma viscosity creates a surface depression (-500 m) right above the apex and two topographic shoulders (+200 m) above the external margins of the apex.

Figure 11a: High resolution experiment with two apexes – initial stages. Case of a minimum rheological contrast of $2\ 10^4$. Effective shear stress (left) and strain-rate field (right). Only one of the two apexes is faulted.

Figure 11b: Same results as for Figure 11a are obtained with two tall apexes. See text for explanations.

References

[1] J.D. Lowell, J.M. Guilbert, Lateral and vertical alteration-mineralization zoning in porphyry ore deposits, Econ. Geol. 65 (1970) 373-408.

[2] R.H. Sillitoe, The tops and bottoms of porphyry copper deposits, Econ. Geol. 68 (1973) 799-815.

[3] S.R. Titley, R.E. Beane, Porphyry copper deposits, Econ. Geol. 75th anniversary vol. (1981) 214-235.

[4] R.H. Sillitoe, Gold-rich porphyry copper deposits: geological models and exploration implications, in: R.V. Kirkham, W.D. Sinclair, R.I. Thorpe, J.M. Duke (Eds), Mineral Deposits Modeling, Geol. Assoc. Canada, Spec. Pap. 40, 1993, pp. 465-478.

[5] H. Shinohara, K. Kazahaya, J.B. Lowenstern, Volatile transport in a convecting magma column: implications for porphyry Mo mineralization, Geology 23 (1995) 1091-1094.

[6] A.H.G. Mitchell, Distribution and genesis of some epizonal Zn-Pb and Au provinces in the Carpathian-Balkan region, IMM Trans. 105 (1996) B127-B138.

[7] R.M. Tosdal, J.P. Richards, Magmatic and structural controls on the development of porphyry Cu ± Mo ± Au deposits, in: J.P. Richards, and R.M. Tosdal (Eds), Structural controls on ore genesis, Society of Economic Geologists, Rev. Econ. Geol., 2001, 14, 157-181.

[8] R.E. Beane, S.R. Titley, Porphyry copper deposits, part II. Hydrothermal alteration and mineralization, Econ. Geol. 75th anniversary vol. (1981) 235-269.

[9] S.R. Titley, Characteristics of porphyry copper occurence in the American Southwest, in: R.V. Kirkham W.D. Sinclair, R.I. Thorpe, J.M. Duke (Eds), Mineral deposit modeling: Geological Association of Canada, Special Paper 40, 1993, pp. 433-464.

[10] G. Sartoris, J.P. Pozzi, C. Philippe, J.L. Le Mouël, Mechanical stability of shallow magma chambers, J. Geophys. Res. 95 (1990) 5141-5151.

[11] H. Koide, S. Bhattacharji, Formation of fractures around magmatic intrusions and their role in ore localisation, Econ. Geol. 70 (1975) 781-799.

[12] R.O. Fournier, Hydrothermal processes related to movement of fluid from plastic into brittle rock in the magmatic-epithermal environment, Econ. Geol. 94 (1999) 1193-1212.

[13] R.V. Kirkham, W.D. Sinclair, Porphyry copper, gold, molybdenum, tungsten, tin, silver, in: O.R. Eckstrand, W.D. Sinclair, R.I. Thorpe (Eds), Geology of Canadian Mineral Deposit Types, Geological Survey of Canada, Geology of Canada 8 (1996) pp. 421-446.

[14] J.B. Lowenstern, Dissolved volatile concentrations in an ore-forming magma, Geology 22 (1994) 893-896.

[15] C.W. Burnham, Energy release in subvolcanic environments: implications for breccia formation, Econ. Geol., 80, (1985) 1515-1522.

[16] J.H. Dilles, Petrology of the Yerington batholith, Nevada; evidence for evolution of porphyry copper ore fluids, Econ. Geol. 82 (1987) 1750-1789.

[17] J.H. Dilles, M.T. Einaudi, Wall-rock alteration and hydrothermal flow paths about the Ann-Mason porphyry copper deposit, Nevada – a 6-km vertical reconstruction, Econ. Geol. 87 (1992) 1963-2001.

[18] J.H. Dilles, J.M. Profett, Metallogenesis of the Yerington batholith, Nevada, in: F.W. Pierce, J.G. Bolm (Eds), Porphyry copper deposits of the American Cordillera, American Geological Society Digest 20, 1995, pp. 306-315.

[19] P.S. Heithersay, J.L. Walshe, Endeavour 26 North: a porphyry copper-gold deposit in the Late Ordovician, shoshonitic Goonumbla Volcanic Complex, New South Wales, Australia, Econ. Geol. 90 (1995) 1506-1532.

[20] R.B. Carten, E.P. Gerhaghty, B.M. Walker, J.R. Shannon, Cyclic development of igneous features and their relationship to high-temperature hydrothermal features in the Henderson porphyry molybdenum deposit, Colorado, Econ. Geol. 83 (1988) 266-296.

[21] D.L. Norton, Fluid and heat transport phenomena typical of copper-bearing pluton environments, in: S.R. Titley (Ed.), Advances in Geology of the Porphyry Copper Deposits, Univ. Arizona Press, Tucson, Arizona, 1982, pp. 59-72.

[22] Borcos, M., and S. Vlad, Plate tectonics and metallogeny in the East Carpathians and Apuseni Mounts, Field trip guide, June 7-19, published by the Geological Institute of Romania, 1994, 43pp.

[23] R. Crahmaliuc, A. Crahmaliuc, Modélisation géophysique du pluton Néogene des Monts Gutai (Roumanie) ; implications métallogéniques (written communication), 1995.

[24] A. Gudmundsson, Formation and development of normal-fault calderas and the initiation of large explosive eruptions, Bull. Volcanol. 60 (1998), 160-170.

[25] T.L. Heidrick, S.R. Titley, Fracture and dike patterns in Laramide plutons and their structural and tectonic implications, in: S.R. Titley (Ed.), Advances in Geology of the Porphyry Copper Deposits, Univ. Arizona Press, Tucson, Arizona, 1982, pp. 73-91.

[26] S.R. Titley, R.C. Thompson, F.M Haynes, S.L. Manske, L.C. Robison, J.L. White, Evolution of fractures and alteration in the Sierrita-Esperanza hydrothermal system, Pima County, Arizona, Econ. Geol. 81 (1986) 343-370.

[27] E. Cubellis, G. Di Donna, G. Luongo, A. Mazzarella, Simulating the mechanism of magmatic processes in the Campi Flegrei area (Southern Italy) by the Lorenz equations, J. Volcanol. Geotherm. Res. 115 (2002), 339-349.

[28] N. Petford, A.R. Cruden, K.J.W. McCaffrey, J.-L. Vigneresse, Granite magma formation, transport and emplacement in the Earth's crust, Nature 408 (2000) 669-673.

[29] E.B. Burov, C. Jaupart, L. Guillou-Frottier, Ascent and emplacement of buoyant magma bodies in brittle-ductile upper crust, J. Geophys. Res. 108 (2003), B4, 2177, doi :10.1029/2002JB001904.

[30] E.B. Burov, L. Guillou-Frottier, Thermo-mechanical behavior of large ash-flow calderas, J. Geophys. Res. 104 (1999) 23081-23109.

[31] G. Brandeis, Crystallization and convection in cooling magma chambers, in: Ben Amar et al. (Eds), Growth and Form, Plenum Press, New York, 1991, pp. 473-482.

[32] H.E. Huppert, The fluid mechanics of solidification, J. Fluid Mech. 212 (1990) 209-240.

[33] S.H. Davis, Hydrodynamic interactions in directional solidification, J. Fluid Mech. 212 (1990) 241-262.

[34] S.H. Davis, U. Müller, C. Dietsche, Pattern selection in single-component systems coupling Bénard convection and solidification, J. Fluid Mech. 144 (1984) 133-151.

[35] R.C. Kerr, A.W. Woods, M.G. Worster, H.E. Huppert, Disequilibrium and macrosegregation during solidification of a binary melt, Nature 340 (1989) 357-362.

[36] C. Jaupart, S. Tait, Dynamics of differentiation in magma reservoirs, J. Geophys. Res. 100 (1995) 17615-17636.

[37] X. Hill, La convection sous un solide: applications aux planètes, Doctoral Thesis (1996) Université Paris 7, 146 pp.

[38] H.E. Huppert, M.G. Worster, Dynamic solidification of a binary melt, Nature 314 (1985) 703-707.

[39] D.N. Riahi, Effect of surface corrugation on convection in a three-dimensional finite box of fluid saturated porous material, Theoret. Comput. Fluid Dynamics 13 (1999) 189-208.

[40] M.A. Mehrabian, R. Poulter, G.L. Quarini, Hydrodynamic and thermal characteristics of corrugated channels: experimental approach, Exp. Heat Transfer 13 (2000) 223-234.

[41] A.N.B. Poliakov, P. Cundall, Y. Podlachikov, V. Laykhovsky, An explicit inertial method for the simulation of visco-elastic flow: an evaluation of elastic effects on diapiric flow in two- and three-layers models, in: D.B. Stone, S.K. Runcorn (Eds),

Flow and Creep in the Solar System: Observations, Modelling and Theory, Dynamic Modelling and Flow in the Earth and Planets, Kluwer, Holland, 1993, pp.175-195.

[42] P.A. Cundall, Numerical experiments on localization in frictional materials, Ing. Arch. 59 (1989) 148-159.

[43] L. Guillou-Frottier, E.B. Burov, J.-P Milesi, Genetic links between ash-flow calderas and associated ore-deposits as revealed by large-scale thermo-mechanical modeling, J. Volcanol. Geotherm. Res. 102 (2000) 339-361.

[44] E.B. Burov, A. Poliakov, Erosion and rheology controls on synrift and postrift evolution: verifying old and new ideas using a fully coupled numerical model, J. Geophys. Res. 106 (2001) 16461-16482.

[45] W.F. Brace, D.L. Kohlstedt, Limits on lithospheric stresses imposed by laboratory experiments, J. Geophys. Res. 85 (1980) 6248-6252.

[46] E.B. Burov, M. Diament, Flexure of the continental lithosphere with multilayered rheolohy, Geophys. J. Int. 109 (1992) 449-468.

[47] B. Parsons, J.G. Sclater, An analysis of the variation of ocean floor bathymetry and heat flow with age, J. Geophys. Res. 32 (1977) 803-827.

[48] D. Turcotte, G. Schubert, Geodynamics: Applications of Continuum Physics to Geological Problems, Wiley, New York, NY, 1982, 450 pp.

[49] J.R. Lang, T. Baker, Intrusion-related gold systems: the present level of understanding, Mineral. Deposita 36 (2001), 447-489.

[50] M. Gerbault, A.N.B. Poliakov, M. Daignières, Prediction of faulting from the theories of elasticity and plasticity: what are the limits ?, J. Struct. Geol. 20 (1998) 301-320.

[51] S.H. Kirby, A.K. Kronenberg, Correction to “Rheology of the lithosphere: selected topics”, Rev. Geophys. 25 (1987) 1680-1681.