

Substitution mechanism of Al ions in MgSiO₃ perovskite under high pressure conditions from first-principles calculations

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Abstract

Large-scale first-principles density functional theory (DFT) calculations have been carried out to investigate how Al³⁺ can be incorporated into MgSiO₃ perovskite under high pressure and to study the resultant change in the compressional mechanism of MgSiO₃ perovskite. We examined two types of MgSiO₃ models with 6.25 mol% Al₂O₃: charge-coupled substitution and oxygen-vacancy mechanisms. Five pressure points from 0 to 100 GPa have been considered. At each pressure point, we have calculated five models of the oxygen vacancy and five models of the charge-coupled mechanisms. We also change the internal positions of the substituted Al in the calculated cells, which have 80 atoms. Our free energy calculations show Al³⁺ replaces the nearest-neighbor cation pairs (Mg²⁺ and Si⁴⁺) at all pressures investigated. The calculated bulk modulus of the most energetically favorable model is 3.4% lower than that of the Al-free MgSiO₃ perovskite. These results may have important implications for discriminating between thermal and compositional effects of 1-D Earth models and the possible influence of aluminum perovskite.

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1. Introduction

The structural and elastic properties of materials under high pressure and temperature condi-

tions have been extensively studied both experimentally [1] and theoretically [2]. These kinds of studies have provided the geophysical community with fundamental knowledge, such as the phonon spectra, of many material properties under physical conditions characteristic of the deep mantle, and improve the modeling efforts of mantle convection with realistic parameters for the equation of state. Most geoscientists believe that the dominant mineral phases of the Earth's lower mantle are essentially MgSiO₃ with the perovskite structure and magnesiowüstite [3]. Therefore, the high-

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pressure behavior of MgSiO_3 perovskite pertains to a wide range of geophysical problems, and has been widely studied both experimentally [4,5] and theoretically [6–10].

Recently, it was reported that small amounts of Al ions incorporated into MgSiO_3 perovskite can drastically change the bulk properties [11,12]. After ferric and ferrous iron, Al^{3+} is the most abundant cation in solid solution in this phase. About 4–5 mol% Al_2O_3 is present in all proposed lower mantle compositions, and it is believed that all Al ions are incorporated into MgSiO_3 perovskite under lower mantle pressure and temperature conditions [13]. When MgSiO_3 perovskite contains 5–10 wt% Al_2O_3 , high pressure experiments suggest the bulk modulus becomes significantly lower, -10% or more, than that of pure MgSiO_3 perovskite [11,12].

Two mechanisms of substitution have been proposed for Al ions incorporation into MgSiO_3 perovskite [14]. For a charge-coupled mechanism (CCM), two Al^{3+} ions replace pairs of Mg^{2+} and Si^{4+} ions in nearest-neighbor sites. No oxygen vacancies are required for charge balance. The second one is an oxygen-vacancy mechanism (OVM), $2\text{Al}^{3+} \rightarrow 2\text{Si}^{4+} + \text{O}^{2-}$. In this case, two Al^{3+} ions replace two Si^{4+} ions and an oxygen ion. It is necessary to remove an oxygen ion to balance charge. Recent Al XAFS study [15] suggested CCM should be the most possible mechanism, and more clearly Stebbins et al. reported the NMR results [16], in which only CCM was detected for the substitution mechanism of Al^{3+} into MgSiO_3 perovskite. The empirical molecular mechanics simulations [14] also favor CCM over OVM at pressures characteristic of the lower mantle. Recently, Brodholt reported the first-principles DFT (density functional theory) [17] results on aluminous perovskite [18], in which OVM should be favored at shallower parts of the lower mantle up to about 30 GPa. The small sizes of cells, i.e. $(\text{Mg}_3\text{Al})(\text{Si}_3\text{Al})\text{O}_{12}$ perovskite and $\text{Mg}_2\text{Al}_2\text{O}_5$ brownmillerite, used in Brodholt's calculations can only represent highly ordered Al-incorporated structures. High concentrations of Al_2O_3 , 25 mol%, are considered for CCM in $(\text{Mg}_3\text{Al})(\text{Si}_3\text{Al})\text{O}_{12}$ perovskite, and no Si was taken into account for OVM in $\text{Mg}_2\text{Al}_2\text{O}_5$ brownmillerite.

In order to investigate how Al^{3+} is incorporated into the MgSiO_3 perovskite structure and how the compressional mechanism of Al-incorporated MgSiO_3 perovskite changes, we carried out first-principles DFT calculations with large supercells consisting of 80 atoms and corresponding to 6.25 mol% Al_2O_3 incorporation. We find that the CCM is more favorable than the OVM at all pressures up to 100 GPa. Although our present calculations cannot explain the experimental reduction of bulk modulus (-10% [11]) due to Al^{3+} incorporation, both of these mechanisms reveal the same tendencies, as found in the experiments [11].

2. Computational procedures

Our calculational methods call for the use of DFT [17] and the plane-wave pseudopotential method [19]. The effective exchange-correlation functional GGA-PBE [20] and Vanderbilt-type ultrasoft pseudopotentials [21] were used with a plane-wave cutoff of 380 eV. A Monkhorst–Pack k -point grid [22] with a spatial resolution of 0.1 \AA^{-1} was used for all structures. All the calculations were performed using the academic version of CASTEP (CAMbridge Serial Total Energy Package) code [17] at 0 K. In these calculations, three-dimensional periodic boundary conditions are taken into account.

At first, a $2 \times 2 \times 1$ super-cell with $Pbnm$ symmetry, which contains 80 atoms, was used for Al-free MgSiO_3 , in which all the atomic positions and cell parameters were allowed to relax. This computation allows direct comparison of the bulk properties of Al-rich and Al-free perovskite. We optimized the cell parameters and the enthalpies, i.e. Gibbs' free energies at 0 K ($E+PV$), at 0, 25, 50, 75 and 100 GPa. Because there are several possible atomic configurations in the cells for Al-incorporation through OVM and CCM, we calculated five different OVMs and five different CCMs changing their atomic configurations in order to determine which atomic configuration is most favorable. In OVM1, a nearest-neighbor pair of two Si^{4+} is replaced by two Al^{3+} ions and O^{2-} between these two Si^{4+} is removed to keep charge

Table 1
Initial internal atomic positions of substituted ions for Al-incorporated MgSiO₃ perovskite

| | Si ⁴⁺ → Al ³⁺ | Si ⁴⁺ → Al ³⁺ | Oxygen vacancy |
|------|-------------------------------------|-------------------------------------|--------------------|
| OVM1 | (0.25, 0.5, 0.5) | (0.5, 0.25, 0.5) | (0.4, 0.4, 0.45) |
| OVM2 | (0.25, 0.5, 0.5) | (0.5, 0.25, 0.5) | (0.65, 0.35, 0.55) |
| OVM3 | (0.25, 0.5, 0.5) | (0.5, 0.25, 0.5) | (0.6, 0.1, 0.55) |
| OVM4 | (0, 0.75, 0.5) | (0.5, 0.25, 0.5) | (0.6, 0.1, 0.55) |
| OVM5 | (0, 0.75, 0.5) | (0.5, 0.25, 0.5) | (0.45, 0.77, 0.75) |
| | Si ⁴⁺ → Al ³⁺ | Mg ²⁺ → Al ³⁺ | |
| CCM1 | (0.5, 0.25, 0.5) | (0.51, 0.47, 0.75) | |
| CCM2 | (0.5, 0.25, 0.5) | (0.74, 0.22, 0.75) | |
| CCM3 | (0.5, 0.25, 0.5) | (0.49, 0.53, 0.25) | |
| CCM4 | (0.5, 0.25, 0.5) | (0.24, 0.72, 0.75) | |
| CCM5 | (0.5, 0.25, 0.5) | (0.01, 0.97, 0.75) | |

balance. In OVM2 and OVM3 two Al³⁺ are substituted as OVM1, although an oxygen vacancy is created at positions (0.65, 0.35, 0.55) and (0.6, 0.1, 0.55), respectively, which are attached to one of the Al³⁺ ions. In OVM4, two distant Si⁴⁺ ions are replaced by two Al³⁺ ions and an oxygen vacancy is created at (0.6, 0.1, 0.55), which is attached to one of the incorporated Al³⁺. In OVM5, two Al³⁺ ions are substituted as in OVM4, although an oxygen vacancy is created at (0.45, 0.77, 0.75), which does not attach to any Al³⁺ ions. In CCM1, a nearest-neighbor pair of Mg²⁺ and Si⁴⁺ ions is replaced by two Al³⁺ ions, whereas distant pairs of Mg²⁺ and Si⁴⁺ are replaced by two Al³⁺ ions in CCM2–CCM5. Distances between two substituted Al³⁺ ions for CCM1, CCM2, CCM3, CCM4 and CCM5 are 2.79, 2.91, 3.24, 5.54 and 5.72 Å, respectively. Table 1 lists all the internal atomic positions for the substituted ions in the calculated super-cells. First-principles DFT calculations for these OVMs and CCMs were done in the same manner as for pure MgSiO₃ perovskite.

To compare the enthalpy difference between those with and without Al ions, we also performed the first-principles DFT calculations for Al₂O₃ and MgO, in which the unit cells of Al₂O₃ and MgO with corundum- and NaCl-type structures were considered, respectively.

All the calculations were carried out using the FUJITSU VPP700E vector parallel supercomputer installed in RIKEN. Although the CASTEP

code here employed has been highly optimized for this vector computer, it still takes about 20 h of wallclock time with eight processors to derive the optimal crystallographic structure of a single model. Here we optimized 11 models with different crystallographic structures at five pressure points, which required about 1000 h wallclock time in total with eight processors in use on this vector parallel supercomputer.

3. Results

Table 2 summarizes the calculated cell parameters, volumes and enthalpies of Al-free MgSiO₃ perovskite as a function of pressure. In this table, the cell parameters and volumes are transferred to the ones for unit cells to compare directly with experiments and other calculations, whereas the enthalpies are for the calculated super-cells. In order to obtain an isothermal bulk modulus, calculated volumes are fitted to the third-order Birch–Murnaghan equation of state with $dB/dP=4$, where B and P are the isothermal bulk modulus and the pressure, respectively. We have found good agreement of the cell parameters and volume for Al-free MgSiO₃ perovskite between experiment [4] and calculation at zero pressure, in which differences in cell parameters and volumes at 0 GPa between these two are less than 0.6% and 0.4%, respectively, whereas the calculated bulk modulus (236 GPa) was –9.6% too low, as is typical with the GGA calculations.

To determine which Al-incorporated model is the most favorable of the five OVMs and five

Table 2

Calculated unit cell parameters, a , b and c , volumes, V , and enthalpies, H^a , as a function of pressure, P , for Al-free MgSiO₃ perovskite

| P (GPa) | a (Å) | b (Å) | c (Å) | V (Å ³) | H (eV) ^a |
|--------------|------------|------------|------------|--------------------------|--------------------------|
| 0 | 4.803 | 4.929 | 6.900 | 163.35 | –38529.58 |
| 25 | 4.655 | 4.802 | 6.673 | 149.18 | –38432.42 |
| 50 | 4.540 | 4.702 | 6.549 | 139.79 | –38342.35 |
| 75 | 4.472 | 4.641 | 6.399 | 132.83 | –38257.31 |
| 100 | 4.381 | 4.570 | 6.319 | 126.52 | –38176.38 |

^a Values for calculated super-cells.

Table 3

Calculated enthalpies, H , of Al-incorporated models, CCM and OVM, as a function of pressure, P

| P (GPa) | H (eV) | | | | |
|--------------|-------------|-----------|-----------|-----------|-----------|
| | OVM1 | OVM2 | OVM3 | OVM4 | OVM5 |
| 0 | -37988.83 | -37988.83 | -37988.64 | -37988.89 | -37988.78 |
| 25 | -37890.70 | -37890.44 | -37890.18 | -37890.49 | -37890.11 |
| 50 | -37800.09 | -37799.69 | -37799.48 | -37799.72 | -37799.13 |
| 75 | -37714.77 | -37714.23 | -37714.02 | -37714.25 | -37713.56 |
| 100 | -37633.63 | -37632.96 | -37632.77 | -37633.02 | -37632.20 |
| | CCM1 | CCM2 | CCM3 | CCM4 | CCM5 |
| 0 | -37559.04 | -37558.93 | -37558.93 | -37558.86 | -37558.96 |
| 25 | -37461.75 | -37461.73 | -37461.69 | -37461.59 | -37461.68 |
| 50 | -37371.70 | -37371.67 | -37371.57 | -37371.35 | -37371.59 |
| 75 | -37286.76 | -37286.70 | -37286.65 | -37286.51 | -37286.56 |
| 100 | -37205.88 | -37205.82 | -37205.75 | -37205.60 | -37205.65 |

CCMs, we have compared the enthalpies of these calculated OVMS and CCMs as a function of pressure (Table 3). Enthalpy differences relative to CCM1 or OVM1 are shown in Fig. 1A,B. Model CCM1 is most favorable at all pressures up to 100 GPa (Fig. 1A). The OVM4 is most stable at 0 GPa, but the OVM1 is most favorable at higher pressures between 25 and 100 GPa within the OVM configuration (Fig. 1B). The enthalpy

differences between OVM1 and other OVMS increase incrementally with pressure as shown in Fig. 1B, while those between CCM1 and the other CCMs do not change much.

Geometrical structures of the calculated supercells optimized at 25 GPa for Al-free and two favorable Al-incorporated models, CCM1 and OVM1, are shown, respectively, in Fig. 2A–C. From the results of enthalpy calculations, in

Table 4

Calculated volumes, V , of Al-incorporated models, CCM and OVM, as a function of pressure, P

| P (GPa) | V (Å ³) | | | | |
|--------------|--------------------------|------------|------------|------------|------------|
| | OVM1 | OVM2 | OVM3 | OVM4 | OVM5 |
| 0 | 165.35 | 165.80 | 165.87 | 165.84 | 165.38 |
| 25 | 150.37 | 150.79 | 150.80 | 150.79 | 151.19 |
| 50 | 140.26 | 140.61 | 140.60 | 140.89 | 140.97 |
| 75 | 132.95 | 132.77 | 132.77 | 133.04 | 133.18 |
| 100 | 126.67 | 126.84 | 126.84 | 126.76 | 126.79 |
| B (GPa) | 220(−6.8%) | 218(−7.6%) | 217(−8.1%) | 218(−7.6%) | 215(−8.9%) |
| | CCM1 | CCM2 | CCM3 | CCM4 | CCM5 |
| 0 | 163.69 | 163.45 | 163.53 | 163.59 | 163.60 |
| 25 | 149.49 | 149.39 | 149.45 | 149.49 | 149.55 |
| 50 | 139.66 | 129.43 | 139.95 | 139.72 | 139.69 |
| 75 | 132.19 | 132.28 | 132.29 | 132.33 | 132.39 |
| 100 | 126.17 | 126.11 | 126.35 | 126.42 | 126.48 |
| B (GPa) | 228(−3.4%) | 230(−2.5%) | 232(−1.7%) | 230(−2.5%) | 232(−1.7%) |

Volumes are 1/4 of calculated super-cells for the direct comparison with experiments and calculations for Al-free MgSiO₃ perovskite. Bulk moduli, B , are obtained with third-order Birch–Murnaghan fitting. Differences in bulk moduli relative to the calculated one for Al-free perovskite are expressed as percent in parentheses.

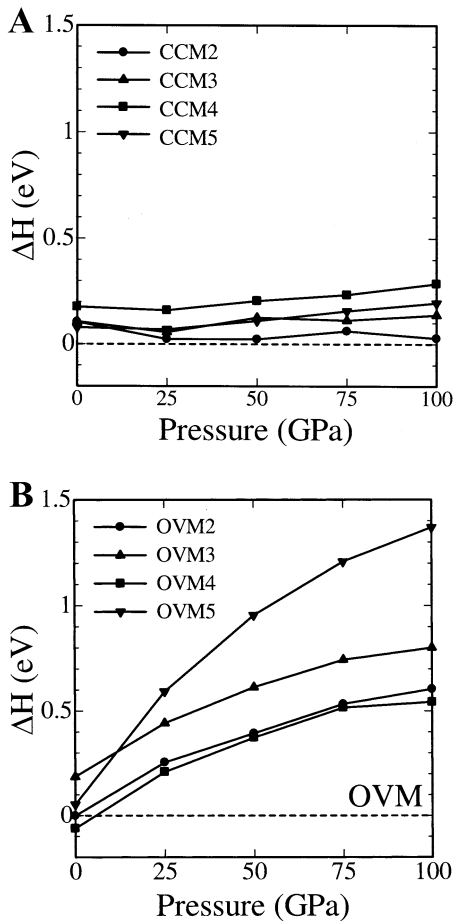


Fig. 1. Enthalpies of Al-incorporated models for (A) CCM and (B) OVM as functions of pressures relative to (A) CCM1 and (B) OVM1, respectively.

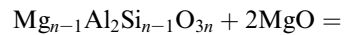
both favorable substitutions, we find that the OVM1 and CCM1, nearest-neighbor pairs of Si^{4+} ions or of Mg^{2+} and Si^{4+} , are replaced by two Al^{3+} ions at all pressures beyond 25 GPa.

Volumes (Table 4) of the most favorable models at zero pressure, i.e. CCM1 and OVM4, are +0.2% and +1.5% larger than that of pure MgSiO_3 , respectively, both of which suggest the same tendencies as the experimental results (+0.5%) [11]. The calculated bulk moduli of all Al^{3+} -incorporated models (bottom line in Table 4) are -1.7% to -8.9% lower than that of Al-free MgSiO_3 perovskite. The calculated bulk moduli of OVMS are much lower than CCMs (Table 3). These calculated reductions of the bulk moduli

due to Al substitutions show the same tendencies as the experiments, but our present calculations cannot entirely explain the experiments, which provide a larger reduction (-10%) [11].

Calculated equilibrium cell parameters at zero pressure and bulk modulus of MgO are 4.270 Å and 156 GPa, respectively, which reproduce the experimental cell parameter (4.213 Å) and bulk modulus (160 GPa) [23] very well. Calculated enthalpies of MgO as a function of pressure are summarized in Table 5.

In order to evaluate the stability of Al^{3+} incorporation into MgSiO_3 perovskite, we have assumed a mineral reaction given by:



We have chosen $2 \times 2 \times 1$ super-cells in which we substituted two Al^{3+} ions, thus $n = 16$. Eq. 1 represents the reaction (from left to right) from charge-coupled substitution to oxygen-vacancy substitution. In order to investigate how Al ions are incorporated into MgSiO_3 perovskite, we calculated the enthalpy differences, ΔH , for Eq. 1, i.e. $\Delta H = \{H(\text{Mg}_{16}\text{Al}_2\text{Si}_{14}\text{O}_{47}) + H(\text{MgSiO}_3)\} - \{H(\text{Mg}_{15}\text{Al}_2\text{Si}_{15}\text{O}_{48}) + 2H(\text{MgO})\}$, in which enthalpies of CCM1 for all pressures, of OVM4 for zero pressure and of OVM1 for pressures between 25 and 100 GPa are employed. Results for enthalpy changes as a function of pressure are shown in Fig. 3. The enthalpy difference ΔH is positive at all pressures from 0 to 100 GPa as shown in Fig. 3. This means Al ions should be able to go into MgSiO_3 perovskite through CCM, which is more favored than OVM at all pressures up to 100 GPa. Recent Al XAFS [15] and NMR [16] experiments suggested that CCM should be the most likely mechanism, and the empirical molecular-mechanics simulations [14] also suggested that the CCM mechanism is more favorable than OVM at lower mantle pressure ranges. In contrast, the recent first-principles DFT results by Brodholt [18] suggested OVM should be favored at shallower parts of the lower mantle up to about 30 GPa, and CCM is favorable at higher pressures. In his calculations, however, he discussed the enthalpy change of a reaction expressed by:

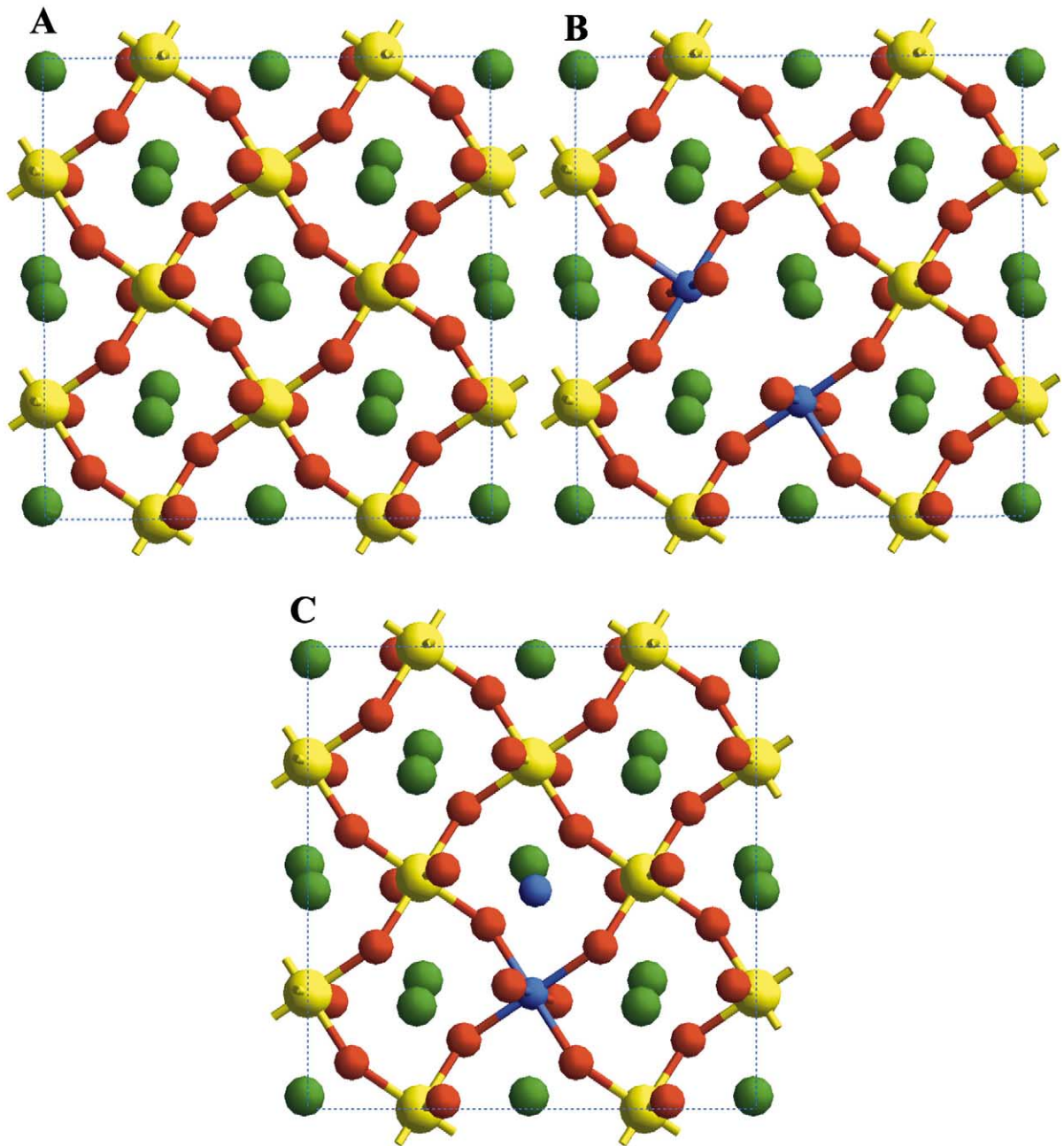


Fig. 2. Projections along the *c* axis of the calculated super-cells optimized at 25 GPa for (A) Al-free and two models of aluminous perovskites, (B) OVM1 and (C) CCM1. Yellow, green, red and blue balls denote Si, Mg, O and Al ions, respectively. Nearest-neighbor pairs of two Si^{4+} ions and of Mg^{2+} and Si^{4+} are replaced by two Al^{3+} ions in these two Al-incorporated models, OVM1 and CCM1, respectively. Note that the oxygen vacancy exists between the two substituted Al^{3+} ions (blue balls) in panel B, whereas no oxygen vacancy exists in panel C.

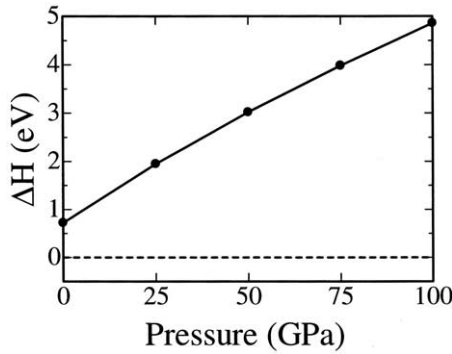
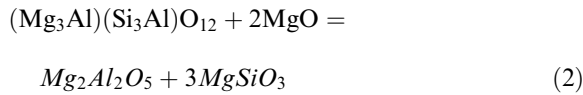


Fig. 3. Calculated enthalpy difference, ΔH , for the reaction in Eq. 1 as a function of pressure.



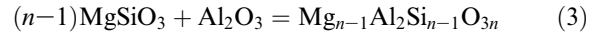
in which $(\text{Mg}_3\text{Al})(\text{Si}_3\text{Al})\text{O}_{12}$ perovskite corresponding to 25 mol% Al_2O_3 incorporation was employed for CCM and no Si was taken into account for OVM in the $\text{Mg}_2\text{Al}_2\text{O}_5$ brownmillerite. Moreover, only highly ordered substitution can be manifested with these $(\text{Mg}_3\text{Al})(\text{Si}_3\text{Al})\text{O}_{12}$ perovskites and $\text{Mg}_2\text{Al}_2\text{O}_5$ brownmillerites. In our first-principles DFT calculations, a larger super-cell with 80 atoms was employed and a more realistic concentration, i.e. 6.25 mol% Al_2O_3 incorporation, was examined, and they showed lower-oriented substitutions. Therefore, our calculations yield a different result in that CCM for incorporating Al^{3+} is found to be more favorable than OVM at lower mantle pressures. This finding completely supports the experimental findings [15,16].

Table 5
Calculated enthalpies, H , as a function of pressure, P , for MgO and Al_2O_3

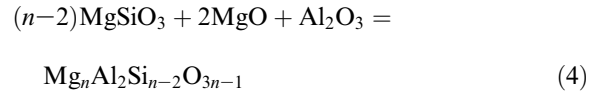
| P (GPa) | $H(\text{MgO})$ (eV) | $H(\text{Al}_2\text{O}_3)$ (eV) |
|--------------|-------------------------|------------------------------------|
| 0 | -1419.31 | -1438.90 |
| 25 | -1416.46 | -1432.86 |
| 50 | -1413.90 | -1427.25 |
| 75 | -1411.54 | -1421.94 |
| 100 | -1409.32 | -1416.89 |

Enthalpies are transferred to the ones for unit chemical formulae.

To examine the feasibility of the Al^{3+} ion to be substituted into MgSiO_3 perovskite via CCM and OVM, we calculated the enthalpy differences for the two reactions expressed by:



for CCM substitution, and



for OVM substitution. Here we choose $2 \times 2 \times 1$ cells, which yield $n=16$. The enthalpy difference, ΔH , between left and right hand sides of Eqs. 3 and 4, i.e. $\Delta H = H(\text{Mg}_{15}\text{Al}_2\text{Si}_{15}\text{O}_{48}) - \{H(15\text{MgSiO}_3) + H(\text{Al}_2\text{O}_3)\}$ for CCM and $\Delta H = H(\text{Mg}_{16}\text{Al}_2\text{Si}_{14}\text{O}_{47}) - \{H(14\text{MgSiO}_3) + 2H(\text{MgO}) + H(\text{Al}_2\text{O}_3)\}$, are summarized in Table 6. We have used the enthalpy for Al_2O_3 with the corundum structure, which is shown in Table 5 together with the one for MgO . All the enthalpy differences, which correspond to the formation enthalpies of aluminous Mg-perovskite via CCM and OVM, are positive, as shown in Table 6, in which ΔH does not change much in the case of CCM, while it increases in the case of OVM with greater pressure. The calculated enthalpy differences for CCM are smaller than those for OVM at all pressures. This result also shows that Al is more soluble in Mg-perovskite via CCM than OVM.

Table 6
Calculated enthalpy differences, ΔH , between left and right hand sides of Eqs. 3 and 4 for CCM and OVM, respectively

| P (GPa) | ΔH (eV) | | | | |
|--------------|--------------------|------|------|------|------|
| | OVM1 | OVM2 | OVM3 | OVM4 | OVM5 |
| 0 | 2.07 | 2.07 | 2.26 | 2.01 | 2.12 |
| 25 | 3.45 | 3.89 | 3.89 | 3.66 | 4.04 |
| 50 | 4.52 | 5.13 | 5.13 | 4.89 | 5.47 |
| 75 | 5.40 | 6.15 | 6.15 | 5.92 | 6.61 |
| 100 | 6.23 | 7.10 | 7.10 | 6.84 | 7.66 |
| | CCM1 | CCM2 | CCM3 | CCM4 | CCM5 |
| 0 | 1.34 | 1.45 | 1.45 | 1.52 | 1.42 |
| 25 | 1.51 | 1.53 | 1.56 | 1.67 | 1.58 |
| 50 | 1.51 | 1.53 | 1.63 | 1.71 | 1.62 |
| 75 | 1.42 | 1.48 | 1.53 | 1.67 | 1.57 |
| 100 | 1.36 | 1.39 | 1.50 | 1.65 | 1.56 |

Our calculations have been performed with the static first-principles method. However, entropy considerations could have a significant effect on relative stabilities of the various models at real mantle temperatures. Although a Monte Carlo or molecular dynamics simulation might be more appropriate to account for configurational contributions to the entropy, the first-principles static calculations at 0 K cannot take the thermal effects into consideration. Although our calculations show that CCM is more favorable than OVM at all pressures, as shown in Fig. 3, the calculated enthalpy difference between these two substitutions at lower pressures is not so large. Hence OVM may coexist with CCM at lower pressures if we take into account the entropic contributions.

4. Conclusions

In this paper we have performed large-scale density functional calculations to investigate in detail the substitution mechanism of Al ions being incorporated into MgSiO_3 perovskite and also the change in the compressional mechanism of aluminous perovskite in the lower mantle. To study the substitution mechanism, we considered 10 substitution configurations for OVM and CCM at five pressure points between 0 and 100 GPa, i.e. 50 calculating points are examined in total. These calculations show that the CCM1, in which a nearest pair of Mg^{2+} and Si^{4+} is replaced by two Al^{3+} ions, should be more favorable than any other CCMs and OVMs at pressures up to 100 GPa. This result is consistent with the experimental XAFS [15] and NMR [16] results. The calculated bulk modulus of the most favorable Al-incorporated MgSiO_3 perovskite, CCM1, is -3.4% relative to that of Al-free perovskite. Our result shows the same trend as the experiments, although our first-principles calculations are not able to explain fully the reduction of 10% in the bulk modulus [11]. We have raised the interesting question regarding the sensitivity of the elastic parameters to the presence of Al-perovskite in the lower mantle. Recent results by Deschamps and Trampert [24] have found that if Al-perov-

skite is present only in 3–4% in volume, then the mid-mantle geotherm can be lowered by 150–200 K. Future ab initio calculations incorporating lattice dynamics effects [25] are needed to verify this effect of Al-perovskite on the determination of the mantle geotherm.

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