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Using an anisotropic gyrotropic model for estimating gyration and attenuation constants

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Abstract

A new approach to estimating of shear-wave attenuation is proposed. The method is based on an assumption that an anisotropic attenuating geological medium possesses gyrotropic properties. Doing so allows to obtain attenuation constant together with gyrotropic constant. Some examples of experimental determination of gyration and attenuation constants are given. A presentation of the new way of determining shear-wave attenuation constant is preceded by a brief elucidation of the main aspects of the phenomenological theory of gyrotropy. The most interesting and important phenomenon inherent to gyrotropy—rotation of shear-wave polarization plane—is illustrated.

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1. Introduction

Geological media have discrete structure and possess some elements of symmetry. Though discrete elements are much less than wavelength, they influence elastic wave propagation in rocks. Seismic anisotropy is the most prominent example of this action. Anisotropic models are effective models, i.e., long-wave equivalents of rocks with such discrete elements as thin layers and sets of fractures.

Recently it was found that rocks can possess not only anisotropic properties but also gyrotropic ones (Obolentseva, 1992, 1996). For seismic gyrotropy, the discrete elements of the medium are the same as those for seismic anisotropy, but they are situated in the

other way in space; as for seismic anisotropy, these elements are situated in the other way in space (Chichinina, 1998, 2000; Chichinina and Obolentseva, 1998b). The necessary condition for the appearance of gyrotropy is the absence of a symmetry center in the medium; an additional requirement to the arrangement of discrete elements is a prevalence in the medium of right or left orientations of elements, in this case the medium will be enantiomorphous (as, for example, is a crystal of quartz). In ‘right’ or ‘left’ media the phenomenon of the rotation of shear-wave polarization plane may occur, and it is the most interesting and important phenomenon inherent to gyrotropy, and to seismic gyrotropy in particular.

Seismic gyrotropy was introduced by analogy with acoustical gyrotropy (Andronov, 1960; Portgal and Burstein, 1968) and well known optical gyrotropy. Anisotropic gyrotropic models of geological media describe shear-wave polarizations in a most general

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way. The purely gyrotropic shear-wave polarization should be discriminated from the effect of large- and medium-scale heterogeneity and from azimuthal anisotropy produced by preferred orientation of vertical and subvertical microcracks. The work by Chichinina and Obolentseva (1998a) illustrates the main features of the gyrotropic propagation in comparison with the azimuthally anisotropic wave propagation.

The present paper is devoted to a new approach to estimating of shear-wave attenuation constant. The method is based on the assumption that a geological medium is attenuating and possesses gyrotropic properties. This allows to obtain attenuation constant together with gyration constant using two-component records of shear waves at a given point. The way differs from the commonly used when the attenuation is determined from the wave amplitude decreasing while it is passing a given distance.

2. What is gyrotropy about?

The most interesting and important phenomenon inherent to gyrotropy is rotation of shear-wave polarization plane. Let us consider how it takes place and why.

What is gyrotropy about? Gyrotropy is an exhibition of the first-order spatial dispersion of elastic properties (Portigal and Burstein, 1968). Spatial dispersion means representation of the stiffness tensor c_{ijkl} in the form of a series as a function of wave vector \mathbf{k} :

$$c_{ijkl}(\mathbf{k}) = c_{ijkl} + ib_{ijklm}k_m + d_{ijklmn}k_mk_n + \dots \quad (1)$$

Accounting of the two first terms $c_{ijkl} + ib_{ijklm}k_m$ of the stiffness-tensor expansion (1) means consideration of the first-order spatial dispersion. Let us take these two terms and substitute them into the Hooke's law $\boldsymbol{\sigma}(\mathbf{k}) = \mathbf{c}(\mathbf{k})\boldsymbol{\varepsilon}(\mathbf{k})$, where stresses $\boldsymbol{\sigma}$ and strains $\boldsymbol{\varepsilon}$ are represented in the form of a plane harmonic wave $\exp[i\mathbf{k}(\mathbf{n}\cdot\mathbf{r} - Vt)]$, $\mathbf{r}=(x_1, x_2, x_3)$. Then making inverse Fourier transform $\mathbf{k} \rightarrow \mathbf{r}$ and taking into account that $ik_m\varepsilon_{kl}(\mathbf{k}) \rightarrow \partial\varepsilon_{kl}/\partial x_m$, one can obtain the following expression for the Hooke's law in gyrotropic media:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + b_{ijklm}\frac{\partial\varepsilon_{kl}}{\partial x_m}.$$

The modified Hooke's law gets an additional term, which includes a spatial derivative of strain multiplied by the fifth-rank order gyration tensor b_{ijklm} .

The corresponding wave equations acquire the third partial derivative of a displacement with respect to the spatial coordinates:

$$c_{ijkl}\frac{\partial^2 u_k}{\partial x_j\partial x_l} + b_{ijklm}\frac{\partial^3 u_k}{\partial x_j\partial x_l\partial x_m} = \rho\frac{\partial^2 u_i}{\partial t^2},$$

$$i = 1, 2, 3.$$

Substituting a displacement vector in the form of a plane harmonic wave

$$\mathbf{U}(\mathbf{r}, t) = \mathbf{A}\exp[i\mathbf{k}(\mathbf{n}\cdot\mathbf{r} - Vt)]$$

yields the corresponding Christoffel equations

$$[c_{ijkl}n_jn_l + i\omega V^{-1}b_{ijklm}n_jn_l n_m]A_k = \rho V^2 A_i, \quad (2)$$

$$i = 1, 2, 3.$$

The expression in the brackets is *complex* Christoffel tensor. The notations are the following: V is phase velocity, A_i ($i=1, 2, 3$) are the components of a polarization vector \mathbf{A} , n_i ($i=1, 2, 3$) are the components of a wave normal \mathbf{n} .

3. Shear wave propagation along the symmetry axis

Let us consider a solution of Christoffel equations (Eq. (2)) in the case of wave propagation along the vertical symmetry axis of a transversely isotropic medium (VTI). Three eigenvalues of Christoffel tensor are squared velocities of three waves. One of them corresponds to the P-wave velocity and is the same as in a non-gyrotropic VTI medium. The other two eigenvalues represent two shear-wave velocities, which are equal to each other in the symmetry-axis direction of a nongyrotropic VTI medium. In a gyrotropic medium, symmetry-axis S-wave velocities differ, they are *almost equal* to the symmetry-axis velocity V_0 in a non-gyrotropic VTI medium:

$$V_1 = V_0 + a, \quad V_2 = V_0 - a, \quad (3)$$

where $V_0 = \sqrt{c_{44}/\rho}$, ($c_{44} = c_{2323}$), $a = \omega b_{543}/(2c_{44})$ and $b_{543} = b_{13233}$ is a gyration constant (see Obolent-

seva, 1992; Chichinina, 2000). In acoustical gyrotropy, $a \ll V_0$ (Sirotnin and Shascolskaya, 1979). At seismic frequencies, this inequality is also valid, corresponding experimental estimates will be presented later.

In Fig. 1, phase velocity difference for α -quartz is illustrated (Pine, 1970). The two shear-wave symmetry-axis velocities are not equal to each other. The velocity difference $2a$ is equal approximately to one percent of the S -wave symmetry axis velocity in a non-gyrotropic VTI medium, i.e., $a = 0.005V_0$.

Polarization vectors $\mathbf{A}^{(r)}$, $r = 1, 2, 3$ ($1 \rightarrow S_1$, $2 \rightarrow S_2$ and $3 \rightarrow P$) are three eigenvectors of Christoffel complex matrix, and that is why polarization becomes complex. In the general case of wave propagation, polarizations are elliptical. In the case of propagation along the symmetry-axis, the polarization is circular for shear waves:

$$\mathbf{A}^{(1)} = \frac{\sqrt{2}}{2}(\mathbf{e}_x + i\mathbf{e}_y), \quad \mathbf{A}^{(2)} = \frac{\sqrt{2}}{2}(\mathbf{e}_x - i\mathbf{e}_y) \quad (4)$$

and linear for P-wave: $\mathbf{A}^{(3)} = \mathbf{e}_z$ (where \mathbf{e}_x , \mathbf{e}_y , and \mathbf{e}_z are unit vectors oriented along corresponding coordinate axes). Note, that y -components of the two shear-wave polarizations (Eq. (4)) are of the opposite sign, i.e., polarizations $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ become counter-clockwise and clockwise, accordingly.

The differences of symmetry-axis wave propagation in a non-gyrotropic anisotropic medium and in gyrotropic one are the following. The first difference.

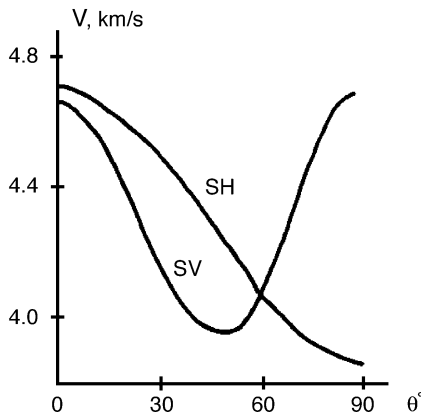


Fig. 1. Shear-wave phase velocities via wave-normal direction for α -quartz. θ° is an angle of a wave normal, degrees.

In a nongyrotropic anisotropic medium, polarizations of the two shear waves are linear, but in a gyrotropic medium they are circular (along a direction of threefold or higher symmetry). And the second difference. There are two shear waves with different velocities propagating along the symmetry axis in gyrotropic media, while in anisotropic case is only one shear wave.

4. Non-gyrotropic VTI medium

It is well known that along the VTI-symmetry axis only one S -wave can propagate. Let it be a plane harmonic S -wave propagating along the symmetry axis z with a linear polarization $\mathbf{A} = \mathbf{e}_x$ and amplitude U :

$$\mathbf{U}(t) = U \exp[i\omega(t - z/V)]\mathbf{e}_x.$$

The idea of superposition of the two oppositely circular-polarized plane shear waves (\mathbf{U}_1 and \mathbf{U}_2) does not contradict it, because the sum of them $\mathbf{U}_1 + \mathbf{U}_2$ is the compound shear wave with linear polarization $\mathbf{A} = \mathbf{A}^{(1)} + \mathbf{A}^{(2)}$, where $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ are given by Eq. (4). Let it be shown, following, e.g., Crawford (1970).

Assume that polarizations $\mathbf{A}^{(1)}$, $\mathbf{A}^{(2)}$ of the two harmonic shear waves $\mathbf{U}_{1,2}(t) = \sqrt{2} U/2 \mathbf{A}^{(1),(2)} \exp[i\omega(t - z/V)]$ are circular of opposite sign (counter-clockwise and clockwise), see formulae (4). It yields

$$\mathbf{U}_1 = U/2(\mathbf{e}_x + i\mathbf{e}_y)(\cos\omega(t - z/V) + i\sin\omega(t - z/V));$$

$$\mathbf{U}_2 = U/2(\mathbf{e}_x - i\mathbf{e}_y)(\cos\omega(t - z/V) + i\sin\omega(t - z/V)).$$

Taking the real parts of the waves \mathbf{U}_1 and \mathbf{U}_2 one can obtain the following circular oscillations:

$$\left. \begin{aligned} \mathbf{u}_1 &= U/2(\cos\omega(t - z/V)\mathbf{e}_x - \sin\omega(t - z/V)\mathbf{e}_y) \\ \mathbf{u}_2 &= U/2(\cos\omega(t - z/V)\mathbf{e}_x + \sin\omega(t - z/V)\mathbf{e}_y) \end{aligned} \right\}, \quad (5)$$

where $\mathbf{u}_1 = \text{Re}(\mathbf{U}_1)$ and $\mathbf{u}_2 = \text{Re}(\mathbf{U}_2)$.

As one can see from Eq. (5), x -component of the compound oscillation $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ is equal to the sum of the two identical cosines, and y -component is equal to the difference of the two identical sines, and so y -component is zero. Then the compound oscillation has to be with linear polarization:

$$\mathbf{u} = U \cos \omega(t - z/V) \mathbf{e}_x.$$

The superposition of the circular oscillations $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ for a fixed time t is shown in Fig. 2.

The same could be done with the imaginary parts $\tilde{\mathbf{u}}_1$ and $\tilde{\mathbf{u}}_2$ of the waves \mathbf{U}_1 and \mathbf{U}_2 , so one can obtain

$$\tilde{\mathbf{u}} = \tilde{\mathbf{u}}_1 + \tilde{\mathbf{u}}_2 = U \sin \omega(t - z/V) \mathbf{e}_x.$$

And it means that the shear wave $\mathbf{U} = \text{Re}(\mathbf{U}) + i\text{Im}(\mathbf{U})$ is

$$\begin{aligned} \mathbf{U} &= \mathbf{u} + \tilde{\mathbf{u}} = U[\cos \omega(t - z/V) + i \sin \omega(t - z/V)] \mathbf{e}_x \\ &= U \exp[i\omega(t - z/V)] \mathbf{e}_x. \end{aligned}$$

Thus, the result of summing up of the two shear waves with the opposite circular polarizations and equal phase velocities V is the S -wave with linear polarization \mathbf{e}_x . When $t = z/V = 0$, $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 = (U/2)\mathbf{e}_x + (U/2)\mathbf{e}_x = U\mathbf{e}_x$.

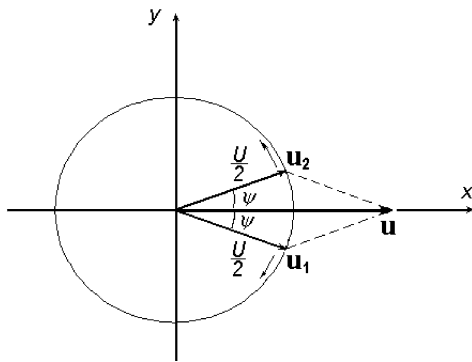


Fig. 2. For a fixed time t , two oscillations $\mathbf{u}_1(t)$ and $\mathbf{u}_2(t)$ with opposite circular polarizations and equal phase velocities. The resulting compound oscillation \mathbf{u} is represented as a vector-sum $\mathbf{u}_1 + \mathbf{u}_2$. Amplitudes of the oscillations \mathbf{u}_1 and \mathbf{u}_2 are equal to the circle radius, i.e., to $U/2$. The angles denoted by ψ are the phases ψ_1 and ψ_2 ($\psi_1 = \psi_2 = \psi$) of the oscillations \mathbf{u}_1 and \mathbf{u}_2 , given by Eq. (5), i.e. $\psi = \omega(t - z/V)$.

5. Gyrotropic VTI medium

In the gyrotropic VTI medium, symmetry-axis shear-wave velocities V_1 and V_2 are not equal to each other, as we have already shown. In this case, one can modify the expressions (Eq. (5)) for the circular oscillations \mathbf{u}_1 and \mathbf{u}_2 as the following:

$$\left. \begin{aligned} \mathbf{u}_1 &= U/2(\cos \omega(t - z/V_1) \mathbf{e}_x - \sin \omega(t - z/V_1) \mathbf{e}_y) \\ \mathbf{u}_2 &= U/2(\cos \omega(t - z/V_2) \mathbf{e}_x + \sin \omega(t - z/V_2) \mathbf{e}_y) \end{aligned} \right\}.$$

The expressions become more complicated now, and y -component of the compound wave $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ is not equal to zero now, i.e., $\mathbf{u} = u_x \mathbf{e}_x + u_y \mathbf{e}_y$, where

$$u_x = U/2[\cos \omega(t - z/V_1) + \cos \omega(t - z/V_2)],$$

$$u_y = U/2[\sin \omega(t - z/V_2) - \sin \omega(t - z/V_1)].$$

The last expressions can be simplified using the trigonometric transforms and the following denotations:

$$\varphi_1 = \omega z/V_1, \quad \varphi_2 = \omega z/V_2, \quad (6)$$

where φ_1 and φ_2 are phases of displacements \mathbf{u}_1 and \mathbf{u}_2 . It yields

$$u_x = U \cos[(\varphi_2 - \varphi_1)/2] \cos[\omega t - (\varphi_1 + \varphi_2)/2], \quad (7)$$

$$u_y = U \sin[(\varphi_2 - \varphi_1)/2] \sin[\omega t - (\varphi_1 + \varphi_2)/2]. \quad (8)$$

Let us analyze expressions (7) and (8). Consider wave propagation from $z=0$ to $z=h$ (see Fig. 3). At the beginning of the wave path $z=0$, therefore $\varphi_1 = \varphi_2 \equiv 0$ and $u_y = 0$, see Eqs. (6) and (8). So, at the wave-path beginning the compound wave has only x -component, as it is illustrated in Fig. 3.

Imagine the compound wave propagating along the vertical symmetry axis z . From expressions (7) and (8), it is clear that if $z=h$, the compound wave gets not only x -, but also the additional y -component. It is illustrated in Figs. 3 and 4.

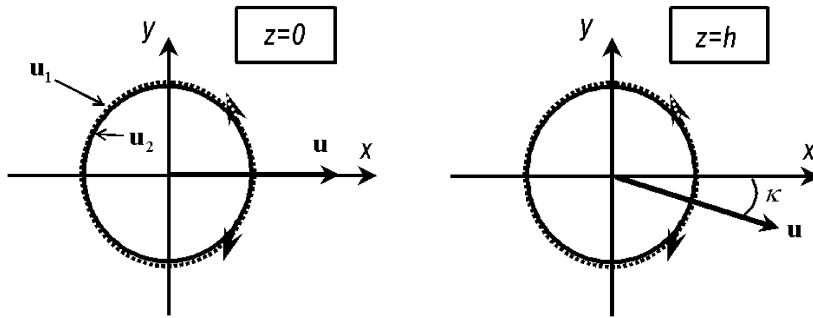


Fig. 3. The displacement vector \mathbf{u} of the compound shear wave in the beginning of the wave path $z=0$ and on propagating to a point $z=h$. κ is rotation angle. \mathbf{u}_1 and \mathbf{u}_2 are shear waves with the opposite circular polarizations (counter-clockwise (dashed line) and clockwise (solid line)) and different phase velocities.

One can compare left and right pictures in Fig. 3 and notice that the displacement vector of the compound wave $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ “rotates” and that the tangent of the rotation angle κ can be expressed by the ratio of u_y to u_x :

$$\tan \kappa = u_y / u_x. \tag{9}$$

If we divide expression (8) for u_y by expression (7) for u_x , we obtain

$$u_y / u_x = \tan[(\varphi_2 - \varphi_1) / 2]. \tag{10}$$

From the last two formulae, one can infer that the rotation angle κ is equal to the half of phase difference

$$\kappa = (\varphi_2 - \varphi_1) / 2.$$

If the wave path is equal to h , we can express rotation angle κ in the terms of the wave path using the expressions (Eq. (6)) for the phases:

$$\kappa = 1/2(1/V_2 - 1/V_1)\omega h. \tag{11}$$

Thus, it was shown that the more wave path, the more rotation angle. The compound displacement vector \mathbf{u} rotates while wave is travelling along the symmetry axis of VTI medium. This is so-called phenomenon of the rotation of shear wave polarization plane.

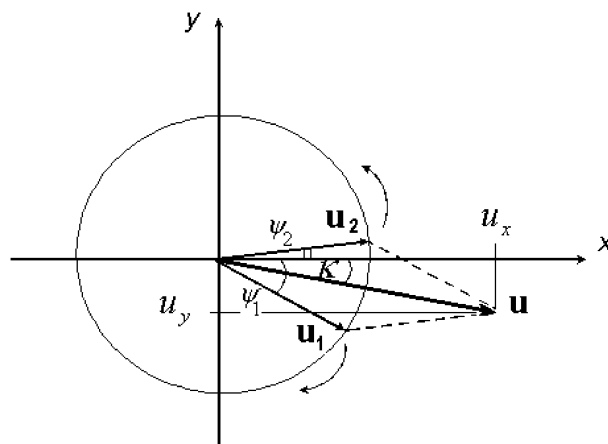


Fig. 4. For a fixed time t , two oscillations $\mathbf{u}_1(t)$ and $\mathbf{u}_2(t)$ with opposite circular polarizations and different phase velocities. The compound oscillation $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ has u_x and u_y components. Displacement vector of the compound wave \mathbf{u} has turned by the angle κ . The angles denoted by ψ_1 and ψ_2 are the arguments of the oscillations \mathbf{u}_1 and \mathbf{u}_2 , i.e., $\psi_1 = \omega(t - z/V_1)$ and $\psi_2 = \omega(t - z/V_2)$.

6. Gyrotropic anisotropic attenuating medium

In the attenuating medium, phase velocities get an imaginary part. We can write a plane harmonic wave in terms of complex slowness $S - i s = 1/V$:

$$\mathbf{u}(t) = U \mathbf{A} \exp(-\omega s z) \exp[i(\omega S z - \omega t)].$$

The amplitude $E = \exp(-\omega s z)$ contains the imaginary part of slowness s , and the attenuation coefficient is $\alpha = \omega s$, where ω is frequency. The real part of the slowness S is included in the wave phase:

$$\varphi = \omega S z.$$

In the case of the wave propagation along the symmetry-axis of the VTI attenuating gyrotropic medium, complex shear-wave velocities are different:

$$V_{1,2} = V_0 \pm a - ib \quad (12)$$

due to the difference ($\pm a$) of the real parts of the symmetry-axis velocities V_1 and V_2 , see formula (3).

The complex slownesses $1/V_1 = S_1 - i s_1$ and $1/V_2 = S_2 - i s_2$ are different, too. The real parts and the imaginary parts of the complex slownesses of the two shear waves may be written in the terms of gyration parameter a and attenuation parameter b as

$$S_{1,2} = (V_0 \pm a) / [(V_0 \pm a)^2 - b^2], \quad (13)$$

$$s_{1,2} = b / [(V_0 \pm a)^2 - b^2]. \quad (14)$$

Therefore, amplitudes and phases of the two shear waves are different, too:

$$E_{1,2} = \exp(-\omega s_{1,2} z), \quad (15)$$

$$\varphi_{1,2} = \omega S_{1,2} z. \quad (16)$$

Let us consider the superposition of the two circularly (oppositely) polarized waves with the dif-

ferent amplitudes E_1 and E_2 and different real phase velocities $V_1 = V_0 - a$ and $V_2 = V_0 + a$:

$$\left. \begin{aligned} \mathbf{u}_1 &= E_1 U / 2 (\cos \omega(t-z/V_1) \mathbf{e}_x - \sin \omega(t-z/V_1) \mathbf{e}_y); \\ \mathbf{u}_2 &= E_2 U / 2 (\cos \omega(t-z/V_2) \mathbf{e}_x + \sin \omega(t-z/V_2) \mathbf{e}_y). \end{aligned} \right\} \rightarrow \mathbf{u} \quad (17)$$

The compound wave (Eq. (17)) may be presented as

$$\mathbf{u}(t) = u_x(t) \mathbf{e}_x + u_y(t) \mathbf{e}_y, \quad (18)$$

$$u_x(t) = A_1 \cos(\omega t + \delta_1), \quad u_y(t) = A_2 \cos(\omega t + \delta_2), \quad (19)$$

and curve (18) and (19) represents an ellipse. Ellipse parameters A , B (large axis length, small axis length) and the turn angle κ (the angle between the large axis of an ellipse and the x -axis) are the functions of the amplitudes A_1 , A_2 and phase difference $\delta = \delta_2 - \delta_1$ of the oscillations (Eq. (19)).

It is illustrated in Fig. 5 that the sum of the oscillations (Eq. (19)) represents the ellipse, which rotates, as the compound wave propagates. The large axis length A and the small axis length B are

$$A = E_1 + E_2; \quad B = E_1 - E_2, \quad (20)$$

where E_1 and E_2 are given by formula (15).

Rotation angle is equal to the half of phase differences, as it was shown in the previous case of a non-attenuating gyrotropic medium:

$$\kappa = (\varphi_2 - \varphi_1) / 2,$$

but here φ_1 and φ_2 are given by formula (16).

Ratio of amplitudes E_1/E_2 and phase difference $\varphi_1 - \varphi_2$ provide the following system of equations:

$$\begin{cases} \exp[-\omega z(s_1 - s_2)] = (1 + B/A) / (1 - B/A); \\ \omega z(S_2 - S_1) = 2 |\kappa|, \end{cases} \quad (21)$$

where $s_{1,2}$ and $S_{1,2}$ are given by expressions (13) and (14) as functions of gyration parameter a and attenuation parameter b . Knowing ellipse parameters—the

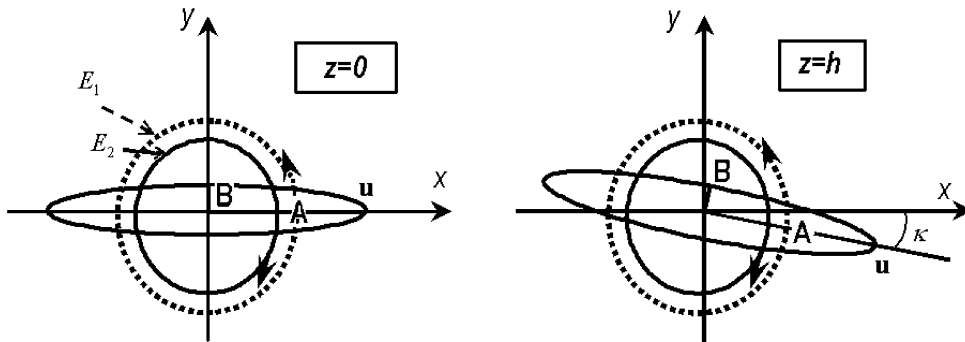


Fig. 5. Elliptical polarization of the compound wave u at the beginning of the wave path ($z=0$) and while propagating to a point ($z=h$). A is the large axis length, B is the small axis length, κ is the turn angle. E_1 and E_2 are the amplitudes of the two circularly polarized shear waves (clockwise (solid line) and counter-clockwise (dashed)).

turn angle κ and the ratio of axis B length to axis A length—one can find from the system of Eq. (21) parameters a and b characterizing gyrotropic rotation and attenuation.

7. Methodology for estimation gyration and attenuation constants

In experiments, the polarization of shear waves was studied for vibro-seismic oscillations directly and for impulse records by using the parameters of harmonics $S_x(\omega)$ and $S_y(\omega)$ spectra of x and y components of shear-wave displacements.

Harmonic oscillations (Eq. (19)) may be presented in a complex form:

$$K(\omega) = r(\omega)\exp[i\delta(\omega)]; \tag{22}$$

for impulse oscillations $r(\omega) = |S_y(\omega)| / |S_x(\omega)|$, $\delta(\omega) = \arg|S_y(\omega)| - \arg|S_x(\omega)|$, where S_x , S_y are spectra of $u_x(t)$ and $u_y(t)$ displacements.

Knowing amplitude ratio r and phase difference δ from spectra S_x and S_y , one can determine at every frequency ω ellipse parameters B/A and κ :

$$\frac{B}{A} = \frac{1 + r^2 - [(1 + r^2)^2 - 4r^2 \sin^2 \delta]^{1/2}}{2r \sin \delta};$$

$$\tan 2\kappa = 2r \cos \delta / (1 - r^2).$$

To determine the angle κ uniquely, one needs to account for the following formulae:

$$\sin 2\kappa = 2r \cos \delta / \sqrt{(1 - r^2)^2 + 4r^2 \cos^2 \delta},$$

$$\cos 2\kappa = (1 - r^2) / \sqrt{(1 - r^2)^2 + 4r^2 \cos^2 \delta}.$$

After determining ellipse parameters B/A and κ , gyration parameter a and attenuation parameter b can be found from the system of Eq. (21). The solution of the system of equations is given by

$$b = [(M^2 + 4V_0^2 P)^{1/2} - M] / (2P),$$

$$a = (V_0^2 - b^2 - Mb)^{1/2},$$

$$\text{where } M = 4|\kappa|V_0/L, \quad L = \ln \frac{1 + B/A}{1 - B/A},$$

$$P = 1 + [(M^2 + 4V_0^2)L / (4\omega h V_0)]^2.$$

8. Experimental data

Here are some examples of experimental determination of turn angle and gyration and attenuation constants.

The experiments were performed in wells drilled in the upper part of the subsurface, in sandy-clayey sediments of Tomsk region, Russia (Obolentseva, 1992, 1996). The shear waves were excited by X -

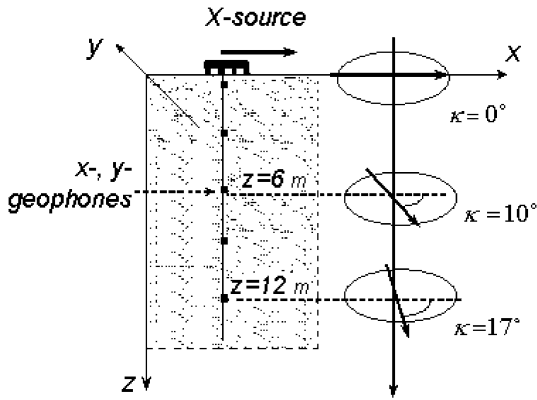


Fig. 6. Scheme of the field experiment and of the rotation of polarization vector.

vibrator and by impulse X-source at the head of the well and were registered by the two-component (x,y) receivers at the different depths in the well, as it is illustrated in Fig. 6. At the depth equal to 6 m, turn angle is equal to 10°, and at the depth of 12 m it has become 17°, that is, it increases directly with path of the compound wave, as it follows from the theory (see Eq. (11)), i.e., turn angle κ is in direct ratio to a wave path z .

The results of determining gyration and attenuation constants are presented in Fig. 7. The gyration constant a , that is a real part of the addition to the

Table 1

Attenuation coefficient α via frequency f determined from the experimental data

f (Hz)	60	90	120	150
α (m^{-1})	0.57	1.23	1.83	2.13

velocity V_0 , is small—in the average, it makes up to 3% from the velocity V_0 :

$$a \approx 5 \text{ m/s}; \quad V_0 = 155 - 175 \text{ m/s.}$$

The imaginary part of the velocity, characterizing the attenuation, is equal, in the average, to 35% from V_0 :

$$b \approx 60 \text{ m/s.}$$

The function $a(f)$ is decreasing.

One can find attenuation coefficient as

$$\alpha = \omega s,$$

where $s \approx b/[V_0^2 - b^2]$, see formula (14). The following values of the attenuation coefficient α were found for frequencies of 60–150 Hz in the depth interval 0–12 m, see Table 1. These data are fairly realistic for the considered medium and are similar to those calculated using the decreasing of main component amplitudes in the corresponding depth intervals.

The sandy–clayey sediments also have been investigated by the acoustic measurements at frequen-

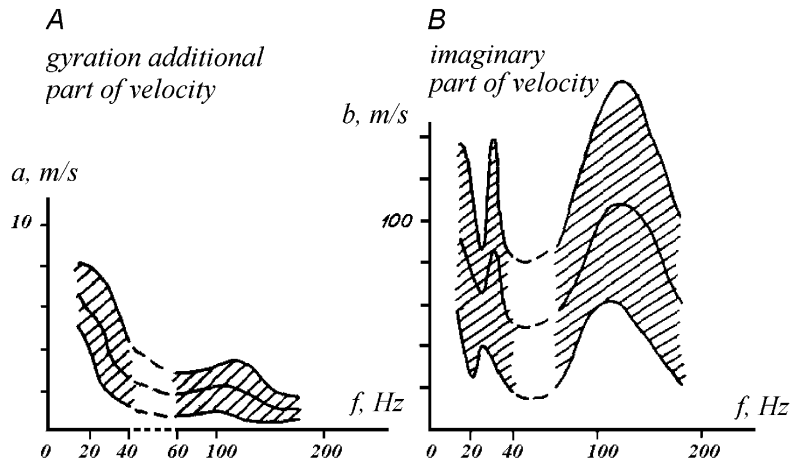


Fig. 7. Gyration (A) and attenuation (B) parameters as a function of frequency in the depth interval 0–12 m.

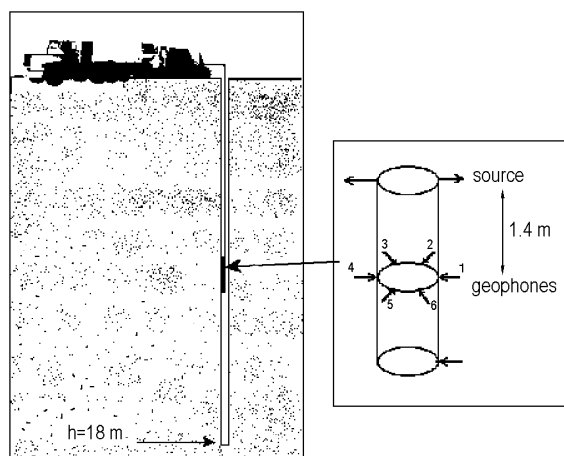


Fig. 8. Scheme of the field experiment and of the special contact sonde.

cies of about 400 Hz in the wells up to 18 m deep (Obolentseva et al., 2000). A special contact sonde with piezoelectric transducers was used (see Fig. 8). The source and receiver piezoelements were oriented radially, the distance between the source circle and the circle with six receivers was equal to 1.4 m.

The polarization processing of the obtained oscillation spectra allowed to estimate both the azimuthal frequency characteristics of the symmetrical wave-field part and the parameters of the ellipses characterizing the dissymmetric part of the wave-field due to gyrotropy. There were determined the following gyration and attenuation constants a and b :

$$a \approx 3.5 \text{ m/s}; \quad b \approx 75 \text{ m/s}; \quad (V_0 \approx 350 \text{ m/s}).$$

A comparison of the obtained parameters with those determined for similar deposits at seismic frequencies (shown at the previous example) shows that gyration constants at frequencies of about 400 Hz are somewhat smaller ($a/V_0 = 0.01$) than those at the frequencies of 10–180 Hz, but ellipse rotation angles are greater, being proportional to frequency. The attenuation constant ($b/V_0 = 0.21$) changes in the same limits as in the previous experiments.

9. Conclusions

A new way of estimating of shear-wave attenuation is proposed. The method is based on the assumption

that a geological medium possesses gyrotropic properties. The gyration and attenuation constants are determined from ellipse rotation angles (relative to source direction) and ellipticity, that is to say, ellipse axes ratio.

Due to the presence of gyrotropy, we have the opportunity to determine the attenuation of shear waves. This way differs from the commonly used when the attenuation is determined by decreasing of wave amplitude at the given path.

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