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Geodetic versus geophysical perspectives of the 'gravity anomaly'

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SUMMARY

A 'gravity anomaly' is essentially the difference between the gravitational acceleration caused by the Earth's masses and that generated by some reference mass distribution. However, there are numerous subtleties to the definition and, moreover, to the practical realization of a 'gravity anomaly'. An attempt is made here to clarify the definition of a 'gravity anomaly' from the geodetic and geophysical perspectives, point out some of the key differences in terminology and philosophy and to identify some of the problems remaining in its practical realization from a variety of observation types. It is argued that if the 'gravity anomaly' is defined and realized in a rigorous and consistent manner, this may lead to the improvement of its use in both geodesy and geophysics.

Key words: ellipsoid, geodesy, geoid, geophysics, gravity anomaly, gravity disturbance, height.

1 INTRODUCTION

Anomalies in the Earth's gravitational field play important roles in both geodesy and geophysics. In geodesy, gravity anomalies are used to define the figure of the Earth, notably the geoid (the equipotential surface of the Earth's gravity field that corresponds most closely to mean sea level). In geophysics, gravity anomalies are used to deduce variations in mass-density and hence subsurface geological structure for a wide variety of applications. To these ends, the geophysicist's aim is to remove gravity effects that mask the local anomalies that are of interest, whereas the geodesist is interested in using a gravity anomaly that preserves the mass of the Earth.

In both disciplines, however, it appears that the definition and practical realization of a 'gravity anomaly' remains open to question, despite it being subject to ongoing investigation (see the majority of the reference list). Of some concern is that investigators in these disciplines seem to be unaware of the other's work, which is demonstrated by even a cursory inspection of the literature cited in each. This is unsatisfactory since (often parallel) advances being made in each discipline are not being used, or even acknowledged, in the other.

This paper, though not exhaustive in its own literature survey, will attempt to identify some of the main differences between the geodetic and geophysical perspectives of a 'gravity anomaly'. It will briefly review the definitions of the *gravity anomaly* and *gravity disturbance*, and relate these to new observation types, such as airborne gravimetry, airborne gravity gradiometry and satellite gravity gradiometry. Throughout the paper, several questions are posed that should be answered to unambiguously define and realize a 'gravity anomaly' in both disciplines.

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2 BACKGROUND AND DEFINITIONS

At the broadest conceptual level, a gravity anomaly is the difference between the Earth's gravitational acceleration (i.e. gravitation and rotation) observed, or estimated, at some reference level, and the gravitational acceleration generated by a simple mass distribution, such as a biaxial ellipsoid of revolution, at the same or some other reference level. In geodesy, the reference level is normally the geoid, but in geophysics, where relative differences in gravity are often all that is important, the reference level can be chosen at an arbitrary height, such as the mean elevation of the area of interest. From a geophysical perspective, a gravity anomaly is better thought of as the difference between a measured value and a predicted value for the *same point* derived from some theoretical reference model (*cf.* Chapin 1996). In the remainder of this paper, the reference gravity field (i.e. normal gravity) is assumed to be generated by the Geodetic Reference System 1980 (GRS80; Moritz 1980).

2.1 The vector and scalar gravity anomaly

The geodetic gravity anomaly is classically defined (e.g. Heiskanen & Moritz 1967) as the difference between gravity *on the geoid* and normal gravity *on the surface of the reference ellipsoid* for the appropriate observation latitude.

Since gravitational acceleration is a vector quantity, this definition needs further clarification, especially as an airborne gravimetry (e.g. Glennie *et al.* 2000) or the use of auxiliary geoid information (e.g.

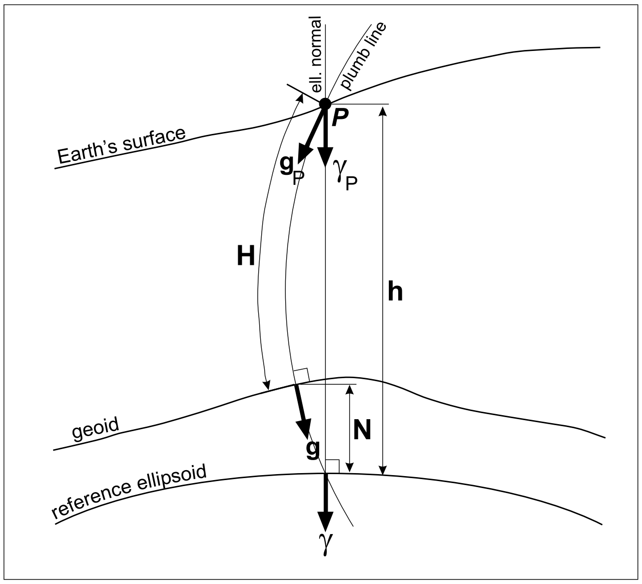

Figure 1. Parameters used to define gravity anomalies and gravity disturbances. **g** is the gravity vector at the geoid, $\boldsymbol{\gamma}$ is the normal gravity vector at the surface of the ellipsoid. $\mathbf{g}_P$ is the gravity vector and $\boldsymbol{\gamma}_P$ is the magnitude of normal gravity, both at point P (in this case, the Earth's topographic surface). H is the orthometric height along the curved plumbline, h is ellipsoidal height along the ellipsoidal surface normal and N is the geoid–ellipsoid separation.

Featherstone *et al.* 2000) provide information on the Earth's gravity vector. Therefore, the vector gravity anomaly is

$$\Delta\mathbf{g} = \mathbf{g} - \boldsymbol{\gamma}, \tag{1}$$

where **g** is the Earth's gravity vector *at the geoid,* which, if required, has been appropriately up/downward-continued from the measurement level and $\boldsymbol{\gamma}$ is the normal gravity vector *at the surface of the ellipsoid* at the same *geocentric* geodetic latitude as the gravity observation (Featherstone & Dentith 1997) summarize the differences between geodetic and geocentric latitude and exemplify their effect on the computation of normal gravity.

Since the Earth's gravity vector is not always observed in practice, the scalar gravity anomaly is defined here as

$$\Delta g = g - \gamma, \tag{2}$$

where g is the magnitude of the gravity vector ($g = |\mathbf{g}|$) *at the geoid* (appropriately up/downward-continued) and γ is the magnitude of the normal gravity vector ($\gamma = |\boldsymbol{\gamma}|$) *at the surface of the ellipsoid* at the same *geocentric* geodetic latitude as the gravity observation. The scalar gravity anomaly is a much simpler quantity to realize in practice.

Importantly, the vector and scalar gravity anomalies are defined only at the geoid. To evaluate them from gravity observations made on or above the Earth's surface, elevations with respect to the geoid (i.e. orthometric heights, H), measured along the curved plumbline of the Earth's gravity field, are required (Fig. 1). This information is used to reduce or downward-continue the observed value of gravity to the geoid for use in eqs (1) and (2).

2.2 The vector and scalar gravity disturbance

The gravity disturbance is a very well-known quantity in geodesy, but appears to be less well known in geophysics, or at least acknowledged (based on the authors' literature reviews). However, the gravity disturbance may be more useful and conceptually more logical for geophysical investigations. As with the gravity anomaly, the gravity disturbance can be either a vector or a scalar quantity. The vector gravity disturbance at point P is

$$\delta\mathbf{g}_P = \mathbf{g}_P - \boldsymbol{\gamma}_P, \tag{3}$$

where $\mathbf{g}_P$ is the Earth's gravity vector at the point P and $\boldsymbol{\gamma}_P$ is the normal gravity vector at the *same* point (Fig. 1). The value of $\boldsymbol{\gamma}_P$ is determined from that computed on the surface of the ellipsoid by subtracting a correction determined from the vertical gradient of normal gravity over the ellipsoidal height (Section 3). In this approach, however, care must be exercised to account for the curvature of the normal gravity plumbline from the ellipsoidal surface normal (e.g. Jekeli 1999). This is to ensure the proper orientation of the normal gravity vector, which does not point along the ellipsoidal normal outside the reference ellipsoid.

Again, assuming that observations of the Earth's gravity vector are not available, the scalar gravity disturbance is

$$\delta g_P = g_P - \gamma_P, \tag{4}$$

where g_P is the magnitude of the vector gravity at the point P and γ_P is the magnitude of the normal gravity vector at the *same* point. The latter value is computed by correcting the magnitude of normal gravity from the surface of the ellipsoid to the point of interest. This is simply the upward application of the free-air correction over the ellipsoidal height (see Section 3).

Choosing P to be the gravity observation point avoids the need to apply (numerically unstable) downward continuation of the gravity observation through the topography. Although Fig. 1 shows point P to be at the Earth's topographic surface, the gravity disturbance can be defined at any point. However, the vector and scalar gravity disturbances can only be computed if the ellipsoidal height (h) of point P, measured from the surface of the ellipsoid along the ellipsoidal surface normal, is known.

2.3 Coordinate systems used in gravimetry

The 3-D position of a gravity observation is required to compute the gravity anomaly and the gravity disturbance. As stated, the orthometric height (H) is required to compute the gravity anomaly and the ellipsoidal height (h) is required to compute the gravity disturbance. However, ellipsoidal heights were not readily available in the past. Instead, orthometric heights were readily available at benchmarks already established on the local (mean sea level-based) vertical datum. This probably explains why gravity anomalies have been more widely adopted as the realizable quantity.

However, algebraically adding a geoid model to orthometric heights yields ellipsoidal heights (i.e. $h = H + N$; see Fig. 1), thus permitting the practical realization of gravity disturbances. Importantly, this can be achieved for existing and new gravity observations. The EGM96 global geoid model (Lemoine *et al.* 1998) can be used for this purpose, for example, but if available, a regional geoid model should be used in preference. Alternatively, ellipsoidal heights compatible with GRS80 are provided directly from GPS-coordinated gravity surveys. Gravity disturbances can then be computed directly using the GPS-derived or geoid-derived ellipsoidal heights.

In addition to choosing the correct height system, there are numerous different realizations of vertical and horizontal geodetic datums to contend with. The vertical datums are problematic (e.g. Heck 1990) because they are based on mean sea level, and due to the effects of sea surface topography (*cf.* land topography) may differ from one another by as much as 2 m (e.g. Rapp 1994). This causes long-wavelength errors in the computed gravity anomalies and gravity disturbances. While these may not appear problematic in regional and local geophysical studies (because regional trends are normally removed before localized interpretations), they are much more problematic in geodesy.

Local horizontal geodetic datums, used for regional surveying and mapping, are not normally geocentric and these latitudes must not be used to compute normal gravity (see Section 3.1). Since different horizontal datums have been chosen in different parts of the world, a single point can have more than one set of coordinates by virtue of the datum used. For example, coordinates in some countries can differ from geocentric coordinates by up to 1 km (Defense Mapping Agency 1997). This can be significant when computing the normal gravity, where a ± 100 m error in latitude generates a ± 80 μGal error in normal gravity at 45° latitude.

In addition, co-registration errors occur when combining gravity data coordinated and reduced with respect to different geodetic datums and ellipsoids. Featherstone & Dentith (1997) and Featherstone (1997) describe the more common procedures to transform local horizontal geodetic coordinates to a geocentric datum. The user must also ensure that the correct datum and ellipsoid parameters are used, since there are several different options. The key is to be consistent and to carefully document the techniques used.

2.4 The indirect effect

The term indirect effect requires clarification. In geodesy, it is the correction applied to the co-geoid computed from gravity anomalies that have first been reduced (i.e. up/downward continued) to the geoid. These reductions effectively change the Earth's gravity potential, which is accounted for by applying the indirect effect to the computed co-geoid to give the geoid. Several models are available for this process (e.g. Wichiencharoen 1982; Martinec & Vaníček 1994a; Sjöberg & Nahavandchi 1999). Importantly, the geodetic indirect effect computed must be consistent with the technique used to reduce the gravity data.

In geophysics, the indirect effect is (rarely) used as a 'correction' for the separation between the geoid and the reference ellipsoid (*cf.* Chapman & Bodine 1979). The source of this *geophysical* indirect effect is clear in the context of the gravity disturbance—a gravity measurement is made at some point, and the anomalous component of that measurement is estimated by determining a theoretical gravity value at that *same* point. This theoretical value is determined by applying the free-air (and sometimes Bouguer) corrections to normal gravity at the ellipsoid. Therefore, it is not appropriate to use the orthometric height (H), as is commonly the case. Failure to use the ellipsoidal height (h) leads to either an undercorrection or overcorrection of normal gravity, depending on the algebraic sign of the geoid–ellipsoid separation, N.

The gravity anomaly (as strictly defined in Section 2.1) can be converted to the gravity disturbance by application of the free-air (and usually the Bouguer) correction over the geoid–ellipsoid separation (e.g. Jung & Rabinowitz 1988; Talwani 1998). Gravity disturbances erroneously computed using orthometric heights can be corrected in an identical manner.

The geophysical indirect effect can be computed using the geoid heights available from global geopotential models, such as EGM96 (Lemoine *et al.* 1998), or regional geoid models, such as AUSGeoid98 (Featherstone *et al.* 2001). Globally, the (undulating) geoid and the ellipsoid are separated by up to ~100 m. This difference is equivalent to a maximum indirect effect of ~30 mGal when computing the free-air disturbance from the free-air anomaly or

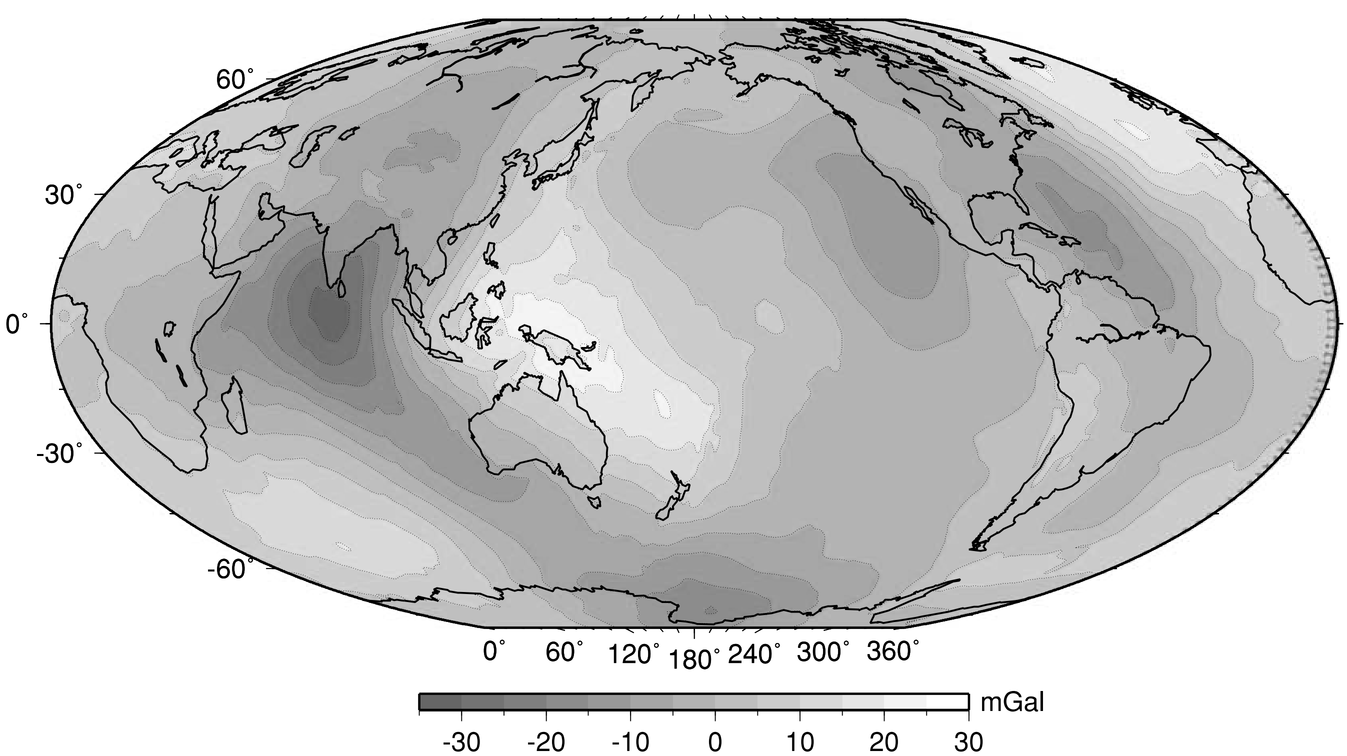

Figure 2. Global map showing the magnitude of the geophysical indirect effect. The free-air indirect effect is computed offshore and the Bouguer indirect effect onshore, hence the discontinuities at some coasts. Calculations are based on geoid heights from the EGM96 global geopotential model (Lemoine *et al.* 1998).

∼20 mGal when computing the Bouguer disturbance from the Bouguer anomaly (Fig. 2).

Since the geophysical indirect effect can be applied using the free-air or Bouguer gravity corrections over the geoid height (Talwani 1998), this raises some question as to what quantity should be computed and interpreted in geophysical studies (*cf.* Jung & Rabinowitz 1988). Therefore, some further work is required to clarify the role of the geophysical indirect effect and how it affects the subsequent interpretation.

3 'CORRECTIONS' TO GRAVITY OBSERVATIONS

Much of the geodetic and geophysical literature describes the process of computing gravity anomalies as a reduction process (e.g. Bullard 1936), where observed gravity is reduced to some datum surface, usually the geoid (using the orthometric height). However, as pointed out by many authors (e.g. Hipkin 1988; LaFehr 1991b; Chapin 1996; Li & Götze 2001; Vanicek *et al.* 2001), the gravity value is not simply reduced to a different level. Determining gravity at the geoid from surface data strictly requires up/downward continuation, a process that is generally a necessity in geodesy.

To compute the gravity anomaly (at the geoid), the up/downward continuation requires knowledge of the vertical gradient (along the plumb line) of the Earth's gravity field ($\delta g/\delta H$) interior and sometimes exterior (e.g. for airborne data) to the Earth's gravitating masses (*cf.* Hammer 1970; LaFehr & Chan 1986). In practice, however, this vertical gravity gradient along the plumb line is difficult to estimate accurately, especially inside the topography (*cf.* Vanicek *et al.* 1996; Wang 1997; Sun & Vanicek 1998). Instead, the vertical gradient of normal gravity ($\delta\gamma/\delta h$), which is recognized as the free-air correction, and the Bouguer gradient are usually used as an approximation. This approximation is a poor one. In Britain, for example, the difference between these vertical gradients leads to corrections that differ by as much as 20 mGal (Hipkin 1988).

Because the vertical gradient of the Earth's gravity field is extremely difficult to determine, a surface-referenced quantity (the gravity disturbance) is a more logical quantity to use in geophysical applications (Chapin 1996; Li & Götze 2001). Large-scale gravity effects are removed by subtracting a theoretical value of gravity from the measured value of gravity. This theoretical value is determined by 'correcting' the value of normal gravity on the ellipsoid to the measurement level. This only requires knowledge of the mathematically defined vertical gradient of the normal gravity field ($\delta\gamma/\delta h$).

3.1 Normal gravity (latitude correction)

The magnitude of the normal gravity field is the largest term in the gravity anomaly and gravity disturbance (eqs 1–4). The Somigliana–Pizetti closed formula (Moritz 1980), which is a standard in geodesy, rather than the Chebyshev approximations often used in geophysics (*cf.* Chapin 1996; Li & Götze 2001), should always be used to compute the normal gravity (or latitude correction). This is because it is exact to 1 μGal, and arguments for computational convenience in favour of the Chebyshev formulae are no longer justifiable (e.g. Featherstone & Dentith 1997). The Somigliana–Pizetti formula gives the magnitude of normal gravity *on the surface* of a geocentric reference ellipsoid:

$$\gamma = \gamma_e \frac{1 + k\sin^2\phi}{\sqrt{1 - e^2\sin^2\phi}}, \tag{5}$$

where k is the normal gravity constant (not to be confused with the universal gravitational constant, G), γ_e is normal gravitational acceleration at the equator and e^2 is the square of the first numerical eccentricity of the ellipsoid (Table 1). Importantly, the *geocentric* latitude (ϕ), which is compatible with GRS80, must be used in eq. (5). Finally, since the normal gravity vector is orthogonal to the surface of the normal ellipsoid (which is both an equipotential surface and a geometrical figure), the normal gravity vector is simple to orient given the dimensions of the ellipsoid and the geocentric latitude.

Table 1. Parameters used to compute normal gravity using the Somigliana–Pizetti formula (eq. 5) and the second-order free-air correction (eq. 6). Values for the GRS80 ellipsoid are from Moritz (1980).

Parameter	Definition	GRS80 value
a	Ellipsoid semi-major axis	6378 137 m
b	Ellipsoid semi-minor axis	6356 752.3141 m
γ_e	Equatorial normal gravity	9.780 326 771 5 m s^{-2}
γ_p	Polar normal gravity	9.832 186 3685 m s^{-2}
k	$(b\gamma_p/a\gamma_e) - 1$	0.001 931 851 353
e^2	$(a^2 - b^2)/a^2$	0.006 694 380 022 90
f	$(a - b)/a$	0.003 352 810 681 18
m	$(\omega^2 a^2 b)/GM$	0.003 449 786 003 08
ω	Angular velocity	7292 115 × 10^{-11} rad s^{-1}
GM	Geocentric gravitational constant	3986 005 × 10^{8} m^3 s^{-2}

3.2 The corrections to normal gravity

3.2.1 Free-air correction

In geodesy, the free-air 'reduction' is used to partly downward- or upward-continue observed gravity to the geoid using the vertical gradient of normal gravity as an approximation (i.e. $\delta\gamma/\delta h$, the first derivative of eq. (5) with respect to ellipsoidal height). In geophysics, the same quantity is used to correct normal gravity to the level of the measurement. In both cases, a second-order free-air correction is more realistic than the linear approximation of 0.3086 mGal m^{-1} since it takes into account the oblate elliptical shape of the Earth. From Featherstone & Dentith (1997), this second-order free-air correction, which can be derived from eq. (5), is

$$\delta g_F = \frac{\delta\gamma}{\delta h} = \frac{2\gamma_e}{a}(1 + f + m - 2f\sin^2\phi)h - \frac{3\gamma_e}{a^2}h^2, \tag{6}$$

where f is the geometrical flattening of the ellipsoid, m is the geodetic parameter, which is the ratio of gravitational and centrifugal forces at the equator and a is the semi-major axis length (equatorial radius) (Table 1). Note that because this equation is derived from eq. (5), the ellipsoidal height (h) is the elevation that should strictly be used. The difference between the linear and second-order free-air corrections reaches −5.7 mGal at the summit of Mt Everest ($\phi \approx 27°58'$, $H \approx$ 8848 m). Though this difference may appear small, it is a systematic effect and is very simple to compute.

3.2.2 Bouguer correction

In geodesy, the Bouguer correction is used as a means to smooth the gravity field in order to reduce aliasing during gridding and prediction of gravity data. In order to preserve the mass of the Earth, the Bouguer reduction is subsequently restored before computation of the co-geoid (*cf.* Featherstone & Kirby 2000).

In geophysics, the Bouguer reduction is used in an attempt to remove the gravitational effect of topographic masses and hence the high degree of correlation between the free-air gravity anomaly and elevation. The Bouguer gravity anomaly appears to be the standard quantity that is interpreted in geophysics. However, based on the series of short notes and correspondence in the journal *Geophysics* (e.g. LaFehr 1991a,b, 1998; Chapin 1996; LaFehr & Chan 1986; Talwani 1998), it is evident that the application of the Bouguer correction is generally not applied according to a uniform standard. This also applies to geodesy, albeit occurring more recently (e.g. Vanicek *et al.* 2001).

The simple Bouguer correction (Bullard A correction) is commonly used as a standard model to attempt to remove the gravitational attraction of topographic masses from gravity observations. This must be applied in conjunction with the free-air correction to compute the simple Bouguer gravity anomaly. However, there are two models for this process—a planar model (i.e. the Bouguer plate) and a spherical model (i.e. the Bouguer shell and cap).

The Bouguer plate correction is widely used, probably because it is relatively straightforward to derive. Thus, it appears in many geophysical and geodetic textbooks. However, an infinitely extending plate approximates the shape of the Earth very poorly (e.g. Qureshi 1976). Accordingly, several authors in both geodesy and geophysics (e.g. Karl 1971; Hensel 1992; LaFehr 1992; Chapin 1996; Talwani 1998; Smith *et al.* 2001; Vanicek *et al.* 2001) advocate the use of a spherical model for the simple Bouguer correction, which is obviously more realistic.

The role of the simple Bouguer correction is a notable example of where geodesy and geophysics appear to have diverged. Viewing the majority of references cited in the 'geophysical' papers (e.g. Karl 1971; LaFehr 1991a,b, 1992, 1998; Hensel 1992; Chapin 1996; Talwani 1998; Li & Götze 2001) and the 'geodetic' papers (e.g. Smith 2000; Smith *et al.* 2001; Vanicek *et al.* 2001) provides evidence of lack of 'communication' between these groups. An earlier example is given in LaFehr (1991b) 'Although it is somewhat puzzling that Bullard (1936) chose not to avail himself of the Lambert (1930) formula. . . '. Clearly, this needs redressing.

One particular ambiguity (or area of confusion) is the application of the spherical Bouguer correction over a cap of 166.7 km (the outer radius of the Hayford–Bowie system), which is approximately equal to the Bouguer plate correction ($2\pi\ G\rho\ h$ mGal m^{-1}), depending on elevation (*cf.* LaFehr 1998; Talwani 1998). This is compounded by the role of the Bullard B (curvature) correction (LaFehr 1991a). Therefore, further work is required to unambiguously define and hence realize the simple (plate, shell and/or cap) Bouguer correction, both in geodesy and geophysics. Meanwhile, recommendations for standardization (e.g. LaFehr 1991b) are particularly relevant.

Irrespective of the use of the plate/shell/cap Bouguer models of the topography (which must also be embedded in the terrain correction for consistency; described next), the overriding limitation in this correction is the accurate estimation of the topographic mass-density (e.g. LaFehr 1991b; Vanicek *et al.* 1999; Huang *et al.* 2001). Incorrect estimation of the topographic mass-density causes distortions in the Bouguer gravity anomalies. These are usually highly correlated with geological structures, thus causing problems in geophysical interpretations, and causing aliasing in gravity gridding and prediction in geodesy.

The former raises the question of whether the Bouguer correction should even be used for quantitative modelling in geophysics. Given the numerous model- and data-based problems in practically realizing the Bouguer gravity anomaly or disturbance, it may be preferable to construct forward and inverse geological models that include topography and generate free-air gravity anomalies or disturbances at the observation points (*cf.* Fullargar *et al.* 2000). In this way, the topographic mass variations that are the reason for the Bouguer correction can be included directly in the interpretation. However, when presented in map form, the high degree of correlation between gravity and topography means that the free-air anomaly is difficult to interpret. If gravity data are to be used purely in map form for interpreting geology, then the complete Bouguer anomaly incorporating variable topographic density does have demonstrable advantages (e.g. Flis *et al.* 1998).

3.2.3 Terrain correction

In geophysics, the complete/refined Bouguer correction comprises the simple Bouguer correction and the terrain correction. This correction is applied to the free-air gravity anomaly or disturbance to yield the complete/refined Bouguer gravity anomaly or disturbance, respectively. The terrain correction (Bullard C correction) is used to model and *remove* the gravitational effects of the topography residual to the Bouguer plate/cap/shell. Of course, the terrain correction used must be consistent with the spherical or planar Bouguer model (*cf.* LaFehr 1991a; Takin & Talwani 1966; Vanicek *et al.* 2001).

In geodesy, the terrain correction is used as part of a 'condensation' reduction (normally according to Helmert's second method) to replace the gravitational effect of the *in situ* topographic masses with an equivalent layer situated (condensed/compressed) at the geoid (e.g. Martinec & Vanicek 1994b). Alternatively, the masses can be moved mathematically inside the geoid. This condensation reduction is required to make the gravity anomaly field a harmonic function, thus permitting the solution of the geodetic boundary-value problem by Stokes' method. Essentially, the terrain correction is applied to the free-air gravity anomaly to yield the Faye gravity anomaly, which is an approximation of Helmert's gravity anomaly. The terrain correction is also used during the gridding and prediction of gravity data to reduce aliasing prior to computation of the geoid.

There appears to be much ambiguity, confusion and debate over the application of the 'terrain correction' (*cf.* Hammer 1982), especially as each discipline appears to use the same terminology to describe slightly different processes. Moreover, some of the terrain correction models used in geodesy are equivalent to the geophysical terrain correction under certain assumptions (e.g. Moritz 1968; Li & Sideris 1994; Martinec *et al.* 1993).

Historically, terrain corrections were computed using Hammer (1939) charts about each gravity computation point. This is an extremely time-consuming process, and was thus neglected for all but the most rugged topography. For instance, terrain corrections that reach 29 mGal have only been applied to the Australian gravity database in Tasmania (e.g. Murray 1998). However, terrain corrections can now be computed very efficiently from the regular grid of elevations provided by digital elevation models (DEMs), after some approximations, using the fast Fourier transform (e.g. Forsberg 1985; Sideris 1985; Schwarz *et al.* 1990; Parker 1995, 1996; Kirby & Featherstone 1999). However, the use of DEMs, which normally describe only the mean elevation in a geographical cell, omit near-station effects that, depending on the resolution of the DEM and the roughness of the topography, can reach several mGal (*cf.* Leaman 1998; Nowell 1999).

Irrespective of the terrain correction method or discipline, there are several common problems, which can be summarized as follows:

(1) correct estimation of near-metre terrain effects, which may reach several tens of mGal (e.g. Leaman 1998; Nowell 1999);

(2) avoidance of numerical instabilities due to planar approximations (e.g. Moritz 1968; Martinec *et al.* 1996; Kirby & Featherstone 1999);

(3) avoidance of weak singularities in the terrain correction kernel (e.g. Klose & Ilk 1993);

(4) the use of more realistic models of the topographic morphology than the flat-topped prisms given in a DEM (e.g. Blais & Ferland 1984; Ma & Watts 1984; Cogbill 1990; Barrows & Fett 1991; Smith 2000);

(5) the effect of errors in DEMs on the computed terrain corrections (Kirby & Featherstone 1999, 2001);

(6) the computation of terrain corrections over the whole globe (e.g. Danes 1982; Smith 2002).

As for the simple Bouguer correction, the role of topographic mass-density variations in the terrain correction should also be considered. Importantly, these must be consistent with the density model used for the simple Bouguer correction. A related consideration is the use of the gravitational constant (*cf.* LaFehr 1998; Talwani 1998), the numerical value of which is not well known (e.g. Schwarzschild 2002).

Clearly, there is a need for a systematic study of the terrain correction in both geodesy and geophysics, including theoretical and numerical comparisons of the terrain correction algorithms with one another. Importantly, such a study should source all the literature from each discipline. This should help to further clarify the differences and ambiguities between the geodetic terrain correction and the geophysical terrain correction.

3.2.4 Atmospheric correction

An additional consideration is that the parameters that define the normal gravity field are determined using satellite-derived geodetic data. As such, normal gravity includes a component due to the mass of the Earth's atmosphere, whereas gravity observed on or above the Earth's surface does not. Therefore, the atmospheric gravity correction (e.g. Ecker & Mittermayer 1969; Sjöberg 1999) must be added to the gravity anomaly and gravity disturbance. This correction term is typically small (<1 mGal) and for geophysical surveys it is usually insignificant. In effect, the atmospheric correction term can be considered to be a bias that is unimportant to geophysical exploration, but is important in geodesy where the mass of the Earth must be preserved.

4 IMPLICATIONS FOR OTHER GRAVITY MEASUREMENT TYPES

4.1 Global geopotential models

A global geopotential model is a representation of the Earth's gravitational field (geoid heights, gravity anomalies, gravity disturbances and vertical deflections) in terms of spherical harmonic basis functions. These are classified as satellite-only models, derived from orbital analysis, combined models, derived from terrestrial gravimetry, satellite altimetry and orbital perturbations, or tailored models, where additional data are used to refine existing models (Kearsley & Forsberg 1990; Wenzel 1998). Lambeck & Coleman (1983), Nerem *et al.* (1995), Rapp (1997a) and Featherstone (2002) review global geopotential models. These are likely to be improved significantly with the advent of dedicated satellite gravity field missions.

When computing gravity anomalies (at the geoid) from a global geopotential model, it is important to recognize that they are based on the assumption that the gravity field is a harmonic function, which only applies outside the gravitating masses. Therefore, additional corrections need to be applied if gravity anomalies or gravity disturbances at the geoid are required. Some techniques to apply these corrections are given in Rapp (1997b) and Sjöberg (1996).

4.2 Satellite radar altimetry

Satellite radar altimetry can be used to determine gravity anomalies and gravity disturbances at the geoid in ocean areas. This is often achieved by taking along-track gradients of stacked sea-surface heights (measured by the altimeter) to yield vertical deflections, which are then converted to gravity anomalies. Alternatively, the inverse Vening Meinesz integral can be used (Hwang 1998). Knudsen & Andersen (1998), Hwang *et al.* (1998) and Sandwell & Smith (1997) have computed grids of gravity anomalies in open ocean areas from a combination of satellite radar altimetry missions.

Gravity disturbances can also be determined in ocean areas from satellite altimetry. Interestingly, however, no such grids have been computed (or at least documented as such). Since the satellite altimeter actually measures the instantaneous sea surface, these data can be averaged (stacked) to estimate the mean sea surface shape. After application of a sea surface topography model, this gives the geoid. This can be used to determine normal gravity at the geoid (and to apply the Bouguer correction for the density of seawater), the geophysical indirect effect (Section 2.4) and subsequently the gravity disturbances at the geoid.

4.3 Airborne gravimetry

Scalar and vector gravity measurements are now being made from aircraft using a variety of techniques (e.g. Schwarz & Wei 1995; Boedecker & Neumayer 1996; Hein 1996; Schwarz & Glennie 1998; Childers *et al.* 1999; Glennie *et al.* 2000; Brozena & Childers 2001). Essentially, the kinematic accelerations of the aircraft are measured and, together with Eötvös effects, are removed from the acceleration measured by a modified shipboard gravimeter or inertial measurement unit, which sense both the gravitational field and aircraft acceleration.

Since GPS is used routinely in airborne gravimetry, ellipsoidal heights are directly available. Scalar and vector gravity disturbances are, therefore, easy to compute at the aircraft altitude. The free-air correction is applied over the ellipsoidal height to correct normal gravity on the ellipsoid up to the level of the measurement point. However, to compute gravity anomalies (to merge with terrestrial gravimetry, for example) downward continuation is required (*cf.* Tscherning *et al.* 1998). This is straightforward when applied down to the topographic surface (ignoring atmospheric effects), but suffers the same limitations as terrestrial gravimetry when downward-continued through the topography.

Because gravity disturbances are easily determined from airborne gravity measurements, it is proposed here that geophysical forward models and interpretations of airborne gravity data use gravity disturbances at the mean flight height or at the maximum height of the topography over which the survey is flown. Forward models could also be constructed that are consistent with both gravity disturbances at the flight height and at the Earth's surface, thereby providing improved constraints on geophysical inversions of gravity data.

4.4 Airborne gravity gradiometry

Scalar and vector gravity measurements can also be derived from airborne gravity gradiometry (e.g. Jekeli 1993; Swain 2001). The advantage of using a gravity gradiometer in an aircraft, over airborne gravimetry, is that the kinematic acceleration of the aircraft is essentially removed when measuring the gravity gradient. Again, GPS is often used to coordinate the observations, which allows for the computation of gravity disturbances at the flight elevation. If gravity anomalies or disturbances are required at a lower altitude, the data must be downward-continued (Tziavos *et al.* 1988; Forsberg & Kenyon 1996), which is subject to the same limitations as for airborne gravimetry.

As proposed for airborne gravity, geophysical forward models can be constructed that generate gravity gradients at the aircraft altitude (*cf.* Dransfield 1994). Again, this avoids problems with the downward continuation of airborne gradiometry data. The forward models can also be constructed to generate gravitational accelerations so that other measurements can be used, such as land or marine gravimetry.

4.5 Dedicated satellite gravimetry

Three satellite gravimetry missions (GRACE, CHAMP and GOCE) will be in operation during the next 7 years (e.g. Ilk 2000; Rummel *et al.* 2002; Featherstone 2002). Only the GOCE mission will use a dedicated space-borne gravity gradiometer, but the GRACE and CHAMP missions deduce gravity gradients using satellite-to-satellite tracking. The gravity models derived from these missions should provide an order of magnitude improvement in our knowledge of the global gravity field to wavelengths greater than ~200 km. The new models are likely to be expressed in terms of surface spherical harmonic basis functions and so gravity disturbances can be computed for all points exterior to the gravitating masses. If gravity anomalies are required, downward continuation through the topography is required and the limitations are the same as for other measurement techniques.

As a result of the spatial resolution of these models, they are likely to be of more use in geodesy and global geophysics. However, the improved precision will make them more useful for removing regional trends from local gravity data prior to geophysical modelling and interpretation. Another advantage is that the new satellite data provide totally independent gravity field information, thus removing the problems associated with correlation between surface gravity data used in existing global geopotential models.

5 SUMMARY AND RECOMMENDATIONS

From this cursory review of the geodetic and geophysical literature, it is suggested that there needs to be an integrated examination of the 'gravity anomaly' from both the geophysical and geodetic perspectives, quantifying the key similarities and differences. Importantly, this must include a complete review of both the geodetic and the geophysical literature, which appears not to have been conducted before. This should eliminate the ambiguities (confusion?) between these disciplines, a notable example being the role of the 'terrain correction'.

From this review (and hopefully a generalization), answers to questions in geophysics such as 'should we be computing gravity anomalies or gravity disturbances and at what point', and 'should we be interpreting gravity disturbances on or above the Earth's surface' can be sought. The use of gravity disturbances outside the gravitating masses avoids hypotheses concerning the topographic mass density and the ambiguities in the mathematical models of the Bouguer gravity anomaly. Instead, these can be embedded in a forward model.

In conclusion, the 'gravity anomaly' is an ambiguous quantity for many reasons and the terminology needs to be re-examined using some unified geodetic and geophysical approach. It is therefore proposed that a working party, or similar, be established comprising both geophysicists and geodesists to resolve these differences and to define uniform standards for dealing with gravity data.

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