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Sensitivity analysis and determination of streambed leakance and aquifer hydraulic properties

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Abstract

A nonlinear regression method is used to calculate the hydraulic parameters of a stream-aquifer system using pumping test data. Five parameters, including the horizontal hydraulic conductivity (K_x), aquifer anisotropy (K_a), streambed leakance l, aquifer specific storage S_s, and specific yield S_y, can be calculated. MODFLOW, coupled to the regression method, simulates the groundwater flow that is affected by streams. Sensitivity analyses indicate that for a given stream-aquifer system, the quality of the stream-aquifer test data can be improved through a careful selection of observation and pumping wells, as well as an appropriate test duration. An optimal location of an observation well is where the magnitude of the sensitivities is enhanced and the correlation of the transient sensitivities of two parameters is reduced. Generally, a longer pumping period will increase the sensitivity for l and K_x and reduce the correlation between S_y and K_x and between S_y and l. Results from hypothetical examples and a field test suggest that a two-well analysis of pumping test data can significantly reduce the correlation of sensitivity coefficients; as a result, convergence occurs faster and the estimated standard errors are reduced.

Keywords: Streambed leakance; Sensitivity analysis; Parameter estimation; Pumping test

1. Introduction

Characterization of the hydraulic connection between stream and aquifer requires knowledge of streambed conductance, as well as the aquifer hydraulic conductivity and anisotropy. The level of stream-aquifer connectivity affects the rate of stream depletion, the migration process of infiltrated stream water towards a pumping well, and the bank storage.

Examples of the investigation of streambed hydraulic conductivity include permeameter tests in channels (de Lima, 1991; Duwelius, 1996; Chen, 2000; Landon et al., 2001), slug tests (Landon et al., 2001; Rus et al., 2001), and groundwater modeling (Hunt, et al., 2001; Yager, 1993). Streambed tests using permeameters often provide the vertical hydraulic conductivity of the streambed at a shallow depth and in a small volume of sediment, while slug tests provide the horizontal hydraulic conductivity.

Hunt et al. (2001) proposed a method to determine the aquifer transmissivity T, the storage coefficient S, and a streambed leakage parameter λ using aquifer tests conducted near a canal or a narrow stream. The analysis of a 6-h pumping test near a drain by Hunt et al. (2001) provided the results of T, S, and λ. The method of Hunt et al. (2001) was

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derived from an analytical solution (Hunt, 1999) that describes the hydraulic head distribution around the pumping well and stream. The Hunt solution (1999) is appropriate for simplified stream-aquifer systems where the pumping well fully penetrates and is fully screened over the whole thickness of an isotropic aquifer and the aquifer is assumed to have a uniform thickness and to extend infinitely in the horizontal direction. Application of the Hunt solution (1999) does not provide the vertical hydraulic conductivity of the aquifer because the vertical flow component in the aquifer is considered negligible.

Yager (1993) used a nonlinear regression method to inversely calculate the parameters of a streambed and aquifer from a 23-h pumping test near the Susquehanna River in New York. In addition to the horizontal hydraulic conductivities and the anisotropy of two aquifer layers, he determined the vertical hydraulic conductivity of the stream sediment. MODFLOW (McDonald and Harbaugh, 1988) was used to simulate the groundwater flow. Convergence occurred for the hydraulic conductivities of two layers of alluvial sediments, but the coefficient of variation (CV) for the streambed hydraulic conductivity exceeds 2000, indicating a large degree of uncertainty in this estimated parameter. Aquifer specific yield and storage coefficient were specified in the calculation. Calibration using the trial-and-error approach also provided the estimates of the streambed and aquifer parameters (Yager, 1993).

Inverse methods have been commonly used in the determination of aquifer hydraulic parameters. A great number of publications on this subject are available. Yeh (1986) provided an excellent review of parameter identification procedures in groundwater hydrology. His review indicated that the inverse methods were mainly used to determine T (transmissivity) and S (storage coefficient) for two-dimensional flow systems. Carrera and Neuman (1986a) identified the horizontal and vertical hydraulic conductivities of a three-layer alluvial aquifer of the Colorado River. As indicated by Yeh (1986), most researchers used the Gauss–Newton method in parameter identification. Other methods were also used (Carrera and Neuman, 1986b). Cooley and Naff (1990) gave a comprehensive discussion of the statistical features of the determined parameters from inverse computation. Depending on the scope of research or the need of a practice, a variety of analytical or numerical groundwater flow models can be coupled with inverse methods for parameter identification. The parameter-estimate process of MODFLOW-2000 (Hill et al., 2000) is considered a comprehensive package for parameter identification because the inverse approach is integrated with MODFLOW, which provides flexibility for incorporation of complex aquifer and boundary conditions. For a given groundwater flow model, the quality of the parameters determined from inverse approaches largely depends on the quality of the observation data. Determination of streambed leakance using inverse parameter estimation methods is not commonly reported.

This study applies an inverse method to determine the streambed leakance and aquifer hydraulic properties using pumping tests near a stream. This method is coupled with MODFLOW, as well as with the Hunt solution (1999); these models are used to simulate groundwater flow near streams. Sensitivity analyses in this paper offer a guide for selecting optimal observation locations that should provide more reliable hydraulic parameter estimates. This method was used to analyze a pumping test on an island bounded by two channels of the Platte River and to determine the hydraulic parameters of the stream-aquifer system.

2. Groundwater flow and stream infiltration

Groundwater pumping creates a cone of depression (Fig. 1); its diameter increases with pumping time. The cone of depression will eventually intercept a nearby stream if the pumping well is situated at an appropriate location. For a losing stream, the pumping increases the rate of stream infiltration to the aquifer; for a gaining stream, the pumping may decrease the hydraulic gradient, or even reverse the gradient in the streambed and induce stream infiltration.

The groundwater flow in the area of the pumping well and the stream can be expressed such that

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_s \frac{\partial h}{\partial t} - R, \quad (1)$$

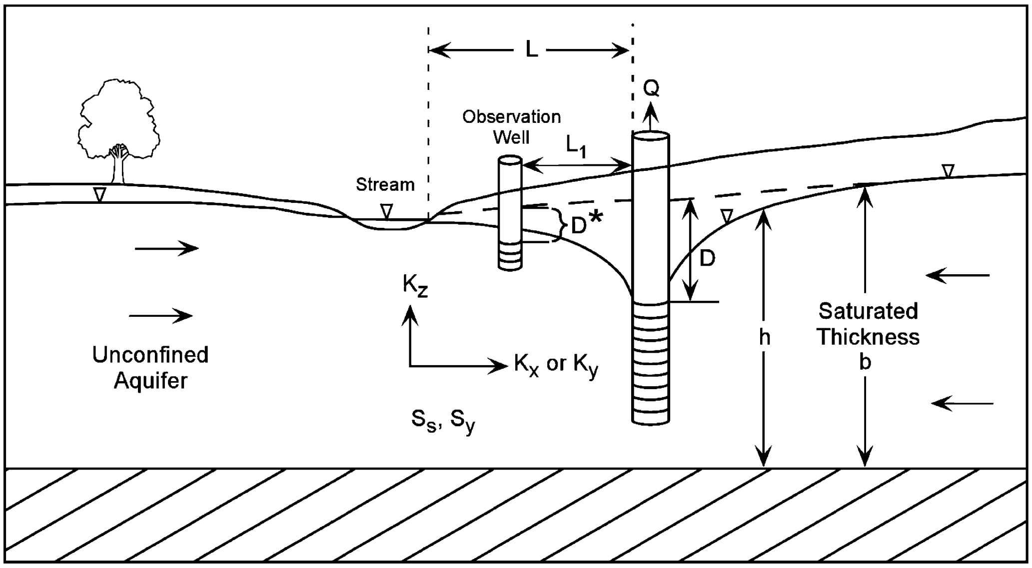

Fig. 1. Schematic diagram showing stream-aquifer tests near a stream. D is the depth to the top of the screen of the pumping well, and D^* is the depth to the top of the screen of the observation well below the initial water table.

where h is the hydraulic head; K_x, K_y, and K_z are the hydraulic conductivities along the x, y, and z directions; S_s is the specific storage of the aquifer (storage coefficient, S, divided by the aquifer thickness); and R is a general sink/source term. Numerical solutions of Eq. (1) can be obtained from MODFLOW (McDonald and Harbaugh, 1988). For a partially screened pumping well, a strong vertical flow could exist around the pumping well; as can be expected, a vertical flow also occurs in the aquifer below a stream after the induced stream infiltration begins (Fig. 1). K_z and the vertical hydraulic conductivity of the streambed play a major role in controlling the vertical flow components. For an unconfined aquifer near the stream, the pumping leads to dewatering. The amount of water released from the aquifer is largely dependent on the specific yield of the aquifer, S_y.

The rate of the induced stream infiltration to the aquifer for a unit length of the reach is calculated from the difference in hydraulic heads in the stream and the adjacent aquifer cell (McDonald and Harbaugh, 1988)

$$Q_{\rm riv} = C^*(H_{\rm riv} - h), \tag{2}$$

where $Q_{\rm riv}$ is the flow between the stream and the aquifer, $H_{\rm riv}$ is the stream stage, h is the head at the node in the cell underlying the stream reach, and C^* is the hydraulic conductance of the stream-aquifer interconnection for a unit length of reach and is defined as

$$C^* = \frac{K_{\rm riv} W}{M}, \tag{3}$$

where $K_{\rm riv}$ is the vertical hydraulic conductivity of the streambed material, W is the width of the stream channel in the cell, and M is the thickness of the riverbed material. C^* is equivalent to the parameter λ of Hunt (1999). It is apparent that the rate of stream infiltration is a function of the stream conductance and the difference of the hydraulic head between the stream and the aquifer. Among the three variables in the unit stream conductance, W can be measured in the field, but determination of $K_{\rm riv}$ and M directly in channels is more challenging. The value of $K_{\rm riv}/M$ indicates the leakance (l) of the streambed, and it is an unknown variable.

For a pumping test near a stream, drawdown data can be collected from observation wells located between the pumping well and the stream (Fig. 1). For a given aquifer with known saturated thickness and pumping rate, drawdown is a function of the aquifer hydraulic parameters K_x, K_y, K_z, S_s, and S_y, as well as the streambed leakance. In this study, $K_x = K_y$ is assumed, and $K_a = K_x/K_z$ represents the ratio of horizontal to vertical hydraulic conductivity. It is our hypothesis that K_x, K_a, S_s, S_y and l can be inversely calculated from a pumping test that is appropriately designed and conducted near streams.

3. Inverse method

Studies where MODFLOW is coupled to the Gauss–Newton method include those by Yager (1993), Hill (1992), and Hill et al. (2000). Although MODFLOW-2000 (Harbaugh et al., 2000) provides the parameter estimation process, the learning period for a new user to apply this package to the analysis of a pumping test is often long. The inverse method used in this study is a nonlinear regression procedure described by Chen (1998) and is coupled with MODFLOW and the Hunt solution (1999), specifically for the analysis of pumping tests conducted near streams. This inverse method minimizes the squared difference between the observed and calculated hydraulic heads

$$E = \sum_{i=1}^{N} (h_{oi} - h_i^*)^2, \tag{4}$$

where h_{oi} and h_i^* are the observed and calculated hydraulic heads, respectively, at pumping time t_i, $i = 1, 2, 3, \ldots, N$ denotes individual observations, and N is the total number of observations. McElwee (1987) described in detail an inverse method for the analysis of pumping tests in a confined aquifer.

In this study, MODFLOW-2000 (Harbaugh et al., 2000) is used to calculate the hydraulic head in the area near the stream. The source codes of MODFLOW-2000 (Harbaugh et al., 2000) were combined with our inverse computational program as a subroutine. The sensitivity coefficients ($\partial h/\partial K_x$, $\partial h/\partial K_a$, $\partial h/\partial S_s$, $\partial h/\partial S_y$ and $\partial h/\partial l$) at a given time t are determined numerically using the forward finite difference method such that

$$\frac{\partial h}{\partial P_k} = \frac{h(P_k + \delta P_k) - h(P_k)}{\delta P_k}, \tag{5}$$

where P_k represents one of the five unknown variables, δP_k is the increment of the kth variable. The value of δ was 0.001 in the calculations of the sensitivities for K_x, K_a, and l, and 0.002 for S_s and S_y. Bard (1970) recommended that lower and upper limits be placed on the values such as $10^{-5} \le \delta \le 10^{-2}$. For each change in the parameter value of K_x, K_a, S_s, and S_y, respectively, the input data file of the layer-property flow (LPF) Package (Harbaugh et al., 2000) is updated accordingly and MODFLOW is re-run. For calculation of the effect of stream conductance, the input file of the River Package is updated for a change in the value of the conductance.

A powerful aspect of using nonlinear regression is that useful statistics can be generated to evaluate the parameter reliability (Hill, 1998). The estimates of the five parameters from the iteration can be considered the average values of the variables. The estimated standard errors (ESE), which could be considered to be the standard deviation of the estimates, are given by

$$\mathrm{ESE}(P_k) = \sqrt{c_{k,k}}, \qquad k = 1, 2, \ldots, 5, \tag{6}$$

where P_k is the kth parameter of the five unknowns, $c_{k,k}$ is the diagonal element in the covariance matrix **C**. **C** is defined as

$$\mathbf{C} \approx (M^{\mathrm{T}}M)^{-1}\hat{\sigma}^2, \tag{7}$$

where $M^{\mathrm{T}}M$ is a 5 by 5 sensitivity matrix and its elements are the products of the sensitivity coefficients for the five parameters, and $\hat{\sigma}^2$ is the estimated head variance from

$$\hat{\sigma}^2 = \frac{\underline{(h_0 - h)}^{\mathrm{T}}\underline{(h_0 - h)}}{N - p}, \tag{8}$$

where $h_0 - h$ is the $N \times 1$ vector representing the difference between measured and calculated hydraulic heads, h_0 is the measured head, h is the calculated head, N is the total number of observations, and p is the number of unknown parameters. It is assumed that the differences between measured and calculated head values are uncorrelated and have zero mean and constant variance (Beck and Arnold, 1977).

MODFLOW provides flexibility in consideration of vertical flow components, stream geometry, and other boundary conditions. The trade-off for using MODFLOW in the analysis of a pumping test is that a significant amount of time is needed for the preparation of input files and computer execution, because the hydraulic heads must be calculated for every node in the modeled domain. On the other hand, an analytical solution, often appropriate for simplified stream-aquifer systems, needs a much shorter amount of time for the analysis of a pumping test. In this study, the Hunt solution (1999) is also coupled with the nonlinear regression for estimation of three parameters: the streambed leakance, the horizontal hydraulic conductivity, and the specific yield.

4. Results

4.1. Sensitivity analysis

Large magnitude and independence of sensitivity coefficients are key criteria for increasing the convergence rate of each iteration. Nonconvergence may result from using collected data (the hydraulic head or drawdown) that are not very sensitive to the hydraulic parameters; in other words, the measured drawdown carries very little information about the parameters. Difficulty encountered in convergence is also frequently due to dependence between sensitivity coefficients, and thus suggests that for effective nonlinear estimation, careful examination of these sensitivity coefficients is imperative. In the analysis of slug tests, McElwee et al. (1995) showed that the best estimates for aquifer transmissivity (T) and storage coefficient (S) are obtained by minimizing the correlation between the sensitivity coefficients for T and S and sampling at points of maximum sensitivity. Both test duration and observation locations affect the correlation and magnitude of the sensitivity coefficients.

In the following sections, we will analyze the relationship among the sensitivity coefficients of the five parameters from several hypothetical examples of stream-aquifer systems. The river, 0.6 m deep and 18.3 m wide, partially penetrates the very top part of an aquifer 20.4 m in thickness. The aquifer was divided into six layers, and the grid size near the river and pumping well was 6 m. The screen lengths of the observation and pumping well were 9.1 m. The pumping rate was 5454 m^3 day^{-1}.

Fig. 2 shows the normalized sensitivities for the four variables l, K_x, K_a, and S_y. For convenience of presentation, a normalized sensitivity (McElwee et al., 1995), defined as $(\partial h/\partial P_k)P_k$, is used; its unit is the same as that of hydraulic head h. Among the four parameters, l, K_x, and K_a are associated with the hydraulic conductivities of aquifer and streambed sediments. As shown in Fig. 2, the magnitude of the sensitivity is a function of pumping time. The sensitivities for l and K_x increase with time. However, the hydraulic head is not sensitive to l for $t < 0.1$ day. This is because the pumping has induced only a very small stream infiltration (Fig. 3) and the aquifer behaves as if the river did not exist. The sensitivity for K_a (the absolute value) is larger between approximately 0.005 and 0.1 day. During this period, the effects of gravity drainage are seen, and the vertical hydraulic conductivity has an important role in the downward movement of water. The sensitivity for K_a becomes smaller and nearly constant when $t > 1$ day. During the period of gravity drainage, the sensitivity for K_x is nearly constant.

S_s and S_y are parameters indicative of the capacity of aquifer storage. The hydraulic head is sensitive to S_s only in the early pumping time; the sensitivity becomes very small for times >0.01 day. Thus, the sensitivity coefficients for S_s are not presented. In an unconfined aquifer, the gravity drainage releases much more water than elastics storage does. This is why the sensitivity for S_s is very low during most of the pumping time. In contrast, the magnitude of the sensitivity for S_y is larger, and the sensitivity increases rapidly with time for t is less than about 0.7 day but decreases thereafter (Fig. 2(d)). The decrease in this sensitivity in the late time is associated with the increasing infiltration of stream water to the aquifer. This agrees with the observation of Yager (1993) that the sensitivity for streambed hydraulic conductivity would increase for longer pumping periods with declining releases from aquifer storage. A longer pumping time also suggests the groundwater flow is closer to a steady state condition; thus, the sensitivities for both S_s and S_y approach zero. The three values of l show only minor impact on the sensitivities for K_a and S_y in late time. In these simulations, the observation well is located 24.4 m from the stream, and 36.6 m from the pumping well. The values of the hydraulic parameters for the calculation of the sensitivity coefficients are shown in Fig. 2; these values are representative of the alluvial aquifers in the Platte River valley, Nebraska (Chen, 1998; McGuire and Kilpatrick, 1998; Chen et al., 2003).

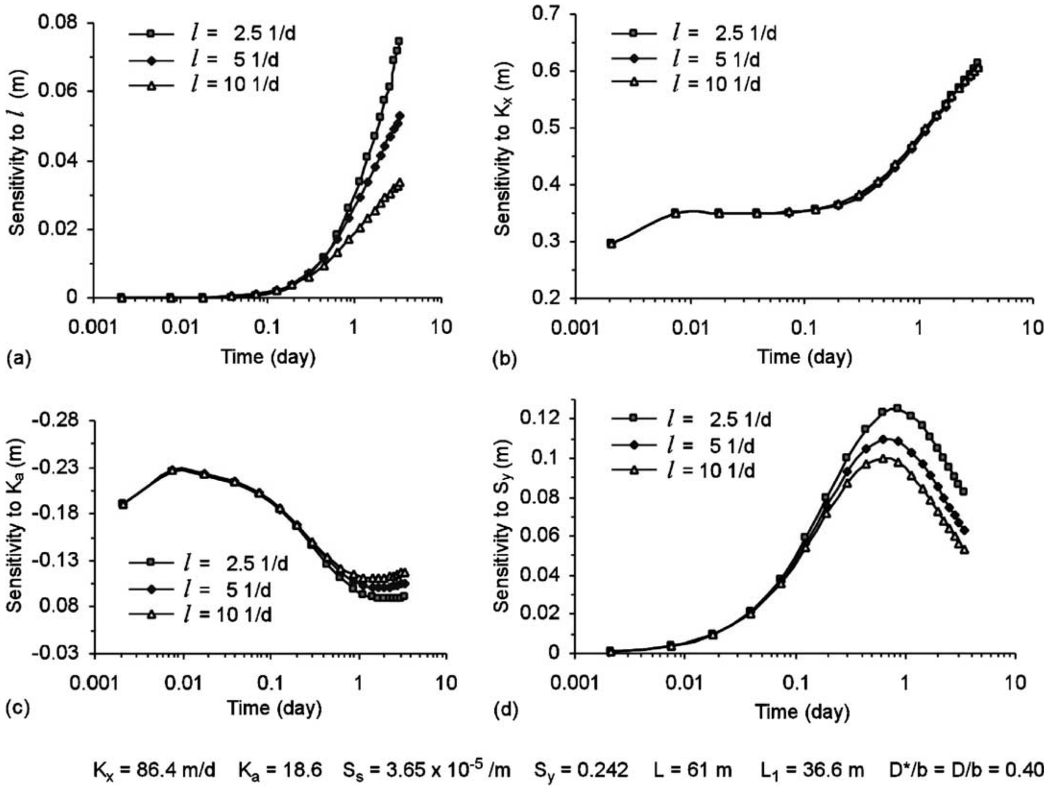

Fig. 2. The magnitude and shape of normalized sensitivities to l, K_x, K_a, and S_y related to three stream conductances.

Simple visual examination indicates a strong correlation between the sensitivities of l and K_x. We calculated the correlation coefficients between the sensitivity curves for pairs of parameters. The correlation coefficients between l and K_x for each of the three streambed leakances are greater than 0.99. The linear correlation between S_y and l and between S_y and K_x would be strong if the pumping terminated at t is about 0.7 day (see Fig. 2 for the shapes of the sensitivity curves). The decreasing trend of the sensitivity for S_y in later time has reduced its correlation with l and K_x. The correlation coefficients between S_y and l and between S_y and K_x are 0.49 and 0.51, respectively. If the correlation between all the parameters is strong (all correlation coefficients >0.99), inverse estimation of the parameters will fail to converge. However, this is not observed for the five parameters used here.

Fig. 4 shows that for a given distance between the stream and pumping well (L), the magnitudes of the sensitivity coefficients are largely dependent on the location of the observation well. Although the shape of the sensitivity curves is similar for each L_1 (L_1, the distance between the pumping and observation wells, see Fig. 1), an observation closer to the stream (a larger L_1) gives a larger sensitivity for l but

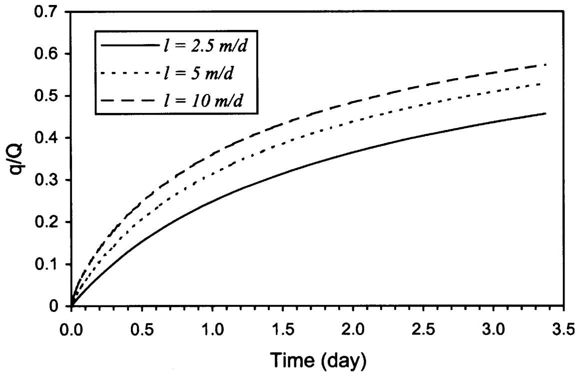

Fig. 3. Stream infiltration (q) induced by a pumping well at the rate of Q. q/Q is less than 10% for the three l values when $t < 0.1$ day.

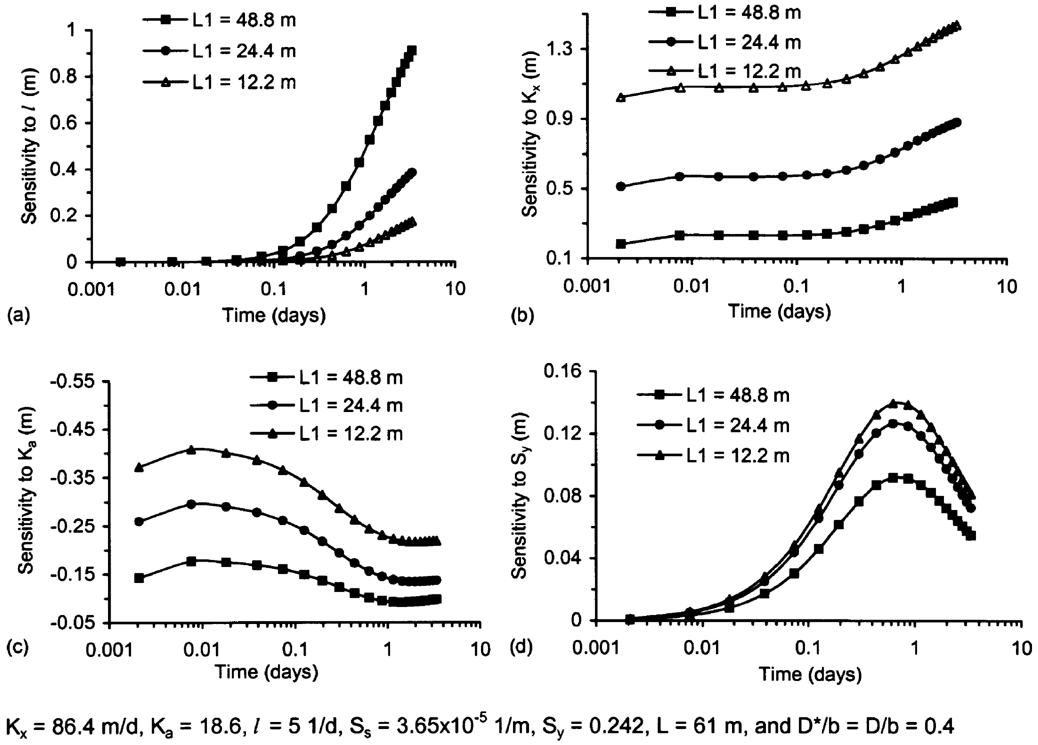

Fig. 4. The magnitude and shape of normalized sensitivities to l, K_x, K_a, S_s, and S_y related to three observation locations.

a smaller sensitivity for K_x, K_a and S_y. This is because the stream has more impact on the hydraulic head in the part of aquifer near the stream, while K_x, K_a and S_y have a strong effect on the hydraulic heads around the pumping well. The hydraulic head is not sensitive to l for t is less than about 0.1 day, even for $L_1 = 48.8$ m. The correlation coefficient between l and K_x is greater than 0.99 for all the three cases. The correlation between other parameters ranges from -0.26 to 0.87.

Fig. 5 shows the sensitivity curves for three distances (L) between the stream and the pumping well. The observation well is located at $L_1/L = 2/3$. It is clearly demonstrated in Fig. 5 that a pumping well farther from the stream reduces the sensitivities for l, K_x, and K_a. For $L = 244$ m, the sensitivity to l does not occur until the pumping has continued for about a day. For $L = 122$ m, it shows some sensitivity at $t = 1$ day but the magnitude is small. This insensitivity to l may result in some difficulty regarding convergence for the inverse procedure in the analysis of short-term pumping tests for the stream leakance l. On the other hand, it may lead to only a small deviation of the estimates of the four aquifer parameters (K_x, K_a, S_s, and S_y), even if the stream infiltration is neglected.

As observed in Fig. 2, the declining of the sensitivity for S_y in late time is also closely associated with the magnitude of the sensitivity for l. When L is larger, for example, $L = 244$ m, it takes a much longer time to induce the stream infiltration. Consequently, the model is not very sensitive to l until pumping is longer than 1 day and the magnitude of this sensitivity is very small for $t < 1$ day. Accordingly, the sensitivity for S_y does not decrease even at the end of the pumping for $L = 244$ m, whereas it decreases after t is about 0.7 day for $L = 61$ m (Fig. 5(d)). For $L = 244$ m, the correlation coefficients between S_y and l and between S_y and K_x are 0.84 and 0.81, respectively, compared to 0.48 and 0.51, respectively, for $L = 61$ m. This higher correlation for the former results from a strong similarity in the shape of

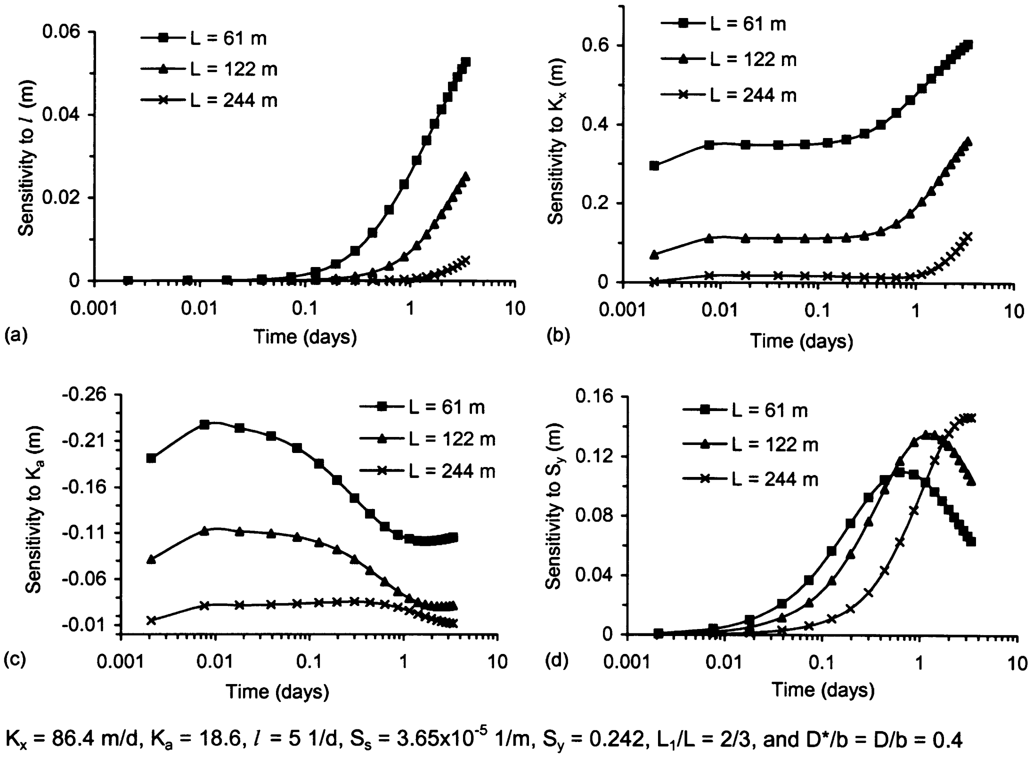

Fig. 5. The magnitude and shape of normalized sensitivities to l, K_x, K_a, and S_y related to three distances between stream and pumping well.

the sensitivity curves. The correlation coefficients between l and K_x and between S_y and K_a are greater than 0.9.

Simulations were also conducted to analyze the relationship between the magnitudes of the sensitivities and the depth of the pumping well. Three depths for the pumping well ($D/b = 0.25$, 0.4 and 0.55) were considered. The length of the well screen is 9.1 m. The depth of the observation well was the same as that of the pumping well for each simulation. For a given L ($L = 61$ m for this case), a shallower pumping well increases the sensitivity to S_y but decreases the sensitivity to K_x, K_a and S_s. A shallower pumping well also slightly increases the sensitivity for l. This is because a shallower pumping well has a stronger hydraulic connection to the stream and enhances the infiltration rate of the stream. The well depth does not affect the trend of the sensitivity curves, nor their relationship. The correlation coefficients between S_y and l and between S_y and K_x are 0.26 and 0.3, respectively, for $D/b = 0.25$. They increase to 0.61 and 0.63, respectively, for $D/b = 0.55$.

Pumping rate can also affect the sensitivity of the parameters. For a given time, a large pumping is particularly needed to increase the sensitivity for l. That is because a large pumping rate can induce a large quantity of stream water infiltrated into the aquifer.

We calculated the correlation coefficients between the sensitivities using two observation wells located at $L_1 = 12.2$ and 48.8 m (see Fig. 4). The correlation between l and K_x, l and K_a, and K_x and K_a is much lower than that using only one location. Additional calculation further confirms that the correlation from any two of the three locations in Fig. 4 is much lower than that for one observation well.

For a given stream-aquifer system, a general guidance for selecting an observation well is a location that provides large sensitivities for the stream and aquifer parameters but low correlation of their sensitivities. Analyses of the sensitivities and their correlation prior to a pumping test near a stream should offer valuable information useful for selecting locations for both pumping and observation locations where high-quality data can be obtained.

4.2. Reliability of estimates

The reliability of the parameter estimates can be evaluated using the ESEs (see Eq. (6)) and the correlation coefficients between the five parameters. A larger ESE gives a wide range for the confidence interval and thus lowers the reliability of the associated estimate. A larger ESE may have contributions from several sources, including a poor design of the stream-aquifer tests that results in low-sensitivity data and an introduced error in the measurements.

The inverse method was used to calculate the five unknowns. All the simulation cases discussed in Section 4.1 were used as hypothetical examples of parameter estimation. First, hydraulic heads at the observation well for each hypothetical example were generated. These heads were then used as 'observed data', and the five parameters were calculated. Convergence occurred for all, although the flexibility in choosing the initial values varies greatly between the cases. A wider range of initial values can be chosen for the cases with lower parameter correlations.

Changes in the observation well location, the distance between stream and pumping well, and the depth of the pumping well result in different ESE values. Although their differences are not significant for these hypothetical cases, a larger effect may be expected for real cases. For example, the ESEs of l for $L_1 = 48.8$ and 12.2 m are 0.0019 and 0.0024 day^{-1}, respectively (see Fig. 4). When the hydraulic head data from the two observation locations were analyzed simultaneously, the ESE of l was reduced to 8.2×10^{-5} day^{-1}.

To further determine the behavior of the ESEs, we introduced an error into each of the hydraulic heads; the error magnitude was 1% of the drawdown at a given time. A positive error, followed by a negative one, was added to the data series. The ESEs for the five parameters are summarized in Table 1. As indicated in this table, the computation using data from the well at $L_1 = 48.8$ m with errors gives very large values of ESEs. In contrast, the analysis using the two observation locations gives much smaller ESEs, and the estimates of the five parameters are much closer to the 'true values'. We assumed the true values are those for the case where errors were not introduced. Fig. 6 shows that the calculated drawdowns fit the observed data, generated by adding 1% of error to the model data. These hypothetical

Table 1
Estimates and ESEs of the l, K_x, K_a, S_s, and S_y from hypothetical examples

		l (day^{-1})	K_x (m day^{-1})	K_a	S_s (m^{-1})	S_y
$L_1 = 48.8$ m (no errors)	True values	5	86.4	18.6	3.65×10^{-5}	0.242
	Estimates	5	86.4	18.7	3.65×10^{-5}	0.242
	ESEs	1.9×10^{-3}	6.9×10^{-3}	1.9×10^{-3}	1.00×10^{-6}	1.4×10^{-5}
	CV	0.038	0.008	0.01	2.7	0.006
$L_1 = 48.8$ m (with errors)	Estimates	10.1	76.2	15.9	3.25×10^{-5}	0.26
	ESEs	27.1	43.3	11.2	6.56×10^{-6}	0.073
	CV	267.3	56.8	70.4	20.2	28.1
$L_1 = 48.8$ and 12.2 m (no errors)	Estimates	5	86.4	18.6	3.65×10^{-5}	0.242
	ESEs	8.2×10^{-5}	1.0×10^{-4}	7.5×10^{-5}	1.00×10^{-6}	2×10^{-6}
	CV	0.0016	0.0001	0.0004	2.7	0.001
$L_1 = 48.8$ and 12.2 m (with errors)	Estimates	4.9	86.3	18.7	4.26×10^{-5}	0.245
	ESEs	0.45	0.56	0.41	6.56×10^{-6}	1.2×10^{-2}
	CV	9.2	0.6	2.2	7.7	4.9

Coefficient of variation(CV) = Estimate(P_k)/ESE(P_k) $\times$ 100.

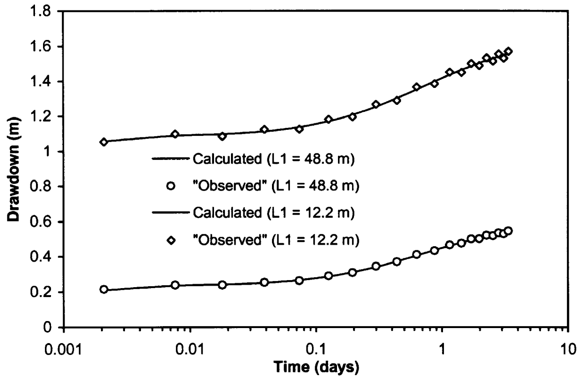

Fig. 6. A hypothetical example showing observed and calculated hydraulic heads that were used to determine the aquifer and streambed parameters. Error was introduced to each hydraulic head before the analysis. The two sets of data were analyzed simultaneously.

examples demonstrate that measurement errors in the drawdown (or hydraulic head) during data collection results in large ESEs of the estimated parameters and thus a lower reliability. This phenomenon is much more obvious in the inverse computations for one well.

The CV, $[\mathrm{ESE}(P_k)/P_k] \times 100$, $k = 1, 2, \ldots 5$, describes the quality of the estimates. As shown in Table 1 for the two-well analysis, the CV for l is the largest and for K_x the smallest. The sequence from high to low of the coefficients of variation for the two-well analysis is $l > S_s > S_y > K_a > K_x$ (Table 1).

The pumping rate (Q) was 5454 m^3 day^{-1} for these hypothetical examples. We also analyzed the effects of lower pumping rates on the estimates. When the pumping rate was reduced to half of the original pumping rate and pumping continued for 2 days, the CV value for l increased from 0.038% (see Table 1 for the single-well analysis) to 0.091%; the reliability of the estimate for l was affected very little. When the pumping rate was one third of the original one ($Q = 1818$ m^3 day^{-1}), the CV value for l increased to 28.97%, and the CV for other four parameters increased to values ranging from 4.2 to 6.5%. The inverse computation did not converge for a pumping rate = 1/4 of the original rate ($Q = 1363$ m^3 day^{-1}). Thus, a lower pumping rate can give a higher uncertainty to the estimates for a given pumping duration.

While a maximum pumping rate for a specific well is a function of well construction features and aquifer hydraulic properties, the pumping time can be easily adjusted for a pumping test. When $Q = 1818$ m^3 day^{-1} and the pumping time increased to 3 days, the CV value for l was 0.24% (compared to 28.97% for the 2-day pumping). The CV values for other four parameters were also very small. Thus, this 3-day pumping test increased the reliability of the estimates. When pumping continued for 6 days at the same rate, the CV value for l was 0.14%. This 6-day pumping test did not improve much the reliability of the estimates of the five parameters because a 3-day pumping had resulted in relatively reliable estimates. For $Q = 1363$ m^3 day^{-1} and pumping time = 6 days, the inverse computation converged but there were large differences between the estimates and the true values, varying from 25 to 34%. These estimates were not reliable.

A high CV of the estimates is often associated with the correlation coefficients between the parameters. The approximate correlation coefficient is computed from the covariance matrix **C** (see Eq. (7)) and is given by

$$r_{k,l} = \frac{c_{k,l}}{\sqrt{c_{k,k} c_{l,l}}}, \qquad k, l = 1, 2, \ldots, 4. \tag{9}$$

Beck and Arnold (1977) pointed out that whenever the absolute values of the off-diagonal elements of matrix $M^{\mathrm{T}}M$ exceed 0.9 in magnitude, the parameter estimates are highly correlated, and thus, tend to be inaccurate. Hill (1998) indicated that when the values of the correlation coefficients are close to -1 and 1, the parameter values cannot be uniquely estimated with the observations in the regression. For a comparison of the single and two-well cases, Table 2 lists the correlation coefficients for the computations using heads to which errors were added. More than

Table 2
Correlation coefficients for two hypothetical examples with introduced errors (lower triangle for one well, upper triangle for two wells)

	l	K_x	K_a	S_s	S_y	
l		-0.1580	-0.2526	-0.0912	-0.5197	
K_x	-0.9997		0.7976	0.0659	-0.4554	For two
K_a	-0.9996	0.9998		0.3143	0.0184	wells
S_s	-0.3013	0.3007	0.3027		0.2254	
S_y	0.9871	-0.9898	-0.9878	-0.2860		
	For one well					

half of the correlation coefficients for the single-well analysis, calculated using Eq. (9), are above 0.999; in contrast, the correlation coefficients for the two-well analysis are all below 0.8. The lower correlation is associated with smaller ESEs and thus gives more reliable results.

4.3. Applications

The method was applied to the analysis of a pumping test conducted on an island in the Platte River near Kearney, Nebraska, USA. The island, approximately 150 acres, is a braided sand bar of the river. Two river channels (the north and the middle, Fig. 7) form the north and south boundaries of the island. The streambed sediments of the channels in the study area consist mainly of sand and gravel, as well as local silt and clay layers. The pumping test was conducted by the Layne-Western Company (1983) for the city of Kearney.

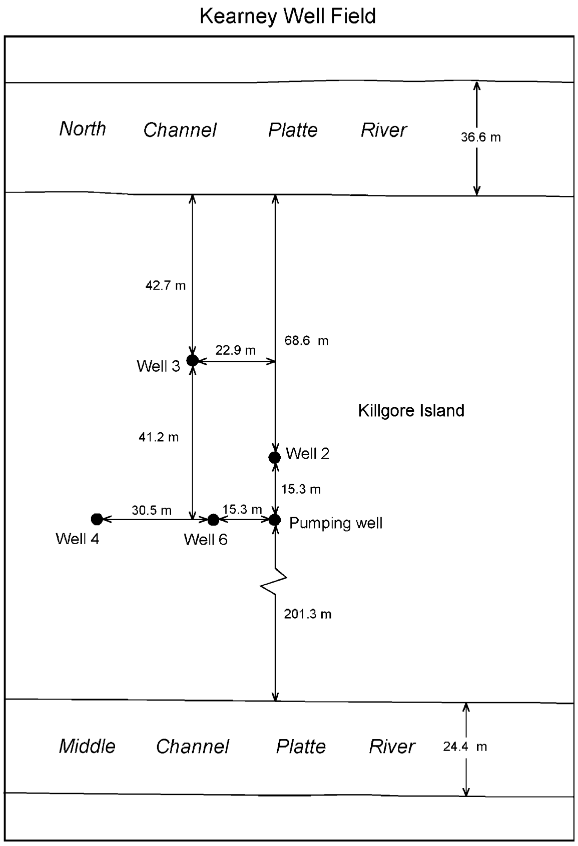

Fig. 7. The locations of pumping and observation wells in the test site, Killgore Island, near Kearney, Nebraska.

The pumping well was located about 83.9 m from the north channel and 201.3 m from the middle channel of the Platte River. There were four observation wells (Fig. 7). Observation wells 2 and 3 were located between the pumping well and the north channel; wells 4 and 6 were located west of the pumping well. Both pumping and observation wells were screened in the lower part of the alluvial sediments of sand and gravel and were at similar depths. The saturated thickness was about 14.3 m and depth to water was about 1.8 m; the shale underlying the alluvial sediment was treated as an impermeable base. The screen length was 7.6 m, and the length of casing between the water table and the top of the well screen was 6.7 m. The screens of the observation wells and the pumping well were at the same levels. The pumping test continued for 48 h with an average pumping rate of 8346 $m^3\ day^{-1}$ (1530 gal/min).

A two-layer model was developed to analyze the pumping test. The screens of the wells were located in the lower layer and their casings were in the upper layer of the model. The north and middle channels were included in the model and were oriented parallel in the west–east direction. The grid spacing varies from 3 to 6 m in the area near the pumping and observation wells. The inverse method described earlier was used to calculate the five parameters. The streambed leakance for the two channels was assumed the same.

Analysis was conducted for individual wells, as well as for a pair of wells simultaneously. The calculated results are summarized in Table 3. The results from the single-well and the two-well analyses show some difference for K_x, K_a, S_s and S_y, but are reasonable. The difference between the two-well analysis and single well analysis is more significant for l. The CV for the two-well analysis are often lower; the value in the CV is generally the highest for l and lowest for K_x. The sequential order of the CV values is $l > S_s > S_y > K_a > K_x$, which is the same as that for the hypothetical example (Table 1). Fig. 8 shows the calculated and measured hydraulic head for the two-well analysis. Because the middle channel was 201 m from the pumping well, its effect on

Table 3
Estimates and ESEs of the l, K_x, K_a, S_s, and S_y from a stream-aquifer test in Killgore Island, near Kearney, Nebraska

		l (day^{-1})	K_x (m day^{-1})	K_a	S_s (m^{-1})	S_y
Well 2	Estimates	3.00	258.4	9	2.34×10^{-4}	0.139
	ESEs	1.03	7.3	0.41	2.62×10^{-5}	0.01224
	CV (%)	34.3	2.8	4.6	11.2	8.8
Well 3	Estimates	4.52	287.5	11.9	1.60×10^{-4}	0.157
	ESEs	1.44	16.9	0.51	2.95×10^{-5}	0.0093
	CV (%)	32.0	5.9	4.3	18.4	5.9
Well 4	Estimates	2.4	323.1	7.3	2.46×10^{-4}	0.179
	ESEs	0.5	7.8	0.17	5.90×10^{-5}	8.82×10^{-3}
	CV (%)	21.5	2.4	2.4	24.0	4.9
Well 6	Estimates	1.5	263.0	12.5	1.59×10^{-4}	0.101
	ESEs	0.3	5.2	0.45	1.64×10^{-5}	7.17×10^{-3}
	CV (%)	19.3	2.0	3.6	10.3	7.1
Wells 2 and 3	Estimates	22.78	237.8	8.2	2.36×10^{-4}	0.174
	ESEs	3.39	1.12	0.15	3.28×10^{-5}	0.0036
	CV (%)	14.9	0.5	1.8	13.9	2.1

the drawdown may be much smaller than the north channel.

The correlation coefficients for the two-well analysis are all lower than 0.8; the highest correlation (0.8) is between K_x and l, and the lowest correlation (0.019) is between l and S_y. In contrast, about half of the correlation coefficients for the single-well analyses are greater than 0.9; the other correlation coefficients can be as low as -0.025.

The results of aquifer tests in the alluvial aquifer in adjacent areas have been reported by Chen (1998), Ayers et al. (1998), McGuire and Kilpatrick (1998), and Chen et al. (2003). These values are shown in Table 4 and indicate a value of K_r around 100 m day^{-1} and the range for K_a from 10 to 50. We believe that compaction of the alluvial materials results in a lower K_r and a larger K_a value than the values for the sediments at the test site, which is a braided sand bar of the Platte River.

We also used the Hunt solution (1999) to calculate the hydraulic head for the inverse analysis of the pumping test. Utilization of the Hunt solution to this test violated several assumptions, including an isotropic aquifer, full penetration of the pumping well, and a relatively narrow stream (compared to the distance between the stream and the pumping well). The purpose of the analysis was to evaluate the differences in the parameter estimates when a mathematical model does not properly represent the stream-aquifer conditions. In the analysis, we used only the drawdown data collected for $t > 105$ min because the sensitivity analyses for K_a (see Figs. 2, 4, and 5) indicate that the role of K_z in drawdown has started to decrease after that time. As a result, the degree of violation of the assumptions of an isotropic aquifer and a fully penetrating well is reduced to a lower level. Analysis was conducted for each well and the results are given in Table 5. The inverse computation for each well converged and provided the values of λ, K_x, and S_y; the value in l was

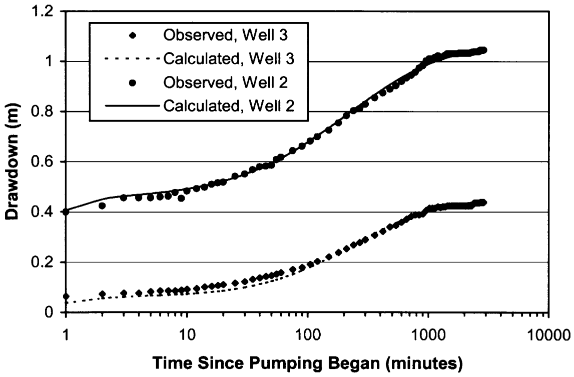

Fig. 8. Calculated and measured hydraulic heads for the pumping test conducted on an island of the Platte River near Kearney, Nebraska: simultaneous analysis of the two wells.

Table 4
Hydraulic conductivity and anisotropy of the alluvial aquifer near the test site as measured by aquifer tests

Test site	Saturated thickness (m)	K_x (m day^{-1})	K_a	References
Grand Island Platte River valley, NE	30.5	108.1	59.7	Chen (1998)
Shelton Platte River valley, NE	13.5	38–227	2.5–562	Ayers et al. (1998)
MSEA Platte River valley, NE	13	103.1 and 118.9	9.1 and 20	McGuire and Kilpatrick (1998)
Wood River Platte River valley, NE	23.5	104.2	11	Chen et al. (2003)

determined on the basis of the width of the north channel. In the analyses where both channels were considered, the λ value for the middle channel was equal to 75% of the λ value for the north channel. As indicated in Table 5, the order of the CV values is $\lambda(l) > S_y > K_x$.

Analyses were also conducted considering only the north channel because it is much closer to the pumping and observation wells. Convergence occurred for each well. The results for S_y and K_x are slightly different from those for the two-channel model, but the effect on the λ value is more significant. Again, the order of the CV values is $\lambda(l) > S_y > K_x$. However, the CV values are larger than those in the two-channel model. The two-well analysis did not converge for either the one-channel or the two-channel model. Although the Hunt solution is an approximation of the stream-aquifer condition at the test site, the results are in reasonable agreement with those determined using MODFLOW to simulate the stream-aquifer conditions.

Because the local silt and clay layers did not form a uniform thickness of layer covering the whole width of the two channels, the treatment of the streambed in the above analyses is only an approximate representation of the channel conditions. Given that the sediments in the streambed consist mostly of sand and gravel, the hydraulic connection between the river and aquifer is very strong. An alternative representation of the channels in the numerical model is to give an extremely large l value; the number of unknown parameters is thus reduced to four (K_x, K_a, S_s, and S_y). We reanalyzed the

Table 5
Estimates and ESEs of the parameters λ, K_x, and S_y from a stream-aquifer test in Killgore Island, Near Kearney, Nebraska using the Hunt solution (1999)

		Considering both channels				Considering the north channel only			
		λ (m day^{-1})[a]	l (day^{-1})	K_x (m day^{-1})	S_y	λ (m day^{-1})	l (day^{-1})	K_x (m day^{-1})	S_y
Well 2	Estimates	32.6	0.89	257.2	0.081	59.2	1.62	244.2	0.094
	ESEs	6.8		6.81	0.008	16.6		7.74	0.0105
	CV (%)	20.9		2.6	9.9	28.0		3.2	11.2
Well 3	Estimates	87.7	2.40	327.7	0.162	131.7	3.60	307.1	0.172
	ESEs	17.2		13.6	0.0083	33.7		14.7	0.0089
	CV (%)	19.6		4.1	5.1	25.6		4.8	5.2
Well 4	Estimates	22.2	0.61	305.2	0.073	37.8	1.03	286.8	0.078
	ESEs	2.8		8.0	3.4×10^{-3}	6.8		10.6	4.5×10^{-3}
	CV (%)	12.6		2.6	4.7	17.9		3.7	5.8
Well 6	Estimates	17.7	0.48	239.2	0.049	34.3	0.94	226.6	0.058
	ESEs	2.9		7.1	4.4×10^{-3}	8.3		7.1	6.8×10^{-3}
	CV (%)	16.1		3.0	9.0	24.3		3.1	11.7
Wells 2 and 3		Did not converge				Did not converge			

[a] This λ value is for the north channel; the λ value for the middle channel is 75% of the λ value for the north channel.

Table 6
Estimates and ESEs of the K_x, K_a, S_s, and S_y from the stream-aquifer test in Killgore Island by giving a very large l value ($=2500\ \mathrm{day}^{-1}$)

		K_x (m day^{-1})	K_a	S_s (m^{-1})	S_y
Well 2	Estimates	230.6	10.0	1.5×10^{-4}	0.18
	ESEs	0.9	0.2	2.62×10^{-5}	0.004
	CV (%)	0.4	1.8	17.5	2.1
Well 3	Estimates	230.4	7.4	2.58×10^{-4}	0.19
	ESEs	0.7	0.1	2.95×10^{-5}	0.005
	CV (%)	0.3	1.5	11.5	2.5
Well 4	Estimates	273.0	6.5	2.88×10^{-4}	0.23
	ESEs	1.1	0.1	5.90×10^{-5}	0.005
	CV (%)	0.4	2.1	20.5	2.2
Well 6	Estimates	201.9	11.6	1.38×10^{-4}	0.13
	ESEs	0.6	0.2	1.64×10^{-5}	0.003
	CV (%)	0.3	1.4	11.9	2.5
Wells 2 and 3	Estimates	228.0	7.5	2.42×10^{-4}	0.19
	ESEs	0.6	0.1	2.95×10^{-5}	0.004
	CV (%)	0.2	1.4	12.2	1.9

pumping test data by assigning $l=2500\ \mathrm{day}^{-1}$ in MODFLOW; the calculated values of K_x, K_a, S_s, and S_y are summarized in Table 6. The values of K_x are smaller, but the values of S_y are larger (compared to the results shown in Table 3). The ESEs for K_x, K_a, and S_y are all smaller for these analyses. These smaller ESE values suggest that this stream-aquifer model with an extremely large l is probably closer to the hydrologic condition of the test site. Nevertheless, streambed leakance (l) can be inversely calculated for a stream using data of a pumping test that is appropriately designed. Table 6 shows that the ESEs for the two-well analysis are smaller than those for a single-well analysis. The correlation coefficients for the two-well analysis are also smaller.

5. Summary and conclusions

The method presented here can calculate the five parameters, K_x, K_a, S_s, S_y, and l, of a stream-aquifer system simultaneously using pumping test data. Larger magnitudes and lower correlation of sensitivity coefficients between parameters are the basic criteria to enhance the convergence of the inverse procedure. Pumping duration affects both the magnitude and the shape of sensitivity coefficients, thus affecting the correlation and the reliability of the estimates. A longer pumping duration generally increases the magnitude of the sensitivity coefficients for K_x and l and reduces the correlation between K_x and S_y and between l and S_y. The sensitivity analyses indicate that analysis of a pumping test, which is shorter than 1 day and the pumping well of which is located more than 150 m from the stream, can neglect the effect of the river in the determination of the aquifer parameters.

After a pumping test continues beyond a given time (0.7 day in most of the hypothetical examples), the sensitivity for S_y begins to decrease; this decrease is associated with an increase in the rate of induced stream infiltration. This decreasing trend in the sensitivity for S_y reduces its correlation with K_x and l. Both the hypothetical examples and field test suggest that the CV for l is the largest and for K_x the smallest, among the five parameters considered.

Careful selection of the locations for pumping and observation wells can also increase the sensitivity to a number of parameters. In order to increase the sensitivity to l, and obtain a reliable value, one needs to site a pumping well close to the stream; the pumping needs to continue for a long period (several days). A pumping test with a large pumping rate can enhance the sensitivity of l. When pumping rates are small, pumping time often needs to be increased for improving the reliability of the parameter estimates.

Results from hypothetical examples and aquifer-stream tests suggest that two-well analysis often lowers the correlation between these stream-aquifer parameters, reduces the ESEs, and significantly improves the reliability of the parameter estimates compared to a single-well analysis. It is highly recommended that water-level data be collected from multiple locations in a stream-aquifer test and be analyzed simultaneously. Although the Hunt solution (1999) is generally intended for a simpler stream-aquifer system, the analysis of the pumping test at the Killgore Island, eliminating the drawdown data for $t<105$ min, provided approximate results of K_x, l and S_y that are comparable with those obtained using MODFLOW.

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