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Horizontal well hydraulics in leaky aquifers

Hongbin Zhan*, Eungyu Park

Department of Geology and Geophysics, Texas A&M University, College Station, TX 77843-3115, USA

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Abstract

This paper presents a general study of horizontal well hydraulics for three aquifer types: a leaky confined aquifer, a leaky water table aquifer, and a leaky aquifer under a water reservoir. Semi-analytical solutions are obtained for cases that exclude and include the aquitard storage. The type curves and derivative type curves for these different conditions are provided. A graphically integrated MATLAB program named HW_LEAK is written to facilitate numerical calculations and generation of the type curves and derivative type curves. This study shows that (1) derivative type curves are more sensitive to the aquitard parameters than the type curves; and that (2) drawdown is sensitive to the aquitard/aquifer thickness ratio and the hydraulic conductivity ratio at the intermediate and later time. Both curves are less sensitive to the aquitard/aquifer specific storage ratio, while the degree of sensitivity of the drawdown to the aquitard parameters is high in a leaky confined aquifer, moderate in a water table aquifer, and low in an aquifer under a water reservoir.

Keywords: Groundwater flow; Horizontal well; Leaky aquifer; Type curves; Derivative type curves

1. Introduction

Horizontal wells have gained significant interest among hydrogeologists and environmental scientists and engineers in recent years because of their many advantages over conventional vertical wells. The study of horizontal wells in hydrologic sciences dates back to Hantush and Papadopulos (1962). During the last decade, groundwater flow to horizontal wells was studied in various aspects (Tarshish, 1992; Cleveland, 1994; Murdoch, 1994; Falta, 1995; Sawyer and Lieuallen-Dulam, 1998; Zhan, 1999; Hunt and Massmann, 2000; Kawecki, 2000; Zhan and Cao, 2000; Steward and Jin, 2001; Zhan et al., 2001; Zhan and Park, 2002; Park and Zhan, 2002; Zhan and Zlotnik, 2002).

Nevertheless, a general theory of groundwater flow to a horizontal well in a leaky aquifer is not yet available and will be the focus of this paper. New solutions for groundwater flow in a leaky confined aquifer, a leaky aquifer under a water reservoir, and a leaky water table aquifer will be presented. Both type curves and derivative type curves will be generated for these different aquifer conditions, where the type curve is defined as the dimensionless drawdown versus dimensionless time on a log–log scale, and the derivative type curve is defined as the first derivative of the dimensionless drawdown over the logarithmic dimensionless time as a function of the dimensionless time on a log–log scale. Graphically integrated MATLAB programs were written to facilitate the calculation of

* Corresponding author. Tel.: +1-979-862-7961; fax: +1-979-845-6162.
E-mail address: zhan@hydrog.tamu.edu (H. Zhan).

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Nomenclature

d	thickness of the main aquifer
d', d''	thickness of the aquitard
d'_D	dimensionless thickness of the aquitard
$F(x_{0D})$	$= [(x_D - x_{0D})^2 + y_D^2]^{1/2}$
h	hydraulic head in the main aquifer
h_0	initial hydraulic head in the main aquifer
H_0	the horizontal function used in solution (17) ($n = 0$)
H_n	the horizontal function used in solution (17) ($n > 0$)
K_x, K_y, K_z	principal hydraulic conductivities of the main aquifer
K', K''	hydraulic conductivity of the aquitard
$K_0(x)$	the zero-order, second kind modified Bessel function
L	screen length of the horizontal well
L_D	dimensionless screen length of the horizontal well
p	Laplace transform parameter corresponding to the dimensionless time
Q	pumping rate
r_D	$= [(x_D - x_{0D})^2 + (y_D - y_{0D})^2]^{1/2}$, radial distance from the point sink/source
s	drawdown
s_D	dimensionless drawdown due to a point sink
s_{HD}	dimensionless drawdown near a pumping horizontal well
$\bar{s}_D$	dimensionless drawdown near a point sink in the Laplace domain
$\bar{s}_{HD}$	dimensionless drawdown near a pumping horizontal well in the Laplace domain
$\bar{s}'_D$	dimensionless drawdown in the aquitard in the Laplace domain
S_s	specific storage of the main aquifer
S'_s	specific storage of the aquitard
S_y	specific yield of the main aquifer
t	time
t_D	dimensionless time
$W(u,v)$	leaky well function
x, y, z	coordinates of the monitoring point or piezometer
x_0, y_0, z_0	coordinates of the sink/source point
x_D, y_D, z_D	dimensionless coordinates of the monitoring point or piezometer
x_{0D}, y_{0D}, z_{0D}	dimensionless coordinates of the sink/source point
z_w	distance from the horizontal well to the lower boundary
z_{wD}	dimensionless distance from the horizontal well to the lower boundary
α_1	delayed index of the water table
α_{1D}	dimensionless delayed index of the water table
γ	dimensionless term related to the specific storage ratio of the aquitard and aquifer, defined in Table 1
$\delta(u)$	Dirac delta function
λ	leaky parameter defined in Table 1
$\bar{\lambda}$	modified leaky parameter defined in Table 1
μ	dimensionless term related to the hydraulic conductivity ratio of the aquitard and aquifer, defined in Table 1
σ	dimensionless term related to the ratio of specific storage and specific yield, defined in Table 1
ω_n	spatial frequency used in solution
Γ	dimensionless leakage term
Ψ	a dimensionless term defined in Eq. (30)
Ω_n	a dimensionless term defined in Eq. (22)
τ	dummy variables used in the time integration

drawdowns and the generation of type curves and derivative type curves. These type curves and derivative type curves are useful tools for interpreting horizontal well pumping tests. By combining these solutions with previous studies (Zhan et al., 2001; Zhan and Zlotnik, 2002), one gains a better understanding of horizontal well hydraulics under various aquifer conditions.

2. Mathematical model I: no aquitard storage

2.1. Problem statement

When the leaky aquitard is thin, the storage water from the aquitard is limited and its influence upon flow inside the aquifer is negligible. A mathematical model for a case that neglects the aquitard storage effect is developed in this section. Fig. 1 shows three schematic diagrams of horizontal pumping wells in a leaky confined aquifer, a leaky aquifer under a water reservoir, and a leaky water table aquifer, respectively. The sufficiently distant lateral boundaries do not influence the flow.

We start with a simple leaky aquifer, when one aquitard exists either above or below the main aquifer,

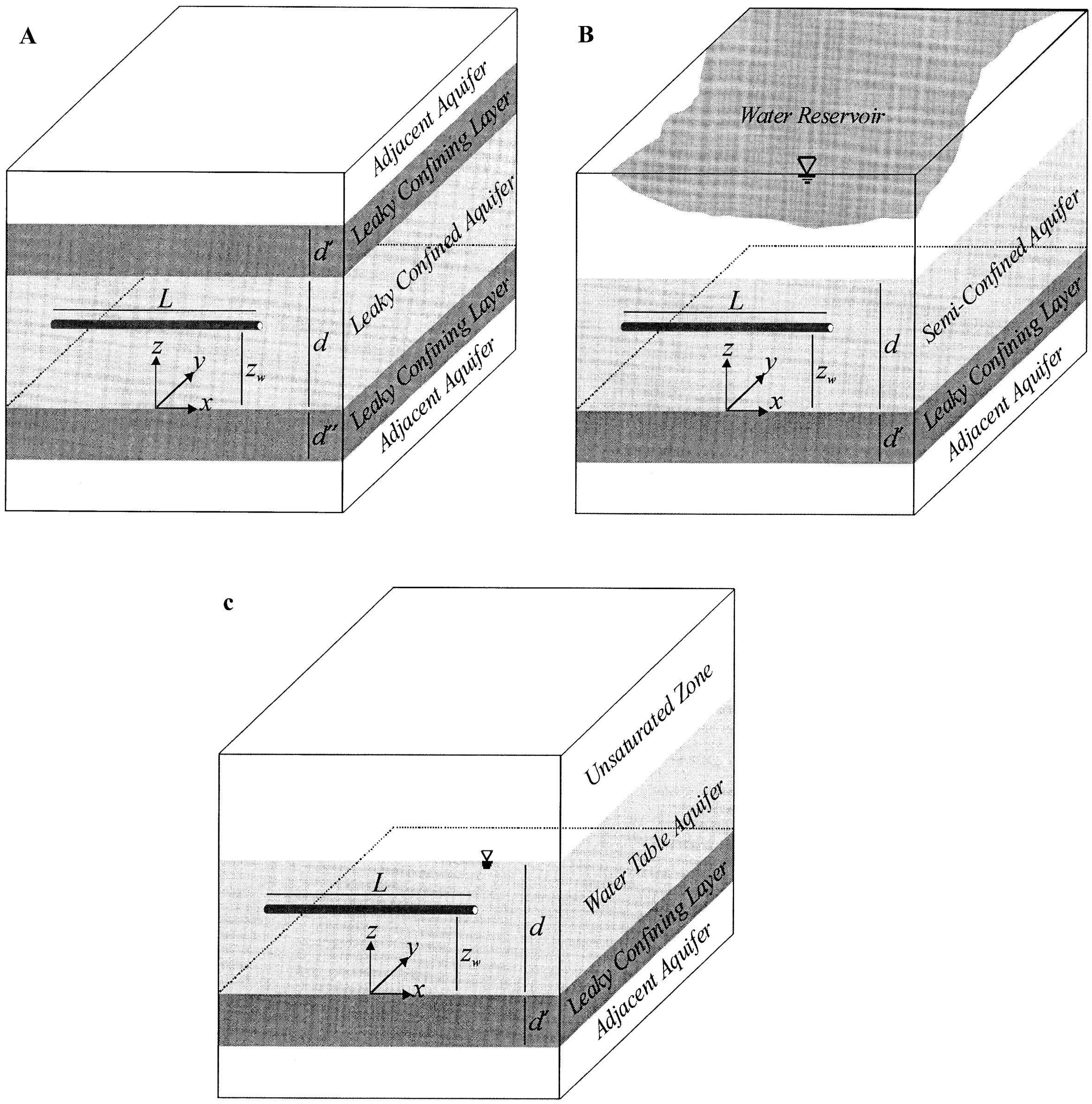

Fig. 1. The schematic diagrams of the horizontal pumping wells. The origin of the coordinate system is at the lower boundary with the x- and y-axes along the horizontal directions and the z-axis along the vertical direction. (A) A leaky confined aquifer; an aquitard is at the upper and/or lower boundary. (B) A leaky aquifer under a water reservoir; an aquitard is at the lower boundary. (C) A leaky water table aquifer; an aquitard is at the lower boundary.

and the adjacent aquifer that leaks water through the aquitard to the main aquifer holds a constant hydraulic head (h_0). This implies that before pumping, the multiaquifer system is hydrostatic with a common hydraulic head h_0. We derive an analytical solution for the problem and gain physical insights into the problem. After developing the mathematical model for this simple conceptual model, this study will show that solutions for more general leaky aquifer conditions can be easily obtained by using certain modifications.

For an aquifer under a water reservoir, the upper boundary is treated as a constant head and the bottom is bounded by an aquitard through which leakage can occur. For a water table aquifer, the upper boundary is the free water table and the bottom is bounded by an aquitard. Hantush (1960) has done an extensive research on leakage from the upper and/or lower aquitards.

We use the Cartesian system of coordinates with the origin at the bottom of the aquifer. The well is positioned along the x-axis and is centered at $(0, 0, z_w)$, where z_w is the distance from the well to the lower boundary. The horizontal well is treated as a line sink and the flux distribution along the well axis is assumed uniform. Zhan et al. (2001), and Zhan and Zlotnik (2002) have provided detailed explanations on the suitability of these assumptions. These assumptions can provide sufficiently accurate approximations to the actual solutions when the horizontal well pumping rate is not extremely large, which is usually true in environmental applications where small pumping rates are favored.

However, if a horizontal well is pumped with a large pumping rate, the flow inside the horizontal well could become so strong that significant hydraulic head losses will exist in the in-well flow. Under these circumstances, different flow states such as laminar, transitional, and turbulent flows can co-exist inside the wellbore and the problem must be treated as a coupled well-aquifer hydraulics problem. A closed-form analytical solution in this case is not possible and a numerical solution will be needed.

First, we will derive a solution for groundwater flow to a point sink, and then superimpose the point solutions along the axis of the horizontal well with uniform strength. Groundwater flow to a point sink in a leaky confined aquifer without the aquitard storage is described as follows (Hantush, 1964; p. 349)

$$S_s \frac{\partial h}{\partial t} = K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} + K' \frac{h_0 - h}{dd'} - Q\delta(x - x_0)\delta(y - y_0)\delta(z - z_0), \quad (1)$$

$$h(x, y, z, t = 0) = h_0, \quad (2)$$

$$\partial h(x, y, z = 0, t)/\partial z = 0, \quad (3)$$

$$h(x = \pm\infty, y, z, t) = h(x, y = \pm\infty, z, t) = h_0, \quad (4)$$

where S_s is the specific storage (m^{-1}); h is the hydraulic head (m); t is time (s); K_x, K_y, K_z are values of principal hydraulic conductivity (m/s) in the x-, y-, and z-directions, respectively; K' is the hydraulic conductivity of the aquitard (m/s); Q is the pumping rate (m^3/s) ($Q > 0$ for pumping and $Q < 0$ for injection); δ is the Dirac delta function (m^{-1}); h_0 is the initial hydraulic head (m); d is the aquifer thickness (m); d' is the thickness of the aquitard; and (x_0, y_0, z_0) are the sink coordinates.

A few assumptions are used in formulating Eq. (1) (Hantush, 1964, p. 348–349): (1) the aquifer is homogeneous and anisotropic; (2) flow in the homogeneous aquitard is vertical; (3) the leakage crossing the interface is assumed to be generated within the main aquifer and is approximated by the fourth term on the right-hand side of the equation; and (4) only one aquitard is considered in Eq. (1). When two aquitards are presented as shown in Fig. 1A, another term related to leakage should be added on the right-hand side of Eq. (1). The solution for this two-aquitard case can be modified easily for the one-aquitard solution. The detail is discussed in Section 2.2.4.

Three different upper boundaries are considered

$$\partial h(x, y, z = d, t)/\partial z = 0, \quad (5)$$

for a leaky confined aquifer

$$h(x, y, z = d, t) = h_0, \quad (6)$$

for a leaky aquifer under a water reservoir that is treated as a constant head boundary; and

$$K_z \partial h(x,y,z=d,t)/\partial z + S_y \partial h(x,y,z=d,t)/\partial t = 0, \quad (7)$$

for a leaky water table aquifer with an instantaneous drainage, or

$$K_z\frac{\partial h(x,y,z=d,t)}{\partial z}+\alpha_1 S_y\int_0^t\frac{\partial h(x,y,z=d,t')}{\partial t'}\times\exp[-\alpha_1(t-t')]\partial t'=0, \quad (8)$$

for a delayed yield drainage, where S_y is specific yield, and α_1 is the empirical constant (Moench, 1995).

We should point out that because the leakage is treated as a term generated within the main aquifer, so the upper and lower boundaries of the leaky confined aquifers are assumed to be impermeable. The boundary conditions (3) and (5) are suitable for cases when leaking aquitard is at the top or base or at both (Fig. 1A).

Changing the variable from h to drawdown $s=h_0-h$, defining all the dimensionless parameters in Table 1, and conducting the Laplace transform, one obtains the following

$$\frac{\partial^2\bar{s}_D}{\partial x_D^2}+\frac{\partial^2\bar{s}_D}{\partial y_D^2}+\frac{\partial^2\bar{s}_D}{\partial z_D^2}-(p+\lambda)\bar{s}_D=-\frac{4\pi}{p}\delta(x_D-x_{0D})\delta(y_D-y_{0D})\delta(z_D-z_{0D}), \quad (9)$$

$$\partial\bar{s}_D(x_D,y_D,z_D=0,p)/\partial z_D=0, \quad (10)$$

$$\partial\bar{s}_D(x_D,y_D,z_D=1,p)/\partial z_D=0, \quad (11)$$

$$\bar{s}_D(x_D=\pm\infty,y_D,z_D,p)=\bar{s}_D(x_D,y_D=\pm\infty,z_D,p)=0, \quad (12)$$

where the subscripts 'D' denote dimensionless terms, $\bar{s}_D$ refers to the dimensionless drawdown in the Laplace domain, the dimensionless term λ is the leaky parameter defined in Table 1, and p is the Laplace transform parameter corresponding to dimensionless time t_D.

The three upper boundary conditions are

$$\partial\bar{s}_D(x_D,y_D,z_D=1,p)/\partial z_D=0, \quad (13)$$

for a leaky confined aquifer

$$\bar{s}_D(x_D,y_D,z_D=1,p)=0, \quad (14)$$

for a leaky aquifer under a water reservoir that is treated as a constant head boundary

$$\sigma\partial\bar{s}_D(x_D,y_D,z_D=1,p)/\partial z_D+p\bar{s}_D(x_D,y_D,z_D=1,p)=0, \quad (15)$$

Table 1
Dimensionless variables

$d'_D=\frac{d'}{d}$	$\alpha_{1D}=\frac{S_s d^2\alpha_1}{K_z}$
$L_D=\frac{L}{d}\sqrt{\frac{K_z}{K_x}}$	$\gamma=\frac{S'_s K_z}{S_s K'}$
$s_D=\frac{4\pi\sqrt{K_xK_y}d}{Q}s$	$\lambda=\frac{K'd}{K_z d'}$
$t_D=\frac{K_z}{S_s d^2}t$	$\bar{\lambda}=\left(\frac{K'}{d'}+\frac{K''}{d''}\right)\frac{d}{K_z}$
$\sigma=\frac{S_s d}{S_y}$	$\mu=\frac{K'}{K_z}$
$x_D=\frac{x}{d}\sqrt{\frac{K_z}{K_x}}$, $y_D=\frac{y}{d}\sqrt{\frac{K_z}{K_y}}$,	
$z_D=\frac{z}{d}$, $x_{0D}=\frac{x_0}{d}\sqrt{\frac{K_z}{K_x}}$,	
$y_{0D}=\frac{y_0}{d}\sqrt{\frac{K_z}{K_y}}$, $z_{0D}=\frac{z_0}{d}$,	
$z_{wD}=\frac{z_w}{d}$.	

for a leaky water table aquifer with instantaneous drainage; or

$$\sigma\frac{\partial\bar{s}_D(x_D,y_D,z_D=1,p)}{\partial z_D}+\frac{p\alpha_{1D}}{p+\alpha_{1D}}\bar{s}_D(x_D,y_D,z_D=1,p)=0, \quad (16)$$

for delayed yield drainage, where the dimensionless specific ratio σ and dimensionless delay index α_{1D} are also defined in Table 1.

2.2. Solution of drawdown

The above equations can be solved using the Fourier transform in the z-direction first. Superimposing the point sink solutions along the horizontal well axis will yield the solution (Zhan et al., 2001; Zhan and Park, 2002; Park and Zhan, 2002). Solution of Eq. (9) is:

$$\bar{s}_D=\sum_{n=0}^{\infty}H_n(x_D,y_D,p)\cos(\omega_n z_D). \quad (17)$$

Substituting Eq. (17) into the upper boundary conditions (13)–(16) will lead to the determination of ω_n. Substituting Eq. (17) into Eq. (9) will result in equations of H_n, which are then solved subject to boundary condition (12).

2.2.1. Drawdown in a leaky confined aquifer without aquitard storage

Appendix A shows that the dimensionless drawdown near a pumping horizontal well, denoted as s_{HD}, is

$$s_{\mathrm{HD}}(t_{\mathrm{D}})=\frac{\sqrt{\pi}}{L_{\mathrm{D}}}\int_0^{t_{\mathrm{D}}}\frac{1}{\sqrt{\tau}}\left[\mathrm{erf}\left(\frac{L_{\mathrm{D}}/2+x_{\mathrm{D}}}{2\sqrt{\tau}}\right)+\mathrm{erf}\left(\frac{L_{\mathrm{D}}/2-x_{\mathrm{D}}}{2\sqrt{\tau}}\right)\right]\exp\left(-\frac{y_{\mathrm{D}}^2}{4\tau}\right)\times\left[\exp(-\lambda\tau)+2\sum_{n=1}^{\infty}\cos(n\pi z_{\mathrm{wD}})\cos(n\pi z_{\mathrm{D}})\times\exp(-[n^2\pi^2+\lambda]\tau)\right]\mathrm{d}\tau, \tag{18}$$

where L_{D} is the dimensionless horizontal-well screen length defined in Table 1, and $\mathrm{erf}(x)$ is the error function. When $\lambda=0$, there is no leakage and Eq. (18) reduces to the solution for a confined aquifer by Zhan et al. (2001).

2.2.2. Drawdown in an aquifer under a water reservoir without aquitard storage

Substituting Eq. (17) into Eq. (14) results in:

$$\omega_n = n\pi + \frac{\pi}{2}, \qquad n = 0, 1, 2, 3, \ldots \tag{19}$$

Following the procedure in Appendix A, the dimensionless drawdown in an aquifer under a water reservoir is obtained

$$s_{\mathrm{HD}}(t_{\mathrm{D}})=\frac{\sqrt{\pi}}{L_{\mathrm{D}}}\int_0^{t_{\mathrm{D}}}\frac{1}{\sqrt{\tau}}\left(\mathrm{erf}\left[\frac{\frac{L_{\mathrm{D}}}{2}+x_{\mathrm{D}}}{2\sqrt{\tau}}\right]+\mathrm{erf}\left[\frac{\frac{L_{\mathrm{D}}}{2}-x_{\mathrm{D}}}{2\sqrt{\tau}}\right]\right)\times\exp\left[-\frac{y_{\mathrm{D}}^2}{4\tau}\right]2\sum_{n=0}^{\infty}\cos\left[\left(n\pi+\frac{\pi}{2}\right)z_{\mathrm{D}}\right]\times\cos\left[\left(n\pi+\frac{\pi}{2}\right)z_{\mathrm{wD}}\right]\times\exp\left[-\left\{\left(n\pi+\frac{\pi}{2}\right)^2+\lambda\right\}\tau\right]\mathrm{d}\tau, \tag{20}$$

where $\lambda\neq 0$ refers to a case in which a leaky aquitard exists at the bottom of the aquifer. When $\lambda=0$, there is no leakage, and the solution then refers to an aquifer with a water reservoir at the upper boundary and a no-flow boundary at the bottom.

2.2.3. Drawdown in a water table aquifer without aquitard storage

Using a similar procedure as that employed by Zhan and Zlotnik (2002) for a water table aquifer (Appendix A), one can obtain the dimensionless drawdown in the Laplace domain for a water table aquifer with a leaky aquitard at the bottom as follows

$$\bar{s}_{\mathrm{HD}}(p)=\frac{4}{pL_{\mathrm{D}}}\sum_{n=0}^{\infty}\frac{\cos[\omega_n z_{\mathrm{D}}]\cos[\omega_n z_{\mathrm{wD}}]}{1+0.5\sin(2\omega_n)}\times\int_{-L_{\mathrm{D}}/2}^{L_{\mathrm{D}}/2}K_0[\Omega_n F(x_{0\mathrm{D}})]\mathrm{d}x_{0\mathrm{D}}, \tag{21}$$

where

$$\Omega_n^2=\omega_n^2+p+\lambda,\ F(x_{0\mathrm{D}})=[(x_{\mathrm{D}}-x_{0\mathrm{D}})^2+y_{\mathrm{D}}^2]^{1/2}, \tag{22}$$

and ω_n is defined as

$$\omega_n\tan(\omega_n)=p/\sigma,\ n=0,1,2,\ldots \tag{23}$$

for an instantaneous drainage, and

$$\omega_n\tan(\omega_n)=p\alpha_{1\mathrm{D}}/[\sigma(p+\alpha_{1\mathrm{D}})],\ n=0,1,2,\ldots \tag{24}$$

for a delayed drainage.

When $\lambda = 0$, there is no leakage from the lower aquitard and Eq. (21) becomes identical to that of Zhan and Zlotnik (2002). Inversion of Laplace transform of Eq. (21) will result in the drawdown in the real time domain.

2.2.4. Generalization of the solution of drawdown

The above semi-analytical solutions can be generalized by considering more realistic leaky conditions. Eq. (1) considers one aquitard that is either above or below the main aquifer. If there are two aquitards, one above and one below the aquifer with hydraulic conductivity K' and K'' and thickness d' and d'' for the upper and lower aquitards, respectively, then Eq. (1) is modified by adding one term $K''(h_0 - h)/dd''$ on the right-hand side of the equation if the adjacent aquifers have the same hydraulic head (h_0). An effective leakage parameter

$$\bar{\lambda} = \left(\frac{K'}{d'} + \frac{K''}{d''}\right)\frac{d}{K_z}$$

can be introduced for this case. Substituting $\bar{\lambda}$ for λ in the derived solution will result in the solution for this generalized case.

3. Mathematical model II: with aquitard storage

3.1. Problem statement

When the leaky aquitard is thick, the storage water from the aquitard can be significant, and its influence upon flow inside the aquifer is non-negligible. A mathematical model for such a case that includes the aquitard storage effect is developed in this section. The governing Eq. (9) is modified for this case as follows

$$\frac{\partial^2 \bar{s}_D}{\partial x_D^2} + \frac{\partial^2 \bar{s}_D}{\partial y_D^2} + \frac{\partial^2 \bar{s}_D}{\partial z_D^2} + \Gamma - p\bar{s}_D = -\frac{4\pi}{p}\delta(x_D - x_{0D})\delta(y_D - y_{0D})\delta(z_D - z_{0D}), \quad (25)$$

where the leakage term Γ is defined as

$$\Gamma = \mu \left.\frac{\partial \bar{s}'_D}{\partial z_D}\right|_{z_D=1}, \quad (26)$$

where $\bar{s}'_D$ refers to the dimensionless drawdown in the aquitard in the Laplace domain, and μ is related to the hydraulic conductivity ratio between the aquitard and aquifer (Table 1). The associated boundary conditions are identical to what has been discussed in the case without aquitard storage.

Only the vertical flow is considered in the aquitard. Using a leaky confined aquifer as an example and assuming that the aquitard is above the aquifer (Fig. 1), Hantush (1960, Eq. (36)) provided a solution that relates the drawdown of the aquitard to the aquifer as

$$\bar{s}'_D = \bar{s}_D \frac{\sinh([1 + d'_D - z_D]\sqrt{\gamma p})}{\sinh(d'_D\sqrt{\gamma p})}, \quad (27)$$

where d'_D is the dimensionless thickness of the aquitard, and γ is related to the specific storage ratio between the aquitard and aquifer. Both are defined in Table 1. Thus, the leakage term becomes:

$$\Gamma = -\mu\sqrt{\gamma p}\coth(d'_D\sqrt{\gamma p})\bar{s}_D. \quad (28)$$

Substituting Eq. (28) into Eq. (25) results in the following governing equation

$$\nabla^2 \bar{s}_D - \Psi^2 \bar{s}_D = -\frac{4\pi}{p}\delta(x_D - x_{0D})\delta(y_D - y_{0D})\delta(z_D - z_{0D}), \quad (29)$$

where

$$\Psi = [p + \mu\sqrt{\gamma p}\coth(d'_D\sqrt{\gamma p})]^{1/2}. \quad (30)$$

A slight modification is needed if the aquitard is at the bottom of the aquifer extending from $z = -d'$ to $z = 0$ (Fig. 1A). For this case, $\bar{s}'_D$ becomes:

$$\bar{s}'_D = \bar{s}_D \frac{\sinh([d'_D + z_D]\sqrt{\gamma p})}{\sinh(d'_D\sqrt{\gamma p})}. \quad (31)$$

The leakage term Γ for this case becomes

$$\Gamma = -\mu \left.\frac{\partial \bar{s}'_D}{\partial z_D}\right|_{z_D=0}.$$

Equations for Γ and Ψ for this case are identical to Eqs. (28) and (30), respectively.

Appendix B shows the details of calculation for this case.

4. Type curve and derivative type curve analyses

A graphically integrated MATLAB program HW_LEAK for calculation of Eqs. (18) and (20) and the spatial integration and the inverse Laplace transform of Eq. (21) was developed. The integration is carried out numerically using the Gaussian Quadrature method (Abramowitz and Stegun, 1972, p. 916; Press et al., 1989), and the numerical inverse Laplace transform utilizes the Stehfest (1970) algorithm. The MATLAB program is available from the authors.

The default values of the parameters used in the following analyses are shown in Table 2. The piezometer is located at (1, 1, 5 m). Examples of analyses based on these solutions are presented below. In general, the derivative type curves, $ds_D/d(\ln t_D)$, are more sensitive to the change of the aquitard parameters than the type curves, and will be used as the diagnostic tool if the type curves fail to show the details of the transient aquifer responses.

Table 2
The default values used in this study

Parameter	Default value
d	10 m
d'	1 m
K_x, K_y, K_z	0.0001 m/s
K'	0.000001 m/s
L	100 m
Q	0.001 m^3/s
S_s	0.00002 m^{-1}
S'_s	0.001 m^{-1}
S_y	0.2
z_w	5 m
α_1	∞
Location of the piezometer	(1, 1, 5 m)

4.1. Type curves and derivative type curves in a leaky confined aquifer

Figs. 2 and 3 show the sensitivity of the type curves and the derivative type curves to the aquitard thickness in a leaky confined aquifer without the aquitard storage. Four different cases with dimensionless aquitard thickness 0.01, 0.1, 1, and 10 are presented and compared with the case without leakage. As expected, these type curves become flat at the later time when the leakage becomes the major water source. The type curve for a thin aquitard case ($d'_D = 0.01$) is substantially different from that of a confined aquifer case. The differences among various cases are clearly shown in the derivative type curves (Fig. 3). The derivative type curves of the leaky aquifer drop sharply to zero at the later time while the counterpart of the confined aquifer becomes flat. This difference can be used to distinguish the confined aquifers from the leaky aquifers.

Figs. 4 and 5 are identical to Figs. 2 and 3 except that the aquitard storage is considered. A few interesting observations follow from comparing Figs. 4 and 5 to Figs. 2 and 3. First, when aquitard storage is considered, the aquitard will release water from storage soon after the pumping starts. Thus, the type curves of the leaky aquifer case will deviate from the confined aquifer type curve earlier than their counterparts of the non-storage case.

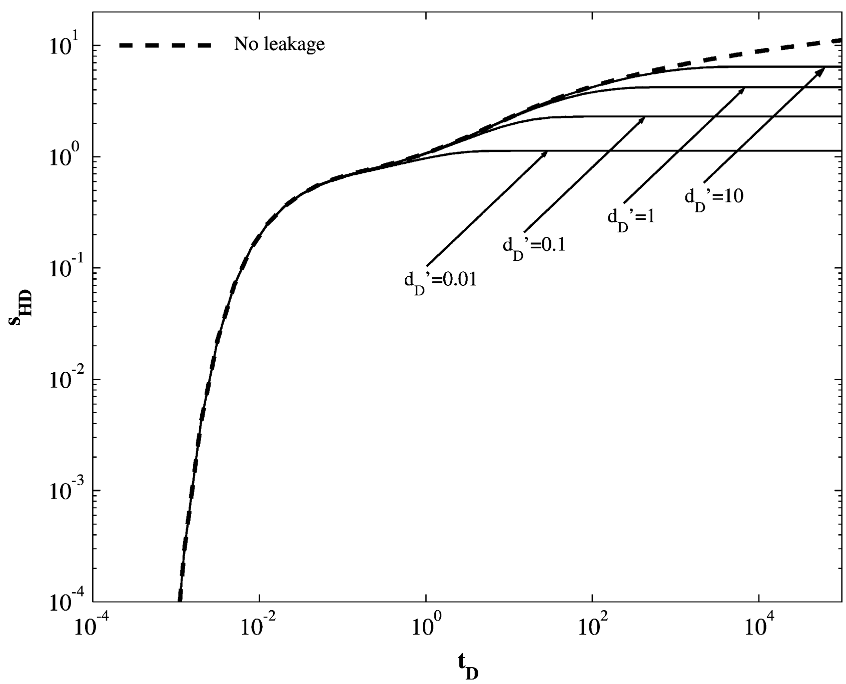

Fig. 2. Comparison of the type curves between a confined aquifer and leaky confined aquifer conditions for different aquitard thickness ($d'_D = 10$, 1, 0.1, and 0.01) without aquitard storage.

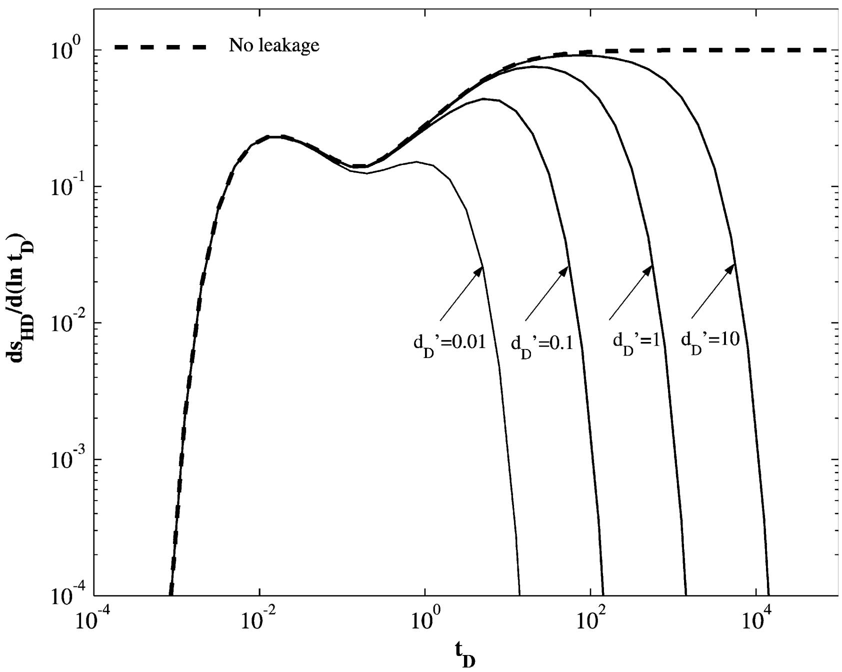

Fig. 3. Comparison of the derivative type curves $ds_{HD}/d(\ln t_D)$ between a confined aquifer and leaky confined aquifer conditions for different aquitard thickness ($d'_D = 10$, 1, 0.1, and 0.01) without aquitard storage.

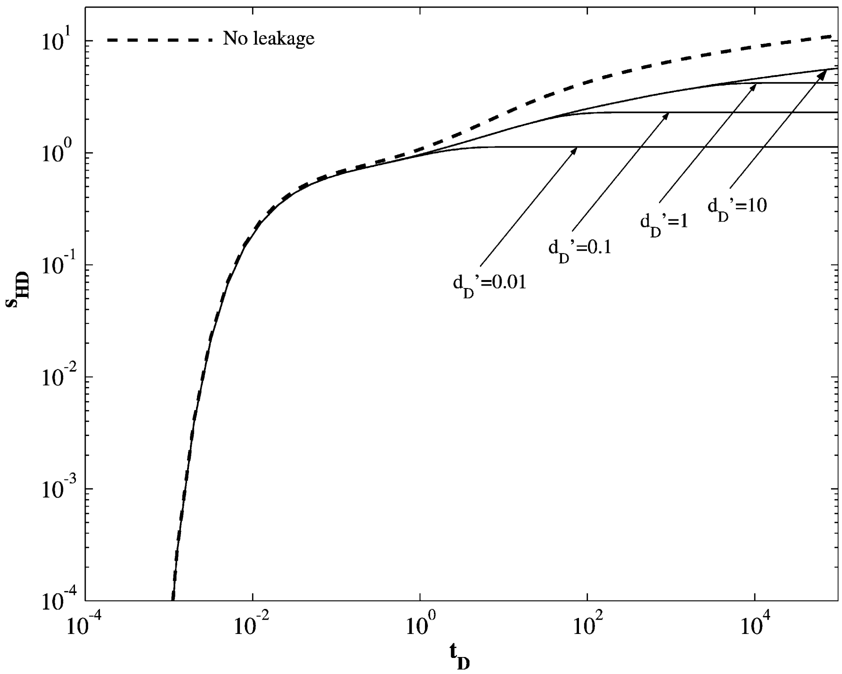

Fig. 4. Comparison of the type curves between a confined aquifer and leaky confined aquifer conditions for different aquitard thickness ($d'_D = 10$, 1, 0.1, and 0.01) with aquitard storage ($S'_s/S_s = 50$).

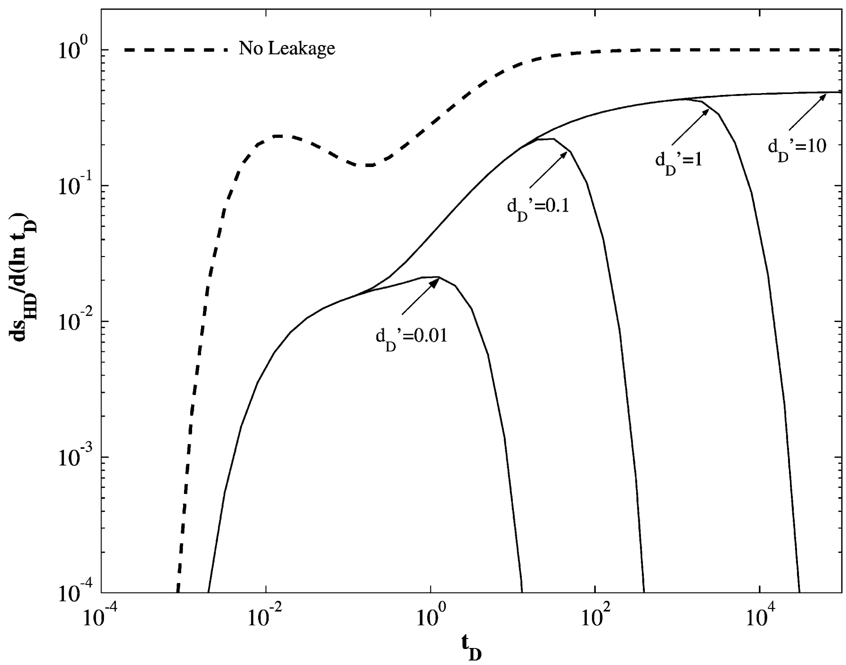

Fig. 5. Comparison of the derivative type curves $ds_{HD}/d(\ln t_D)$ between a confined aquifer and leaky confined aquifer conditions for different aquitard thickness ($d'_D = 10$, 1, 0.1, and 0.01) with aquitard storage ($S'_s/S_s = 50$).

This is shown in Figs. 3 and 5. Second, because the aquitard storage releases water soon after the pumping starts, the aquitard now serves as a buffer zone for the leakage. Thus, the leakage effect is delayed if compared to the case without storage. Fig. 5 shows that because the aquitard storage releases water to the aquifer around the same time as the aquifer releases its storage water, the drawdown in the aquifer is smaller than that observed in a confined aquifer. Furthermore, when the aquitard thickness increases, the aquitard storage effect becomes greater (Figs. 3 and 5).

Figs. 6 and 7 show the sensitivity of the dimensionless drawdown to the aquitard/aquifer specific storage ratio, S'_s/S_s. As expected, a higher ratio implies greater amounts of water released from the aquitard, resulting in a larger deviation from the case without storage ($S'_s = 0$). The specific storage of the aquitard mostly affects the intermediate time drawdown. Fig. 6 indicates that the type curves are generally not very sensitive to S'_s/S_s.

Analysis of the increasing aquitard/aquifer hydraulic conductivity ratio, K'/K_z, is found to be identical to that caused by increasing d'_D when the aquitard storage is negligible, because the drawdown only depends on $\lambda = K'd/K_zd'$. Thus decreasing K'/K_z is equivalent of increasing d'_D. Figs. 8 and 9 show the sensitivity of the dimensionless drawdown to K'/K_z when the aquitard storage is considered. Eqs. (27)–(30) indicate that the drawdown depends on d'_D, S'_s/S_s, and K'/K_z for this case. The case without aquitard storage is different; decreasing K'/K_z is not exactly equivalent to increasing d'_D (Figs. 4, 5, 8 and 9). A greater K'/K_z implies a rapid release of the storage water from the aquitard and a faster speed of leakage across the aquitard, which is a departure from the non-leaky curve.

4.2. Type curves and derivative type curves in a leaky aquifer under a water reservoir

In general, type curves and derivative type curves in a leaky aquifer under a water reservoir

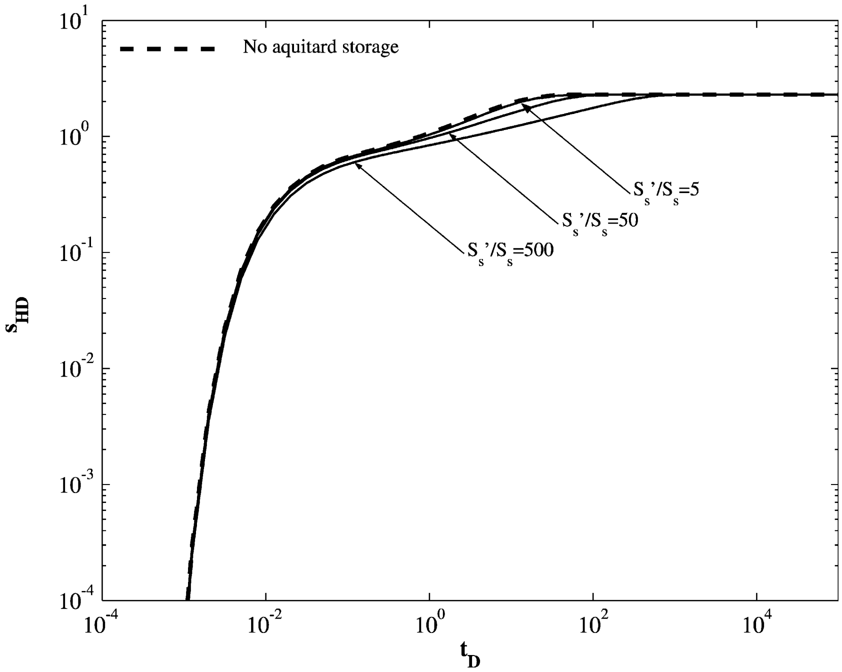

Fig. 6. Comparison of the type curves between a case without the aquitard storage and cases with different aquitard storage coefficients ($S_s'/S_s = 500$, 50, and 5) in leaky confined aquifers.

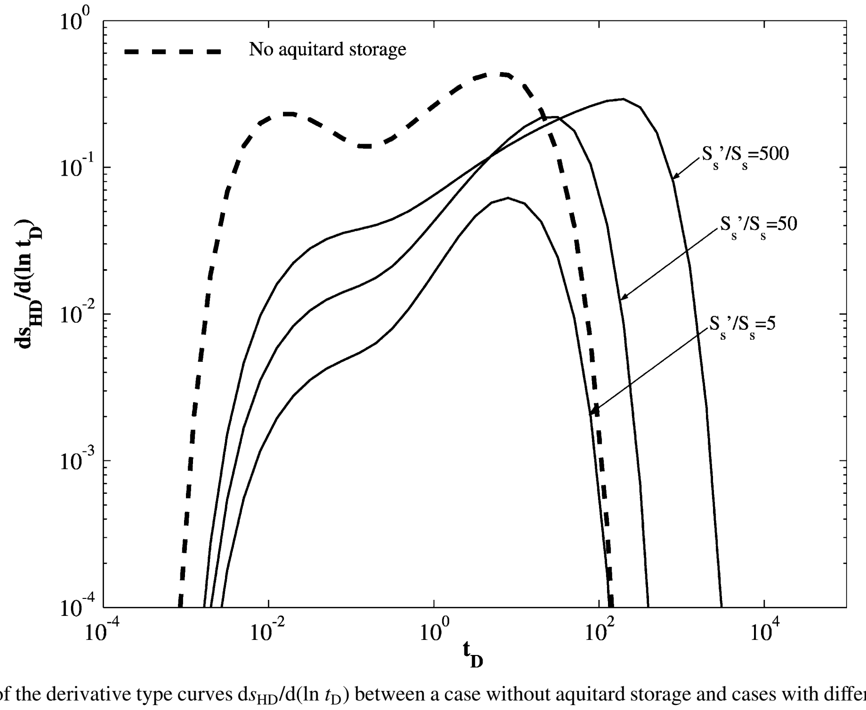

Fig. 7. Comparison of the derivative type curves $ds_{HD}/d(\ln t_D)$ between a case without aquitard storage and cases with different aquitard storage coefficients ($S_s'/S_s = 500$, 50, and 5) in leaky confined aquifers.

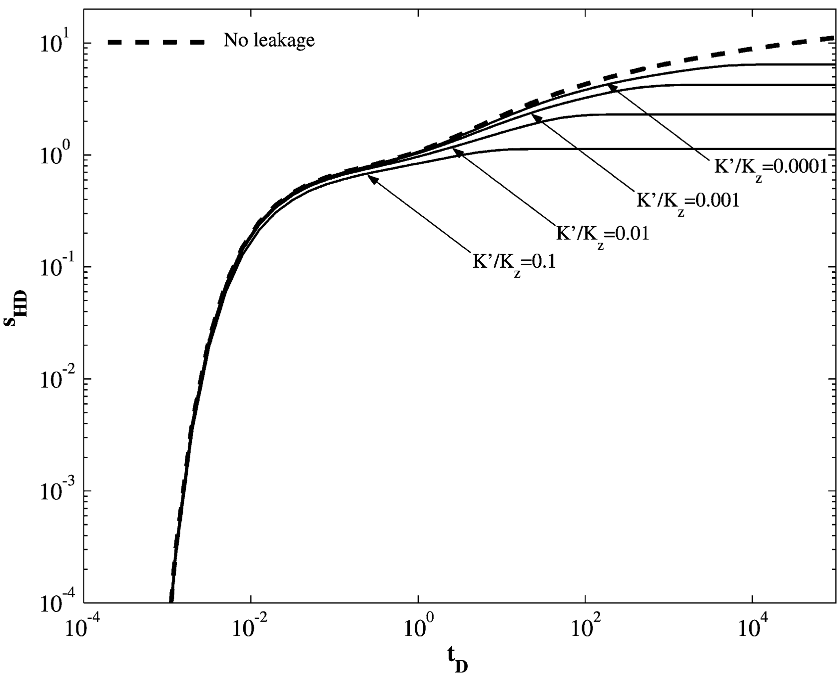

Fig. 8. Comparison of the type curves between a confined aquifer and leaky confined aquifers with different aquitard hydraulic conductivities ($K'/K_z = 0.1$, 0.01, 0.001, and 0.0001) with aquitard storage ($S_s'/S_s = 50$).

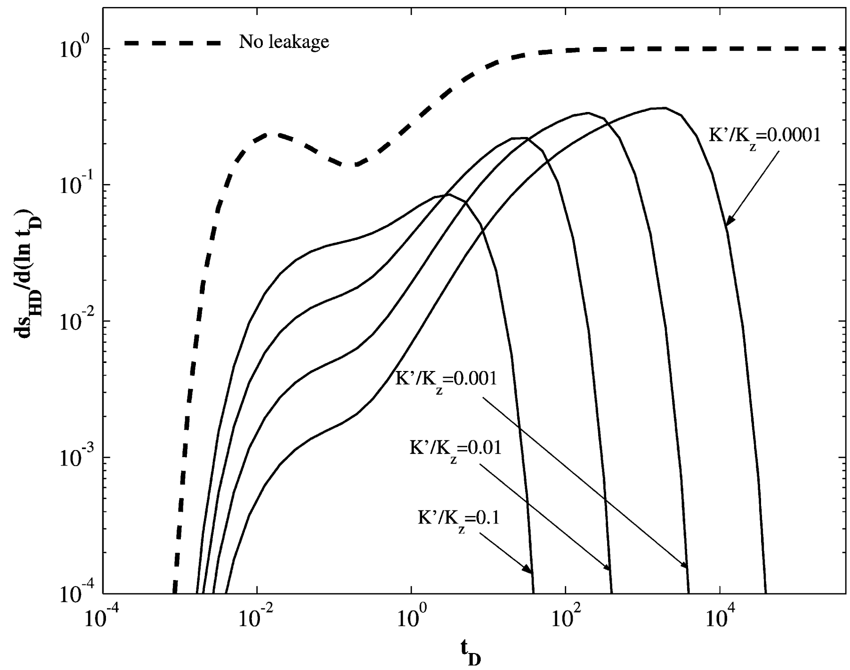

Fig. 9. Comparison of the derivative type curves $ds_{HD}/d(\ln t_D)$ between a confined aquifer and leaky confined aquifers with different aquitard hydraulic conductivities ($K'/K_z = 0.1$, 0.01, 0.001, and 0.0001) with aquitard storage ($S_s'/S_s = 50$).

are less sensitive to the aquitard parameters compared to a case of a leaky confined aquifer. This is because the aquifer can be fed by two water sources in addition to the water in the aquifer storage: one from the upper water reservoir, and another from the lower adjacent layer. The water reservoir is the second important water source after the aquifer storage, because it is in direct contact with the aquifer. One can see only small changes to the type curves during the intermediate time when varying S_s' from 0.01 to 0.0001 (Figs. 10 and 11).

4.3. Type curves and derivative type curves in a leaky water table aquifer

The response of a leaky water table aquifer to the pumping combines traits of responses of a leaky confined aquifer and a leaky aquifer under a water reservoir. In this case, water drained under the moving water table becomes the dominated water source at the later time, thus the leakage from the lower aquitard becomes less important. Respectively, the type curves are less sensitive to the aquitard parameters when compared to the leaky confined aquifer. However, water drainage from the water table yields less water than the upper water reservoir that is treated as constant head. Thus, the type curves in a water table aquifer are more sensitive to the aquitard parameters than in the case of an aquifer under a water reservoir (Figs. 12–15).

Figs. 12 and 13 show weak sensitivity to the dimensionless aquifer thickness when the aquitard storage is not considered, and this sensitivity is less profound than that in Figs. 2 and 3. Figs. 14 and 15 show similar sensitivity to the dimensionless aquitard thickness as Figs. 12 and 13 when the aquitard storage is considered. In fact, the difference between Figs. 12 and 14 or between Figs. 13 and 15 is at the early and intermediate time. When including the aquitard storage, the aquitard will release water soon after the pumping starts. Therefore, the drawdown at the early time will be smaller than that in the non-storage aquitard case.

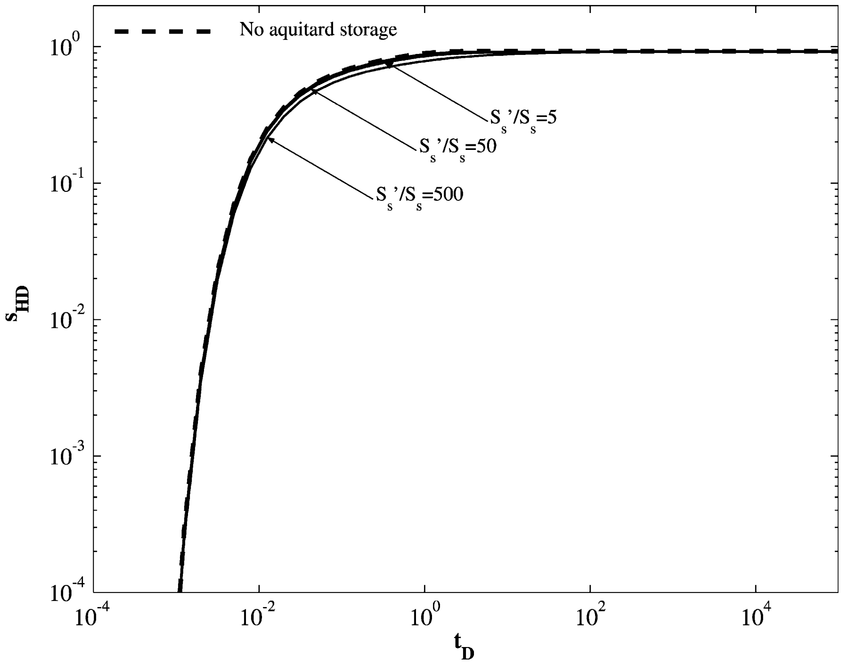

Fig. 10. Comparison of the type curves between a case without the aquitard storage and cases with different aquitard storage coefficients ($S_s'/S_s = 500$, 50, and 5) for leaky aquifers under water reservoirs.

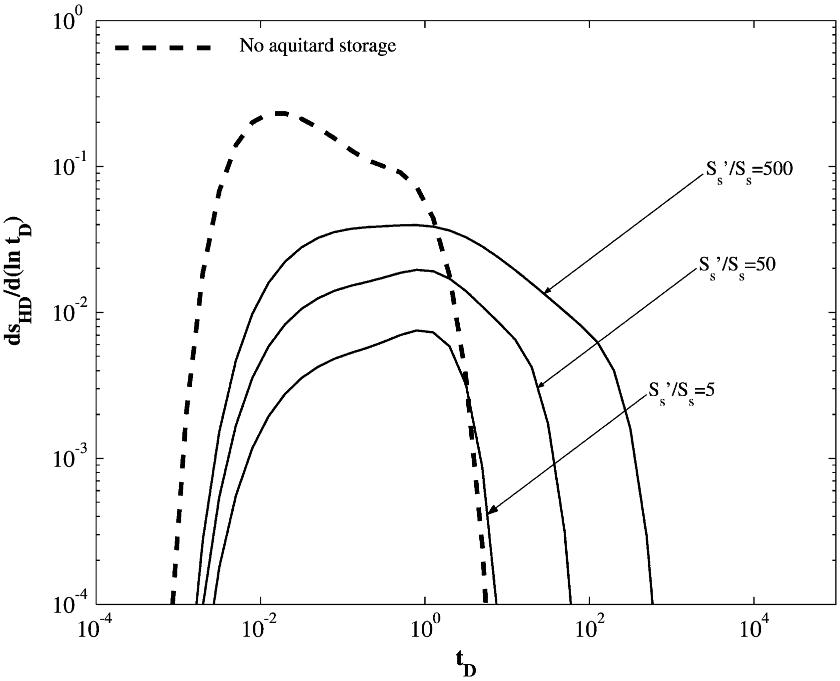

Fig. 11. Comparison of the derivative type curves $ds_{HD}/d(\ln t_D)$ between a case without aquitard storage and cases with different aquitard storage coefficients ($S_s'/S_s = 500$, 50, and 5) for aquifers under water reservoirs.

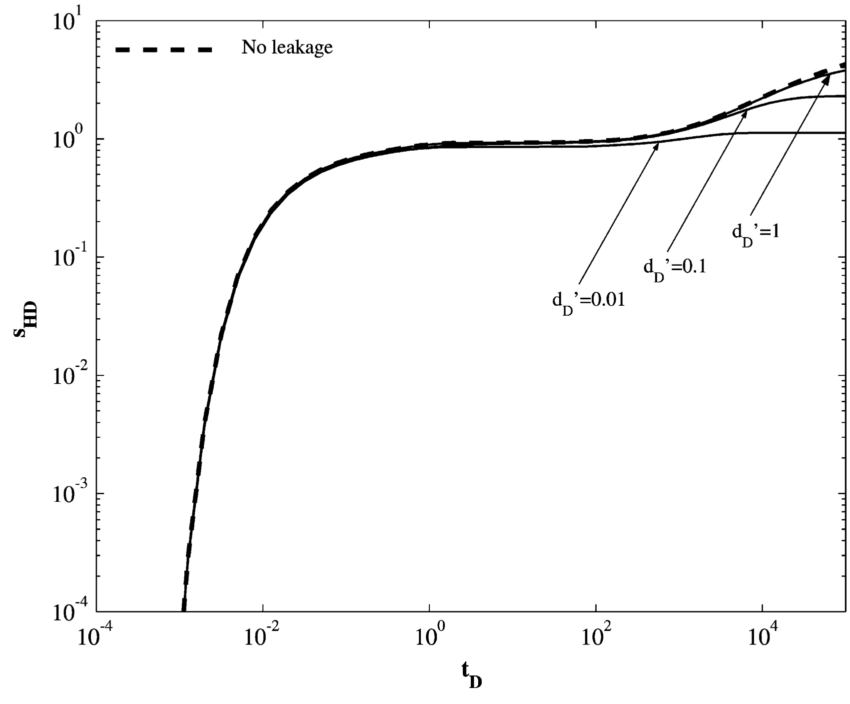

Fig. 12. Comparison of the type curves between a water table aquifer and leaky water table aquifers with different aquitard thickness ($d_D' = 1$, 0.1, and 0.01) without aquitard storage.

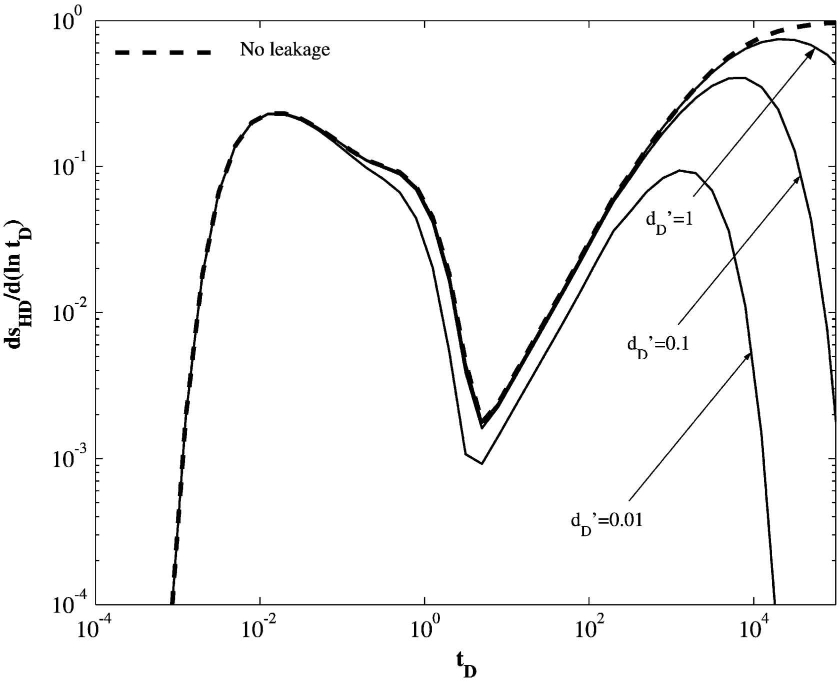

Fig. 13. Comparison of the derivative type curves $ds_{HD}/d(\ln t_D)$ between a water table aquifer and leaky water table aquifers with different aquitard thickness ($d'_D = 1$, 0.1, and 0.01) without aquitard storage.

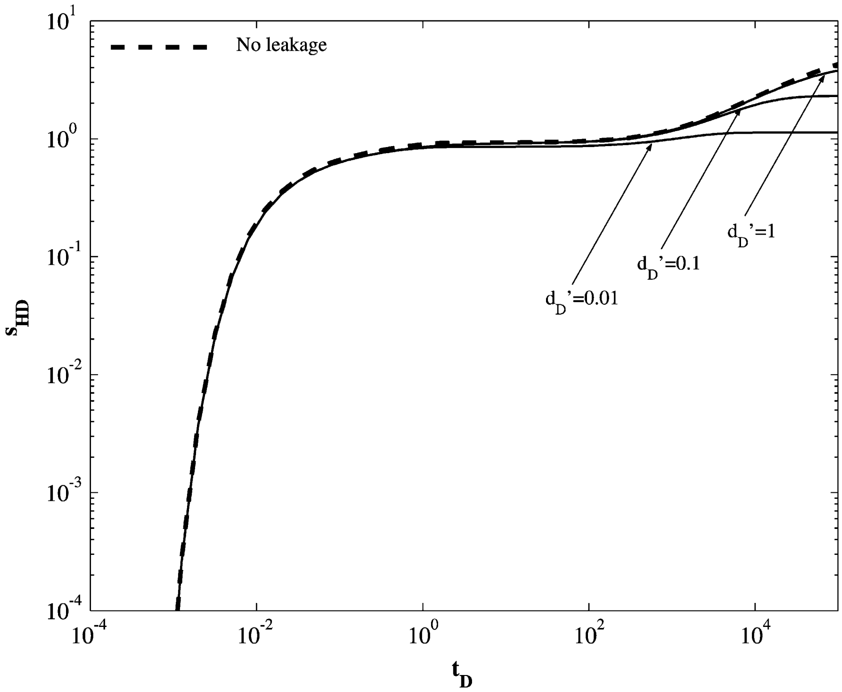

Fig. 14. Comparison of the type curves between a water table aquifer and leaky water table aquifers with different aquitard thickness ($d'_D = 1$, 0.1, and 0.01) with aquitard storage ($S'_s/S_s = 50$).

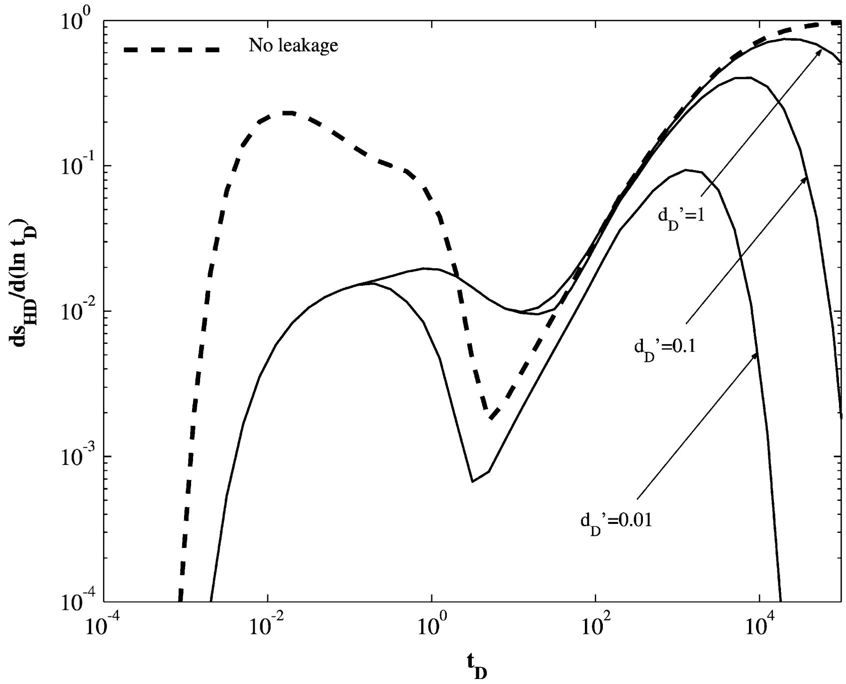

Fig. 15. Comparison of the derivative type curves $ds_{HD}/d(\ln t_D)$ between a water table aquifer and leaky water table aquifers with different aquitard thickness ($d'_D = 1$, 0.1, and 0.01) with aquitard storage ($S'_s/S_s = 50$).

5. Conclusions

This study provides a general computational tool for studying groundwater flow to a horizontal well in a leaky confined aquifer, an aquifer under a water reservoir and a leaky aquitard at the bottom, and a water table aquifer with a leaky aquitard at the bottom. It considers two different approaches in treatment of the aquitard properties: with and without the aquitard storage. The solution for a point sink is first derived and then the superposition technique is used for derivation of the solution for a horizontal well.

The type curves and derivative type curves of groundwater flow to a horizontal well under different aquifer conditions are analyzed. A graphically integrated MATLAB program (HW_LEAK) is used for the generation of the type curves and derivative type curves. These curves are important tools for interpretation of the horizontal well pumping data in leaky aquifers. The sensitivity analyses of the type curves and derivative type curves show the following conclusions:

1. In general, the derivative type curves are more sensitive to the aquitard parameters than the drawdown type curves. They can be used as diagnostic tools to the drawdown data when the type curves fail to recognize the aquifer properties.
2. The type curves and derivative type curves are usually sensitive to the aquitard/aquifer thickness ratio and the hydraulic conductivity ratio at the intermediate and later times. They are generally less sensitive to the aquitard/aquifer specific storage ratio. The most noticeable effect of the aquitard storage occurs at intermediate time.
3. Type curves and derivative type curves are most sensitive to the aquitard parameters in a leaky confined aquifer case. They are less sensitive to those parameters in a leaky aquifer under a water reservoir because the water reservoir serves as the major water source soon after the pumping start, and it substantially surpasses the contribution from the aquitard leakage. The sensitivity to the aquitard parameters in a leaky

water table aquifer case is between cases of a leaky confined aquifer and a leaky aquifer under a water reservoir.

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Appendix A. Drawdown in a leaky confined aquifer without aquitard storage

Substituting Eq. (17) into Eq. (13) results in:

$$\omega_n = n\pi, \qquad n = 0, 1, 2, 3, \ldots \tag{A1}$$

Substituting Eq. (17) into Eq. (9), multiplying by $\cos(\omega_n z_D)$, and integrating from 0 to 1 in the z_D direction results in the following two equations

$$\frac{\partial^2 H_0}{\partial x_D^2} + \frac{\partial^2 H_0}{\partial y_D^2} - (p+\lambda)H_0 + \frac{4\pi}{p}\delta(x_D - x_{0D})\delta(y_D - y_{0D}) = 0, \tag{A2}$$

and

$$\frac{\partial^2 H_n}{\partial x_D^2} + \frac{\partial^2 H_n}{\partial y_D^2} - (p+\lambda+\omega_n^2)H_n + \frac{8\pi}{p}\delta(x_D - x_{0D}) \times \delta(y_D - y_{0D})\cos(\omega_n z_{wD}) = 0, \quad n > 0, \tag{A3}$$

where z_w and z_{wD} are the dimensional and dimensionless distances from the horizontal well to the aquifer lower boundary, respectively.

The solutions to above problems are similar to what has been discussed by Zhan et al. (2001), and Zhan and Park (2002), and are given below

$$H_0 = \frac{2}{p}K_0\left(r_D\sqrt{p+\lambda}\right), \tag{A4}$$

$$H_n = \frac{4\cos(\omega_n z_{wD})}{p}K_0\left(r_D\sqrt{p+\omega_n^2+\lambda}\right), \quad n > 0, \tag{A5}$$

where $r_D = [(x_D - x_{0D})^2 + (y_D - y_{0D})^2]^{1/2}$, and K_0 is the zero-order, second kind modified Bessel function. The point sink solution in Laplace domain becomes:

$$\bar{s}_D = \frac{2}{p}K_0(r_D\sqrt{p+\lambda}) + \sum_{n=1}^{\infty}\frac{4\cos(\omega_n z_{wD})\cos(\omega_n z_D)}{p}K_0\left(r_D\sqrt{p+\omega_n^2+\lambda}\right). \tag{A6}$$

The point sink solution in the real time domain is obtained by conducting the inverse Laplace transform of Eq. (A6)

$$s_D(t_D) = W\left(\frac{r_D^2}{4t_D}, r_D\sqrt{\lambda}\right) + 2\sum_{n=1}^{\infty}\cos(\omega_n z_{wD}) \times \cos(\omega_n z_D)W\left(\frac{r_D^2}{4t_D}, r_D\sqrt{\omega_n^2+\lambda}\right), \tag{A7}$$

where W is the leaky well function used by Hantush (1964). The solution to a horizontal well is obtained by integrating Eq. (A7) along the well axis

$$s_{HD}(t_D) = \frac{1}{L_D}\left[\int_{-L_D/2}^{L_D/2} W\left(\frac{r_D^2}{4t_D}, r_D\sqrt{\lambda}\right)dx_{0D} + 2\sum_{n=1}^{\infty}\cos(\omega_n z_{wD})\cos(\omega_n z_D) \times \int_{-L_D/2}^{L_D/2} W\left(\frac{r_D^2}{4t_D}, r_D\sqrt{\omega_n^2+\lambda}\right)dx_{0D}\right], \tag{A8}$$

where s_{HD} refers to the drawdown generated by a horizontal well. Integration of Eq. (A8) is similar to what has been discussed in Zhan et al. (2001), and Zhan and Park (2002):

$$s_{HD}(t_D) = \frac{\sqrt{\pi}}{L_D}\int_0^{t_D}\frac{1}{\sqrt{\tau}}\left[\mathrm{erf}\left(\frac{L_D/2 + x_D}{2\sqrt{\tau}}\right) + \mathrm{erf}\left(\frac{L_D/2 - x_D}{2\sqrt{\tau}}\right)\right]\exp\left(-\frac{y_D^2}{4\tau}\right)\left[\exp(-\lambda\tau) + 2\sum_{n=1}^{\infty}\cos(n\pi z_{wD})\cos(n\pi z_D) \times \exp(-[n^2\pi^2+\lambda]\tau)\right]d\tau. \tag{A9}$$

Appendix B. Drawdown in a leaky confined aquifer with aquitard storage

Substituting Eq. (17) into Eq. (29), multiplying by $\cos(\omega_n z_D)$, and integrating from 0 to 1 in the z_D direction results in the equation of H_n. These solutions are then solved using the method provided by Zhan et al. (2001), and Zhan and Park (2002). The point sink/source results in Laplace domain are as follows:

In a leaky confined aquifer,

$$\bar{s}_D = \sum_{n=0}^{\infty} \frac{4\cos(n\pi z_{wD})\cos(n\pi z_D)}{p} K_0\left(r_D\sqrt{n^2\pi^2 + \Psi^2}\right). \tag{A10}$$

In a leaky aquifer under a water reservoir,

$$\bar{s}_D = \sum_{n=0}^{\infty} \frac{4\cos\left(\left[n\pi + \frac{\pi}{2}\right]z_{wD}\right)\cos\left(\left[n\pi + \frac{\pi}{2}\right]z_D\right)}{p} \times K_0\left(r_D\sqrt{\left(n\pi + \frac{\pi}{2}\right)^2 + \Psi^2}\right). \tag{A11}$$

In a water table aquifer,

$$\bar{s}_D = \frac{4}{p}\sum_{n=0}^{\infty} \frac{\cos(\omega_n z_{wD})\cos(\omega_n z_D)}{1 + 0.5\sin(2\omega_n)} K_0\left(r_D\sqrt{\omega_n^2 + \Psi^2}\right), \tag{A12}$$

where ω_n and r_D are the same as that used in the case without the aquitard storage.

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