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Possible influences of core processes on the Earth's rotation and the gravity field

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Abstract

In this work, we review the processes in the Earth's core that influence the Earth's rotation on the decadal time scale. While core–mantle coupling is likely to be responsible for the decadal length-of-day variations, this hypothesis is controversial with respect to polar-motion variations. The electromagnetic-coupling torques are strongly dependent on the assumed electrical conductivity of the lower mantle, while the topographic torques are influenced by the topography of the core–mantle boundary (CMB). Because no comprehensive theoretical framework for determining the topographic and material parameters of the CMB region is currently available, the modeled results about the coupling torques can only be verified by their consistency with the observed variations of the Earth's rotation and the geomagnetic field. A second path of investigation is to consider the relative angular momentum of the core. Recently, the axial angular-momentum balance has been found to coincide with observed variations in the geomagnetic field and the length of day. However, with respect to polar-motion variations, the angular-momentum balance is not yet closed. We also discuss the role of an irregular motion of the figure axis of the oblate inner core with respect to the outer core and mantle in the excitation of polar motion. In particular, we assume that the associated changes of the Earth's inertia tensor cause the observed decadal variations in polar motion. From this assumption, we can derive the temporal variation of the orientation of the figure axis of the inner core from polar-motion data. Finally, we calculate the gravity variations caused by this relative inner-core motion and compare them with the accuracy of current and planned satellite missions.

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1. Introduction

Jochmann and Greiner-Mai (1996) and Greiner-Mai and Jochmann (1998) showed that the observed decadal variations of the length of day, ΔLOD, cannot be explained by atmospheric excitation (the atmospherically excited part of the main variations is about 14%). A similar result is also obtained for polar motion excitation (Section 3). Jochmann (1999) investigated the contribution of ground water storage and concluded that its effect is marginal over the decadal time scale. The contribution of ocean circulation is not sufficiently known because it has not been monitored for a sufficiently long period of time.

The correlation between decadal variations of the geomagnetic field and ΔLOD indicate that the motions near the core surface causing the geomagnetic variations are also responsible for the excitation of ΔLOD. Core–mantle coupling has been known for some decades as a mechanism by which an exchange of angular momentum between the core and mantle is possible. Lorentz forces and (e.g. geostrophic) pressure variations at the core–mantle boundary (CMB) cause torques on the mantle.

If the lower mantle is conductive, an electrical current **j** can cross the core surface or be induced in the mantle by temporal variations of the magnetic field. The interaction of **j** with the magnetic field in the mantle produces electromagnetic (EM) torques. To excite the torques necessary to explain ΔLOD (e.g. Greiner-Mai, 1987), the electrical conductivity of the mantle σ_M must be sufficiently high [the equivalent conductance G is suggested by Holme (1998b) to be of the order of 10^8 Sm^{-1}]. Unfortunately, the values of σ_M by which the observed ΔLOD can be explained are one or two orders lower than those necessary to explain polar motion variations by EM coupling (e.g. Greiner-Mai, 1993; Greff-Lefftz and Legros, 1995 and this paper, Fig. 2).

The genesis of a lower-mantle conductivity of sufficient magnitude has not previously been adequately explained, and the discussions about its values are controversially. From laboratory experiments, it is concluded that σ_M increases exponentially towards the core-mantle transition zone and reaches a maximum value of ca. 10 Sm^{-1} (e.g. Shankland et al., 1993). However, based on this σ_M model, the associated EM torques are not sufficiently high. Although a consistent model for the conductivity and structure of the transition zone has not yet been produced, the possibility of thin shells of high conductivity has been discussed. Because of this situation, we assume an a priori model of the mantle conductivity with a conductance producing the necessary torques. Determining the correct conductivity model from observed Earth-rotation values is an inverse geophysical problem that cannot be solved unambiguously without additional geophysical informations from other disciplines.

In Section 2, we will review our own investigations of EM coupling. For topographic coupling and the associated pressure torque, we will refer to the literature (e.g. Hinderer et al., 1987, 1990; Jault and Le Mouël, 1989; Hide et al., 1996; Hulot et al., 1996).

In Section 3, we will review concepts of how to explain Δm by internal mass redistributions. This is based on the angular momentum approach with an assumed relative motion of the figure axis of an ellipsoidal rigid inner core. In addition, such motions will cause gravity variations, the testability of which by modern gravity measurements is discussed also in this paper.

Indications of a possible torque approach of the estimated inner-core rotations are discussed at the end of Section 3.1.

2. Electromagnetic core–mantle coupling

In addition to the assumption of a suitable conductivity model, the computation of the torques requires the following problems to be solved:

(1) solving the induction equation for the mantle to derive the geomagnetic field, **B**, in the mantle and at the CMB from its spherical-harmonic expansion at the Earth's surface (inverse problem) (2) determining the velocity field **u** at the core surface by inverting the frozen-field equation and (3) reducing the observed Earth-rotation variations by incorporating the atmospherically excited parts.

The first part follows from the description of the Lorentz torque, **L**, on the mantle:

$$\mathbf{L} = \frac{1}{\mu_0}\int_{V_{\mathrm{M}}} \boldsymbol{r} \times \left(\mathrm{curl}\mathbf{B} \times \mathbf{B}^0\right)\mathrm{d}V, \tag{1}$$

where μ_0 is the vacuum permeability and V_{M} the conducting part of the mantle volume. In previous studies of EM coupling (e.g. Stix and Roberts, 1984; Greiner-Mai, 1993), a perturbation method is applied to solve the induction equation for **B**. The basic idea of this method is to divide **B** into the potential field, $\mathbf{B}^0$, and a sequence of perturbation terms, $\mathbf{B} = \mathbf{B}^0 + \mathbf{B}^1 + \ldots$. The convergence of the series must be checked for the respective physical problem to be solved (e.g. it depends on the electrical conductivity, the time scale and the spatial dimension of the variations).

For decadal variations of **B**, only the first order torque $\mathbf{L}^1$ is conventionally computed, while the higher orders are neglected. This is expressed by:

$$\mathbf{L}^1 = \frac{1}{\mu_0}\int_{V_{\mathrm{M}}} \mathbf{r} \times \left(\mathrm{curl}\mathbf{B}^1 \times \mathbf{B}^0\right)\mathrm{d}V. \tag{2}$$

$\mathbf{B}^1$ is then determined by the solution of the first order induction equation:

$$\mathrm{curl}\left(\frac{1}{\mu_0\sigma_{\mathrm{M}}(r)}\mathrm{curl}\mathbf{B}^1\right) = -\dot{\mathbf{B}}^0, \tag{3}$$

in which the inhomogeneity is the secular variation field continued into the mantle by potential theory. The vectorial equation is usually transformed to a scalar equation by defining the poloidal ($\mathbf{B}_{\mathrm{p}}$) and toroidal ($\mathbf{B}_{\mathrm{t}}$) parts of **B**,

$$\mathbf{B} = \mathbf{B}_p + \mathbf{B}_t = \mathrm{curl\ curl}(\mathbf{r}S) + \mathrm{curl}(\mathbf{r}T),$$

by the scalar fields S and T, obtaining the scalar induction equations

$$\Delta S = \mu_0\sigma_M(r)\dot{S},\ \ \Delta T - \frac{1}{r\sigma_M}\frac{\mathrm{d}\sigma_M}{\mathrm{d}r}\frac{\partial}{\partial r}(rT) = \mu_0\sigma_M(r)\dot{T}. \tag{4}$$

Using the first-order perturbation theory, S and $\dot{S}$ are replaced by S^1 and $\dot{S}^0$ respectively whereas T^1 is substituted for T in Eq. (4). An approach for the higher-order scalar equations is given by Benton and Whaler (1983).

The toroidal field cannot be observed outside of a conductor and must be computed from boundary values of **B** and the velocity field **u** at the CMB ($r=R_c$). The "advection part" of T^1 (R_c) depends on the electromotive force $\mathbf{u}\times\mathbf{B}^0$, i.e. on the model used for **u**, which is determined by inverting the frozen-field equation

$$\dot{B}_r + \nabla_h(\mathbf{u}B_r) = 0; r = R_c \tag{5}$$

as mentioned in (2). In Eq. (5) ∇_h is the horizontal divergence and $\mathbf{B}_r$ the radial component of **B**. In general, Eq. (5) cannot be solved uniquely without an additional constraint for **u**, which is either obtained from a dynamical concept of the outer-core motions such as geostrophy (e.g. Jault and Le Mouël, 1989) or given by an a priori assumption to be purely toroidal (e.g. Gubbins, 1982) or piecewise stationary (Voorhies, 1986) or stationary in a drifting frame (Holme and Whaler, 2000). The assumption of purely toroidal fields is valid in our investigations. The used velocity fields are

- rigid axial rotation of an upper core shell relative to the mantle (Greiner-Mai, 1986, 1987, 1993), [$\mathbf{u} = \Omega \times \mathbf{r}, \Omega = (0, 0, \omega)$]
- rigid non-axial relative rotation (Greiner-Mai, 1990a), [$\mathbf{u} = \Omega \times \mathbf{r}, \Omega = (\omega_1, \omega_2, \omega)$]
- zonal motions to the third degree (Greiner-Mai, 1990b), [$\mathbf{u} = \left(0, 0, u_\varphi\right)$, $u_\varphi = \sum_{l=1}^{3} q_1^0(R_c)P_1^1(\cos\vartheta)$].

A complete description of the EM coupling problem and explicit expressions (based on the conductivity model of Stix and Roberts, 1984) has been given in Greiner-Mai (1989). Therefore, no further formalisms will be provided in this work. The reduction of the observed ΔLOD by the atmospherically excited parts [point (3)] is described in Jochmann and Greiner-Mai (1996) and Greiner-Mai and Jochmann (1998).

A re-examination of L_z^1 by the use of new data for **B** and a comparison with the "mechanical" torque derived from a new ΔLOD time series (L_z^{mech}) is given by Liao and Greiner-Mai (1999). The corresponding results are shown in Fig. 1 as an example. In addition, we show newly determined comparisons between the mechanical torques derived from polar motion data (atmospheric contributions subtracted) and EM torques in Fig. 2.

The results of our investigations can be summarized as follows. (1) The variations in the EM coupling torques based on the used models of **u** (see points earlier) and a conductance consistent with the observed ΔLOD have amplitudes of the order of 10^{17} Nm. The periods of the variations approximately agree with those found in ΔLOD, however from the phases L_z^{mech} and L_z^1 seem to be negatively correlated (Fig. 1a). (2) The correlations between ΔLOD and the angular velocity, ω, of the outer core (see **u** model in the first point earlier) can be explained by the angular momentum balance between the mantle and an upper-core layer of a thickness of about 250–300 km. (3) The EM coupling torques are too low to excite polar-motion variations and reach the necessary values only if conductivity values are assumed to be of the order of the core conductivity.

Similar results have been found by Holme (1998a,b), who has used zonal geostrophic motions and a highly conducting thin shell of the lowermost mantle, the evidence of which has been found by Buffett (1992) who argued that the associated EM coupling is required for the retrograde annual nutation of the Earth to be out-of-phase with tidal forcing.

For evaluating the angular momentum approach corresponding with the zonal geostrophic motions, these motions are interpreted by coaxially rotating cylinder annulies (e.g. Jault et al., 1988; Jackson et al., 1993). This hypothesis is based on the Taylor theorem. The results show that the angular momenta of the core and the mantle can be well balanced (better than in our shell-model). With respect to the polar-motion variations, for which a non-axial velocity field is need, the model fails because it is strongly axial.

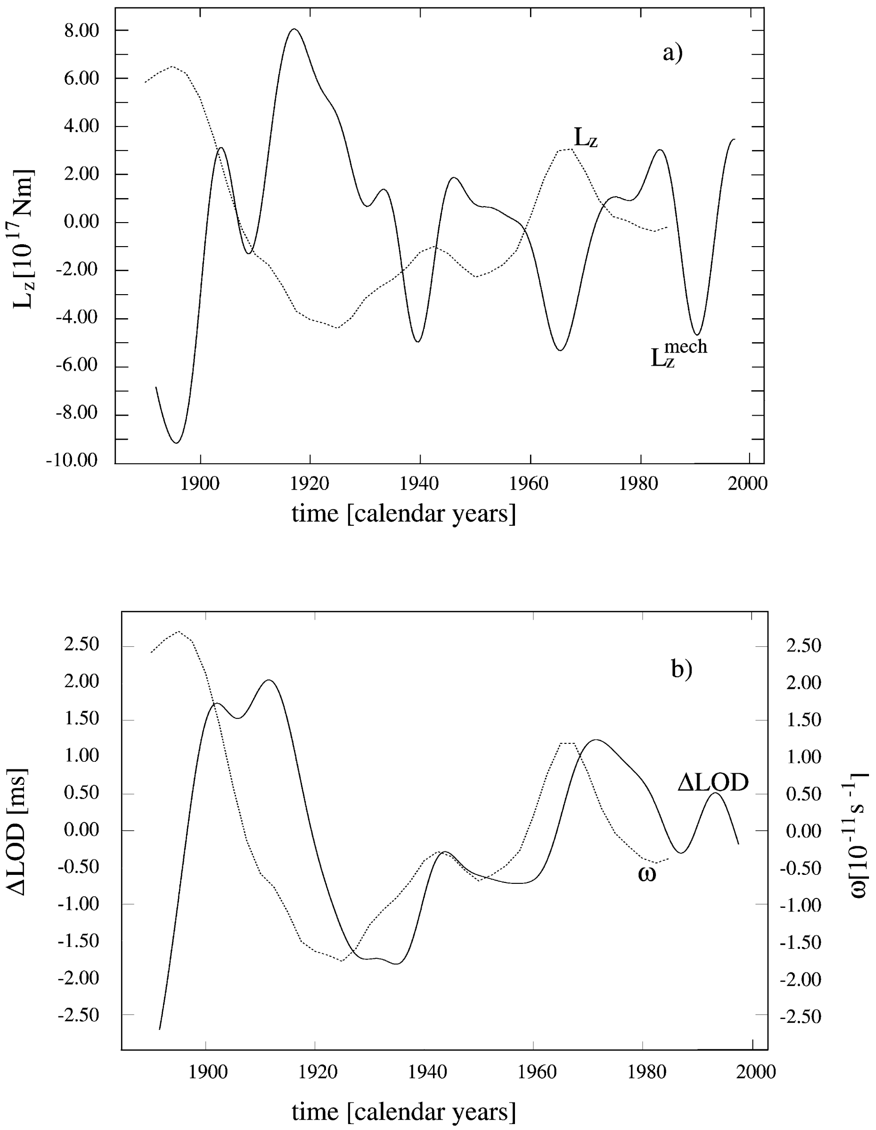

Fig. 1. Results for EM coupling: (a) comparison of the variations in the axial EM torque, $L_z = L_z^1$, with those of the mechanical torque, L_z^{mech}, derived from ΔLOD (b) comparison of ΔLOD with variations in angular velocity ω of the relative rotation of the outermost core, derived from geomagnetic variations by inverting Eq. (5) (Liao and Greiner-Mai, 1999, Fig. 7).

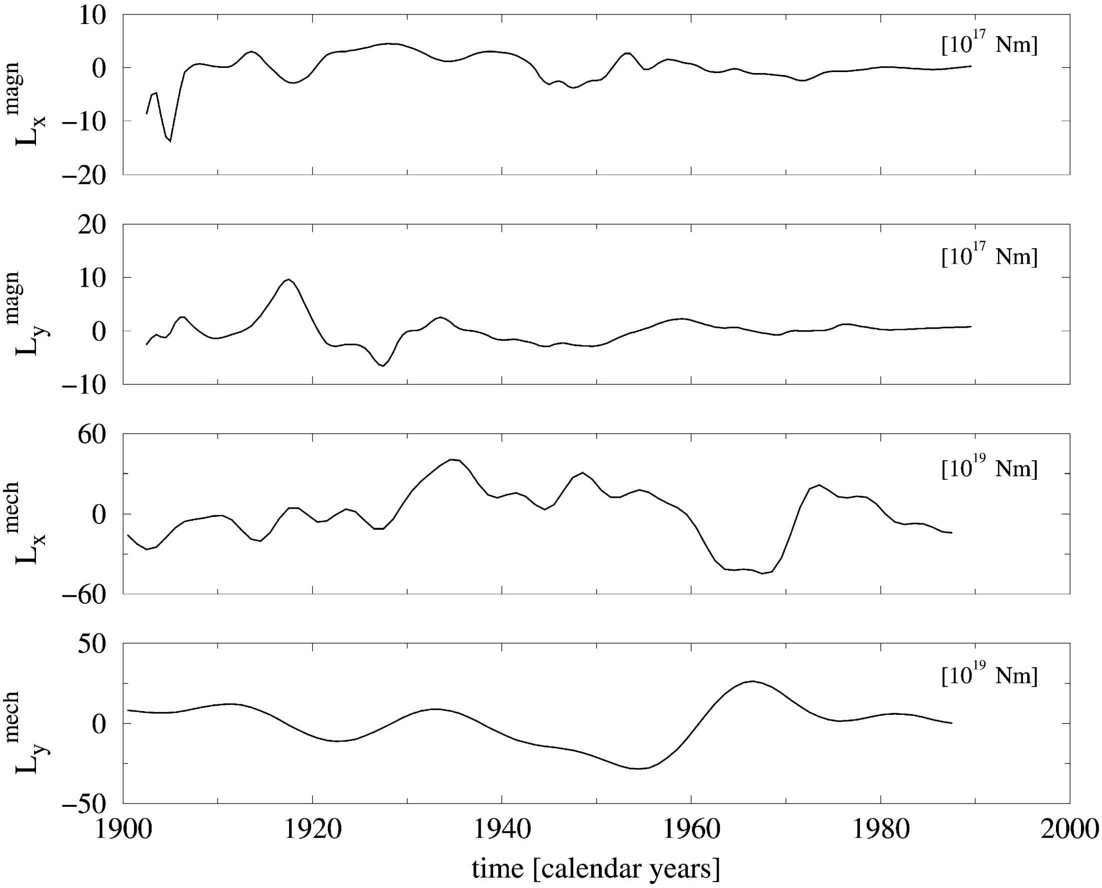

Fig. 2. Equatorial components of the EM ("magn") and mechanical ("mech") torques. The mechanical torques are derived from polar motion data (see Section 3) that has had the atmospheric contributions subtracted. In all curves, the linear trend is removed and only the decadal part is shown.

In both models of relative rotations in the core, the angular momentum budget can be approximately closed with respect to ΔLOD, while the EM and mechanical torques show a negative correlation (see Fig. 1a), which is not the consequence of the phase lag caused by mantle conductivity. According to Holme (1998a), it may arise from an incomplete treatment of the flow non-uniqueness. In particular, the constraints applied here (rotational motions) are not complex enough to allow for a best fit of the flow to both observed quantities, the mechanical torques, L_z^{mech}, and the secular variation, $\dot{\mathbf{B}}$.

Gravitational torques are associated with small density anomalies in the outer core caused by the attraction of mass anomalies in the mantle and with an non-spherical shape of the inner core. Buffett (1996) has found that this coupling can cause oscillations in ΔLOD with periods between 2 and 3 years, the exact value of which depends on the models of density anomalies in the mantle and/or triaxial ellipsoidal shape of the inner core and its motion. The motion of the inner core can be maintained by EM and/or viscous coupling between the inner core and the overlying fluid in the outer core, i.e. its values depend on the models of the outer-core dynamics used or must be derived from Earth's rotation data by an inverse solution of the coupling model. In the decadal time scale, a dynamic model of axially outer-core motions is given by Barginsky's (1970) torsional oscillations associated with above mentioned cylinder rotations inferred from tangentially geostrophic surface motions. Buffett (1998) has investigated free oscillations in ΔLOD arising from the fundamental mode of about 60 years in these torsional oscillations of the core. He studied the angular momentum transfer by EM and topographic coupling between outer core and by gravitational coupling between inner core and mantle. He has found that the most viable coupling mechanism is due to the gravitational torques. In his model, the relative inner-core rotation is excited by the torsional oscillation of the outer-core fluid and fricition at the inner-core boundary, while the restoring gravitational forces constrain this rotation to be nearly locked to the mantle. The role of the inner core for Δm is outlined in Section 3.

Fig. 2 shows that there is probably no reasonable σ_M-model by which the excitation of decadal polar-motion variations, Δm, can be explained by EM core mantle coupling (see also Hide et al., 1996; Greff-Lefftz and Legros, 1995). Hinderer et al. (1987) suggested that Δm can possibly be excited by pressure torques resulting from the interaction of temporally variable flows near the CMB with the CMB-topography (topographic coupling). Jault and Le Mouël (1989) suggested that knowledge of the CMB-topography is inadequate to decide whether this type of coupling is responsible for Δm or not. In addition, such coupling would excite variations of LOD with larger amplitudes than are observed (see also Hinderer et al., 1990). Hide et al. (1996) found that the topographic torque is too small by a factor of 5 to excite the decadal Δm.

Normally, a selfconsistent model is necessary to explain simultaneously the decadal variations in both ΔLOD and Δm. But up to now, it has been difficult to explain the difference of two orders of magnitude between the necessary (mechanical) axial and non-axial torque components by assuming the same model of σ_{M}. With respect to the angular momentum approach, the situation is similar- in models where the balance of the axial angular momentum can be closed fairly well, non-axial components are excluded by the assumptions or are too small.

Recently, we have extended these investigations by the development and application of a new method of field continuation to the CMB (Ballani et al., 1995, 1999, 2002). This procedure inverts the poloidal Eq. (4) for S without approximations. The corresponding mathematical formulation is a well-known ill-posed problem because the boundary values of the geomagnetic field are given only on one side of the sphere by which the mantle is approximated geometrically. The initial-value problem is solved by defining an initial function by the potential field or the perturbation solution at the CMB. Eq. (4) for S in its spherical harmonic decomposition is then transformed into a Volterra integral equation. The integral operator is approximated by a T. Öplitz matrix, and the elements of this matrix are constructed by a forward solution of Eq. (4) for S with prescribed base functions at the CMB. The solution is then obtain by a modified Tikhonov regularization.

The advantages of this method can be demonstrated by comparison with the traditional perturbation method outlined at the beginning of this section. First, the perturbation method is based on an expansion of **B** and solves approximate equations of the type (3) for each term of this expansion in an iterative way [the respective $\dot{\mathbf{B}}^i$ is derived from the solution of the (i−1)th equation, e.g. by numerical differentiation]. On the contrary, the new method solves the complete initial-boundary value problem for Eq. (4) without having to expand the magnetic flux into a converging series of incremental perturbations and to determine separately the time derivatives of the potential field and higher-order terms.

Second, the convergence of the perturbation solution, $\mathbf{B}^0 + \mathbf{B}^1 + \mathbf{B}^2 + \ldots$ depends on scaling arguments derived from the physics of the problem to be modeled. As an example, the convergence is good for smooth decadal and large-scale spatial variations of **B**, but fails for short-period changes. The spatio-temporal scales of the variations to be computed and the magnitude of the assumed conductivity dictate wether the method can be applied or not and how much perturbation terms, $\mathbf{B}^i$, and equations of type (3) must be considered. The great advantage of the new method is that no scaling arguments are necessary and that its applicability is limited only by the stability of the regularization procedure, which depends on the smoothness of the data series.

By this new method, the poloidal field was recently computed at the CMB and in a passive upper layer of the core of 50 km thickness (Ballani et al., 1999, 2002). It also allows Eq. (4) for S to be inverted without scaling arguments about σ_{M} and the time scale of the **B**-variations. Furthermore, the Gauss coefficients of the secular-variation field, $\dot{g}_{nm}$, $\dot{h}_{nm}$, required for the perturbation method are not necessary. Otherwise, the inversion is limited by the noise and the frequency of the variations in the data series. Besides further mathematical developments, one of the next steps will be the application of this method to EM core–mantle coupling and the frozen-field equation.

A first test in Ballani et al. (1999, 2002) showed that the results insignificantly depart from those of the perturbation method, if the decadal time scale is considered and the magnitude of σ_{M} is by about two orders smaller than that of the core. For shorter-period variations, the results at the CMB differ for σ_{M}-values necessary for EM coupling while significant differences in the decadal time scale are obtained for the continuation in regions with core conductivity. The structure of the uppermost core is still unclear, and the continuation of **B** into the core may become important if sedimentation processes of lighter elements stop below the CMB (Buffett et al., 2000) and cause a non-moving highly conductive layer such that the frozen-flux process is located at $r < R_{\mathrm{c}}$.

3. Inner-core motions and decadal fluctuations of polar motion and the gravity field

3.1. Inner-core motions and polar-motion variations

As shown in Section 2, the model of the excitation of decadal variations of m, which is consistent with ΔLOD models, could not yet be reached by either the angular momentum or torque approach.

Therefore, we will investigate an internal process causing changes of mass geometry with respect to the polar axis of the Earth. This may take place within the innermost cylinder of the cylindrical model mentioned in Section 2, which is defined by the diameter of the inner core (IC).

Because of the relatively large density difference between the inner and outer core (AC), a wobble of the oblate IC relative to the AC and mantle may be a mechanism causing the changes of the mass geometry and the inertia tensor of the Earth (Jochmann, 1989). This was postulated as a priori hypothesis by Greiner-Mai and Barthelmes (2001) and used to calculate the necessary motions of the IC, i.e. they assumed that a relative motion of the figure axis of the IC existed, which explains the decadal variations of m from which the atmospheric contributions are subtracted. The objective of those investigations was to derive the changes in the orientation angles, φ_{f} (longitude), ϑ_{f} (co-latitude), of this figure axis in the polar co-ordinate system of the Earth from data of polar motion and the atmospheric excitation function, and to calculate the influence of the relative motion on the gravity field.

While Jochmann (1989) (and later in Greiner-Mai et al., 2000) used a geomagnetic hypothesis to calculate φ_f and ϑ_f, we will only refer to Greiner-Mai and Barthelmes (2001) in this work, who used this a priori hypothesis to explore its consequence for the gravity field, and extended the earlier investigations by including the atmospheric influence on polar motion. With regard to the theoretical derivations, etc., we refer to their paper and will only give a short outline.

Jochmann (1989) used a conventional visco-elastic model of a torque-free Earth and the associated polar motion equation in its approximation valid for decadal variations of m. Greiner-Mai and Barthelmes (2001) extended the polar-motion equation by introducing the atmospheric excitation function explicitly, obtaining

$$\dot{m} + \alpha m = j\sigma_{\mathrm{CH}}\left(m - \frac{\sigma_{\mathrm{EU}}}{\sigma_{\mathrm{CH}}}\psi_i - \psi_{\mathrm{atm}}\right), \tag{6}$$

where $m = m_x + jm_y$ is the complex co-ordinate of the Earth's rotation pole, $\sigma_{\mathrm{CH}} = 5.28\mathrm{a}^{-1}$ and $\sigma\mathrm{EU} = 7.46\ \mathrm{a}^{-1}$ are the Chandler- and Eulerian frequencies, $\alpha = 0{:}05\mathrm{a}^{-1}$ is the damping constant and $\psi_i = \psi_{i,x} + j\psi_{i,y}$ and $\psi_{\mathrm{atm}} = \psi_{\mathrm{atm},x} + j\psi_{\mathrm{atm},y}$ are the complex excitation functions of the relative IC motion and atmospheric circulation respectively $\left(j = \sqrt{-1}\right)$. x and y refer to the geocentrical coordinate system x, y, z, where z is the polar axis of the Earth. Eq. (6) can be solved for ψ_i to show that ψ_i can be derived from data of m and ψ_{atm}. On the other hand, ψ_{i} must be expressed by φ_f and ϑ_f, resulting in two equations for these unknown variables [see Eq. (9)].

The expression for ψ_i as a function of φ_f and ϑ_f is given by Jochmann (1989), who obtained

$$\psi_i(t) = \frac{C_i - A_i}{C - A}\frac{\Delta\rho}{\rho_i}\frac{1}{2}\sin 2\vartheta_f \exp(j\varphi_f) \tag{7}$$

A detailed derivation is given in Greiner-Mai et al. (2000). In Eq. (7), C and A are the principle moments of inertia of the Earth, C_{i} and A_{i} the same for the IC and $\Delta\rho$ is the density difference between IC and AC. Smylie et al. (1984) determined the flattening of the IC by using Clairaut's equation, while $\Delta\rho$ is given by theoretical Earth models. Using Smylie's et al. (1984) flattening value and the density jump according to the model of Bullen and Jeffreys (in Egyed, 1969, p. 197), Jochmann (1989) obtained the following excitation function:

$$\psi_i(t) = 4.3787 \cdot 10^{-5}\sin 2\vartheta_f \exp(j\varphi_f) \tag{8}$$

If the more modern PREM model of Dziewonski and Anderson (1981) is used, the right-hand side of Eq. (8) must be multiplied by the factor 0.32.

The calculation of the atmospheric excitation function, ψ_{atm}, is presented in Jochmann and Felsmann (2001). They derived the matter term of ψ_{atm} (according to the variations of the inertia tensor) from air pressure values published by Vose et al. (1992), while the motion term in atm is insignificant and can be neglected. For polar motion, m, Greiner-Mai and Barthelmes (2001) used IERS (EOP97C01) data. For decadal variations, the term $\dot{m} + \alpha m$ can be neglected in Eq. (6). Consistent with this, the lower-period parts of the time series must be filtered out from the data series. Solving Eq. (6) for ψ_i and inserting the filtered values of m and ψ_{atm}, we obtain the results presented in Fig. 3, where the notation ψ_i^{obs} denotes values of ψ_i derived from observed quantities. Fig. 3 shows that while the atmospheric contribution is significant, it does not explain the observed polar-motion variations.

Solving Eq. (8) for φ_{f} and ϑ_{f}, we obtain

$$\varphi_{\mathrm{f}} = \arctan\frac{\psi_{i,y}^{\mathrm{obs}}}{\psi_{i,x}^{\mathrm{obs}}},\ \ \vartheta_{\mathrm{f}} = \frac{1}{2}\arcsin\, b\sqrt{\left(\psi_{i,x}^{\mathrm{obs}}\right)^2+\left(\psi_{i,y}^{\mathrm{obs}}\right)^2}, \tag{9}$$

where $b = 1{:}107\times10^{-4}$ for the density model of Bullen and Jeffreys and 3.460×10^{-4} for the PREM model, when the excitation function is given in milliarcseconds (mas). The numerical results are shown in Fig. 4.

The linear trend of $\varphi_{\mathrm{f}}(t_i) = \varphi_0 + \omega_f \cdot t_i$ defines a mean relative rotation of the figure axis on a cone about the polar axis of the Earth with the angular velocity ω_{f} . The mean apex angle of the cone is given by the mean value of $\vartheta(t_i)$, i.e. the mean angle between the figure axis of the IC and the polar axis of the Earth. In the following we divide the modeled relative rotation into stationary parts due to the trends, quasi-periodic oscillations and irregular changes.

Typical features of the resulting relative rotation of the IC then are: (1) ϑ_{f} changes between 0.1° and 0.5° for the density model of Bullen and Jeffreys and between 0.4° and 1.5° for the PREM model,

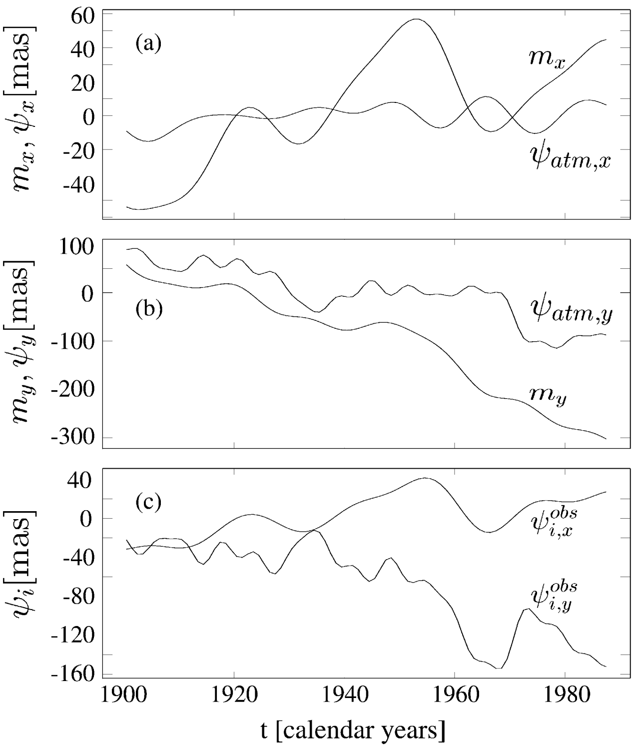

Fig. 3. Decadal variations of the x- and y-components of: (a,b) polar motion, m (filtered IERS data) and atmospheric excitation functions, $\psi_{\mathrm{atm,x}} = \mathrm{Re}(\psi_{\mathrm{atm}})$, $\psi_{\mathrm{atm,y}} = \mathrm{Im}(\psi_{\mathrm{atm}})$ (derived from air pressure), and (c) excitation functions of the relative IC motion, $\psi_{i,x}^{\mathrm{obs}} = = \mathrm{Re}(\psi_i^{\mathrm{obs}}), \psi_{i,y}^{\mathrm{obs}} = \mathrm{Im}(\psi_i^{\mathrm{obs}})$,according to Eq. (6) (Greiner-Mai and Barthelmes, 2001).

(2) the direction of mean relative rotation of the figure axis of the IC is eastward ($\omega_f > 0$), (3) its mean angular velocity is $\omega_f \sim 0.7°$ year^{-1} , and (4) ϑ_f (t) contains more periodicities than φ_f (t) but seems to change its trend at about 1940, i.e. oscillates between 1900 and 1940, increases between 1940 and 1965 by ca. 0.04 year^{-1} and then again oscillates with longer periods. In addition, the orientation of the figure axis of the IC fluctuates with periods of approximately 20, 30 and 70 years.

Although the dynamic approach is not considered in this paper, we will discuss possible causes of this IC motion relative to the mantle by referring to the literature. Glatzmaier and Roberts (1996) showed that an angular velocity of an eastward relative IC rotation (φ) about the polar axis of the Earth of 1° year^{-1} or larger is consistent with recent dynamo models. The earlier mentioned mean rate of change of ω_f (t) is of the same order of magnitude and direction. This suggests that EM torques may exist, hence explaining the linear trend of φ_f (t). But it cannot be expected that long-term dynamo changes cause decadal variations of φ_f (t). Referring to the axial model, Aurnou and Olson (2000) calculated damped oscillations of φ as a consequence of the combined effect of gravitational and EM torques, assuming a triaxial ellipsoid for the shape of the IC (see also Buffett, 1996). Fig. 4 shows a more complicated behavior of φ_f (t), but also elements of a temporary damping.

The major problem is the maintenance of the tilt, ϑ_f (t), versus hydrostatic pressure and gravity by non-axial torques. The temporal behavior of ϑ_f (t) shows elements that are comparable to the

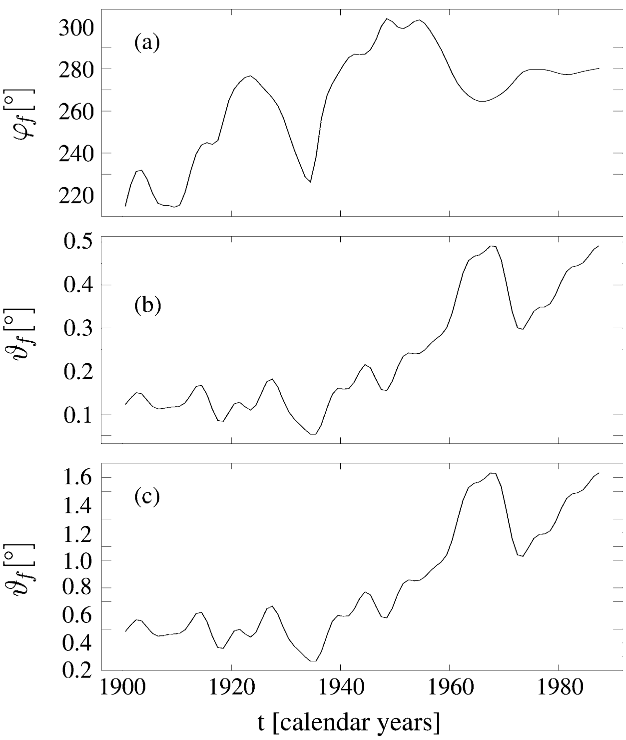

Fig. 4. Resulting orientation angles of the figure axis of the IC (a) φ_f is according to Eq. (9) the same for both density models (b) ϑ_f is from the density model of Bullen and Jeffreys and (c) ϑ_f from the PREM model (Greiner-Mai and Barthelmes, 2001).

axial case in Aurnou and Olson (2000) for φ. In analogy to the axial case, there may be non-axial EM torque components caused by the EM interaction of an axial relative rotation with the non-axial magnetic field. According to Smylie et al. (1984), these non-axial torques must be of the order of 10^{22} Nm to maintain a mean $\vartheta_f(t)=0.5°$. The mechanisms that can generate non-axial EM torques of this magnitude is so far unclear and should be a topic of further investigation.

Finally, if the IC is viscous as proposed by Buffett (1997), its shape would change in a way that corresponds to the combined effect of all forces on it. This effect was included in Buffett's (1998) investigation of free oscillations of ΔLOD by gravitational coupling between IC and mantle. If the deformations also changes the components of the inertia tensor included in ψ_i (see Section 3.1), the figure axis would vary over time scales defined by the IC viscosity. Therefore, it should be mentioned that some irregular parts of the decadal polar-motion, interpreted in this paper by a rigid IC model, may be caused by a viscous reaction of the IC to temporally variable forces on it.

3.2. *Influence of the inner-core motions on the gravity field*

The temporal variations of the orientation of the oblate IC's figure axis will cause variations in the gravity field. They can be calculated using conventional methods and compared with the accuracy of recent gravity models, as well as the models found by current (CHAMP and GRACE) and planned satellite missions (GOCE and LICODY).

The derivation of the corresponding formalism is shown by Greiner-Mai and Barthelmes (2001), and like Section 3.3.1, we will only outline some of the basic steps and illustrate the main results.

Greiner-Mai and Barthelmes (2001) showed that (1) the calculation of the gravity potential can be reduced to an integration about a rotational ellipsoid that is homogeneously filled with mass of density $\rho=\Delta\rho$, (2) the potential in an IC-fixed co-ordinate system is calculated with the polar axis coinciding with the figure axis of the IC and (3) transformed the solution to the mantle-fixed co-ordinate system conventionally used for gravity models.

Usually, the geopotential, V, is expressed by the coefficients C_{nm} and S_{nm} of the spherical harmonic expansion:

$$V(r,\varphi,\vartheta)=\sum_{n=0}^{N}\sum_{m=0}^{n}(C_{nm}\cos m\varphi+S_{nm}\sin m\varphi)P_{nm}(\cos\vartheta). \tag{10}$$

To solve our particular problem, that is the potential difference between an aligned and not aligned IC, ΔV must be computed in the mantle fixed co-ordinate system. The associated coefficients of ΔV, i.e. $\Delta C_{\text{nm}}(t)$ and $\Delta S_{\text{nm}}(t)$, are then functions of $\vartheta_f(t)$ and $\vartheta_f(t)$. To determine their magnitude and dependence on φ_f(t) and $\vartheta_f(t)$, we first calculate the potential coefficients of the oblate IC in the IC-fixed co-ordinate system according to steps (1) and (2). The resulting expression is given by

$$C_{2l,0}=3\frac{M_e}{\mathrm{M}}\left(\frac{a^2-b^2}{R_0^2}\right)^l\frac{(-1)^l}{(2l+1)(2l+3)\sqrt{4l+1}} \tag{11}$$

In Eq. (11), $M_e=\frac{4}{3}\pi\Delta\rho a^2 b$ is the mass of the ellipsoid with semi axes a and b, $\Delta\rho$ is its density and M is the total mass of the Earth. Because the figure of the inner core is approximated by a

Table 1
Predicted rates of change of the second-degree spherical harmonic coefficients of the gravity field caused by the relative wobble of the IC derived from polar motion (according to Greiner-Mai and Barthelmes, 2001)

m	$\frac{\mathrm{d}}{\mathrm{d}t} C_{2m}$[year^{-1}]	$\frac{\mathrm{d}}{\mathrm{d}t} S_{2m}$[year^{-1}]
0	$+0.7\times10^{-12}$	
1	$+3.4\times10^{-12}$	$+16.3\times10^{-12}$
2	$+0.4\times10^{-12}$	$+0.1\times10^{-12}$

rotational ellipsoid, only the coefficients with even degree appear in Eq. (11). Practically, it is sufficient to consider only C_{20} in this co-ordinate system, since the other coefficients are negligible.

The transformation of the spherical-harmonic coefficients $C_{nm}, S_{nm} \Rightarrow \left(C_{np}(\alpha, \beta, \gamma), S_{np}(\alpha, \beta, \gamma)\right)$ into a co-ordinate system rotated by the angles α,β,γ is given by Kautzleben (1965) and Ilk (1983). Its application to a rotation by the angles φ_{f} and ϑ_{f} still results in an extensive expression, given by Greiner-Mai and Barthelmes (2001), [Eqs. (23)–(26)]. Using the parameters of the IC (a and b in Smylie et al., 1984), $\Delta\rho$ according PREM, M according IERS standard and the values of $\varphi=\varphi_{\mathrm{f}}$ and $\vartheta=\vartheta_{\mathrm{f}}$ shown in Fig. 4, Greiner-Mai and Barthelmes (2001) obtained in the IC-fixed coordinate system the following value for C_{20}:

$$C_{20} = -1.240 \times 10^{-8},$$

and for the temporal variations of the transformed coefficients $\Delta C_{nm}(\varphi_{\mathrm{f}}\,(t); \vartheta_{\mathrm{f}}\,(t))$ and $\Delta S_{nm}(\varphi_{\mathrm{f}}\,(t), \vartheta_{\mathrm{f}}\,(t))$ ($n=2$; $m=0$, 1, 2) the curves shown in their Fig. 4. From these values they determined the time derivative of the coefficients by linear regression over the past 10 years, presented in Table 1.

For comparison, Table 2 presents the accuracy of the low-degree coefficients of recent gravity models, e.g. GRIM4 (Schwintzer et al., 1997) and the expected accuracies of the current CHAMP (Reigber et al., 1997) and GRACE (Tapley, 1997) satellite missions.

Because of the small gravity changes caused by the theoretical IC motion, it has been impossible to prove the IC hypothesis until now. Furthermore, the major difficulty will be the separation of gravity variations caused by surface processes. Nevertheless, expected improvements in the current gravity models by the just mentioned satellite missions may allow the IC hypothesis to be verified in future.

Table 2
Estimated standard deviation of low-degree ($n<5$) spherical harmonic coefficients and their mean time derivatives

model	$\sigma(C/S)$	$\sigma(\frac{\mathrm{d}}{\mathrm{d}t}$(C/S)) [year^{-1}]
GRIM4	2×10^{-10}	4×10^{-12}
CHAMP	2×10^{-11}	1×10^{-12}
GRACE	2×10^{-12}	1×10^{-13}

4. Conclusions

With respect to core–mantle coupling, the major problem is to find a uniform model for the parameters and geometrical structure of the core-mantle transition zone that consistently explains the different magnitudes of the axial and non-axial torque components necessary for the excitation of decadal ΔLOD and Δm, respectively.

The situation is similar in the angular momentum approach where the balance of the relative angular momenta of the core and the mantle can be reached for the axial component, while the existence of a sufficiently strong non-axial component is not yet proved.

Finally, we have outlined an alternative model of relative inner-core motion that explains polar-motion variations by changes in mass geometry within the core which can possibly be tested by modern gravity measurements. The major problem is that the variation in the orientation of the figure axis of the inner core and the maintenance of its finite tilt versus known restoring forces cannot be explained by a dynamic model. Respective ideas mentioned at the end of Section 3.1 may be a subject of future investigations.

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