Available online at www.sciencedirect.com
SCIENCE DIRECT®

Journal of Hydrology 278 (2003) 144–152

Journal of Hydrology
www.elsevier.com/locate/jhydrol

Utilization of regression models for rainfall estimates using radar-derived rainfall data and rain gauge data

Zbyněk Sokol*

Institute of Atmospheric Physics, Academy of Science of the Czech Republic, Boční II, 1401, 141 31 Prague 4, Czech Republic

Received 28 January 2002; accepted 28 March 2003

Abstract

The procedure estimating hourly rainfalls by merging radar-derived rainfalls and gauge measurements is developed and tested. It uses simple linear regression, which is complemented by the normalization and correction of distribution. The data from radar Tulsa, Oklahoma, Weather Surveillance Radar-1988 Doppler version and rain gauge data from the radar domain are used. The quality of estimates is evaluated against independent rain gauges by the root-mean-square-error, bias and correlation coefficient in dependence on the density of a gauge network. The results indicate that even a sparse gauge network (about 50 gauges, i.e. 4000 km^2 per one gauge) is sufficient to improve the radar-derived rainfalls. The improvement increases with the number of gauges.

Keywords: Rainfall; Radar; Rain gauge; Regression

1. Introduction

Weather radar provides high-resolution rainfall fields. However, their quantitative use is restricted by errors and uncertainty in derived precipitation estimates. The sources of errors and their relative influence are described at length in the literature (e.g. Joss and Waldvogel, 1990; Collier, 1996; Harrison et al., 2000). There are two basic approaches to the correction of radar-derived rainfalls.

The first approach is based on the identification and correction of vertical profiles of reflectivity (VPR). The analytic methods use radar data from several radar beam elevations to retrieve VPRs by the application of the radar equation. Assuming spatial uniformity of the VPR the data within few tens of kilometres from the radar position are utilised and the derived VPR is applied to correct data from longer ranges (e.g. Andrieu and Creutin, 1995; Borga et al., 1997). The original assumption of spatial uniformity of VPR was later extended to the local spatial uniformity of VPR (Vignal et al., 1999). Other methods rely on physical-statistical models. They utilise additional independent meteorological data and consist of two steps: identification of the VPR and correction of the identified VPR (Kitchen et al., 1994; Joss and Lee, 1995; Kitchen, 1997).

The second approach is based on the adjustment of radar-derived precipitation using gauge data. Most of these methods stem from the gauge-to-radar (G/R) statistical adjustment technique. The aim is to correct radar-derived precipitation to the quantitative level of gauge measurements. The methods differ in input

* Fax: +420-2-72764745.
E-mail address: sokol@ufa.cas.cz (Z. Sokol).

0022-1694/03/$ - see front matter
doi:10.1016/S0022-1694(03)00139-2

data. They either use radar precipitation, and gauge precipitation is utilised only to derive the parameters of the statistical model (e.g. Gabella and Amitai, 2000; Kráčmar et al., 1998), or merge both types of data at each application (e.g. Seo and Breindenbach, 2001; Harrison et al., 2000; Gibson, 2000; Michelson and Koistinen, 2000). The latter approach allows to modify model parameters continuously in relation to recent data. The rain gauge adjustment method deals with all sources of radar errors in a single process. The limited representativeness of the gauge measurements is the main disadvantage of this method (Groisman and Legates, 1994; Collier, 1996).

In this study, a method merging gauge and radar-derived precipitation is presented. The aim is to estimate rainfalls in sites where the gauges are not located. The method is based on the application of a regression model, normalization and correction of distribution. The regression model uses ground precipitation values instead of the ratio G/R as the dependent variable (predictand). The root-mean-square-error between the model results and the rain gauge measurements, which are used as the ground truth, is considered as the basic measure of the accuracy.

This paper is organized in the following way. In Section 2 the input data are described. The rainfall estimation procedure is presented in Section 3 and the ways of its validation are described in Section 4. Section 5 contains the results, and conclusions are briefly summarised in Section 6.

2. Data

In this study, hourly radar rainfall data from radar Tulsa, Oklahoma, Weather Surveillance Radar-1988 Doppler version and hourly rain gauge data under the radar umbrella were used. The radar rainfall data (RADX) were derived by using the algorithm that is described by Fulton et al. (1998). The radar rainfalls cover a square region of 131 × 131 pixels. The size of the pixel is 4 km × 4 km and each rain gauge was assigned to one pixel. There were no pixels containing more than one gauge. For this study, only the pairs of corresponding rain gauge and RADX hourly rainfalls were available. The data covered the period from May to September 1997 and contained only the terms with at least one nonzero rain gauge measurement. There were 1609 terms and more than 300 000 pairs in the data set. It contained data from 211 gauges and only 7% of gauge measurements recorded nonzero rainfalls. The number of rainfall values exceeding 30 and 50 mm was 153 and 23 respectively. The maximum measured value was 83.3 mm.

The data pairs were checked to remove erroneous or significantly anomalous values. The checking procedure consisted of three steps, the parameters of which were subjectively selected with the aim to maintain as many data as possible. In the first step, rain gauges with less than 500 measurements, which represented 31% of the total number of available terms, were excluded. It reduced the total number of gauges from 211 to 197.

In the second step, the pair differences were calculated. Twelve single pairs with the absolute difference exceeding the threshold 50 mm were removed from the data set. The threshold 50 mm was subjectively selected.

In the third step, the measurements of individual gauges G_i, $i = 1,\ldots n$, (index i denotes terms and n is the number of terms) were compared with the corresponding radar data $\mathrm{RADX_i}$. When the correlation coefficient $\mathrm{CC(RADX,G)} < 0.35$ or when $0.3 \le \mathrm{RADX_{mean}}/\mathrm{G_{mean}} \le 3.0$ was not true, where

$$\mathrm{RADX_{mean}} = \frac{1}{n}\sum_{i=1}^{n}\mathrm{RADX}_i,\qquad G_{\mathrm{mean}} = \frac{1}{n}\sum_{i=1}^{n}G_i, \tag{1}$$

$$\mathrm{CC}(\mathrm{RADX},G) = \frac{\sum_{i=1}^{n}(\mathrm{RADX}_i-\mathrm{RADX_{mean}})(G_i-G_{\mathrm{mean}})}{\sqrt{\sum_{i=1}^{n}(\mathrm{RADX}_i-\mathrm{RADX_{mean}})^2\sum_{i=1}^{n}(G_i-G_{\mathrm{mean}})^2}}, \tag{2}$$

then the gauge was excluded from the data set. This requirement reduced the number of rain gauges to 182. The rain gauge data, which had passed through the checking procedure, were considered sufficiently representative for the radar adjustment procedure.

The estimations of small rainfall amounts are not important and therefore the data set was reduced to contain only the terms with at least one gauge measurement ≥5 mm. Moreover, the terms that

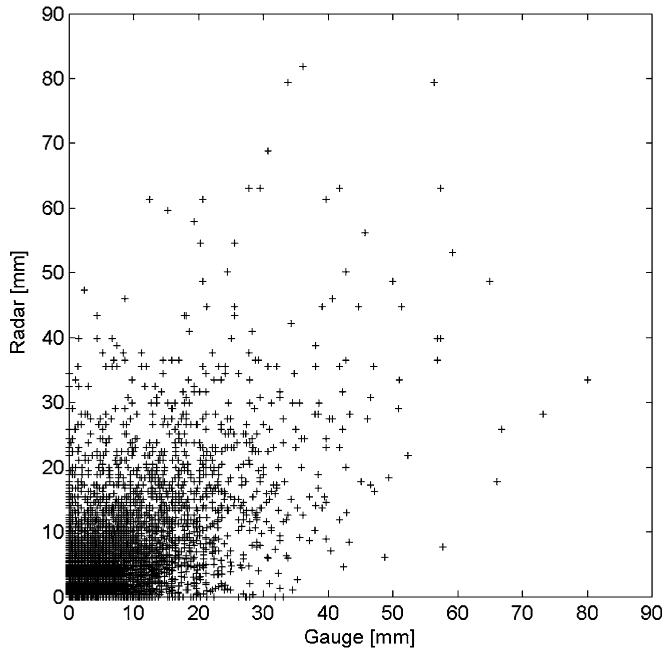

Fig. 1. The scatter plot of all pairs of data used in the study.

contained less than 150 gauge measurements were excluded. The aim of this requirement was to have enough data to study the influence of the rain gauge density on the accuracy of rainfall estimates.

The resultant data set contained 527 terms with 85 092 pairs of gauge and radar rainfalls and 16% of them contained nonzero gauge rainfalls. The scatter plot of all pairs of radar and gauge data (Fig. 1) as well as the mean values of radar and gauge rainfalls at gauge positions (Fig. 2) indicate reasonable quality of both radar and gauge data. The gauge positions are depicted in Fig. 3 (subfigure 1N).

The checked data were divided into two subsets with approximately the same size, which were alternatively used as calibration and verification data sets. The first set (1997a) contained data from May, June, and July (264 terms); the second one (1997b) contained data from July, August, and September (263 terms). The basic statistics of the data sets 1997a and 1997b, which are given in Table 1, were different for both gauge measurements and radar-derived rainfalls. A visible difference is in the ratio of radar and gauge mean rainfalls, which is 0.93 for the first data set and 0.74 for the second one. It follows that the radar underestimated differently gauge rainfalls for the data sets.

3. Rainfall estimation procedure

The rainfall estimation procedure, which is presented in this study, consists of three steps. They are applied successively one after the other: a) application of a regression model, b) normalization, and c) correction of distribution. The regression model provides the first rainfall estimate, which is further corrected by the normalization and by the correction of distribution.

3.1. a) Linear regression model

A multiple linear regression model was applied to describe the relationship between the dependent variable (predictand) and predictors in arbitrary pixel (i,j) and for each term. Two predictors were used: (i) RADX(i,j) and (ii) the rainfall estimate, GINT(i,j), which was calculated as a weighted average of gauge rainfalls

$$\mathrm{GINT}(i,j) = \frac{\sum_{k=1}^{N_S} w_k \mathrm{G}_k}{\sum_{k=1}^{N_S} w_k}, \tag{3}$$

where $N_S = 10$ and G_k are rainfalls from the N_S nearest gauges to the pixel (i,j). The weights w_k

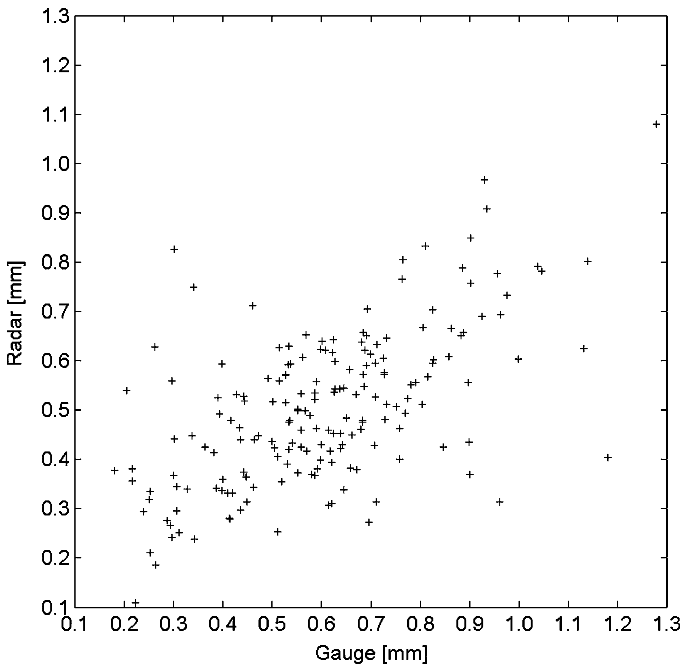

Fig. 2. The mean values of radar-derived rainfalls and related rain gauge measurements over all data used.

depend on the Euclidian distance r_{kij} (in pixels) between the pixel (i,j) and the gauge position (k) by the formula

$$w_k = \exp(-\alpha r_{kij}) \quad (4)$$

The value of α was determined by tests, which compared the accuracy of the interpolation in Eq. (3) for various values of α. For each pixel containing a rain gauge, the interpolation was performed from neighbouring gauge measurements. The interpolated and measured values were compared by using the root-mean-square-error. The parameter $\alpha = 0.5$ yielded the lowest error and therefore it was used in the next calculations.

Besides the RADX and GINT, other predictors, which described the position of the pixel (e.g. distance from the radar, elevation), were also tested. However, they did not appear representative and they were not retained.

The linear regression model was in the form of

$$\mathrm{REG} = a_0 + a_1\mathrm{RADX} + a_2\mathrm{GINT}, \quad (5)$$

where a_0, a_1 and a_2 are regression coefficients. Following the common concept of the ground truth, rainfalls measured by rain gauges G, were used as predictands REG in Eq. (5). The model coefficients were determined to minimize

$$\Theta(a_0,a_1,a_2) = \sum_{s=1}^{M_c}(a_0 + a_1\mathrm{RADX}_s + a_2\mathrm{GINT}_s - G_s)^2, \quad (6)$$

where G_s, $\mathrm{RADX_s}$ and $\mathrm{GINT_s}$, $s = 1,\ldots,M_c$ are all available triplets of values (from the same pixel) from the calibration data set. The $\mathrm{GINT_s}$ was calculated without using the gauge rainfall G_s. When the model in Eq. (5) was applied, the negative values of the REG were set to 0.

3.2. b) Normalization

The aim of the normalization step is to reduce the multiplicative error of the REG. It appears when the ratio of radar and gauge mean rainfalls is different for calibration and verification data sets. The normalization is performed separately for each

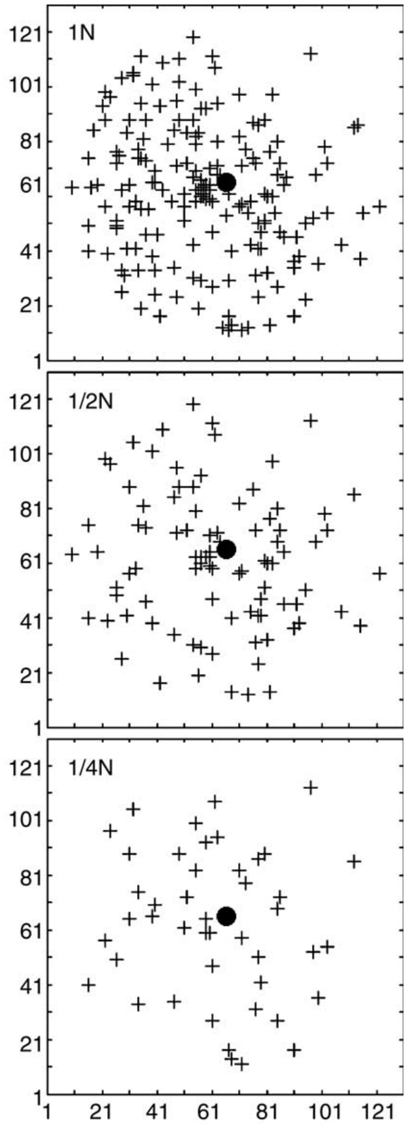

Fig. 3. Rain gauge distributions for 1N, 1/2N and 1/4N networks (crosses) and the radar position (black circle).

term. It consists in the multiplication of the REG by a factor q

$$q = \begin{cases} \dfrac{\sum_{k=1}^{N_\mathrm{T}} G_k}{\sum_{k=1}^{N_\mathrm{T}} \mathrm{REG}_k} & \text{if the number pairs with } G_k > 0 \text{ and } RADX_k > 0 \text{ is greater than } N_\mathrm{P} = 10 \\ 1 & \text{otherwise} \end{cases} \quad (7)$$

Table 1
Mean gauge and radar-derived rainfalls (mm) for specified categories of gauge rainfalls related to the data sets 1997a and 1997b

Category	1997a			1997b		
	All data	>0 mm	>20 mm	All data	>0 mm	≥ 20 mm
Mean–gauge	0.617	4.01	29.57	0.590	3.49	28.24
Mean–radar	0.573	3.44	19.65	0.439	2.40	18.37
N	41908	6454	218	43184	7292	174

N is the number of data pairs.

where N_T is a number of gauge and radar pairs (G_k, REG_k, $k = 1,\ldots,N_\mathrm{T}$) for the given term. The number N_p was subjectively determined by experiments. In order to avoid extreme changes of the REG, the values of q were restricted to the interval $<0.3,3>$. The normalized rainfall estimates REGN $= q$*REG were calculated for both the calibration and verification data sets.

3.3. c) Correction of distribution

The REGN values overestimate low precipitation and underestimate heavy precipitation, which results in the difference between distribution functions of actual and calculated rainfalls. The following algorithm was applied to modify the distribution of REGN rainfalls. All gauge rainfalls from the calibration data set and corresponding REGN values were arranged separately in ascending order. Let $g_1 \leq g_2 \leq \ldots \leq g_{Mc}$ and $e_1 \leq e_2 \leq \ldots \leq e_{Mc}$ be the sequences of gauge rainfalls and REGN values, respectively. Further, let $d_1 < d_2 < \ldots < d_\mathrm{M}$ be the sequence of given rainfall values. Then for each d_i, the corresponding r_i value is found to have the same order in the sequence e_i, $i = 1,\ldots,M_\mathrm{c}$ as d_i in the sequence g_i, $i = 1,\ldots,M_\mathrm{c}$. This can be mathematically expressed as follows $r_i = e_L$, where index $L = \max_s (g_s < d_i)$ The rainfall estimate with the corrected distribution (REGND) was obtained by using the following relationships

$$\mathrm{REGN}' = \mathrm{REGN}, \qquad \text{if} \quad \mathrm{REGN} < r_1, \quad (8a)$$

$$\mathrm{REGN}' = \mathrm{REGN} + d_M - r_M, \quad \text{if} \quad \mathrm{REGN} > r_{M_c}, \tag{8b}$$

$$\mathrm{REGN}' = d_i + (d_{i+1} - d_i)(\mathrm{REGN} - r_i)/(r_i + 1 - r_i), \quad \text{if}\, r_i \le \mathrm{REGN} \le r_{i+1}, \tag{8c}$$

$$\mathrm{REGND} = \left(\sum_{s=1}^{M_c} \mathrm{REGN}_s / \sum_{s=1}^{M_c} \mathrm{REGN}'_s \right) \mathrm{REGN}'. \tag{8d}$$

The aim of the Eq. (8d) is to maintain the total sum of corrected data. In this study five d_i values 0.1, 1, 5, 10 and 30 were used. In order to determine the d_i values the procedure described by Eq. (8a)–(8d) was applied to several subjectively selected sets of d_i values. The resultant REGND values were compared with gauge rainfalls at the verification data sets. The d_i values, which yielded the lowest root-mean-square-error, were further used. The transformation does not change the order of values. Therefore, if the REGN values well reflect the order of actual values, the improvement of the distribution can also improve the accuracy of the estimates.

4. Validation

The rainfall estimations were evaluated against the independent rain gauges by using root-mean-square-error (RMSE), CC(P,G) (see Eq. (2)) and bias (BIAS)

$$\mathrm{RMSE} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (P_k - G_k)^2}, \tag{9}$$

$$\mathrm{BIAS} = \sum_{k=1}^{n} P_k / \sum_{k=1}^{n} G_k, \tag{10}$$

where G_k are gauge rainfalls, P_k are estimated rainfalls (REG, REGN and REGND) and n is the number of considered pairs $(G_k,\ P_k)$. The RMSE, CC and BIAS were calculated at the verification data sets for two rainfall categories: (i) the pairs $(G_k,\ P_k)$, where $G_k >$ 0 mm; (ii) the pairs $(G_k,\ P_k)$, where $G_k > 20\ mm$. In addition the BIAS was also calculated for all the data (from the verification data sets) because it is an important characteristic from the climatological viewpoint. The aim of two rainfall categories was to distinguish between non-zero and heavy rainfalls.

In order to study the influence of the gauge density on the accuracy of the procedures a similar approach was applied as in (Seo and Breindenbach, 2001). The whole network (1N), one-half (1/2N) and one-fourth networks (1/4N) were used in the tests. The 1/2N and 1/4N networks were randomly selected from the 1N network (Fig. 3). The densities of gauges corresponding to 1N, 1/2N and 1/4N networks are approximately 1000, 2000 and 4000 km^2 per one gauge.

In the tests, the measurements from one gauge were excluded from the data, and the corresponding adjusted pixel estimate was compared with these measurements. All gauges were gradually excluded and that enabled the independent verification at each gauge position, regardless of the network used.

5. Results

In this section, the accuracy of the procedure described in Section 3 is addressed, and the effect of the normalization and correction of distribution is evaluated. The results are different for the single verification data sets 1997a and 1997b. The estimates for the 1997b set were slightly worse. However, in order to simplify the comparison, the RMSE and CC were averaged over the verification sets. The BIAS was evaluated separately for both verification sets to avoid the bias compensation by averaging. The RMSE and CC were evaluated in dependence on the gauge network (1N, 1/2N and 1/4N).

The RMSE and CC of the RADX, REG, REGN and REGND are compared in Fig. 4. The REG decreases the RMSE of RADX for non-zero precipitation. However, for heavy precipitation the REG slightly improves the RMSE of RADX only for the 1N network. The normalization (i.e. REGN) yields lower RMSE than both the REG and RADX for all the precipitation categories and gauge networks. The correction of distribution (i.e. REGND) improves estimates of heavy precipitation, while for non-zero precipitation the RMSE of the REGND is slightly worse than that of the REGN. All the models yield higher CC than the RADX with one exception (heavy precipitation and 1/4N network). Although the REGN provides the highest CC in most cases, the results of REGND are very similar.

The BIAS is displayed in Fig. 5 for three categories of actual precipitation. The problem,

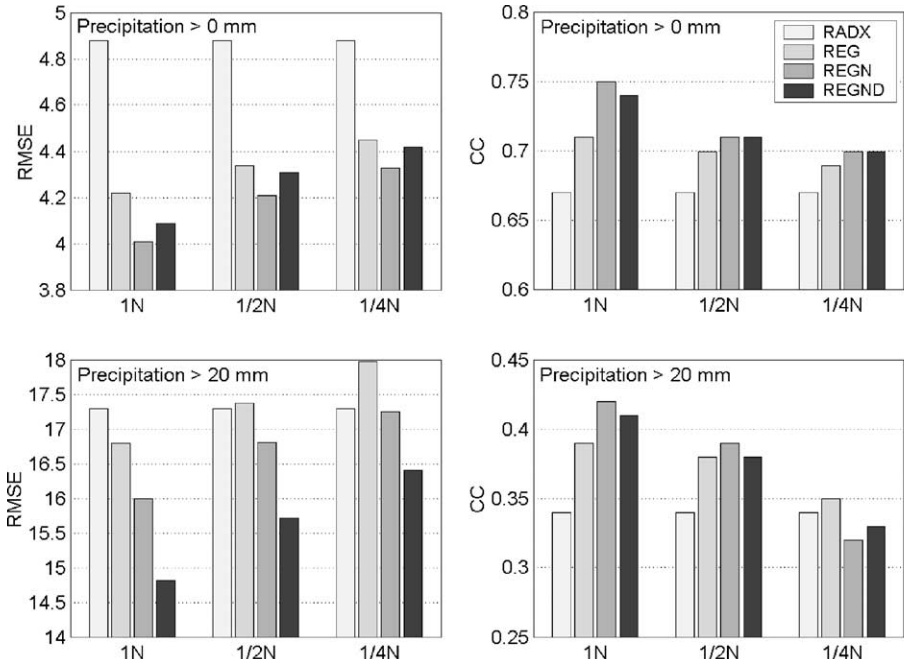

Fig. 4. The comparison of the RMSE and CC of the RADX, REG, REGN and REGND for two precipitation categories in dependence on the rain gauge network.

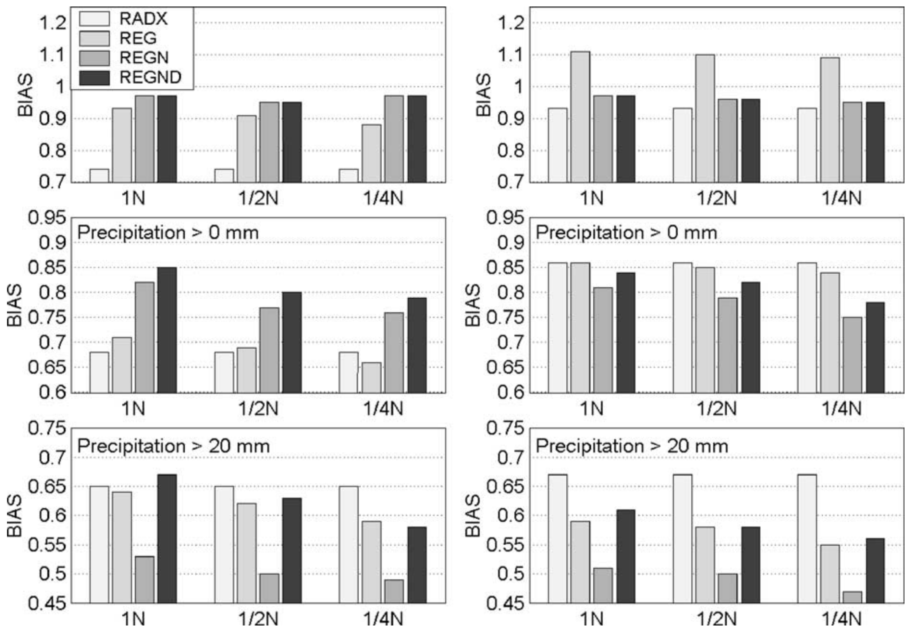

Fig. 5. The BIAS of the RADX, REG, REGN and REGND calculated at 1997b (left column) and 1997a (right column) verification data sets. The BIAS is displayed for all data (upper figures) and two precipitation categories in dependence on the rain gauge network.

Table 2
The regression coefficients of the REG model in dependence on the gauge network

Predictor	1N	1/2N	1/4N
Constant	0.08608	0.09812	0.11653
RADX	0.57952	0.63020	0.66363
GINT	0.32342	0.27255	0.19894

The linear regression model was developed by using 1997a data set.

which can occur by using the regression model, is indicated in the upper subfigures related to the evaluation of all data. The regression model corrects the bias of radar rainfalls at the calibration data. However, if the radar bias is different for the calibration and verification data, which is this case, the regression correction is wrong, as the upper right subfigure of Fig. 5 shows. This is the reason for the application of the normalization. The comparison of the BIAS of the RADX, REG and REGN for all data shows quite good reduction of the bias by the normalization. The results of REGN and REGND are identical for all data because of (8d). For the categories with non-zero and heavy precipitation, the results are different for 1997a and 1997b data sets. The correction of distribution improves the BIAS of the REGN. However, for heavy precipitation the best results are obtained by the RADX.

The REG model coefficients, derived from the 1997a calibration data, are shown in Table 2. The ratio between the coefficients of the RADX and GINT in dependence on the gauge network well describes decreasing reliability of the GINT with the decreasing number of gauges.

6. Conclusions

In this study, the procedure estimating hourly rainfalls in locations without rain gauges and covered by the radar, was developed and tested. The procedure merges radar and gauge data by using the regression technique. The regression model outputs were modified by the normalization and correction of distribution. The independent data were used to evaluate the procedure as well as its single steps.

If the bias of the radar-derived precipitation changes (e.g. it is different for calibration and verification data sets), the regression model is not able to remove it. However, the normalization, which is used in each application, reduces the bias. Consequently, it yields lower root-mean-square-error of rainfall estimates than radar-derived precipitation. When the normalization is complemented by the correction of distribution, the accuracy of the rainfall estimates slightly decreases. However, the accuracy of estimates of heavy precipitation apparently improves.

The root-mean-square-error of rainfall estimates depends on the rain gauge density. For the density 1000 km^2 per one gauge the procedure improves the radar-derived estimates by more than 15%. For the density 4000 km^2 per one gauge the improvement is about 10%.

Acknowledgements

The work was supported by the GA CR under the grant 205/01/1066 and by the GA ASCR under the grant S3042101. The author thanks the NWS, NOAA for providing the data used in this study. The author wishes to thank Jay Breindenbach and D.-J. Seo for helpful discussion during his visit to the NWS NOAA. The author thanks the anonymous referees for numerous comments, which significantly improved the text.

References

Andrieu, H., Creutin, J.D., 1995. Identification of vertical profiles of radar reflectivities for hydrological applications using inverse method. Part 1: formulation. J. Appl. Meteor. 34, 225–239.

Borga, M., Anagnostou, E.N., Krajewski, W., 1997. A simulation approach for validation of a brightband correction method. J. Appl. Meteor. 36, 1507–1518.

Collier, Ch.C., 1996. Applications of Weather Radar Systems, Praxis Publishing Ltd., ISBN 0-471-96013-6; pp. 390.

Fulton, R.A., Breidenbach, J.R., Seo, D.J., Miller, D.A., 1998. The WSR-88D rainfall algorithm. Wea. Forecasting 13, 377–395.

Gabella, M., Amitai, E., 2000. Radar rainfall estimates in an alpine environment using different gage adjustment techniques. Phys. Chem. Earth (B) 25, 927–931.

Gibson, M., 2000. Comparative study of several gauge adjustment scheme. Phys. Chem. Earth (B) 25, 921–926.

Groisman, P.Y., Legates, D.R., 1994. The accuracy of United States precipitation data. Bull. Amer. Soc. 75, 215–227.

Harrison, D.L., Driscoll, S.J., Kitchen, M., 2000. Improving precipitattion estimates from weather radar using quality control and correction techniques. Meteorol. Appl. 6, 135–144.

Joss, J., Waldvogel, A., 1990. Precipitation Measurements and Hydrology. In: Atlas, D., (Ed.), Radar in Meteorology, Amer. Meteor. Society, pp. 577–606.

Joss, J., Lee, R., 1995. The application of radar gauge comparisons to operational precipitation profile corrections. J. Appl. Meteor. 34, 2612–2630.

Kitchen, M., 1997. Towards improved radar estimates of surface precipitation rate at long range. Q. J. R. Meteorol. Soc. 123, 145–163.

Kitchen, M., Brown, R., Davies, A.G., 1994. Real time correction of weather radar for the effects of bright band range and orographic growth in widespread precipitation. Q. J. R. Meteorol. Soc. 120, 1231–1254.

Kráčmar, J., Joss, J., Novák, P., Havránek, P., Šálek, M., First steps towards quantitative usage of data from Czech weather radar network. COST75 Advanced Weather Radar Systems. COST75 Final Seminar, Locarno, Switzerland, 23–27 March. 1998; pp. 91–101.

Michelson, D.B., Koistinen, J., 2000. Gauge-Radar network adjustment for the baltic sea experiment. Phys. Chem. Earth (B) 25, 915–920.

Seo, D.J., Breindenbach, J.P., 2001. Real-Time correction of spatially nonuniform bias in radar rainfall data using rain gauge measurements. Submitted for publication.

Vignal, B., Andrieu, H., Creutin, J.D., 1999. Identification of vertical profiles of reflectivity from volume scan radar data. J. Appl. Meteor. 38, 1214–1228.