Determining Geometrical Spreading from Traveltimes

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Introduction

Geometrical spreading plays an important role for various applications, e.g. true amplitude migration, estimation of Fresnel zones, divergence corrections. Unfortunately, fast algorithms for computing traveltimes like FD techniques do not yield amplitudes. They usually must be calculated by the slower ray methods (dynamic ray tracing).
We make use of the relationship between the wavefront curvature expressed by the propagator matrix and the ray amplitude derived by Hubral et al. (1992). The propagator is determined from traveltimes by a hyperbolic variant of the paraxial approximation and can thus also be used for interpolating traveltimes. This is especially interesting as it can be applied to intermediate shot and receiver locations.

Method

Taylor expansion of the traveltime curve around a central ray yields the following approximation for a paraxial ray:

$$t(s_I,\, g_I) = t_0 - p_I\, s_I + q_I\, g_I + \frac{1}{2}\, s_I\, S_{IJ}\, s_J + \frac{1}{2}\, g_I\, G_{IJ}\, g_J - s_I\, N_{IJ}\, g_J .$$

The source s_I (I=1,2) and receiver positions g_I (I=1,2) are considered to be located on arbitrary reference surfaces, e.g., earth surface or reflectors. p_I and q_I are the slowness vector components of the central ray at the source and geophone in two-component notation (for further details, see Bortfeld (1989)). Furthermore we introduce the second order derivative matrices

$$S_{IJ} = \frac{\partial^2 t}{\partial s_I \partial s_J}, \quad G_{IJ} = \frac{\partial^2 t}{\partial g_I \partial g_J}, \quad N_{IJ} = -\frac{\partial^2 t}{\partial s_I \partial g_J}.$$

Knowing that traveltimes are better approximated by hyperbolae (Ursin, 1982) we square the first equation neglecting spatial terms of higher order than two. This results in (Schleicher et al., 1993)

$$t^2(s_I, g_I) = (t_0 - p_I\, s_I + q_I\, g_I)^2 + t_0(s_I\, S_{IJ}\, s_J + g_I\, G_{IJ}\, g_J - 2 s_I\, N_{IJ}\, g_J)$$

Combining traveltimes for certain source and receiver positions from multi-coverage experiments then leads to solving the hyperbolic equation for p_I, q_I, S_{IJ}, G_{IJ} and N_{IJ}, e.g., Gajewski (1998). In traveltime modeling for migration, multifold data is available. Other than the time information is not needed.

As the paraxial vicinity in which the equation is valid may encompass a number of gridpoints of the model, we can revert to using only every nth point in either direction depending on the model. For example n=10 would lead to save a factor of 1000 in storage for three dimensions. Gridspacing does not need to be the same in every direction which makes resampling of non-uniform grid data unnecessary.

For the computation of geometrical spreading we employ the following expression obtained by Hubral et al. (1992)

$$|\mathcal{L}| = \frac{1}{v_s}\sqrt{\frac{\cos\alpha_s\,\cos\alpha_g}{|\det N|}}$$

where v_s is the velocity at the source. The incidence resp. emergence angles α_s and α_g can be computed from the slowness components, i.e.

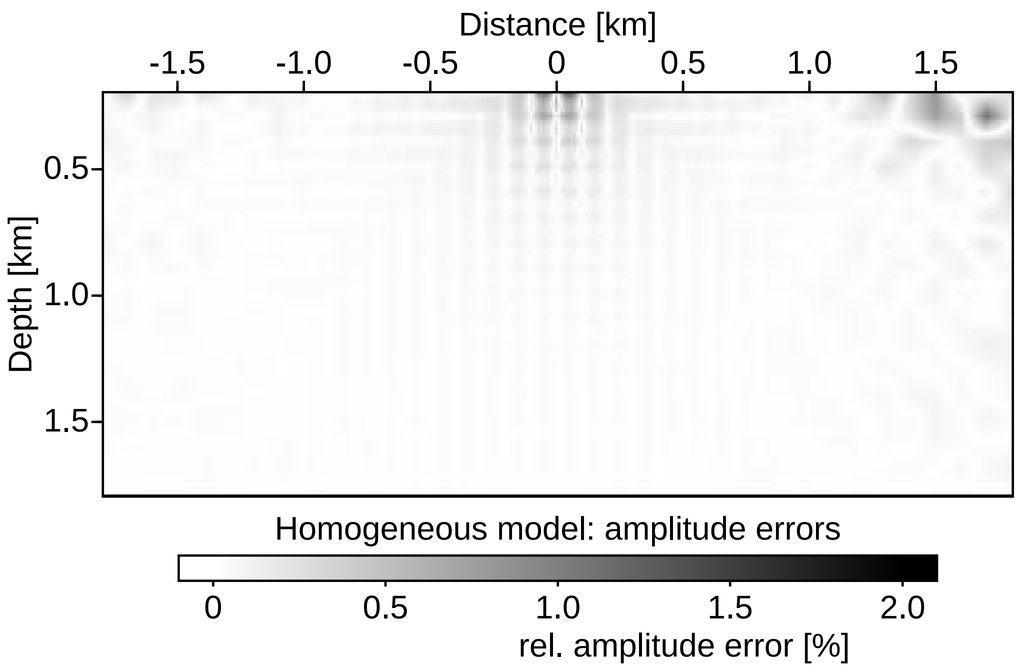
$$\sin\alpha_s = \sqrt{p_1^2 + p_2^2}\cdot v_s.$$

For the 2-dimensional problems discussed below the above equation reduces to

$$|\mathcal{L}| = \frac{1}{v_s}\sqrt{\frac{\cos\alpha_s\,\cos\alpha_g}{|N_{11}|}}.$$

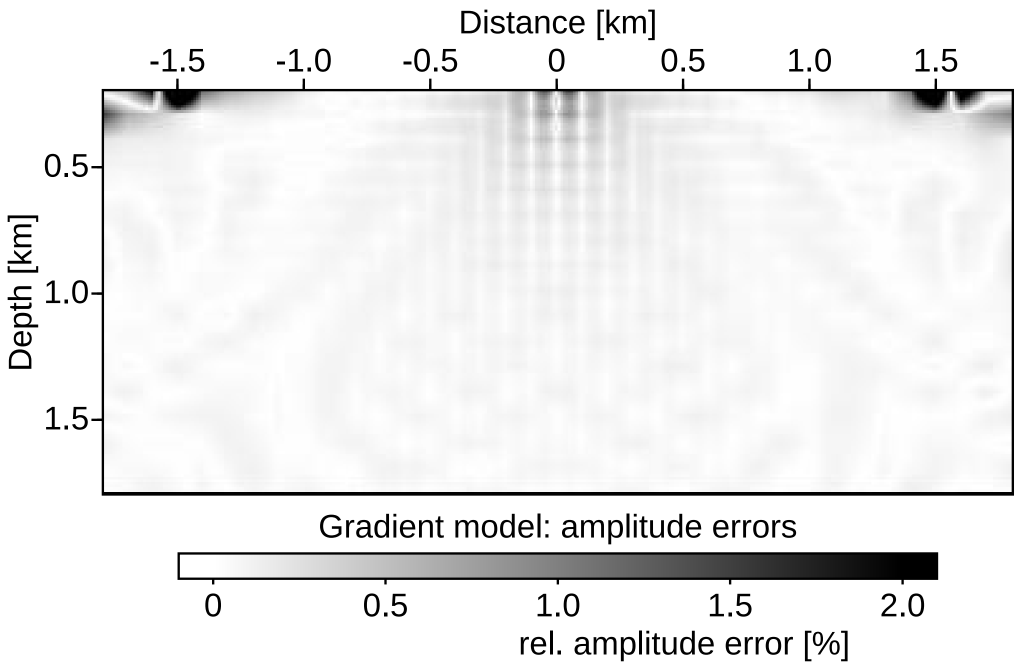
Applications

For the amplitude calculation we present two examples. The first is a homogeneous model with v=3.0kms^{-1} with the source at 0.0km in x- and at 0.11km in z-direction. Traveltimes and reference amplitudes were computed with a wavefront construction routine. The derivatives were computed from traveltimes on a 100m×100m coarse grid and the geometrical spreading was obtained by bilinear interpolation onto a 10m×10m fine grid. The maximum error is 1.3% with a median of 0.02%.



Apart from the area around the source we only find larger errors near z=0 towards higher offsets. They are caused by α_s, α_g approaching 90° as there the cosine is very sensitive to its argument's error. For a migration of reflection data, however, these regions are not important.

The second model has a constant velocity gradient of $\partial v/\partial z$=0.5s^{-1} and $v(z = 0)$=3.0kms^{-1}. The grids and source position are the same as for the homogeneous model. Here the maximum error is 5.6%, the median 0.05%.



The application to complex models like e.g. the Marmousi model is on its way. Owing to the size of heterogeneities, for this model smaller gridspacing is required. However, finer gridspacing needs higher accuracy in the traveltimes to avoid numerical instabilities since the moveout is small for small offsets. Up to a certain degree this problem can be compensated by smoothing the traveltime data. Due to the bilinear interpolation from the coarse onto the fine grid areas including triplications become a problem. This is to be expected as we use different branches of the traveltime curve to compute slownesses and the N_{IJ}, G_{IJ}, S_{IJ}-matrices.

A future application of our work is locating the triplication points using the N_{IJ}, G_{IJ}, S_{IJ}-matrices which is important for the implementation of a hybrid finite-differences and wavefront construction traveltime solver to compute later arrivals efficiently (Ettrich & Gajewski, 1997).

Summary

We presented a technique for computing the geometrical spreading from traveltimes only. The accuracy of the results is sufficient for the calculation of migration weights and divergence corrections. The technique is less time consuming than dynamic ray tracing if we use a finite-differences algorithm to compute traveltimes.

The algorithm has proved useful also in interpolating traveltimes leading to storage savings as well as the ability to interpolate traveltimes for varying source and receiver positions.

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