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Seasonal flow frequency analysis

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Abstract

Numerous hydrologic designs require seasonal discharge estimates. Given the seasonal variation in the distribution of rainfall, significant differences can exist between the annual maximum and seasonal maximum T-yr return period discharges. Recommendations for making seasonal flood frequency analyses for gauged and ungauged locations are presented. When performing a seasonal frequency analysis of gauged data, missing discharge data will generally be a problem. Two potential solutions to the problem of incomplete records are introduced. First, a maximum likelihood approximation that replaces missing discharges below a threshold is developed and tested. Results of simulations indicate that it is more accurate than the method demonstrated in Bulletin 17B. Second, the ratio of the measured instantaneous to mean daily discharge is regionalized. This provides a method of replacing missing, below threshold, discharges. The ratio of measured instantaneous discharges above a threshold and the corresponding mean daily discharges can be used for predicting missing instantaneous discharges in seasonal records. Once regionalized, the method can be used for developing seasonal frequency curves at ungauged locations with either the USGS regression equations or a rainfall-runoff model. The Eastern Coastal Plain data are used to demonstrate the development of regional index ratios.

Keywords: Frequency analysis; Discharge estimation; Incomplete data records; Seasonality; Statistics; Regionalization; Seasonal flows

1. Introduction

Many hydrologic design problems are based on the analysis of annual maximum flood (AMF) series; for example, they are used for establishing the boundaries of the T-yr floodplain and evaluating the effects of instream encroachments. It may not be appropriate to use an AMF analysis for other types of hydrologic design problems, such as cases where the instream activity is of short duration or the event occurs at a specific time of year. For example, the design of a small coffer dam that will be in place for only a couple of months may be over-designed if the design flood is based on the AMF analysis, especially if the period in which the coffer dam is in place is during a drier season of the year. If T-yr flood discharges are needed for the growing season or a period of fish spawning, they should be estimated from a seasonal flow frequency analysis rather than an annual maximum flow frequency analysis. For cases where a seasonal frequency analysis is appropriate, the use of an AMF analysis may cause a design bias, reduce the accuracy of the design, and unnecessarily increase the cost of the project.

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The issue of seasonal flood frequency analysis was identified as early as 1951 (Creager et al., 1951).

Most gauged records include all instantaneous maximum discharges above a threshold. Unfortunately, seasonal flood records are often incomplete in that for many years of the gauged record the largest seasonal flow was below the threshold. If a seasonal flow record includes values for every year of the duration of gauging, then the data could be analyzed using a standard log-Pearson III analysis (IACWD, 1982). A complete seasonal record is unlikely to be the norm unless the season represents a major portion of the year and includes the wet season. Where the seasonal instantaneous maximum flood record is incomplete, estimation of seasonal T-yr discharges requires either filling in the missing values or a method of analysis other than a univariate frequency analysis.

Bulletin 17B (B17B) (IACWD, 1982), which is still the standard procedure for frequency analysis in the US, presents a method for handling incomplete flood records, which is referred to as the conditional probability adjustment. This is based on a procedure by Jennings and Benson (1969). The analysis involves graphing the fitted population curve not with the probabilities based on the LP3 deviate table but with probabilities adjusted by the ratio of the number of measured flows available to the length of gauging record. The 2-, 10-, and 100-yr flows estimated from the adjusted graph are then used to compute synthetic estimates of the population moments from which the final frequency curve is computed.

In addition to the B17B method, other methods have been proposed. Aron and Rachford (1974) provide methods for replacing the missing values including adjusting the ranks of the measured series, generating missing values using regression with another gauging station, and a rank-matching method with another series. Hershfield and Wilson (1960) discussed the problem for rainfall depths, and Waylan and Woo (1982) examined the problem of floods of mixed-process origin. These proposals do not address the issue of estimates at ungauged locations.

Seasonal flow frequency curves are more frequently needed at ungaged locations. Existing procedures, both T-yr peak discharge regression equations and design storm rainfall-runoff models, are intended to yield T-yr annual maximum peak discharges or hydrographs, not seasonal estimates. Regression equations are generally calibrated from annual maximum peak discharges and the intensity–duration–frequency curves used to obtain the rainfall depth for the hydrograph methods are based on annual maximum rainfalls. Therefore, neither can be used directly to obtain seasonal flow frequency discharges.

Two problems have been identified. First, in order to develop seasonal flow frequency estimates at gauged sites, a method for analyzing incomplete flow records must be adopted. Second, since seasonal flow frequency discharges are often needed at ungauged sites, a method is necessary for adjusting annual maximum discharge estimates into seasonal estimates. These two issues are addressed by this work.

2. Moment estimation from incomplete records

Records of measured maximum discharges often include only those values above a threshold, where the threshold is low enough to include every annual maximum; however, the threshold is not usually low enough to enable a complete maximum discharge record to be compiled for each season. This would be the case especially where the rainfall distribution across seasons is characterized by high variation, with some seasons being much below average and some much above average. If seasonal maximum discharge records are incomplete, it is not possible to use sample moments computed from the available data as estimates of the population moments. Thus, a method of obtaining estimates of the population moments from incomplete records is needed. A procedure will be developed using maximum likelihood estimation (Benjamin and Cornell, 1970; McCuen and Snyder, 1986).

The objective is to derive expressions for estimating the population parameters of a normal distribution given an incomplete record. The record length H consists of N measured values above the threshold and M unmeasured values below the threshold discharge X_0, i.e. $H = N + M$. The assumption of normality is made to make the calculations tractable; however, the results can be applied to the logarithms of the data to reflect a log-normal analysis.

The aim of the analysis is to provide estimates of the location μ and scale σ parameters of the normal distribution from an incomplete record. Similar analyses, but with respect to historic and paleoflood information, were made by Stedinger and Cohn (1986) and Cohn et al. (1997). If we denote the discharges above the threshold as X and below the threshold as W, the following likelihood function can be formulated for the stated conditions:

$$L(\mu,\sigma|X_1,X_2,\ldots,X_N,W_1,W_2,\ldots,W_M) = \prod_{i=2}^{N} f[(X_i|X>X_0)|\mu,\sigma]\prod_{j=1}^{M} f[(W_j|W<X_0)|\mu,\sigma] \tag{1}$$

Eq. (1) gives the likelihood of parameters μ and σ given that sampling has yielded N systematic values X_i above the threshold and M unmeasured values W_j below the threshold. The first product gives the likelihood of getting the sample values X_i given population parameters μ and σ. The second product gives the likelihood of experiencing M flows W_j that are below the threshold.

The conditional probabilities of Eq. (1) can be reformulated by explicitly defining the sample spaces. Simplifying the likelihood function on the left-hand side of Eq. (1) to L, Eq. (1) becomes:

$$L = \prod_{i=1}^{N} \frac{f(X_i|\mu,\sigma)}{\int_{X_0}^{\infty} f(X>X_0|\mu,\sigma)\mathrm{d}x} \times \prod_{j=1}^{M} \frac{f(W_j|\mu,\sigma)}{\int_{-\infty}^{X_0} f(W<X_0|\mu,\sigma)\mathrm{d}w} \tag{2}$$

Inserting the normal density function into Eq. (2) gives:

$$L = \left\{ \frac{\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{X_i-\mu}{\sigma}\right)^2\right]}{\int_{X_0}^{\infty} f(X>X_0|\mu,\sigma)\mathrm{d}x} \right\}^N \times \left\{ \frac{\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{W_j-\mu}{\sigma}\right)^2\right]}{\int_{-\infty}^{X_0} f(W<X_0|\mu,\sigma)\mathrm{d}w} \right\}^M \tag{3}$$

The integrals in the denominator of Eq. (3) can be expressed in terms of the cumulative normal distribution $F(\,)$:

$$L = \frac{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^H \left\{\exp\left[-\frac{1}{2}\sum_{i=1}^{N}\left(\frac{X_i-\mu}{\sigma}\right)^2\right]\exp\left[-\frac{1}{2}\sum_{j=1}^{M}\left(\frac{W_j-\mu}{\sigma}\right)^2\right]\right\}}{[1-F(X_0)]^N[F(X_0)]^M} \tag{4}$$

The maximum of a likelihood function occurs for the same population parameters as the maximum of the logarithm of the likelihood function. The natural logarithm of Eq. (4) is:

$$\ln L = -H\ln\sigma - H\ln\sqrt{2\pi} - \frac{1}{2\sigma^2}\times\left[\sum^{N}(X_i-\mu)^2 + \sum^{M}(W_j-\mu)^2\right] - N\ln[1-F(X_0)] - M\ln[F(X_0)] \tag{5}$$

Taking the derivatives of Eq. (5) with respect to the unknowns μ and σ and setting them to zero yields:

$$\frac{\partial(\ln L)}{\partial\mu} = 0 = \frac{1}{\sigma^2}\left[\sum^{N}(X_i-\mu) + \sum^{M}(W_j-\mu)\right] + \frac{\partial F(X_0)}{\partial\mu}\left[\frac{N}{1-F(X_0)} - \frac{M}{F(X_0)}\right] \tag{6a}$$

$$\frac{\partial(\ln L)}{\partial\sigma} = 0 = \frac{-H}{\sigma} + \frac{1}{\sigma^3}\left[\sum^{N}(X_i-\mu)^2 + \sum^{M}(W_j-\mu)^2\right] + \frac{\partial F(X_0)}{\partial\sigma}\left[\frac{N}{1-F(X_0)} - \frac{M}{F(X_0)}\right] \tag{6b}$$

Assuming that the threshold X_0 is small relative to the mean, i.e. X_0 is in the tail of the distribution, the derivatives of Eqs. (6a) and (6b) should be small. This was confirmed by calculation. Thus, the last terms of Eqs. (6a) and (6b) are small relative to the first terms of Eqs. (6a) and (6b), with calculations suggesting that the values of the derivative terms being less than 3% of the sum of the other terms.

Under the assumption that it is reasonable to drop the terms of Eqs. (6a) and (6b) that include the derivatives, the equations reduce to:

$$0=\sum^{N}(X-\mu)+\sum^{M}(W-\mu)=\sum^{N}X+\sum^{M}W-H\mu \quad (7a)$$

and

$$0=-H+\frac{1}{\sigma^2}\left[\sum^{N}(X-\mu)^2+\sum^{M}(W-\mu)^2\right] \quad (7b)$$

Solving Eq. (7a) for μ and Eq. (7b) for the variance σ^2 yields

$$\mu=\frac{1}{H}\left(\sum^{N}X+\sum W\right) \quad (8a)$$

and

$$\sigma^2=\frac{1}{H}\left[\sum^{N}(X-\mu)+\sum^{M}(W-\mu)^2\right] \quad (8b)$$

The obvious problem with Eqs. (8a) and (8b) is that they depend on the unmeasured values below the threshold X_0, which are not part of the flood record.

To obtain the best estimators of the parameters μ and σ, Eqs. (8a) and (8b) suggest several options, as follows: (1) ignore the summations that include W and use H for computing μ and $H-1$ for computing σ^2; (2) ignore the summations that include W, but use N in place of H since the summations with X involve N values; and (3) replace the values of W with the threshold value X_0 and use Eqs. (8a) and (8b). A fourth option is to use the B17B method (IACWD, 1982) for incomplete records, which is referred to as the conditional probability adjustment.

A second problem with the maximum likelihood analysis (MLA) presented is that it is based on the normal distribution, which has zero skew. Since the non-zero skew Pearson III distribution is commonly used for flood frequency analyses, the above analysis would not provide an estimate of the standardized skew coefficient. The concepts suggested by Eqs. (8a) and (8b) would suggest that the following could be used to estimate the population skew coefficient γ:

$$\gamma=\frac{H\left(\sum^{N}(X-\mu)^3+\sum^{M}(W-\mu)^3\right)}{(H-1)(H-2)S^3} \quad (9)$$

The three options expressed above for estimating μ and σ^2 can be applied to the estimator of Eq. (9).

2.1. Simulation of seasonal flow records

To evaluate the three options for estimating the three moments for incomplete seasonal flood records, a Monte Carlo simulation experiment was used to evaluate the bias and accuracy of the estimators. Both estimators are expressed in relative terms, with the bias expressed as a fraction of the mean and the standard error expressed as a fraction of the standard deviation.

A Monte Carlo evaluation was made in which 50,000 log-Pearson III samples were generated and average values of the evaluation criteria computed. The experiments were repeated for different sample sizes (10, 25, and 50), different coefficients of variation (0.1, 0.2, 0.3), different threshold probabilities (5, 10, and 20%), and different skews (−0.5, 0.0, 0.5). The skew of the population did not influence the results. Table 1 provides some typical results for selected combinations. Of the three options, method 3 consistently had the smallest relative biases and relative standard errors. The coefficient of variation had very little effect on the accuracy, possibly because of the log transformation. The statistics did not change significantly as the record length was changed, although the standard error decreased, as expected, as the record length increased. As the threshold probability increased, which would reflect the condition of a greater number of missing discharges, both the bias and standard error tended to increase, as would be expected because the estimates would be based on fewer measured discharge values (i.e. the N values of X). The standard errors for the skew coefficient suggest that Eq. (9) provides a good approximation even through it is extrapolated from the results of a zero-skew, normal distribution maximum likelihood approximation (MLA).

2.2. Incomplete record adjustment: Bulletin 17B

The method developed using a maximum likelihood analysis yields estimates of the population moments without the necessity of graphing the data or assigning probability plotting positions to the data. B17B provides an alternative procedure for incomplete record analysis. It differs from the maximum likelihood approximation presented herein

Table 1
Relative bias (B_r) and relative standard error (S_r) of the estimated statistic (μ, mean; σ, standard deviation; and γ, skew) as a function of the coefficient of variation (C_v), threshold probability (P_t), and record length (H) for three estimation methods

Stat.	C_v	P_t	H	B_r for method			S_r for method		
				1	2	3	1	2	3
μ	0.1	0.9	25	0.020	−0.084	0.005	0.080	0.316	0.057
	0.2	0.9	25	0.040	−0.066	0.010	0.160	0.288	0.114
	0.3	0.9	25	0.059	−0.048	0.014	0.238	0.280	0.170
σ	0.1	0.9	25	−0.213	−0.297	−0.122	0.029	0.033	0.023
	0.2	0.9	25	−0.210	−0.293	−0.120	0.116	0.133	0.092
	0.3	0.9	25	−0.212	−0.295	−0.122	0.262	0.300	0.208
μ	0.2	0.95	25	0.012	−0.013	0.002	0.119	0.150	0.117
	0.2	0.90	25	0.040	−0.066	0.010	0.160	0.288	0.114
	0.2	0.80	25	0.070	−0.146	0.022	0.236	0.516	0.120
σ	0.2	0.95	25	−0.074	−0.098	0.000	0.093	0.094	0.093
	0.2	0.90	25	−0.210	−0.293	−0.120	0.116	0.133	0.092
	0.2	0.80	25	−0.256	−0.413	−0.263	0.136	0.169	0.123
μ	0.2	0.9	10	0.039	−0.066	0.009	0.206	0.388	0.176
	0.2	0.9	25	0.040	−0.066	0.010	0.160	0.288	0.114
	0.2	0.9	50	0.039	−0.066	0.010	0.140	0.247	0.083
σ	0.2	0.9	10	−0.155	−0.251	−0.067	0.161	0.163	0.139
	0.2	0.9	25	−0.210	−0.293	−0.120	0.116	0.132	0.092
	0.2	0.9	50	−0.231	−0.310	−0.140	0.103	0.125	0.076
γ	0.1	0.95	25	−0.181	−0.210	−0.110	0.184	0.211	0.146
	0.1	0.85	25	−0.174	−0.201	0.038	0.189	0.216	0.154
	0.1	0.75	25	−0.153	−0.186	0.028	0.197	0.242	0.202

in that it is a graphical method and that it uses an adjustment of the probability scale. The specific steps in making the B17B incomplete record analysis (note: the notation used here differs from the B17B notation) are:

1. Given N measured flows X_i above the threshold X_0 from a period of record of H years, compute values for the log-Pearson III frequency curve using the standard equation:

$$\log Q = \overline{Q} + K_i S_q \tag{10}$$

   in which $\overline{Q}$ and S_q are mean and standard deviation of the logarithms of the X_i values, respectively; and K_i is the LP3 deviate value for exceedence probability p_i and station skew g.
2. Compute the adjustment ratio $R = N/H$.
3. Multiply the probabilities p_i of step 1 by the ratio R of step 2; denote the adjusted probabilities as P_i. (Note: this adjustment is made to the probabilities associated with the LP3 deviates, not the plotting position probabilities.)
4. Plot the discharges of Eq. (10) versus the adjusted probabilities P_i of step 3 and draw the frequency curve to fit the P_i vs. log X_i values.
5. Using the curve of step 4, obtain estimates of the 2-yr (Q_2), 10-yr (Q_{10}), and 100-yr (Q_{100}) discharges.
6. Compute the synthetic statistics:

$$G_s = -2.5 + 3.12\frac{\log(Q_{100}/Q_{10})}{\log(Q_{10}/Q_2)} \tag{11a}$$

$$S_s = \frac{\log(Q_{100}/Q_2)}{K_{100} - K_2} \tag{11b}$$

$$\overline{X_s} = \log(Q_2) - K_2(S_s) \tag{11c}$$

B17B indicates that the procedure is acceptable as long as the ratio R is 75% or greater and that the skew is between −2.0 and +2.5. B17B does not discuss the accuracy of the method.

To evaluate the accuracy of the B17B procedure and compare it with the maximum likelihood approximation presented above, a Monte Carlo experiment was developed. The relative bias and relative standard errors were computed for a range of each of the following variables: record length $H = \{10, 25, 50\}$, coefficient of variation $C_v = \{0.1, 0.2, 0.3\}$, standardized skew coefficient $g = \{0.5, 0, -0.5\}$, and the truncation probability, $P_0 = \{0.95, 0.85, 0.75, 0.7, 0.6, and 0.5\}$. From the results of the simulations, the effects of the skew, the sample size, the coefficient of variation, and the threshold are analyzed separately.

2.3. Effect of population skew

The maximum likelihood analysis was based on the assumption of a normal distribution for the underlying population. Since a major use of the resulting method for analyzing incomplete records will be for use with the log Pearson type III distribution, the application of the rule of replacing missing values with the threshold to skewed data was investigated by simulations with the values of H, C_v, g, and P_0 previously given. Table 2 shows the results for a record length H of 50, a coefficient of variation of 0.3, and a threshold probability of 95%, and population skews of −0.5, 0.0, 0.5. In all cases, the maximum likelihood approximation showed less bias and better standard errors.

All simulations, including the case shown in Table 2, indicate that the accuracy of the method is insensitive to the population skew. This is also true for the B17B method. All variation of the statistics with the population skew was well within the sampling variation of the simulation. Therefore, other comparisons will only present results for the zero skew case.

2.4. Effect of sample size

Simulations were made for sample sizes of 10, 25, 50 and 75, with the bias and standard errors computed for both the maximum likelihood approximation and the B17B method. Again, simulations were made for the previously stated values of H, C_v, g and P_0. The results for one set of conditions are shown in Table 3 for the case of an 85% threshold probability, a coefficient of variation of 0.3, and a population skew of zero. As should be expected, the biases and standard errors decreased with increases in sample size. In general, and as evident in Table 3, the maximum likelihood approximation provided better results than the B17B method.

2.5. Effect of coefficient of variation

The coefficient of variation of the logarithms was varied from 0.1 to 0.3, and the biases and standard errors computed for both methods. The results in Table 4, which are typical of the results for all simulations, are for a sample size of 25, a threshold probability of 85%, and a skew of zero. The results for both the maximum likelihood analysis and B17B

Table 2
Variation of bias and accuracy of the mean and standard deviation with the population skew for the maximum likelihood approximation (MLA) and the Bulletin 17B method (B17B)

Method	Skew	Mean				Standard deviation			
		Bias	Relative bias	Std. error	Rel. std. error	Bias	Relative bias	Std. error	Rel. std. error
MLA	−0.5	0.015	0.005	0.013	0.002	−0.040	−0.047	0.0064	0.009
	0.0	0.014	0.005	0.013	0.002	−0.040	−0.047	0.0065	0.009
	0.5	0.016	0.006	0.013	0.002	−0.040	−0.047	0.0065	0.009
B17B	−0.5	0.042	0.015	0.014	0.002	−0.081	−0.097	0.0112	0.016
	0.0	0.041	0.015	0.014	0.002	−0.082	−0.097	0.0113	0.016
	0.5	0.043	0.015	0.014	0.002	−0.082	−0.097	0.0113	0.016

Table 3
Variation of bias and accuracy of the mean and standard deviation with sample size for the maximum likelihood approximation and the Bulletin 17B method

Method	Sample size	Mean				Standard deviation			
		Bias	Relative bias	Std. error	Rel. std. error	Bias	Relative bias	Std. error	Rel. std. error
MLA	10	0.092	0.031	0.064	0.007	−0.168	−0.186	0.048	0.059
	25	0.080	0.029	0.026	0.003	−0.148	−0.176	0.028	0.040
	50	0.080	0.029	0.016	0.002	−0.146	−0.174	0.024	0.034
	75	0.077	0.026	0.013	0.001	−0.087	−0.111	0.014	0.018
B17B	10	0.104	0.035	0.091	0.010	−0.201	−0.243	0.071	0.087
	25	0.138	0.049	0.038	0.005	−0.199	−0.237	0.047	0.067
	50	0.132	0.047	0.027	0.003	−0.194	−0.231	0.041	0.058
	75	0.116	0.039	0.023	0.003	−0.192	−0.213	0.041	0.050

methods indicate that the biases and standard errors are poorer for the larger coefficients of variation. This is expected, as the greater internal variation of the samples increases the spread about the mean. The maximum likelihood approximation yields smaller biases and standard errors than the B17B method.

2.6. Effect of threshold proportion

B17B limits the use of its technique such that the missing proportion of the duration of record must not exceed 25 percent, i.e. discharge values must be available for at least 75% of the years for which the site was gauged. The bias and accuracy of the maximum likelihood approximation were evaluated for threshold probabilities of 50, 60, 70, 75, 85, and 95 percent. The threshold probability is the proportion of the underlying population above the threshold discharge. Thus, it reflects the amount of record that is complete. For example, a threshold probability of 60% would indicate that in 40% of the years the largest value was below the threshold.

Ideally, the bias and accuracy would not get poorer as the threshold probability decreases. The values in Table 5 indicate that the quality of the estimated values of the standard deviation decreases with decreases in the threshold probability. At 75%, which is the bound for using the B17B procedure, the maximum likelihood approximation shows a relative under prediction of σ of 24%, although the relative standard error is a reasonable 7.7%. The statistics on the mean are much more encouraging with a small relative bias and relative standard error even for the 50% threshold probability. The results suggest that the maximum likelihood approximation can be used for a 50% threshold probability which is much less constraining than the 75% threshold imposed with the B17B procedure.

Table 4
Variation of bias and accuracy of the mean and standard deviation with the coefficient of variation (C_v) for the maximum likelihood approximation (MLA) and the Bulletin 17B method (B17B) for a sample size of 25

Method	C_v	Mean				Standard deviation			
		Bias	Relative bias	Std. error	Rel. std. error	Bias	Relative bias	Std. error	Rel. std. error
MLA	0.1	0.029	0.009	0.0033	0.0004	−0.053	−0.176	0.004	0.040
	0.3	0.080	0.029	0.0257	0.0033	−0.148	−0.176	0.028	0.040
B17B	0.1	0.039	0.013	0.0045	0.0005	−0.066	−0.221	0.006	0.061
	0.3	0.138	0.049	0.0377	0.0048	−0.199	−0.237	0.047	0.066

Table 5
Variation of bias and accuracy of the mean and standard deviation for selected threshold probabilities (p_0) of the maximum likelihood approximation for a sample size of 25

p_0	Mean				Standard deviation			
	Bias	Relative bias	Std. error	Rel. std. error	Bias	Relative bias	Std. error	Rel. std. error
0.95	0.005	0.0018	0.0033	0.0004	−0.015	−0.051	0.0015	0.0168
0.85	0.029	0.0095	0.0033	0.0004	−0.053	−0.176	0.0036	0.0395
0.75	0.050	0.0166	0.0045	0.0005	−0.079	−0.236	0.0070	0.0773
0.70	0.062	0.0208	0.0056	0.0006	−0.092	−0.305	0.0091	0.1008
0.60	0.092	0.0305	0.0096	0.0011	−0.117	−0.389	0.0143	0.1589
0.50	0.129	0.0429	0.0175	0.0019	−0.141	−0.471	0.0206	0.2290

2.7. Application

The procedure was applied to four watersheds, which ranged in area from 3.85 to 31.9 mi^2 and had annual maximum record lengths of 15, 29, 42, and 42 years. All of the watersheds are in the Eastern Coastal Plain of Maryland (see Table 6). The year was divided into four seasons: October–December, January–March, April–June, July–September. The number of years in which a seasonal discharge greater than the threshold discharge was available is also shown in Table 6. In only one season at one of the four gauges was the percentage greater than the 75% needed to use the B17B method for handling incomplete records. The maximum likelihood approximation was used instead. The percentage of years with a flow above the threshold was as low as 5%, as high as 76%, and averaged 45.1%. Thus, on the average, the threshold discharges were used to fill in more than 50% of the records.

The seasonal records were adjusted by using the threshold discharge when the actual value did not exceed the threshold. The annual maximum series weighted log skew was used with the computed seasonal log mean and log standard deviation (Eqs. (8a) and (8b), respectively) to compute seasonal discharges for six return periods (2, 5, 10, 25, 50, and 100 yr). The discharges for the six return periods, four seasons, and four gauges are given in Table 7 along with the discharges from the annual maximum series analysis. It is interesting to compare the computed 100-yr seasonal discharge to the annual maximum frequency curve and compute the return period of the seasonal 100-yr discharge as if it were an annual maximum discharge. The return periods of the seasonal 100-yr discharges when assessed using the annual maximum frequency curves are given in Table 6. About one-half of the seasonal 100-yr discharges would have a return period of less than 10 years when evaluated with the flood frequency curve of the annual maximum series. Given that the region is characterized by a relatively uniform distribution of rainfall, it is reasonable to expect that

Table 6
Seasonal frequency analysis of four coastal watersheds varying in record length (n) and drainage area

Gauge no.	n	Area (km^2)	Threshold discharge (m^3/s)	Record length for season				Percentage of n for season				Return period (yr) of 100-yr seasonal discharge for season			
				1	2	3	4	1	2	3	4	1	2	3	4
01483200	42	9.86	1.42	14	28	20	24	33	67	48	57	4	5	40	25
01483500	15	23.94	3.40	9	7	7	6	60	47	47	40	25	5	4	18
01483700	42	81.66	2.63	2	19	9	12	5	45	21	29	<2	22	4	22
01484000	29	34.82	3.68	12	18	22	13	41	62	76	45	4	7	5	30

Table 7
Annual and seasonal maximum discharges using a maximum likelihood approximation for adjusting incomplete seasonal records and the four-station mean ratios of the seasonal to annual flows

Gauge no.	Season	Discharge (m^3/s) for return period					
		2-yr	5-yr	10-yr	25-yr	50-yr	100-yr
1483200	Annual	4.02	7.02	9.48	13.22	16.45	20.13
	1	1.81	2.66	3.28	4.16	4.87	5.61
	2	2.10	3.14	3.91	4.98	5.86	6.77
	3	1.95	2.97	3.77	4.84	5.75	6.68
	4	2.32	4.25	5.89	8.44	10.73	13.36
1483500	Annual	5.89	10.99	16.11	25.31	34.71	46.91
	1	4.28	7.36	10.22	15.12	19.84	25.76
	2	3.99	5.32	6.31	7.79	8.98	10.31
	3	3.79	4.84	5.63	6.74	7.64	8.61
	4	3.99	6.37	8.47	11.86	15.03	18.80
1483700	Annual	14.10	24.24	13.70	43.57	53.00	63.17
	1	2.89	4.08	4.87	5.92	6.71	7.50
	2	5.15	10.87	16.05	24.32	31.82	40.54
	3	3.82	7.28	10.19	14.55	18.35	22.57
	4	4.53	9.82	14.72	22.65	29.95	38.48
1484000	Annual	8.21	16.45	24.01	36.41	47.93	61.67
	1	4.53	6.51	7.93	9.88	11.41	13.02
	2	4.95	7.81	9.99	13.11	15.68	18.49
	3	5.27	7.84	9.74	12.34	14.44	16.70
	4	5.72	10.87	15.46	22.74	29.36	37.12
Mean ratio	1	0.483	0.403	0.366	0.330	0.308	0.289
	2	0.542	0.463	0.430	0.401	0.386	0.374
	3	0.511	0.411	0.367	0.327	0.304	0.286
	4	0.567	0.562	0.562	0.563	0.566	0.569

an annual maximum could occur in any season. Thus, the disparity between the seasonal and annual maximum curves is reasonable. The average ratios are shown in Table 7. Therefore, the seasonal frequency curves should be at similar levels and have similar ratios to the annual maximum curves. Within the bounds of sampling variation, the results of these four gauges are very similar, as expected.

3. Regional seasonal frequency analysis

Gauging records often include average daily discharge records and selected instantaneous maximum discharges above a threshold. The threshold is generally low enough to include at least one value per year, which becomes the annual maximum discharge. The annual maximum instantaneous discharge record is therefore adequate to perform a LP3 annual maximum frequency analysis. The record of instantaneous maximum discharges will generally not include a value above the threshold for each season of the year. That is, seasonal frequency records of instantaneous discharges will often be incomplete.

3.1. A regional model for incomplete records

Where records of mean daily and instantaneous maximum discharges are available, the problem of incomplete seasonal frequency records can be overcome using the maximum likelihood approximation. Another approach is to develop relationships for predicting the missing instantaneous value from the recorded mean daily flow for the day on which the instantaneous value occurred. One possible model would be to use the existing records to develop a relationship of the form

$$q_i = bq_d \quad (12)$$

in which q_i is the instantaneous maximum discharge, q_d is the mean daily discharge for the same day, and b is a regression coefficient. To use the relationship of Eq. (12) at sites where instantaneous discharges are not available, the regression coefficient b could be related to watershed characteristics by regionalizing b from watersheds where the necessary data are available.

The regionalization procedure was applied to the USGS delineated (Dillow, 1996) Eastern Coastal Plain of Maryland. Each of the flood records for the gauges in the region were used to develop the relationship of Eq. (12) (see Table 8). The number of discharges above the instantaneous discharge threshold varied from watershed to watershed, with a range from 22 to 167 and a median of 116 for the 18 stations in the region. The 18 values of b varied from 1.08 to 2.74 with a mean of 1.58. When applied to the 18 watersheds, the zero-intercept regression model of Eq. (12) provided accurate estimates, with the correlation coefficients ranging from 0.81 to 0.98, with a mean of 0.92. When correlations are poor, the adjustments of Matalas and Jacobs (1964) should be considered. The relative biases range from −26 to 9%, although 15 of 18 watersheds have relative biases of less than 5% in absolute value. For the three watersheds with large relative biases, the correlation coefficients are 0.87, 0.915, and 0.98, which suggests that, in spite of the biases, the prediction accuracies are still quite good.

Graphical analyses of the data showed that the computed values of b were related to both the drainage area A (km^2) and the percentage of forest cover F, with the relationships showing negative, curvilinear trends. Negative trends would be rational. For smaller drainage areas, the duration of the hydrograph would be shorter and the portion of the hydrograph around the peak would be less. This would yield a relative low mean daily discharge with respect to the peak discharge. Heavily forested areas generally cause significant smoothing of the hydrograph, so the peak will be relatively smaller than for an unforested area. Several models were tried, with the following model providing the best fit and most rational structure:

$$\hat{b} = 1 + 3.2\exp(-0.00195A - 0.025(F + 10) - 0.000273A(F + 10)) \quad (13)$$

The values of b for the 18 watersheds provided a correlation coefficient of 0.913 and a relative bias of zero.

To assess the accuracy of computed instantaneous discharges using Eqs. (12) and (13), the two models were used to predict the 1898 instantaneous maximum discharges for the 18 watersheds in the Eastern Coastal Plain. The correlation coefficient for the model is 0.978, with a relative standard error ratio of 22.6%. Fig. 1 shows the predicted and actual values of the values of the instantaneous maximum discharges. The overall fit is excellent except for the slight under prediction for the very small discharges. The results

Table 8
Characteristics (n, sample size; A, area (km^2); F, precent forest cover) of gauges in the Eastern Coastal Plain of Maryland and results of calibration of Eq. (12) (b, regression slope; R, correlation coefficient; R_b, relative bias; S_e/S_y, standard error ratio)

Gauge no.	n	A	F	b	R	R_b	S_e/S_y
1483200	135	10.0	43	1.924	0.943	0.026	0.334
1483500	37	24.1	21	1.906	0.931	−0.037	0.364
1483700	42	81.7	46	1.222	0.903	0.037	0.430
1484000	99	34.8	35	1.505	0.972	−0.039	0.234
1484100	96	7.2	45	1.525	0.844	0.044	0.536
1484300	22	18.2	54	1.267	0.929	−0.009	0.369
1484500	128	13.3	51	1.398	0.805	0.044	0.594
1485000	116	154.9	30	1.084	0.979	0.012	0.202
1485500	160	114.9	85	1.090	0.979	0.002	0.204
1486000	163	12.3	57	1.807	0.901	0.000	0.433
1487000	142	193.0	40	1.151	0.979	0.024	0.203
1488500	167	112.4	29	1.354	0.870	0.093	0.493
1489000	78	18.2	33	2.280	0.916	0.019	0.401
1490000	29	38.4	50	1.280	0.915	0.081	0.404
1491000	124	289.3	35	1.149	0.985	0.015	0.170
1492000	117	15.1	26	2.191	0.871	−0.039	0.492
1493000	140	57.1	43	1.491	0.909	−0.010	0.235
1493500	102	32.5	8	2.735	0.980	−0.261	0.197
Minimum	22	7.2	8.0	1.08	0.80	−0.26	0.17
Maximum	167	289.3	85.0	2.74	0.99	0.09	0.59
Mean	105	68.1	40.6	1.58	0.92	0.00	0.35
Std dev	47	77.8	16.6	0.47	0.05	0.07	0.13

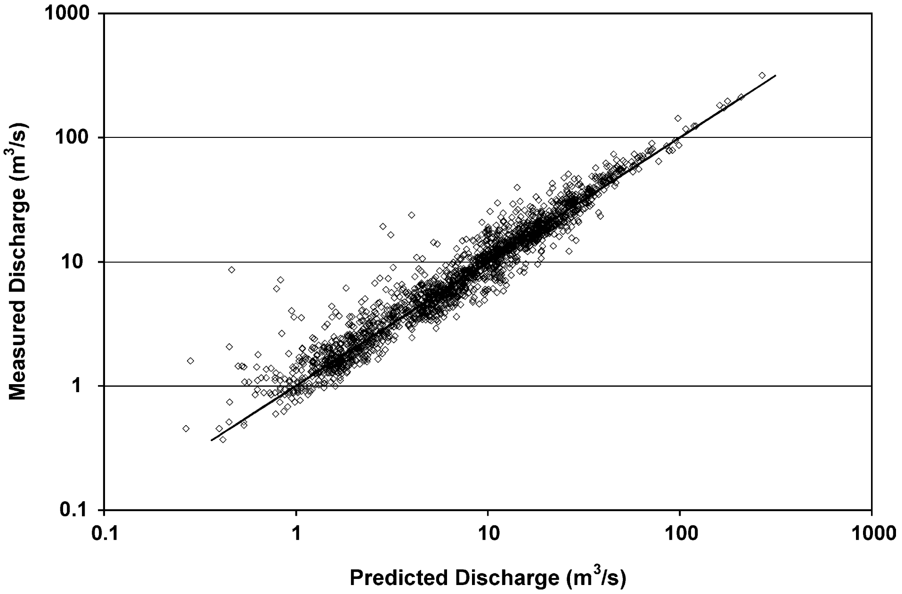

Fig. 1. Measured discharge versus instantaneous discharge predicted with Eqs. (12) and (13).

indicate that, where regional data are available, a regional model such as Eq. (13) is an alternative for filling in incomplete records.

3.2. Regional index ratios for seasonal frequency curves

Each of the streamflow records for the 18 watersheds in the Eastern Coastal Plain region were divided into two seasons, October through March and April through September. The annual maximum and seasonal maximum frequency curves were assembled from the available records of above-threshold instantaneous discharges. Where a seasonal instantaneous discharge above the threshold did not occur in any year, a value was estimated with the regional model of Eq. (13) from the largest daily mean discharge for the season in that year. The percentage of instantaneous discharges that had to be estimated ranged from 15 to 65% for the winter season and from 19 to 50% for the summer season.

Frequency curves were fit to the annual maximum record and the two seasonal records for each of the 18 gauging stations. The B17B procedure was used, with a few low outliers detected, and censored, based on the B17B outlier criterion. The weighted skew for the annual maximum series was used in computing the seasonal frequency curves in order to prevent the seasonal curve from intersecting the annual maximum curve. Log Pearson Type III discharges were then computed for six return periods: 2-, 5-, 10-, 25-, 50-, and 100-yr. For each watershed, the ratio of the T-yr seasonal discharge to the T-yr annual maximum discharge was computed, and then the ratios were averaged over the 18 watersheds. The ratios for the six return periods from the 2- to 100-yr in the winter season are: 0.73, 0.71, 0.70, 0.69, 0.69, and 0.69. The ratios for the summer season are: 0.70, 0.77, 0.82, 0.88, 0.92, and 0.96.

The two sets of ratios show different trends, as well as indicating that the seasonal frequency curve is considerably different from the annual curve. More records included maximum values below the threshold in the summer season, which meant that the low values, which had to be estimated, would cause a larger variation. With a higher standard deviation, the slopes of the curves, in general, increased, which leads to higher ratios for the infrequent (high return period) discharges. For the winter season, the ratios showed a relatively constant ratio of about two-thirds. The variation did not reveal a dominant trend, thus it is reasonable to apply a constant ratio over all return periods.

3.3. Estimation at ungauged locations

Where the gauged data are not adequate to develop a seasonal frequency curve, an index flood approach can be used. The annual maximum frequency curve can be estimated with a model and then seasonal-dependent ratios can be used to transform the T-yr annual maximum discharge to a T-yr seasonal maximum discharge. Where the USGS regressions are applicable, they could be used to estimate the annual maximum frequency curve. Otherwise, an uncalibrated rainfall-runoff model such as HEC-1 or TR-20 could be applied to estimate the annual maximum frequency curve.

The index ratios given above enable the seasonal frequency curve to be developed for ungauged watersheds within the Eastern Coastal Plain region. The annual maximum frequency curve could be computed by multiplying the above index ratios to obtain the seasonal frequency curve for the season of interest:

$$q_{ST} = R_{ST} q_{aT} \tag{14}$$

in which q_{ST} is the seasonal instantaneous maximum discharge for return period T and season S, q_{aT} is the annual maximum instantaneous discharge for return period T, and R_{ST} is the index ratio for season S and return period T.

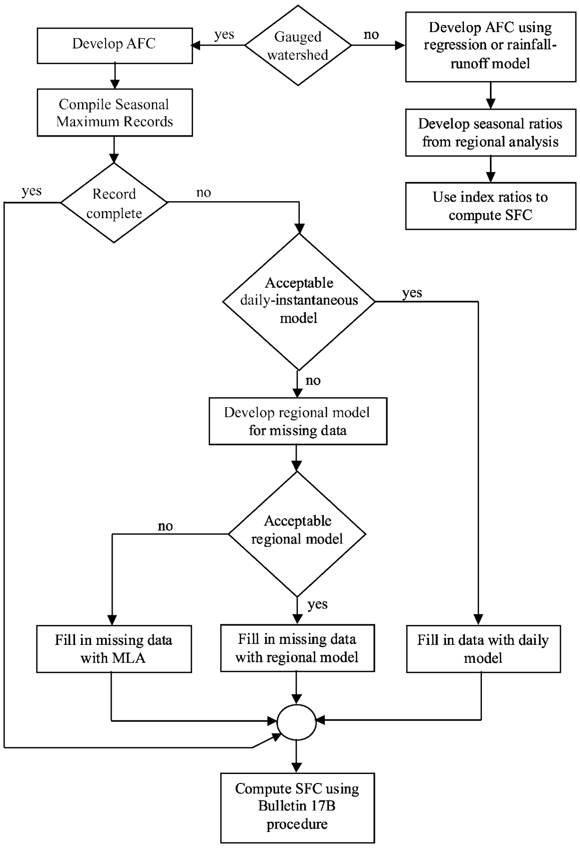

Fig. 2. Flowchart of procedure for developing seasonal frequency curves (SFC) at gauged and ungauged sites.

4. Conclusions

Several procedures related to seasonal flow frequency analysis have been developed and presented. The flow chart of Fig. 2 shows how a seasonal flow frequency curve can be developed for either gauged or ungauged watersheds. If the site is ungauged, then a regional model would need to be developed (Stedinger et al., 1993), such as the model developed herein for the Eastern Coastal Plain of Maryland. In order to develop the regional index ratios, it would be necessary to apply the portion of the flowchart that is applicable to gauged sites.

Three methods for filling in missing data are discussed and illustrated. The maximum likelihood approximation requires the least data and can be used if a regional model is not available or the model for the specific gauged site provides poor correlation, as might be the case for small watersheds. If seasonal analyses will be made at many sites in a region, then the regional model would be preferable in order to provide some stability between the seasonal curves for the individual watersheds in the region.

The maximum likelihood approximation appears to be more accurate than the method provided in B17B for computing a frequency curve from an incomplete record. This may result from the adjustment using the ratio of the number of discharges above the threshold to the record length, $R = N/H$. Jennings and Benson (1969) indicate that this is “the probability in any year of an event that exceeds…a base level…above which flood magnitudes are recorded…” This ignores the fact that the probability of a large flood is different than the probability of a low discharge. If all flood magnitudes had an equal probability, then this would be true. However, it is only an approximation when floods follow a distribution other then the uniform distribution, such as a log-normal or LP3 distribution. For distributions other than a uniform pdf, it is only an approximation. For cases in semiarid climates where data records often have zero-flood values, the maximum likelihood approach could be used to develop a model similar to Eqs. (8a) and (8b).

Annual flood frequency estimates can be adjusted into seasonal estimates for both gauged and ungauged locations. However, missing data that result from separating the annual maximum series into seasons must be addressed. B17B provides one method for filling in missing data but is limited to only 25% missing (threshold probability of 75%), which may not be sufficient for seasonal analyses. For peak flow gauges, which often include all instantaneous maximum discharges above a threshold, a maximum likelihood approximation was developed to overcome the problem of missing data in the individual seasons. Two reason missing data may be a problem are: (1) the available data are primarily used for annual analyses and the thresholds values are set accordingly; and (2) depending on region, the distributions of seasonal rainfall can range from uniform to highly variable (i.e. wet and dry seasons).

To demonstrate the accuracy of the MLA for filling in missing seasonal values, Monte Carlo simulations were used. The simulations, which varied the population skew, sample size, coefficient of variation, and the percentage of missing values, show that the MLA provides better estimates of mean and standard deviation than the method presented in B17B. Additionally, the simulations showed that the MLA could be used below the limiting probability of 75% specified for the B17B method. For a threshold probability of 50%, the MLA still results in acceptable accuracy.

As a case study, a seasonal frequency analysis based on four seasons at four gauges within the USGS delineated Eastern Coastal Plain of Maryland shows the results of the MLA. The results show that the seasonal 100-yr discharges are quite different than the corresponding annual series 100-yr discharge, which shows the value of making seasonal frequency analyses for season-sensitive design problems. Almost half of the seasonal 100-yr discharges were less than their corresponding annual maximum 10-yr discharges.

For ungauged locations, a regional approach must be taken to develop seasonal frequency curves. Gauges that record both instantaneous peaks and daily averaged discharges, were used to develop a relationship between instantaneous and averaged daily flows. The regional relationship presented in this paper, Eqs. (12) and (13), applies to the Eastern Costal Plain and uses both drainage area and forest cover as predictor variables. When tested on 1898 instantaneous maximum discharges from 18 gauges, the relationship yielded accurate estimates having a correlation coefficient of 0.978 and a relative standard error ratio of 22.6%, which indicates that maximum seasonal daily averaged discharges can be used to estimate missing instantaneous data. If a regional model provides accurate estimates and daily data are available, it could be used rather than the maximum likelihood approximation.

A second case study evaluated the use of a regional relationship to estimate missing data for two seasons October–March and April–September. The seasonal frequency curves for the 18 watersheds were calculated, and the ratios of the seasonal T-yr discharge to T-yr annual maximum discharge were calculated. The winter ratios were on average approximately 70% for each return period. The summer ratios, which required estimating a higher fraction of missing values, had higher standard deviations, which resulted in a sloped ratio curve increased from about 70% at the 2-yr return period and to 96% for the 100-yr return period. These ratios represent the winter and summer regional index ratios and could be used for applications at an ungauged location in the region. The seasonal frequency estimates can be determined by multiplying the annual estimates, determined by the USGS regional regression equations or a rainfall-runoff model, by their corresponding T-yr index ratio, as shown by Eq. (13).

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