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Any P-wave kinematic algorithm for vertical transversely isotropic media can be ADAPTed to moderately anisotropic media of arbitrary symmetry type

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Abstract

Most of P-wave anisotropic kinematic algorithms (modeling, processing, and inversion) have been developed for the case of Transverse Isotropy (TI). Does it mean that when dealing with more complex symmetry types (Arbitrarily tilted TI, orthorhombic, monoclinic or even triclinic), all these algorithms are irrelevant? In fact, not at all. It has recently been demonstrated that in 2D geometry any qP-wave TI kinematic algorithm can be simply generalized to the case of monoclinic symmetry using the so-called Azimuthally Dependent Anisotropy Parameter Transformation (ADAPT), assuming moderate anisotropy. The extension of the technique to the case of arbitrary anisotropy type (triclinic) is achieved in this paper. The method is successfully checked for seismic modeling in a full 2D model with layers of contrasted anisotropy types and with arbitrary vertical and horizontal velocity variations (non-constant gradient). Typically, the approximate travel times using ADAPT differ from the exact travel times by a few milliseconds for total travel times of the order of a few seconds. Applications to seismic processing are also described. The simplicity of the procedure and the generality of the applicability of the ADAPT recipe are striking and very convenient for practical applications. They certainly deserve further analysis.

Keywords: P-wave kinematics; Arbitrary anisotropy; Seismic anisotropy

1. Introduction

The presence of seismic anisotropy in sedimentary basins is now commonly accepted (e.g., Winterstein and Paulsson, 1996; Thomsen, 1986). For instance, the fraction of papers involving seismic anisotropy in international conventions such as those of the Society of Exploration Geophysicists (SEG) or of the American Geophysical Union (AGU) is steadily growing, keeping a seismic anisotropy community very active (e.g., Brown and Lawton, 1993; Fjaer et al., 1996; Rasolofosaon, 1998; Ikelle and Gangi, 2000).

Seismic anisotropy, if not correctly taken into account, can strongly affect many steps of seismic processing, such as velocity analysis, dip move-out, time migration, time-to-depth conversion or amplitude versus offset (e.g., Tsvankin, 1996). That is why anisotropy is now often integrated in industrial computer codes. However, the anisotropy type commonly considered is transverse isotropy with a vertical infinite-fold axis of symmetry (VTI). Although VTI is the

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most frequently encountered anisotropy type in sedimentary basins, there may exist some other realistic cases, for instance the bedding planes can sometimes substantially dip (tilted TI medium), or in other cases the presence of one or multiple set of fractures imply that the medium can apparently exhibit a more complicated symmetry type, even the most complicate type (triclinic), in the coordinate system of acquisition of the seismic data (e.g., Helbig, 1994). So a central question arises: Are all the computer codes developed for the VTI case totally irrelevant for more complicate anisotropy types (arbitrarily tilted TI, orthorhombic, monoclinic or even triclinic)? In fact, not at all, at least if one restricts the topic to the kinematics of qP-waves in 2D geometry in moderately anisotropic media. It has recently been demonstrated (Rasolofosaon, 2000b) that in such cases any qP-wave VTI kinematic algorithm can be used, just as it is, in media of monoclinic symmetry with a horizontal symmetry plane, using the so-called Azimuthally Dependent Anisotropy Parameter Transformation (ADAPT), as will be detailed later. This paper is a continuation of this work.

The outline of the paper is as follows. First, I summarize the ADAPT technique. This will be followed by the extension of ADAPT, initially restricted to monoclinic media with a horizontal symmetry plane, to media of arbitrary symmetry type (triclinic). Then the new procedure is checked on a seismic modeling. Some applications to seismic processing, namely velocity analysis, are described in the following section. Finally, I will discuss the results and summarize the most general conclusions.

2. The ADAPT recipe

It has been recently demonstrated that any qP-wave kinematic algorithm in 2D geometry developed for TI media can straightforwardly be adapted to monoclinic media (with a horizontal symmetry plane) (Rasolofosaon, 2000b). The method simply consists in replacing the anisotropy parameters ε and δ of Thomsen (1986) in the TI kinematic equations by their azimuthally dependent counterparts $\varepsilon(\lambda)$ and $\delta(\lambda)$ in the monoclinic equations. These two functions are defined in Appendix A. The transformation $(\varepsilon,\lambda) \rightarrow [\varepsilon(\lambda),\delta(\lambda)]$ is called the Azimuthally Dependent Anisotropy Parameter Transformation (ADAPT) or more simply the ADAPT recipe. The mathematical justification of the method simply comes from the formal similarity between the qP-wave velocity equation in transversely isotropic (TI) media and the corresponding qP-velocity equation in media of monoclinic symmetry with a horizontal symmetry plane. More precisely, assuming moderate anisotropy, in Transversely Isotropic (TI) media the qP-wave velocity equation writes (Thomsen, 1986):

$$\frac{V(\theta)}{V_{\mathrm{P}}^{\mathrm{vertical}}} \approx 1 + \delta\ S_\theta^2\ C_\theta^2 + \varepsilon\ S_\theta^4 \tag{1}$$

For brevity, I use $C_\theta = \cos\theta$ and $S_\theta = \sin\theta$, where θ is angle between the propagation direction and the vertical direction, or colatitude angle. The qP-wave velocity in the vertical direction ($\theta = 0$), or more simply the vertical velocity, is denoted by $V_{\mathrm{P}}^{\mathrm{vertical}}$. The corresponding equation in anisotropic media of symmetry as complex as monoclinic with a horizontal symmetry plane is (Rasolofosaon, 2000a):

$$\frac{V(\theta,\lambda)}{V_{\mathrm{P}}^{\mathrm{vertical}}} \approx 1 + \delta\ (\lambda)\ S_\theta^2\ C_\theta^2 + \varepsilon\ (\lambda) S_\theta^4 \tag{2}$$

where λ is the azimuth angle between the plane containing the vertical axis z and the direction of propagation and the coordinate plane xz. The azimuth dependent functions $\varepsilon(\lambda)$ and $\delta(\lambda)$ are defined in Appendix A.

3. Generalized ADAPT recipe: extension to the case of arbitrary anisotropy type

In principle, the application of the conventional ADAPT recipe is restricted to media of symmetry higher than monoclinic, such as orthorhombic media or TI media with a horizontal or vertical symmetry axis. But this does not mean that nothing can be done when dealing with media of more complicate symmetry types (TI with arbitrarily tilted axis or even triclinic). In such cases, I suggest to operate in steps. Starting from a medium with the most general anisotropy type (triclinic), the first step consists in replacing the initial triclinic medium by its best monoclinic approximation using the method proposed by Arts (1993) and Arts et al. (1991). As described in the two previous references, the "best monoclinic tensor", or target tensor, that best approximates an arbitrary triclinic tensor, or source tensor, is the elastic tensor that minimizes the norm of the difference between the source tensor and the target tensor. The norm of a fourth rank tensor is equal to the sum of the squares of its components. Thus, defined it is independent of the coordinate system. In the second and last step, the ADAPT recipe is applied to the approximate monoclinic medium instead of the initial triclinic medium.

The last step is straightforward and has been described by Rasolofosaon (2000b). The most unclear step, up to now, is the first one which is in fact rather simple. In effect, the approximate monoclinic stiffness matrix is obtained from the triclinic stiffness matrix simply by replacing by zero the elastic coefficients C_{14}, C_{15}, C_{24}, C_{25}, C_{34}, C_{35}, C_{46} and C_{56} as explained in the two previous references. An illustration of this process is given in Appendix B with the gas-saturated dolomite reservoir rock, studied by Rasolofosaon (2000a) for which I give the complete triclinic stiffness matrix and the corresponding approximate monoclinic stiffness matrix.

At first sight, the technique adopted in this first step seems to be quite rough. In fact, as pointed out by Arts (1993) the kinematics of the P-wave, in contrast with S-waves, in a triclinic medium and in its best monoclinic approximation do not differ substantially, especially when the anisotropy strength is moderate. This is clearly illustrated in Figs. 1 and 2 corresponding to the over-mentioned dolomite sample. Fig. 1 shows the complete directional dependence of the exact qP-wave phase velocity in the rock sample as function of the colatitude θ and the azimuth λ. The anisotropy is moderate, for instance typically the P-wave velocity roughly varies from 3.9 to 4.4 km/s. The P-wave velocities in the triclinic rock sample and in the approximate monoclinic medium typically deviates by a few parts per thousands as shown in Fig. 2.

The main conclusion of this part is that the ADAPT recipe initially restricted to monoclinic media have been extended to media of arbitrary anisotropy type,

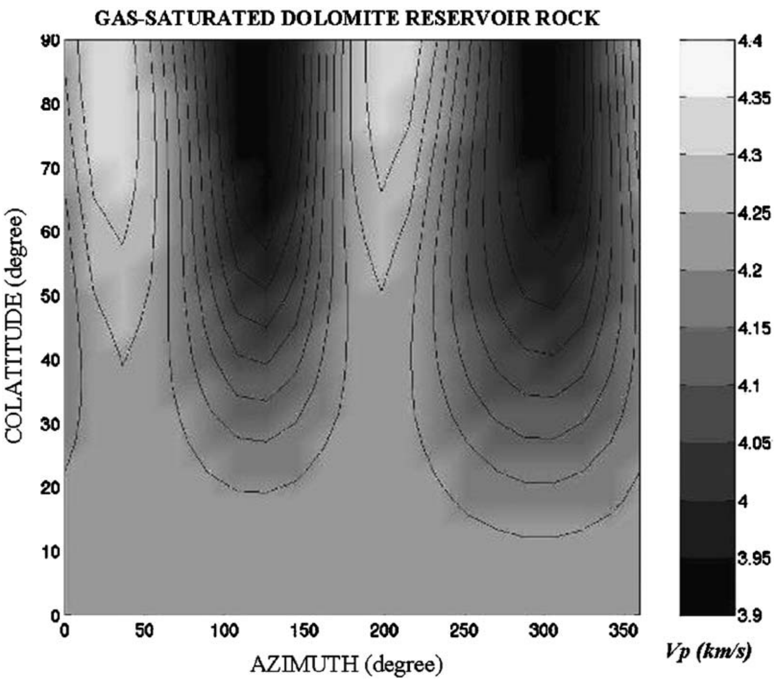

Fig. 1. Complete directional dependence of the exact qP-wave phase velocity in a gas-saturated dolomite reservoir rock as function of the colatitude θ and the azimuth λ. The elastic constants are in Appendix B.

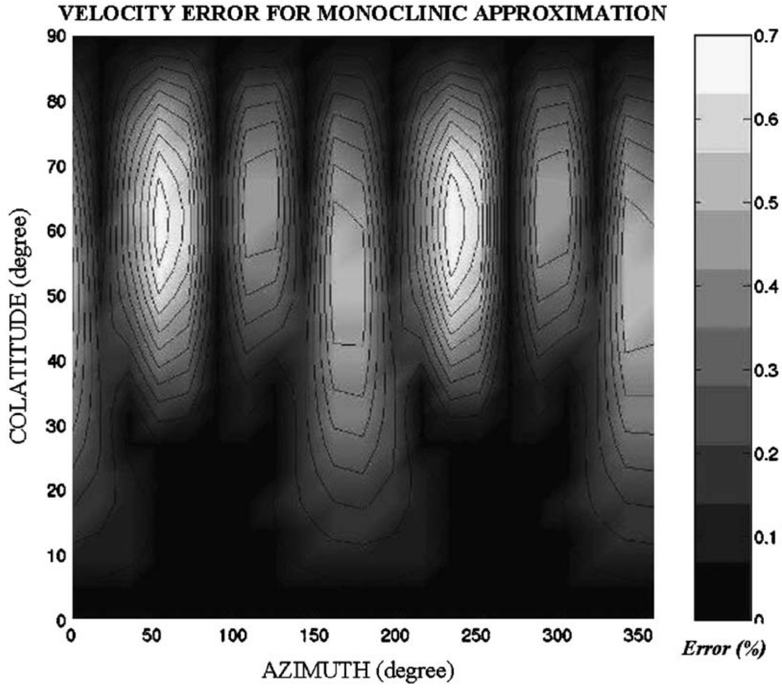

Fig. 2. Complete directional dependence of the deviation between the exact qP-wave phase velocity and its best monoclinic approximation normalized by the exact qP-wave phase velocity as function of the colatitude θ and of the azimuth λ in a gas-saturated dolomite reservoir rock. The elastic constants are in Appendix B.

however of moderate anisotropy strength. As a consequence, any qP-wave kinematic algorithm developed for TI media can straightforwardly be adapted to triclinic media simply by using the modified ADAPT recipe described above. This will be applied to 2D seismic modeling in the next section.

4. Application of the generalized ADAPT recipe to seismic modeling in triclinic media

4.1. General procedure

A simple way to explain the general procedure for the application of the ADAPT recipe to qP-wave kinematics modeling is to start from an example. Assume that one has a computer code for computing the travel times of qP-waves in 2D models constituted by TI media exclusively. At first sight, and in principle, such a computer code cannot deal with models constituted by media of anisotropy type more complicate than TI, such as orthorhombic (outside the symmetry planes), monoclinic or triclinic. In fact this is wrong if the anisotropy strength is moderate. In other words, this computer code can also be used for 2D models constituted by media of arbitrary anisotropy type, as far as the anisotropy strength is moderate. The general procedure, inspired from the previous section, is described by the flow chart in Fig. 3. First of all, let us start with a 2D model made of media of general anisotropy type (top of the diagram). In the first step, while keeping the geometry of the model strictly unchanged, one replaces each triclinic medium constituting the model by its "best monoclinic approximation" with a horizontal symmetry plane as described in the previous section. Without restriction, one can assume that the 2D geometry plane is a vertical, of azimuth λ. Since the anisotropy strength is assumed moderate, the kinematic equations of qP-wave in this plane is formally identical to the kinematic equations of qP-wave in the vertical plane of a VTI medium characterized by the anisotropy parameters $\varepsilon = \varepsilon(\lambda)$ and $\delta = \delta(\lambda)$, and by the same vertical velocity V_P^{vertical},

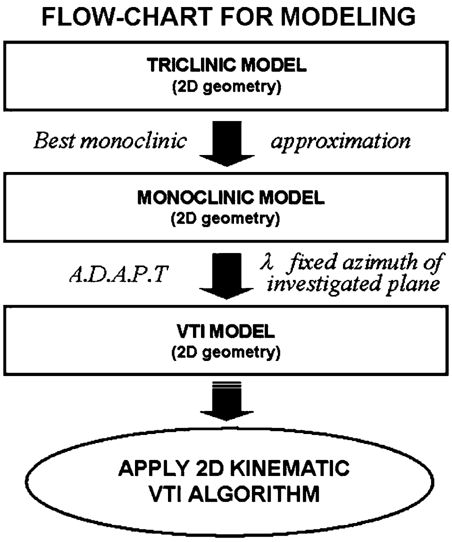

Fig. 3. Flow chart for modeling showing how to replace an anisotropic 2D model made of media of arbitrary anisotropy type by a kinematically equivalent 2D model made of transversely isotropic media with a vertical symmetry axis using the ADAPT recipe.

as illustrated in Fig. 1 and the corresponding comments. As a consequence, the approximate monoclinic model can be replaced by an equivalent VTI model on which the available modeling code can be applied. The numerical test of this procedure is described in the next section.

4.2. Numerical test

The procedure previously described is tested on the model sketched in Fig. 4. The model is two-dimensional (2D) invariant in the *Y* direction. It is 10-layer model including TI media with symmetry axes in general arbitrarily oriented. Since the TI symmetry axis is not aligned with the axes *XYZ* of description of the problem, the media exhibit in appearance more complicate symmetry types, for instance monoclinic or even triclinic (e.g., Helbig, 1994). In other words, it is a model including media with contrasted symmetry types (vertical TI, horizontal TI, orthorhombic, monoclinic and even triclinic). The physical parameters are

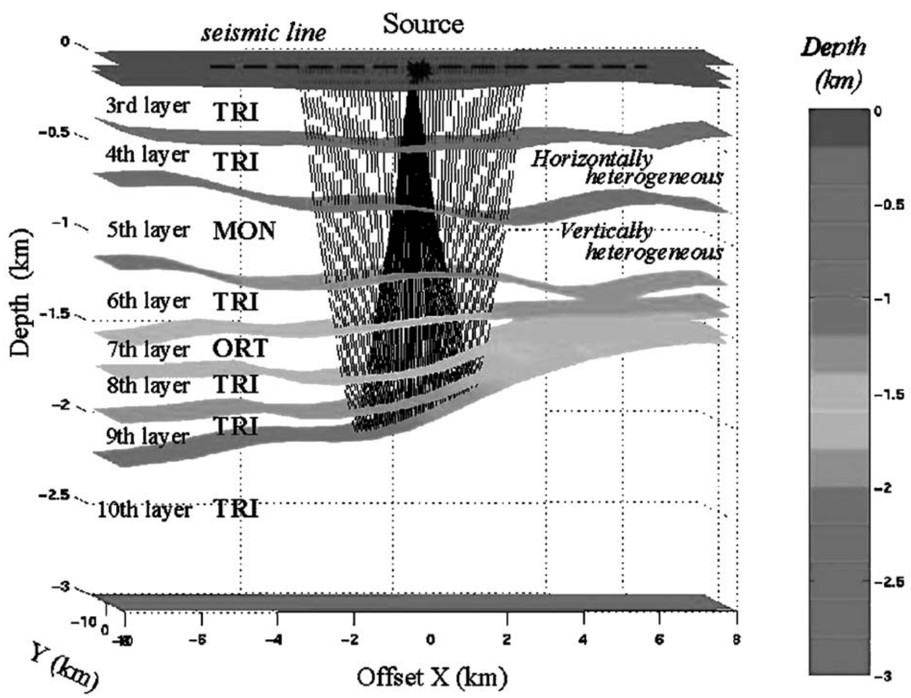

Fig. 4. Geometry of the 2D blocky model containing 10 layers of contrasted anisotropy types. Two layers, namely layers 4 and 5, exhibit arbitrary vertical variation, respectively, horizontal variation, of velocity.

listed in the table on the right side in Fig. 5. The table include the layer number (first column) [note that reflector number N, the free surface being the first reflector, is the interface above layer number N], the apparent anisotropy type (second column), the orientation of the TI axis (referenced by the colatitude θ in column 3, and the azimuth λ in column 4, the conventions being given on the left of the table), the qP-wave vertical velocity $V_P^{vertical}$ (fifth column), the anisotropy parameters ε, δ and γ (columns 6 to 8), the squared ratio $(V_S^{vertical}/V_P^{vertical})^2$ between the S-wave and P-wave vertical velocities (column 9), and the density (column 10). Note that the anisotropy strength is not weak but moderate (with anisotropy parameters as large as 15%, and the anellipticity $\varepsilon - \delta$ can be positive (layers 2, 3, 7 and 10), negative (layers 4, 6 and 9) or equal to zero (layers 5 and 8). Furthermore, the layers 4 and 5 are heterogeneous. The fourth layer exhibits a horizontal velocity variation with a non-constant gradient ($V_P^{vertical}$ = 2.2, 2.05, 1.9, 1.85, 1.8, 1.9, 1.95, 2.05, 2.2 km/s for $X = -8, -6, -4, -2, 0,$ 2, 4, 6, 8 km, respectively). Note that the non-constant lateral gradient varies around 0.5 s^{-1} in the region of interest of the experimental configuration in Fig. 4. The fifth layer is characterized by a vertical velocity variation with a non-constant gradient, ($V_P^{vertical}$ = 2.2, 2.3, 2.5, 2.5, 2.4, 2.4, 2.3, 2.2, 2.2 km/s for $Z = 0.66$, 0.74, 0.82, 0.90, 0.98, 1.06, 1.14, 1.22, 1.30 km, respectively).

The source is located at the origin of the coordinate system and the 60 receivers are on the surface ($Z = 0$) along a seismic line parallel to the X-axis. Offsets range from -3 to 3 km. I mainly focus on the P-wave reflected on the reflectors 3 to 9. The maximum offset to depth ratio roughly ranges from 1.5 (ninth reflector) to 6 (third reflector).

I compare the exact travel times computed for the original model in Fig. 5 with the approximate travel times corresponding to the equivalent Vertical TI model (see Fig. 4 and the corresponding comments in the text). In both cases, I used the ray-tracing code Anisamp of Farra (1989, 1999). The results are illustrated in Figs. 6 and 7. In Fig. 6, the approximate travel times are plotted as functions of the offset. It is worth noting that at the scale of the plot curves corresponding to the exact travel times, not shown on this figure, are hardly separable from the curves corresponding to the approximate travel times. Fig. 7 which shows the travel time error, or the difference between the exact and the approximate travel times, as function of the offset illustrates this. Note the difference between the scales of the y-axis of the two previous figures, namely the travel times are expressed in seconds in Fig. 6 and the travel time

LAYER	ANISOTROPY TYPE	TI AXIS ORIENTATION θ(deg)	TI AXIS ORIENTATION λ(deg)	$V_P^{vertical}$ (km/s)	ANISOTROPY PARAMETERS ε	δ	γ	$\left(\frac{V_S^{vertical}}{V_P^{vertical}}\right)^2$	ρ (kg/m^3)
1	VTI	0	0	1.5	0.05	0.1	0.05	0.01	1000
2	MONOCLINIC	90	30	1.6	0.1	0.05	0.15	0.024	1850
3	TRICLINIC	10	70	1.85	0.15	0.1	0.05	0.062	2000
4	TRICLINIC	60	30	Horizontal Nonlinear Gradient	0.05	0.1	0.15	0.069	2100
5	MONOCLINIC	45	0	Vertical Nonlinear Gradient	0.1	0.1	0.05	0.16	2300
6	TRICLINIC	50	40	2.6	0.05	0.15	0.1	0.189	2100
7	ORTHORHOMBIC	90	0	2.4	0.15	0.05	0.05	0.16	2300
8	TRICLINIC	30	70	2.8	0.1	0.1	0.05	0.189	2250
9	TRICLINIC	40	70	2.7	0.05	0.15	0.1	0.207	2400
10	TRICLINIC	20	10	3	0.1	0.05	0.1	0.25	2300

Fig. 5. Physical parameters of the 10-layer model in Fig. 4. See details in the text.

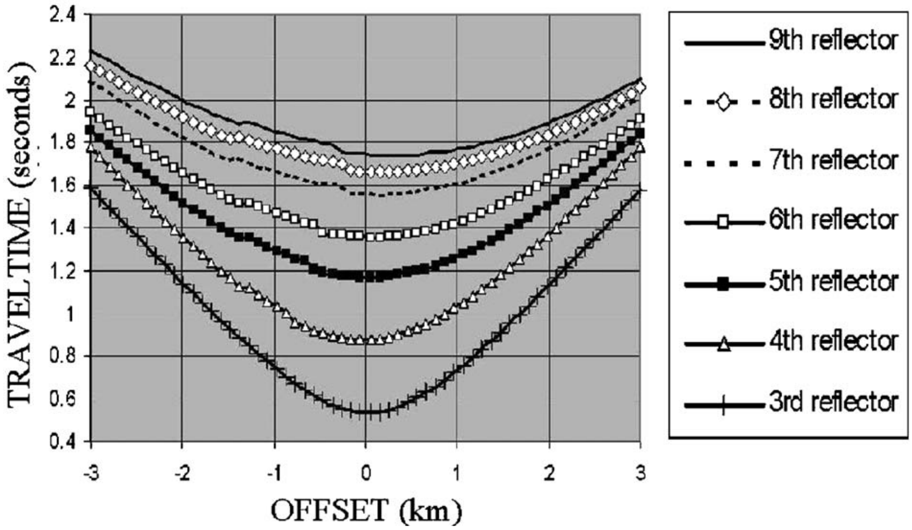

Fig. 6. Travel time as function of the offset in the VTI 2D model kinematically equivalent to the triclinic model in Fig. 4. This model is deduced by the technique described by the flow chart shown in Fig. 3.

errors in milliseconds in Fig. 7. One clearly notice that the travel time errors induced by using the ADAPT recipe is quite small and hardly exceeds 4 ms for travel times of roughly 2 s.

Many different simulations were performed on various types of models. For concision, the results are not reported in this paper but the conclusions are very similar to what has been said here. For 2D kinematic modeling of qP-wave, the VTI approximation sketched in Fig. 4 and based both on the "monoclinic approximations" of the media and on the use of the ADAPT recipe is quite accurate and allows to extend the applicability of modeling codes initially designed for VTI media to the case of media of

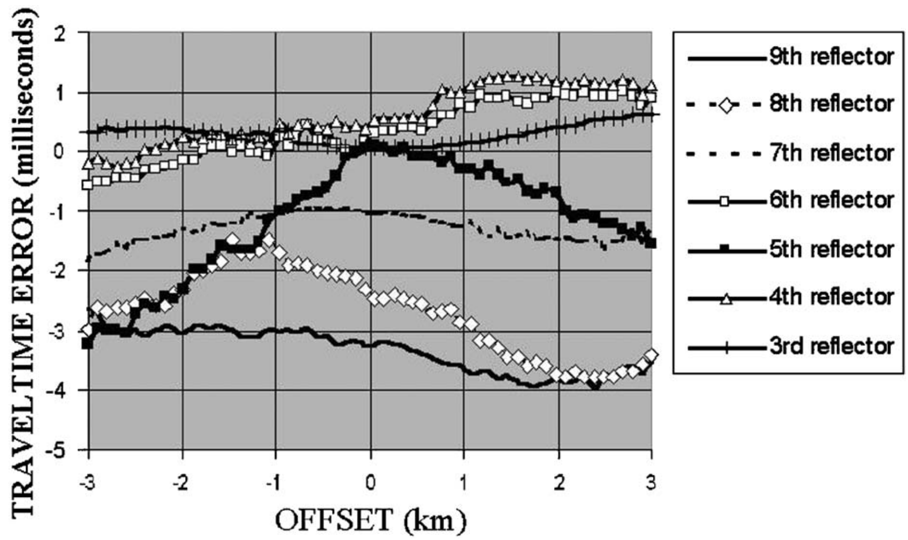

Fig. 7. Difference between the approximate travel time in the equivalent VTI model and the exact travel time in the triclinic model in Fig. 4 as function of the offset.

arbitrary symmetry type (triclinic), as far as the anisotropy strength is moderate and the geometry is two-dimensional. In the next section, some applications of the same idea to seismic processing are described.

5. Some applications to seismic processing

The generalized ADAPT recipe introduced in Section 3 is applicable not only to seismic modeling, as described in the previous section, but also to any ramification of kinematic processing. We illustrate this point by two examples related to a particular aspect of seismic processing, namely velocity analysis.

The first example is Tabti and Rasolofosaon (1998) who derived a quartic-corrected time–distance equation, and its corresponding Dix formalism for interval parameters inversion, for reflected qP-wave in horizontally multilayered anisotropic media of arbitrary anisotropy type:

$$T^2(X,\lambda) = T_0^2 + \frac{X^2}{[V_{\mathrm{nmo}}(\lambda)]^2} - \frac{2\eta(\lambda)X^4}{[V_{\mathrm{nmo}}(\lambda)]^2\{T_0^2[V_{\mathrm{nmo}}(\lambda)]^2 + [1+2\eta(\lambda)]X^2\}} \tag{3}$$

where $T^2(X,\lambda)$ is the squared travel time for the offset X and for the azimuth λ, T_0^2 the squared zero-offset travel time, and $V_{\mathrm{nmo}}(\lambda)$ the Normal Moveout (NMO) velocity for a seismic line along the azimuth λ. The functions $V_{\mathrm{nmo}}(\lambda)$ and $\eta(\lambda)$ are defined by:

$$\begin{cases} (V_{\mathrm{nmo}}(\lambda))^2 = (V_{\mathrm{P}}^{\mathrm{vertical}})^2 \ (1 + 2\,\delta(\lambda)) \\ \eta(\lambda) = \dfrac{(V_{\mathrm{H}}(\lambda))^2 - (V_{\mathrm{nmo}}(\lambda))^2}{2\ (V_{\mathrm{nmo}}(\lambda))^2} \end{cases} \tag{4}$$

where $(V_{\mathrm{H}}(\lambda))^2 = (V_{\mathrm{P}}^{\mathrm{vertical}})^2(1+2\varepsilon(\lambda))$ is the squared qP-wave velocity in the horizontal direction of azimuth λ. Eq. (4) is written for a single layer, the corresponding expression for a stack of layers can be found in the previous reference. In the VTI version of Eq. (3), the denominator of the quartic term contains a correction term in X^2 in order to produce finite errors for X infinitely large. In this limit, one has $T^2 \approx X^2/(V_{\mathrm{H}})^2$. By taking definition (4), the same property is preserved for the case of weakly anisotropic media of arbitrary symmetry type. Eq. (3) is straightforwardly obtained from the corresponding TI equation obtained by Alkhalifah (1997) and Alkhalifah and Tsvankin (1995), using the ADAPT recipe.

Strictly speaking, Eq. (3) is applicable to media of at least monoclinic symmetry with a horizontal symmetry plane. But as pointed out by Rasolofosaon (2000b) this equation is also applicable to media of arbitrary anisotropy type (triclinic) because of the symmetry with respect to the vertical axis of the geometry of the seismic rays in the unperturbed isotropic model (obtained by putting to zero all the anisotropy parameters). However, with the use of the generalized ADAPT recipe the last geometrical property is no longer necessary. In other words the recipe is applicable in any 2D geometry.

The second example is de Bazelaire et al. (2000) who studied the same problem as the one considered by Tabti and Rasolofosaon (1998). They proposed an azimuth-dependent time–distance equation different from Eq. (3). They derived an equation of a time-shifted hyperbola type based on the work of de Bazelaire (1988) and de Bazelaire and Viallix (1994). The details of their approach will not be given here, for reason of conciseness (details can be found in the original paper). They used a long and sophisticated mathematical derivation based on the analogy between geometrical optics and ray seismics. However, their result could have been straightforwardly obtained using the ADAPT recipe. Furthermore, de Bazelaire et al. (2000) restricted their analysis to monoclinic media with a horizontal symmetry plane. In fact by using the generalized ADAPT recipe introduced in this paper, their equation can also be applied to media of arbitrary symmetry type (triclinic).

Considering the generality of the method, as previously said, there are many potential applications to any type of kinematic algorithm. However, for concision this will not be discussed in this paper but will be the topic of a future paper.

6. Discussion

6.1. The striking generality of the adapt recipe and its corollary

The main conclusion of this paper is contained in the title. In 2D geometry, any qP-wave kinematic algorithm (such as velocity analysis, dip move-out, time migration, e.g., Tsvankin, 1996) designed for Transversely Isotropic media with a Vertical symmetry axis (VTI) can be straightforwardly adapted to the case of media of arbitrary anisotropy type, provided that the anisotropy strength is moderate. The corollary of this is that, in practical situations, with qP-wave kinematic data along a single azimuth it is impossible to separate the VTI case from a monoclinic case or a triclinic case. As a consequence, in such cases, it is useless to assume that the studied medium is more complicate than VTI. Only additional seismic profiles in planes of different azimuths will allow to either validate the VTI assumption or to justify the use of more complicate assumptions on the symmetry type (TI with a horizontal symmetry axis, orthotropic, monoclinic, etc.).

6.2. Limitations of the generalized ADAPT recipe and comments

As a reminder, I summarize here the main assumptions for the generalization of the ADAPT recipe to triclinic media proposed in this paper, with some additional comments:

– The anisotropy strength is assumed to be moderate (anisotropy parameters typically smaller than 20%). This is most of the time the case if one refers to field studies (e.g., Berthet et al., 1998; Tabti and Rasolofosaon, 1998).
– In the case of a medium of symmetry as low as triclinic, or of an apparently triclinic medium (with symmetry directions misaligned with the axes of description of the problem), the elastic tensor is approximated by a monoclinic tensor with a horizontal symmetry plane. However, in the case of a moderately anisotropic media the difference in terms of qP-wave velocity is negligible.
– The justification of the ADAPT technique comes from the formal similarity between the qP-wave velocity equation (Eq. (1)) in transversely isotropic (TI) media and the corresponding equation (Eq. (2)) in media of monoclinic symmetry with a horizontal symmetry plane. As a consequence, a natural limitation of ADAPT is imposed by the limitations of Eq. (1) itself, and will not be repeated here for reason of concision (for more details, see Thomsen (1986)).
– Only P-waves are considered. This is a serious limitation. It is known that the S-waves signatures to seismic anisotropy is often more pronounced (e.g., Helbig, 1994) but the theoretical results for S-waves are often much more complicate and, as a consequence, not as easily tractable as those for P-waves (e.g., Mensch and Rasolofosaon, 1997). However, it is worth noting that P-waves constitute the great majority of data acquisition in seismic exploration and as a consequence the essential of available data.
– The geometry of the problem is 2D. This is another serious limitation. However, if one refers to the processing tools available in the industry, 3D tools often actually means multi 2D. In other words, true 3D tools are quite rare.
– Only kinematic aspect is considered in this paper mainly because, compared to dynamic equations, kinematic equations are much easier to generalize. Very few dynamic equations have a similar form in the TI case and in cases of more complicate anisotropy types (Rasolofosaon, 2000b).

6.3. Arbitrary anisotropy hypothesis: a sophistication for experts or a practical necessity?

Considering that seismic anisotropy is a necessary ingredient for better exploiting the field data, those who are quite unfamiliar with seismic anisotropy are encouraged to consult some excellent recent references in the non-specialized literature (e.g., Hake and Helbig, 2000; Thomsen, 2002). Now for those who are familiar with seismic anisotropy, however restricted to the VTI case, I would say that the rewards of getting interest in more complicate type of anisotropy may be great. Arbitrary anisotropy is not only sophistication restricted to experts. Evidently highly degenerate cases, such as isotropy or transverse isotropy (TI), are contained in this general formalism, as special cases. The general formulation is necessary in practice to adapt the sophistication of the model to the quality and to the complexity of the data. For instance, the symmetry axes of the media of propagation are not necessarily aligned with the coordinate axes of acquisition of the seismic data. As a consequence because of the "apparent" triclinic behavior of the media, a triclinic description is necessary (e.g., Helbig, 1994). From another point of view, as previously said, the simple result stating that if one azimuth is investigated on the field it is not necessary to assume a model more complicated than VTI that is evidently unpredictable even by the smartest researcher who restricts himself to the VTI world. Only a general triclinic analysis can achieve such kind of result.

7. Conclusion

In a previous paper (Rasolofosaon, 2000b), it has been demonstrated that in 2D geometry and assuming moderate anisotropy strength any qP-wave kinematic algorithm developed for TI media with a vertical symmetry axis can straightforwardly be adapted to monoclinic media (with a horizontal symmetry plane) simply by using the ADAPT recipe, consisting nothing but the replacement of the anisotropy parameters δ and ε in the TI equations by their azimuthally dependent counterparts $\delta(\lambda)$ and $\varepsilon(\lambda)$ defined in Appendix A. ADAPT is an acronym for the Azimuthally Dependent Anisotropy Parameter Transformation.

Here, an extension of this transformation to media of arbitrary anisotropy type (triclinic), called the generalized ADAPT recipe, is proposed. The technique is based first on the approximation of the initial triclinic media by their "best monoclinic replacement medium" (Arts et al., 1991; Arts, 1993) and then on the application of the conventional ADAPT recipe on this replacement medium. In contrast to the conventional procedure, this technique is applicable even if the seismic rays in the unperturbed isotropic rays from the source to the receivers are not symmetrical with respect to the vertical axis.

The technique is successfully tested in seismic modeling on a complete 2D model with layers exhibiting moderate anisotropy strength (anisotropy parameters typically smaller than 20%), contrasted anisotropy types (orthotropic, monoclinic and triclinic) and complex velocity variations (e.g. horizontal and vertical velocity variations with non-constant gradients). Typically for travel times larger than 2 s, the difference between the exact travel times and the corresponding approximate travel times using the generalized ADAPT recipe do not exceed 4 ms, which is more than reasonable for practical applications.

Some examples of application to velocity analyses are given. The main result is that the kinematic equations found in the literature (de Bazelaire et al., 2000; Tabti and Rasolofosaon, 1998) obtained by complicate algebraic manipulations using very different techniques can be derived straightforwardly using the generalized ADAPT recipe.

As pointed out in a previous publication (Rasolofosaon, 2000b), the simplicity of the procedure and the generality of the applicability of the ADAPT recipe is striking, is very convenient for practical applications and certainly deserve further analyses. We warmly encourage such kind of work.

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Appendix A. Definitions of the anisotropy functions and parameters used for the ADAPT recipe

In the Azimuthally Dependent Anisotropy Parameter Transformation (ADAPT), the transversely isotropic parameters ε and δ of Thomsen (1986) are replaced by azimuthally dependent functions $\varepsilon(\lambda)$ and $\delta(\lambda)$ defined by:

$$\varepsilon(\lambda) = \varepsilon_x C_\lambda^4 + \delta_z C_\lambda^2 S_\lambda^2 + 2C_\lambda S_\lambda(\varepsilon_{16} C_\lambda^2 + \varepsilon_{26} S_\lambda^2) + \varepsilon_y S_\lambda^4 \quad \text{(A.1)}$$

and

$$\delta(\lambda) = \delta_x C_\lambda^2 + 2\chi_z C_\lambda S_\lambda + \delta_y S_\lambda^2 \qquad \text{(A.2)}$$

where λ designates the azimuth and where for brevity I use the notations $S_\lambda = \sin\lambda$, and $C_\lambda = \cos\lambda$. In the two previous equations, all the weighting factors in front of the directional functions $S_\lambda = \sin\lambda$ and $C_\lambda = \cos\lambda$ are new anisotropy coefficients, introduced by Mensch and Rasolofosaon (1997) and Rasolofosaon (2000a), and generalizing Thomsen's anisotropy parameters ε and δ. For convenience, their definitions are given here.

$$\varepsilon_x = \frac{c_{11} - c_{33}}{2c_{33}}, \quad \varepsilon_y = \frac{c_{22} - c_{33}}{2c_{33}},$$
$$\delta_x = \frac{c_{13} - c_{33} + 2c_{55}}{c_{33}}, \quad \delta_y = \frac{c_{23} - c_{33} + 2c_{44}}{c_{33}},$$
$$\delta_z = \frac{c_{12} - c_{33} + 2c_{66}}{c_{33}} \text{ and } \chi_z = \frac{c_{36} + 2c_{45}}{c_{33}},$$
$$\varepsilon_{16} = \frac{c_{16}}{c_{33}}, \quad \varepsilon_{26} = \frac{c_{26}}{c_{33}}.$$

where the c_{ij} are the stiffness coefficients of the considered medium in the conventional two-index notation of Voigt (e.g., Helbig, 1994).

Appendix B. Matrix of the stiffness coefficients of the gas-saturated dolomite reservoir rock

The stiffness matrix of the gas-saturated dolomite reservoir rock (density $\rho = 2300$ kg/m^3) considered in Figs. 1 and 2 is equal to:

$$(C_{IJ})_{\text{TRICLINIC}} = \begin{pmatrix} 41.411 & 7.410 & 7.896 & 0.114 & -0.267 & 1.914 \\ 7.410 & 37.227 & 7.451 & 0.239 & 0.194 & 1.567 \\ 7.896 & 7.451 & 41.267 & 0.276 & -0.138 & 0.337 \\ 0.114 & 0.239 & 0.276 & 14.968 & 0.632 & 0.465 \\ -0.267 & 0.194 & -0.138 & 0.632 & 15.824 & 0.189 \\ 1.914 & 1.567 & 0.337 & 0.465 & 0.189 & 15.235 \end{pmatrix} \text{GPa}$$

After Arts (1993), the monoclinic medium with a horizontal symmetry plane that best approximates this rock sample has the following stiffness matrix:

$$(C_{IJ})_{\text{MONOCLINIC}} = \begin{pmatrix} 41.411 & 7.410 & 7.896 & 0.000 & 0.000 & 1.914 \\ 7.410 & 37.227 & 7.451 & 0.000 & 0.000 & 1.567 \\ 7.896 & 7.451 & 41.267 & 0.000 & 0.000 & 0.337 \\ 0.000 & 0.000 & 0.000 & 14.968 & 0.632 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.632 & 15.824 & 0.000 \\ 1.914 & 1.567 & 0.337 & 0.000 & 0.000 & 15.235 \end{pmatrix} \text{GPa}$$

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