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# Analytical modelling of the formation temperature stabilization during the borehole shut-in period

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## SUMMARY

The problem of formation temperature stabilization during the shut-in period is solved analytically by the approximate generalized integral-balance method. The model accounts for the thermal history of the borehole exploitation, which may include a finite number of circulation and shut-in periods, and different flow regimes during circulation periods. The latter is determined by the temperatures of the circulating fluid and different Biot numbers that depend on intensity of the heat transfer on the bore-face. Normally the temperature fields in the well and surrounding rocks are calculated numerically by the finite-difference and finite-element methods or analytically, by applying the Laplace-transform method. Formulae, analytically obtained by Laplace transform, are rather bulky and require tedious non-trivial numerical evaluations. Moreover, in previous research the heat interactions of the circulating fluid with formation were treated under the condition of constant bore-face temperatures. In the present study the temperature field in the formation disturbed by the heat flow from the borehole is modelled by the heat conduction equation and thermal interaction of the circulating fluid with formation is approximated by the Newton relationship on the bore-face. The problem for circulation and shut-in periods is solved analytically using the heat balance integral method, where the radius of thermal influence, which defines the thermally disturbed domain, is a function of time, which satisfies the algebraic equation. Within this method, the approximate solution of the heat conduction problem is sought in the form of a finite sum of functions which belong to a complete set of the linearly independent functions defined on the finite interval bounded by the radius of thermal influence and satisfy the homogeneous boundary condition on the bore-face. It can be proved theoretically that the approximate solution found by this method converges to the exact one. Numerical results illustrate quite good agreement between the approximate and exact solutions. As a result of its simplicity and accuracy, the derived solution is convenient for geophysical practitioners and can be readily used, for instance, for predicting equilibrium formation temperatures.

**Key words:** fluid circulation, integral-balance method, rock formation temperature, shut-in period, thermal stabilization.

## 1 INTRODUCTION

The reliable assessment of thermal interaction between a borehole and the surrounding rock formation is of considerable interest in a number of geophysical applications. The following applications are worth mentioning: (1) interpretation of electric logs and estimation of the formation temperatures from well logs, which require knowledge of temperature disturbances in the formation produced by circulating fluid during drilling (Luheshi 1983; Jones *et al.* 1984; Shen & Beck 1986); (2) optimal design of the drilling bit cooling system within the high-temperature formation (Saito & Sakuma 2000) require assessment of the heat either delivered from the high-temperature rocks to the drilling bit or transmitted to the formation from the circulating fluid; (3) developing new technologies and methods in the area of geothermal energy production (Morita & Tago 1995; Kanev *et al.* 1997; Fomin *et al.* 2002). Shen & Beck (1986) provided a detailed review of the numerical and analytical approaches for modelling bottom hole temperature stabilization. In the above paper and the earlier study of Lee (1982) the formation temperatures, obtained by the method of Laplace transformation, are rather bulky and require tedious non-trivial numerical evaluations and, therefore, are not very convenient for engineering estimations. Moreover, in these publications and in a number of earlier studies, heat interactions of circulating fluid and with formation were treated under the condition of constant bore-face temperature or heat flux (Edwardson *et al.* 1962; Ramey 1962; Squier *et al.* 1962; Jensen &

Sharma 1989; Arnold 1990). On the basis of these solutions, and with some additional simplifying assumptions, several simple analytical formulae for the rock temperature in the formation and the heat flux on the walls of the well were proposed (Kutasov 1987; Kutasov 1999). However, in reality, the temperature on the wall of the well is an unknown function of time and wellbore depth, and depends on the fluid flow regime. The above-mentioned approach can be used mainly in the case of highly intensive heat transfer between the circulating fluid and surrounding rocks, which takes place for fully developed turbulent flow in the well. In the present study, a Newtonian model of heat transfer on the bore-face during the circulating period is adopted, for which a model with given bore-face constant temperature is a particular case. Moreover, in the present study the initial temperature of formation can be an arbitrary function of spatial coordinates. It depends on the borehole exploitation history and should be obtained from the solution of the heat transfer problem for the previous stage of the borehole exploitation. Similarly to Shen & Beck (1986), the heat transfer model formulated below can be solved using the Laplace transform. However, it would lead to rather awkward formulae. In order to avoid the complexity of the final solution and to obtain a simpler solution convenient for geophysical studies, it would be interesting to employ the approximate analytical integral heat balance method proposed by Goodman (1958, 1964) and later improved by Volkov *et al.* (1988). This method was successfully applied by Fomin *et al.* (1994) for solving the problem of moving a heat source within the borehole in application to melting of the paraffin deposition in the annulus. A simplified solution found by this method could also be beneficial for its further incorporation into the complete model of heat and mass transfer processes for simulating the whole life-cycle of borehole exploitation. The generalized integral heat balance method (Volkov *et al.* 1988) is applied in the present study for solution of the heat conduction problem in the surrounding borehole formation during the circulating and shut-in periods. The fundamentals of this method are briefly described in the Appendix.

## 2 SYMBOLS USED IN THE TEXT

| | |
|---|---|
| $a$ | Equilibrium formation temperature gradient, $\partial t_f(z^*)/\partial z^*$ |
| $a_k(\tau)$ | Functions in approximate solutions (23) and (A1) |
| $B$ | Biot number defined by eq. (5) |
| $b$ | Temperature on the surface, $t_f(0)$ |
| $c_r$, $c_L$ | Specific heat of the rock and fluid, respectively |
| $d_r$ | Heat diffusivity of the formation |
| $f_k(r)$ | Functions in approximate solution (23) and (A1) |
| $f_k'(l)$ | Derivative $df_k(r)/dr$ at $r = l$ |
| $g$ | Flow rate in the borehole during the circulation stage |
| $H$ | Depth of the borehole |
| $h_w$ | Heat transfer coefficient on the bore-face |
| $J_0$, $J_1$ | Bessel functions of the first kind of the order 0 and 1, respectively |
| $q(\tau)$, $q_w(z, \tau)$ | Heat fluxes on the bore-face defined by eqs (18), (19), (21) and (22) |
| $k_r$, $k_L$ | Heat conductivity of the formation and fluid, respectively |
| $l(\tau)$ | Radius of thermal influence |
| $l_c(\tau)$ | Radius of thermal influence during the circulation period |
| $l_s(\tau)$ | Radius of thermal influence during the shut-in period |
| $r_w$ | Radius of the borehole |
| $r$, $z$ | Non-dimensional cylindrical coordinates defined by eq. (5) |
| $T(z, r, \tau)$ | Auxiliary temperature defined by eq. (11) |
| $T_c(z, r, \tau)$ | Modified temperature of the formation during fluid circulation defined by eq. (5) |
| $T_m(z, \tau)$ | Modified mean temperature of the fluid during circulation defined by eq. (5) |
| $T_s(z, r, \tau)$ | Modified temperature of the formation during the shut-in period defined by eq. (10) |
| $t_b$ | Temperature at the bottom of the borehole ($z = 1$) |
| $t_c(z, r, \tau)$ | Temperature of formation during fluid circulation |
| $t_s(z, r, \tau)$ | Temperature of formation during the shut-in period |
| $t_f$ | Equilibrium temperature of formation |
| $t_m(z, \tau)$ | Mean fluid temperature in a borehole |
| $Y_0$, $Y_1$ | Bessel functions of the second kind of order 0 and 1, respectively |
| $\beta$ | Parameter defined by eq. (10) |
| $\tau$ | Time |
| $\tau_c$ | Duration of the circulation period |
| $\tau_s$ | Time for shut-in period |
| *Superscript* | |
| * | Dimensional quantities |
| *Subscripts* | |
| c | Circulation |
| s | Shut-in |
| L | Liquid |
| m | Mean value |
| r | Rock |
| w | Wall of the borehole |

## 3 SYSTEM MODEL AND ANALYSIS

According to Shen & Beck (1986), the temperature in the formation during the shut-in period will be practically the same either assuming that the fluid is a perfect conductor during circulation and shut-in periods (i.e. radially constant fluid temperature in the well) or considering the radial variation of the fluid temperature. On the basis of the above conclusion, we will assume that the borehole fluid is well stirred laterally throughout the circulating and shut-in periods, which allows one to use the cross-sectional average of the fluid temperature $t_m(z^*, \tau^*)$ as a function of axial coordinate and time. This is analogous to the assumption that the borehole fluid remains a perfect conductor in the radial direction. Shen & Beck (1986) validated that the temperature profile computed by the latter approach does not differ significantly from the solution of the exact model except for very small shut-in periods. Circulation of fluid in the borehole, during drilling or production stages of the borehole exploitation, disturbs the initial equilibrium temperature in the rock formation, $t_f(z^*) = az^* + b$. The heat transfer on the bore-face, $r^* = r_w$, can be modelled by Newton's law $-k_r\, \partial t/\partial r^* = h_w(t_m - t)$, where $t$ is the disturbed formation temperature during the borehole exploitation. This model allows one to account for the influence of the flow regime in the borehole during the circulating period, since the heat transfer coefficient, $h_w$, differs to an order of magnitude for the laminar, transient to turbulent and fully developed turbulent flow regimes. Since the radial temperature gradients are typically 100–1000 times greater than temperature gradients in the vertical direction (Luheshi 1983; Shen & Beck 1986), the derivatives of temperature with respect to $z^*$ can be neglected in the governing equations and the temperature distribution $t_c$ in the surrounding rock during the fluid-circulation period in cylindrical coordinates $(r, z)$ can be described by the following non-dimensional mathematical model:

$$\frac{\partial T_c}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_c}{\partial r}\right), \quad 1 < r < \infty, \quad 0 < \tau < \tau_c; \tag{1}$$

$$\tau = 0, \qquad T_c = 0; \tag{2}$$

$$r = 1, \qquad -\partial T_c/\partial r = B[T_m(z,\tau) - T_c]; \tag{3}$$

$$\lim_{r\to\infty} T_c < \infty. \tag{4}$$

The non-dimensional quantities in eqs (1)–(4) are defined by the following relationships:

$$r = r^*/r_w, \quad z = z^*/H, \quad B = h_w r_w/k_r, \quad \tau = \tau^* d_r/r_w^2,$$
$$T_c = t_c - t_f, \quad T_m = t_m - t_f. \tag{5}$$

Eq. (1) describes the temperature distribution in the rock surrounding the borehole, eq. (2) is the initial condition at $\tau = 0$, eq. (3) represents Newton's law of heat transfer on the borehole wall, eq. (4) is the condition of the finite temperature of the rock at $r \to \infty$.

If at the specified time $\tau = \tau_c$, fluid circulation within the borehole is shut down, the disturbed temperature of the formation $t_c$ tends to recover to its initial undisturbed distribution $t_f$. Assuming that the borehole fluid remains a perfect conductor, the process of stabilization of the bottom hole temperature during the shut-in period is governed by the following equations (Shen & Beck 1986):

$$\frac{\partial T_s}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right), \quad 1 < r < \infty, \quad \tau > \tau_c; \tag{6}$$

$$\tau = \tau_c, \qquad T_s = T_c(z, r, \tau_c) \tag{7}$$

$$r = 1, \qquad \frac{\partial T_s}{\partial r} = \beta\frac{\partial T_s}{\partial \tau}; \tag{8}$$

$$\lim_{r\to\infty} T_s = 0, \tag{9}$$

where

$$\beta = \frac{c_L\rho_L}{2c_r\rho_r}, \quad \tau_c = \tau_c^* d_r/r_w^2, \quad T_s = t_s - t_f. \tag{10}$$

In the problem (6)–(9), $T_c$ is the solution of eqs (1)–(4) at the end of circulation period and eq. (8) results from averaging the heat conduction equation over the borehole cross-section and assuming that the borehole fluid is a perfect conductor, $k_L \to \infty$.

From the definition of $T_c(r, \tau)$ it follows that this function should satisfy eq. (6). Hence, if for fixed $\tau_c$ a solution of eqs (6)–(9) is sought in the form

$$T_s(z, r, \tau_s, \tau_c) = T(z, r, \tau_s) + T_c(z, r, \tau_s + \tau_c), \tag{11}$$

where $\tau_s = \tau - \tau_c$, $\tau_s \geq 0$, then the unknown auxiliary temperature $T$ should satisfy the following equations:

$$\frac{\partial T}{\partial \tau_s} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \quad 1 < r < \infty, \quad \tau_s > 0; \tag{12}$$

$$\tau_s = 0, \quad T = 0; \tag{13}$$

$$r = 1, \quad \frac{\partial T}{\partial r} = \beta\frac{\partial T_c}{\partial \tau_s} - \frac{\partial T_c}{\partial r} + \beta\frac{\partial T}{\partial \tau_s}; \tag{14}$$

$$\lim_{r\to\infty} T = 0. \tag{15}$$

The exact solution $T_1$ of eqs (1)–(4) for the temperature $T_m = 1$ is found from Carslaw & Jaeger (1959),

$$T_1(r,\tau) = 1 - \frac{2B}{\pi}\int_0^\infty \frac{e^{-p^2\tau}}{p} \times \frac{\{J_0(pr)[pY_1(p) + BY_0(p)] - Y_0(pr)[pJ_1(p) + BJ_0(p)]\}dp}{\{[pJ_1(p) + BJ_0(p)]^2 + [pY_1(p) + BY_0(p)]^2\}}, \tag{16}$$

where $J_0(p)$, $J_1(p)$ are Bessel functions of the first kind of order zero and one, respectively; $Y_0(p)$, $Y_1(p)$ are Bessel functions of the second kind of order zero and one, respectively.

For an arbitrary $T_m$, due to the Duhamel theorem, the solution of eqs (1)–(4) can be presented in the following form:

$$T_c = \frac{\partial}{\partial \tau}\int_0^\tau T_m(z,p)T_1(r,\tau - p)\,dp \quad (1 < r < \infty, 0 < \tau < \tau_c). \tag{17}$$

The heat flux on the bore-face $q_w$ can be readily computed from (16) and (17) as

$$q_w = -\frac{1}{B}\left.\frac{\partial T_c}{\partial r}\right|_{r=1} = T_m(z,\tau) + \int_0^\tau T_m(z,p)\frac{\partial}{\partial \tau}q(\tau - p)\,dp, \quad (0 < \tau < \tau_c), \tag{18}$$

where

$$q = -\frac{1}{B}\left.\frac{\partial T_1}{\partial r}\right|_{r=1} = \frac{4B}{\pi^2}\int_0^\infty \times \frac{e^{-p^2\tau}\,dp}{\{[pJ_1(p) + BJ_0(p)]^2 + [pY_1(p) + BY_0(p)]^2\}p}. \tag{19}$$

As can be seen, the exact solution (16)–(19) is rather awkward. Applying the approximate generalized integral balance method (Volkov *et al.* 1988), Fomin *et al.* (2002) proposed a simple approximate solution for the problem (1)–(4),

$$T_c = \begin{cases} T_m\dfrac{B\ln(l_c/r)}{1 + B\ln(l_c)}, & r \leq l_c \\ 0, & r > l_c, \end{cases} \tag{20}$$

$$q_w(\tau) = T_m(\tau)q(\tau), \tag{21}$$

where

$$q(\tau) = 1/[1 + B\ln(l_c)], \quad l_c = 1 + \frac{2.084 + 0.704B}{1.554 + 0.407B}\sqrt{\tau}. \tag{22}$$

Comparison with the exact solution (16)–(19) shows that the solution (20)–(22) is sufficiently accurate for simulating the heat flux on the bore-face and temperature field in the formation during the period of fluid circulation (Fig. 1). Substitution of eqs (20) and (21) into the boundary conditions (7) and (14) closes the model for the shut-in period (6)–(9) and the auxiliary problem, eqs (12)–(15), respectively. In view of the simplicity and accuracy of eqs (20)–(22), it is sensible to employ the same generalized integral-balance method for the further analysis of temperature distribution during the shut-in period. For this purpose, it is convenient to convert eq. (14) to the following form:

$$r = 1, \quad \frac{\partial T}{\partial r} = \beta\frac{\partial T_c}{\partial \tau_s} - \frac{\partial T_c}{\partial r} + \beta\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \tag{23}$$

Eq. (23) is obtained from eq. (14) by replacing the time derivative with the second-order spatial derivative, which is permissible for a smooth and differentiable solution $T$ of eq. (12) everywhere within the domain and its boundary $r = 1$. Following the procedure of the generalized integral balance method (see the Appendix), the solution of eqs (12), (13), (15) and (23) to the second order of accuracy is sought in the following form:

$$T = -\left(\left.\frac{\partial T_c}{\partial r}\right|_{r=1} - \beta\left.\frac{\partial T_c}{\partial \tau_s}\right|_{r=1}\right)\ln\frac{r}{l_s} + a_1(\tau_s)f_1(r) + a_2(\tau_s)f_2(r) \tag{24}$$

for $r \le l_s(\tau_s)$, where $l_s$ is the radius of the thermal influence that defines the thermally disturbed region of the rock formation. For $r > l_s(\tau_s)$, the temperature $T$ is assumed to be equal to 0 and at $r = l_s(\tau_s)$,

$$T|_{r=l_s} = \partial T/\partial r|_{r=l_s} = 0. \tag{25}$$

Functions $f_1$ and $f_2$ in eq. (24) should satisfy the corresponding steady-state eq. (A2) with homogeneous boundary conditions $\frac{\partial f_k}{\partial r} = \beta\frac{\partial}{\partial r}(r\frac{\partial f_k}{\partial r})$ at $r = 1$, which follow from eq. (23) and are similar to the conditions (A3) for the problem (1)–(4). It can readily be shown that

$$f_1 = 1,\ f_2 = r^2/4 + (\beta - 0.5)\ln r. \tag{26}$$

Substituting eqs (26) into (24) and using conditions (25) yield

$$T = -\left(\left.\frac{\partial T_c}{\partial r}\right|_{r=1} - \beta\left.\frac{\partial T_c}{\partial \tau_s}\right|_{r=1}\right)\frac{l_s^2}{l_s^2 - 1 + 2\beta} \times\left[\ln\frac{r}{l_s} + 0.5\left(1 - \frac{r^2}{l_s^2}\right)\right]. \tag{27}$$

Multiplying eq. (12) by $r$ and integrating it for $r$ over an interval $(1, l_s)$ leads to the following equation for $\nu = \int_1^{l_s} rT\,dr$:

$$\frac{\partial \nu}{\partial \tau_s} = -\beta\left.\frac{\partial T}{\partial \tau_s}\right|_{r=1} + \left.\left(\frac{\partial T_c}{\partial r} - \beta\frac{\partial T_c}{\partial \tau_s}\right)\right|_{r=1}, \quad \nu|_{\tau_s=0} = 0. \tag{28}$$

Eq. (28) integrates to the following expression:

$$\nu = -\beta T|_{r=1} + \int_0^{\tau_s}\left.\left(\frac{\partial T_c}{\partial r} - \beta\frac{\partial T_c}{\partial \tau_s}\right)\right|_{r=1} d\tau_s. \tag{29}$$

On the other hand, due to eq. (27),

$$\nu = \int_1^{l_s} rT\,dr = \left.\left(\frac{\partial T_c}{\partial r} - \beta\frac{\partial T_c}{\partial \tau_s}\right)\right|_{r=1} \times\frac{1}{l_s^2 - 1 + 2\beta}\,\frac{l_s^4 - 4l_s^2\ln l_s - 1}{8}. \tag{30}$$

Combining eqs (29) and (30), and after rather trivial algebraic evaluations, we obtain an implicit equation for the unknown radius of thermal influence, $l_s$,

$$\frac{l_s^4 - 4(1 - 2\beta)l_s^2\ln l_s - 4\beta l_s^2 + 4\beta - 1}{8\left(l_s^2 - 1 + 2\beta\right)} = \frac{\int_0^{\tau_s}\left.\left(\frac{\partial T_c}{\partial r} - \beta\frac{\partial T_c}{\partial \tau_s}\right)\right|_{r=1} d\tau_s}{\left.\left(\frac{\partial T_c}{\partial r} - \beta\frac{\partial T_c}{\partial \tau_s}\right)\right|_{r=1}}. \tag{31}$$

With $l_s$ defined by eq. (31), the real temperature around the well during the shut-in period is given by the equation

$$T_s(z, r, \tau_c + \tau_s) = T_c(z, r, \tau_c + \tau_s) - \left(\left.\frac{\partial T_c}{\partial r}\right|_{r=1} - \beta\left.\frac{\partial T_c}{\partial \tau_s}\right|_{r=1}\right) \times\frac{l_s^2}{l_s^2 - 1 + 2\beta}\left[\ln\frac{r}{l_s} + 0.5\left(1 - \frac{r^2}{l_s^2}\right)\right] \tag{32}$$

for $r \le l_s$.

The values of $\frac{\partial T_c}{\partial r}|_{r=1}$ and $\frac{\partial T_c}{\partial \tau_s}|_{r=1}$ in eqs (31) and (32) can easily be evaluated from the formulae (20)–(22).

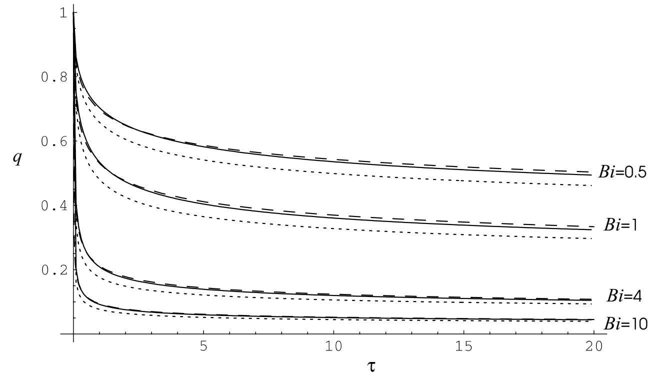


**Figure 1.** Variation of the bore-face heat flux during circulation, $q$, with respect to time ($0 \le \tau \le \tau_c$) for different $B$. Solid line, exact solution (18); dotted line, approximate solution (A17) and (A18); dashed line, proposed solution (21) and (22).

## 4 NUMERICAL RESULTS AND DISCUSSION

Although previous studies related to application of the generalized integral method convincingly validate its accuracy and efficiency for solving heat conduction problems, it would be reasonable (due to the approximate character of the obtained solution) to verify this fact once again. For instance, as is illustrated in Fig. 1 for the circulation period, the heat flux distribution $q_w$ on the bore-face, computed by the approximate formulae (21) and (22) and exact solution (18) and (19), are in good agreement for all $B$ and $\tau$. The satisfactory consistency of the results for the shut-in period, computed with eqs (31) and (32) and obtained experimentally (Proselkov 1975), is illustrated in Table 1.

Thermophysical properties of formation and circulating fluids are well documented. For computations we take (Shen & Beck 1986): $k_r = 2.51$ W m$^{-1}$ K$^{-1}$, $k_L = 0.61$ W m$^{-1}$ K$^{-1}$, $\rho_r c_r = 2.09 \times 10^6$ J m$^{-3}$ K$^{-1}$, $\rho_L c_L = 4.19 \times 10^6$ J m$^{-3}$ K$^{-1}$. Computed temperature distributions with respect to radial distance for different Biot numbers and durations of circulating periods are shown in Figs 2–7. For a relatively short circulation period of $\tau_c = 10$, the extent of the thermally disturbed formation area (radius of thermal influence) and the corresponding temperature distribution vary significantly with time (Figs 2–4). Whereas for a long circulating time, of $\tau_c = 1000$ (Figs 5–7), the changes in the radius of thermal influence are insignificant for all shut-in times $\tau_s$ and temperature profiles differ for each duration of the shut-in period only for $r < l_s/2$. For $r > l_s/2$

**Table 1.** Comparison of the temperatures $T_s$ measured experimentally (data in parentheses were reported by Proselkov 1975) and computed by eqs (31) and (32).

| $\tau_s$ / $r$ | 0 | 0.62 | 1.87 | 3.12 | 6.24 | 12.5 |
|---|---|---|---|---|---|---|
| 1 | 1 | 0.6 | 0.427 | 0.347 | 0.243 | 0.162 |
| | (1) | (0.74) | (0.45) | (0.345) | (0.22) | (0.16) |
| 1.5 | 0.775 | 0.585 | 0.421 | 0.343 | 0.241 | 0.161 |
| | (0.72) | (0.63) | (0.43) | (0.335) | (0.22) | (0.15) |
| 2.4 | 0.517 | 0.501 | 0.389 | 0.324 | 0.232 | 0.157 |
| | (0.47) | (0.49) | (0.38) | (0.31) | (0.21) | (0.135) |
| 3.35 | 0.34 | 0.353 | 0.332 | 0.289 | 0.216 | 0.149 |
| | (0.3) | (0.32) | (0.29) | (0.25) | (0.18) | (0.12) |
| 4.5 | 0.194 | 0.208 | 0.234 | 0.231 | 0.189 | 0.138 |
| | (0.175) | (0.195) | (0.195) | (0.2) | (0.15) | (0.11) |

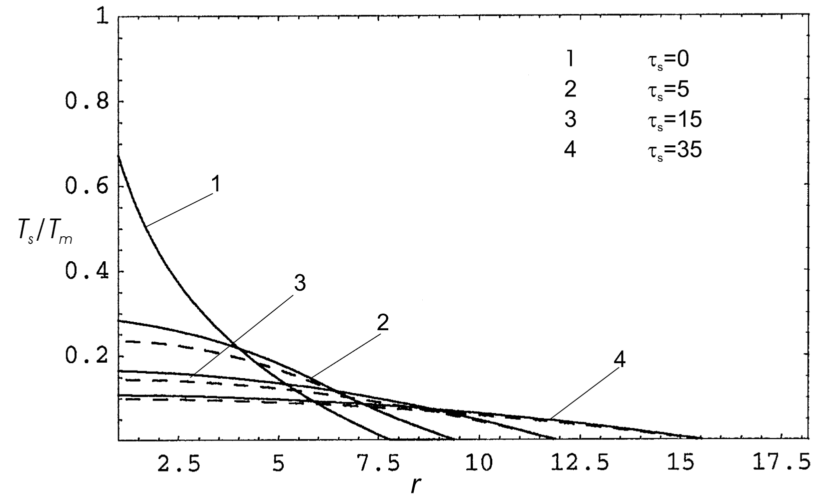


**Figure 2.** Radial temperature distribution for $B = 1$ and $\tau_c = 10$. Solid lines, $\beta = 1$; dashed lines, $\beta = 0$.

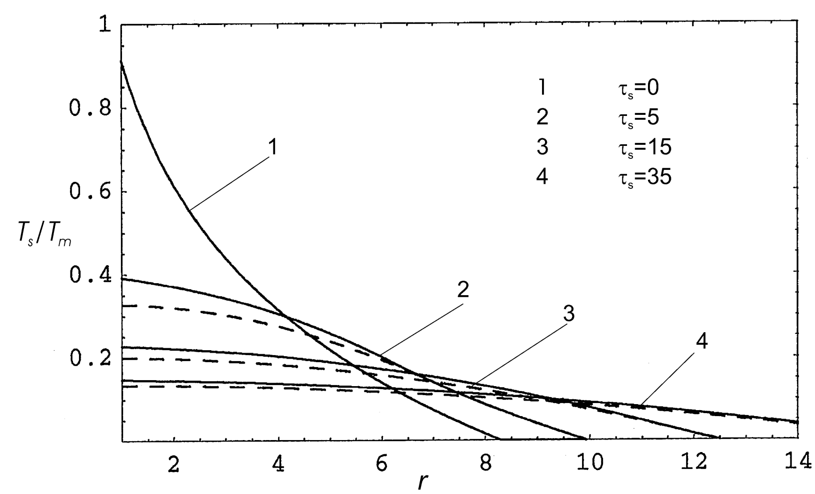


**Figure 3.** Radial temperature distribution for $B = 5$ and $\tau_c = 10$. Solid lines, $\beta = 1$; dashed lines, $\beta = 0$. (1) $\tau_s = 0$, (2) $\tau_s = 5$, (3) $\tau_s = 15$, (4) $\tau_s = 35$.

(in the case of a long circulation time), temperatures are insensitive to the duration of the shut-in period. The dashed lines in Figs 2–7 correspond to $\beta = 0$. As can be seen, these temperature profiles differs insignificantly from the curves evaluated for $\beta = 1$ except for very small distances from the borehole. Moreover, for long circulation times (Figs 5–7) the discrepancies are negligible already at the onset of the shut-in period. Thus we may simplify the final equations (31) and (32) by setting $\beta = 0$. The other parameter that can affect the solution is the Biot number $B$, which characterizes the flow regime during circulation. Without denoting the exact bounds for the parameter $B$ for different regimes, we will simply take the typical values: for the laminar flow $B = 1$ or 0.4 (for this regime $B$ is typically less than unity), $B = 100$ or $B \to \infty$ for the fully developed turbulent flow, and $B = 5$ for the transient from laminar to turbulent flow regime. As has been mentioned above, the previous results of Shen & Beck (1986), Lee (1982), Jones *et al.* (1984) and Luheshi (1983) correspond to constant temperature or flux on the bore-face during circulation. In our model, this means that $B \to \infty$, i.e. turbulent circulation is assumed. As can readily be seen from Figs 2–7, high Biot numbers lead to high temperatures during the circulation period and consequently also during the shut-in periods. In other words, assuming that $B \to \infty$ may result in an overestimation of the formation temperature and, consequently, the time of stabilization.

Figs 8–10 show the formation temperature distribution with respect to the shut-in time for $B = 0.4$ (laminar flow), $B \to \infty$ (turbulent flow) and $B = 5$ (transition from laminar to turbulent flow). As can be expected, temperatures are lower for a shorter circulation time $\tau_c$. The laminar flow regime (Fig. 8) also provides lower temperatures in formation during circulation and shut-in periods and, therefore, the time of temperature stabilization is shorter in this case. Results presented in Figs 8 and 9 correspond to the constant mean temperature within the borehole during circulation, $T_m =$ constant. However, in reality $T_m$ is a function of time $\tau$ and depth $z$. Numerical computations for the unsteady flow regime in the borehole conducted by Raymond (1969) show that shortly after onset of circulation the thermal behaviour of a fluid begins to approach a slow, logarithmic decline. Such a decline suggests that the heat transfer processes in the circulating fluid rapidly becomes pseudo-steady. In this case the fluid temperature can be found analytically in explicit form. For instance, assuming the pseudo-steady regime in the borehole, the mean temperature of the fluid in the production well can

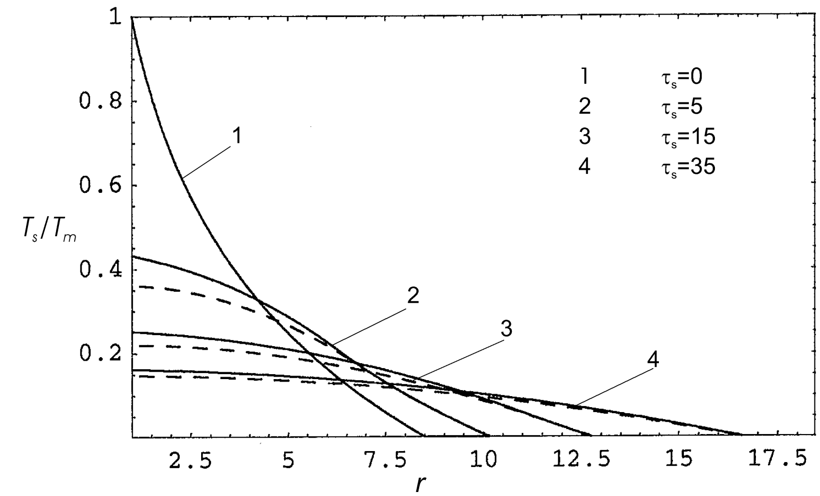


**Figure 4.** Radial temperature distribution for $B \to \infty$ and $\tau_c = 10$. Solid lines, $\beta = 1$; dashed lines, $\beta = 0$. (1) $\tau_s = 0$, (2), $\tau_s = 5$, (3) $\tau_s = 15$, (4) $\tau_s = 35$.

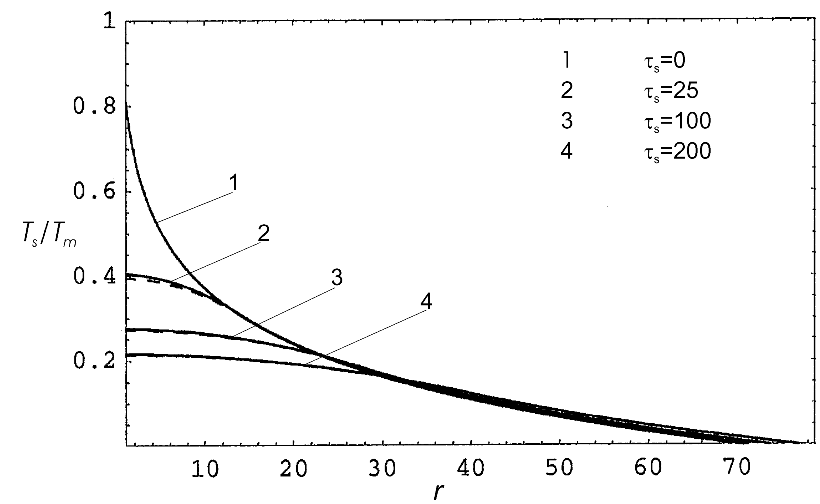


**Figure 5.** Radial temperature distribution for $B = 1$ and $\tau_c = 1000$. Solid lines, $\beta = 1$; dashed lines, $\beta = 0$. (1) $\tau_s = 0$, (2) $\tau_s = 25$, (3) $\tau_s = 100$, (4) $\tau_s = 200$.

be presented as (Ramey 1962; Arnold 1990):

$$T_m = (t_b - b - aH)\exp\left[-\frac{Bq}{G}(1-z)\right] + \frac{aHG}{Bq}\left\{1 - \exp\left[-\frac{Bq}{G}(1-z)\right]\right\}, \quad (33)$$

where $G = (gc_L)/(2\pi Hk_r)$ and $q(\tau)$ is defined by eq. (22).

Results illustrated in Fig. 10 by the dashed lines correspond to the flow regime in the geothermal production borehole at the depth $z = 0.3$ ($z^* = 900$ m), which is characterized by the following data (Jing *et al.* 2000): $t_b = 100\,^\circ\mathrm{C}$, $b = 20\,^\circ\mathrm{C}$, $a = 0.05\,^\circ\mathrm{C\,m^{-1}}$, $H = 3000$ m, $g = 2.4$ kg s$^{-1}$. For comparison, the solid lines in this figure are attributed to the constant fluid temperature, $T_m =$ constant. As can be seen, even though the flow rate $g$ is relatively high and the fluid temperature $t_b$ at the bottom of the borehole ($z = 1$) is much lower than the equilibrium temperature, accounting for the time dependence of the function $T_m$ does not affect the temperatures during the shut-in period significantly. Computations carried out for the other flow regimes also reveal that the thermal effect of the fluid temperature variation with time is of minor importance for the shut-in period. Hence, the assumption of constant temperature $T_m$ can be adopted as quite a reasonable approximation for modelling the temperature field in the formation during the shut-in period. The case when $T_m$ is constant, $\tau_c = 2$ and $B \to \infty$ has been considered by Shen & Beck (1986) and plotted in their fig. 2(a). Comparison of their exact solution with our approximate computations (curve 4 in Fig. 9) once again verifies the sufficient accuracy of the integral-balance method. For these reasons, the approximate solution obtained in the present study can be used reliably for solving the inverse problem of predicting the equilibrium formation temperature. Briefly, if the duration of the circulation period and thermophysical properties of the particular rock and fluid are known then for each specified value of coordinate $z = z_i$ the temperature $T_s(z_i, 1, \tau_c + \tau_s)$ variation versus time $\tau_s$ can be computed using eqs (32), (20) and (33). Hence the borehole temperature $t_s(z_i, 1, \tau_c + \tau_s)$ can be presented as

$$t_s(z_i, 1, \tau_c + \tau_s) = t_f(z_i) + T_s(z_i, 1, \tau_c + \tau_s). \quad (34)$$

On the other hand, several temperature logs during the shut-in period allow one to simulate the real borehole temperature $t_w(\tau_s)$ as a function of time $\tau_s$ with a good degree of accuracy. Substituting the function at the left-hand side of eq. (34) by this, experimentally obtained temperature $t_w(\tau_s)$, defines the equilibrium temperature $t_f$

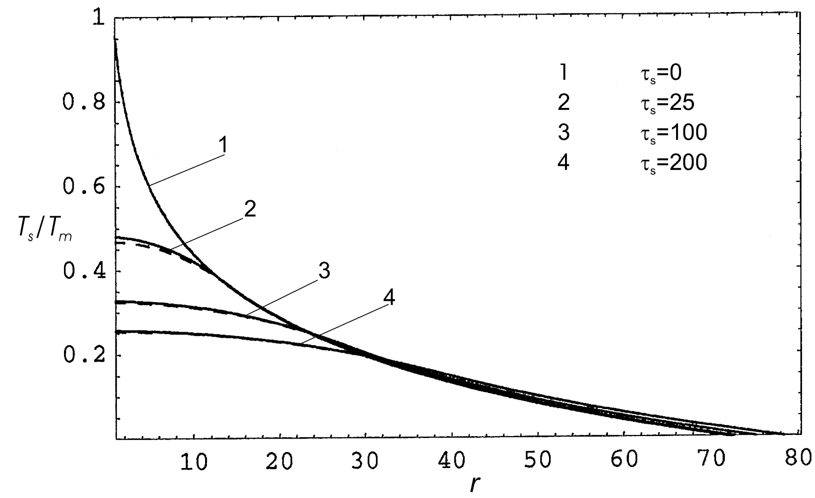


**Figure 6.** Radial temperature distribution for $B = 5$ and $\tau_c = 1000$. Solid lines, $\beta = 1$; dashed lines, $\beta = 0$. (1) $\tau_s = 0$, (2) $\tau_s = 25$, (3) $\tau_s = 100$, (4) $\tau_s = 200$.

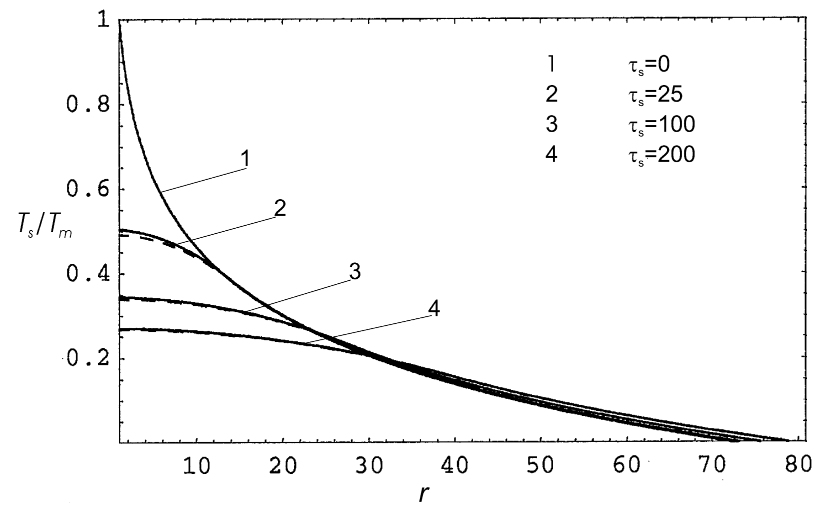


**Figure 7.** Radial temperature distribution for $B \to \infty$ and $\tau_c = 1000$. Solid lines, $\beta = 1$; dashed lines, $\beta = 0$. (1) $\tau_s = 0$, (2) $\tau_s = 25$, (3) $\tau_s = 100$, (4) $\tau_s = 200$.

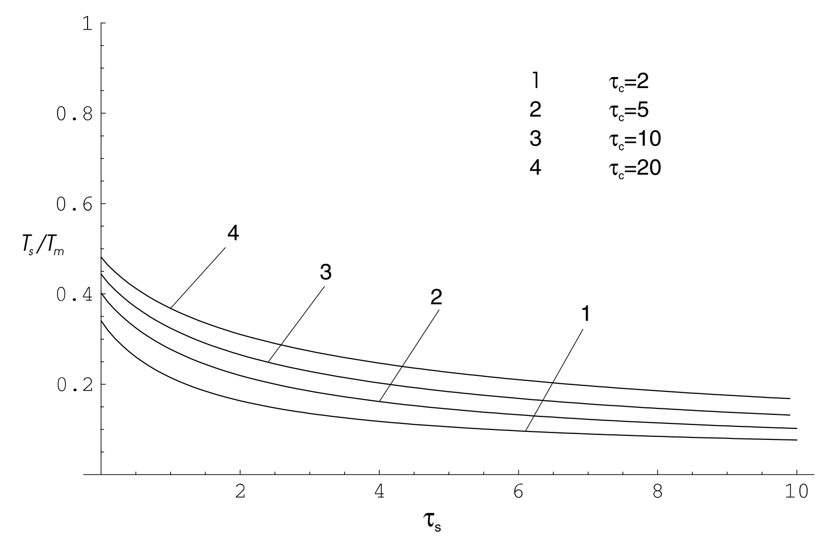


**Figure 8.** Temperature distribution $T_s/T_m$ on a bore-face ($r = 1$) with respect to shut-in time for different circulation times, $B = 0.4$, $\beta = 1$ and $T_m =$ constant.

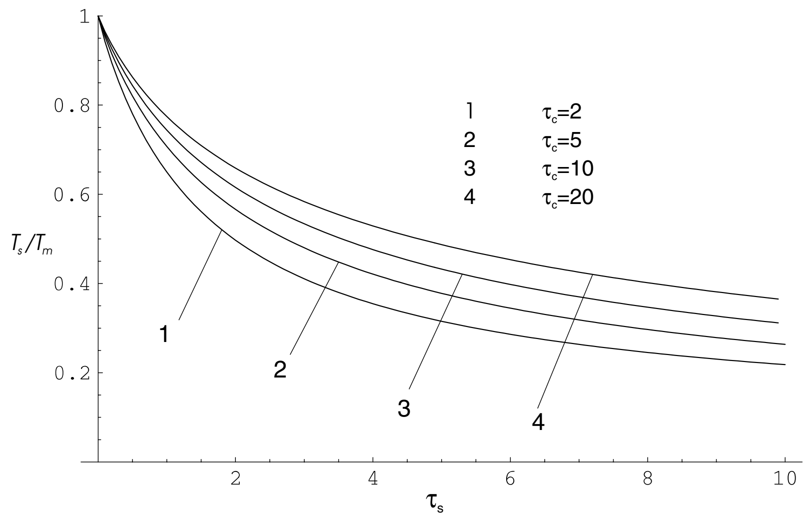


**Figure 9.** Temperature distribution $T_s/T_m$ on a bore-face ($r = 1$) with respect to shut-in time for different circulation times, $B \to \infty$, $\beta = 1$ and $T_m =$ constant

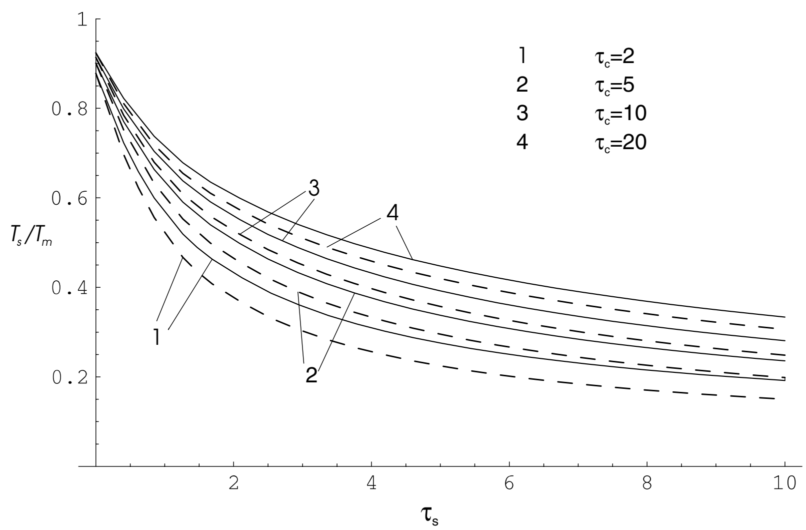


**Figure 10.** Temperature distributions on a bore-face ($r = 1$) with respect to shut-in time for different circulation times, $B = 5$, $\beta = 1$. Solid lines, $T_s/T_m$, where $T_m$ is constant; dashed lines, $T_s/T_m(\tau_c)$, where $T_m$ is defined by eq. (33) at $z = 0.3$.

at $z = z_i$. Repeating the same procedure at the other depths, $z = z_1$, $z_2$, etc., the formation temperature distribution in equilibrium $t_f(z)$ can be readily restored. The detailed description of the inversion procedure for this problem can be found in the above-cited papers by Shen & Beck (1986), Lee (1982) and Luheshi (1983). Furthermore, these papers illustrate how the linear regression method can by applied for identifying the other best-matched parameters that characterize the process.

The main advantages of the solutions obtained in the present work are: (1) their simplicity, which is convenient for geophysical estimations and (2) the capability to handle the situations characterized by the low Biot numbers and, subsequently, by different thermal regimes in the borehole during the circulation period.

As a final remark, we would like to point out the possibility of extending the present methodology to the modelling of the complete drilling cycle, which includes a finite number of circulating and drilling periods. For such an application, solution of eqs (32) and (34), $t_{s,1} = t_s(r, \tau_c + \tau_{s,1})$, at the end of the shut-in period, $\tau_s = \tau_{s,1}$, would become the initial temperature in the formation at onset of the next drilling period instead of the equilibrium temperature. In other words, in eqs (1)–(5), we should denote $t_f = t_{s,1}$ Then, denoting in eqs (5) $T_c = t_c - t_{s,1}$ and $T_m(z, \tau) = t_m - (t_{s,1} - \frac{1}{Bi}\frac{\partial t_{s,1}}{\partial r})|_{r=1}$, we obtain exactly the same non-dimensional governing eqs (1)–(4) for the circulation period. Then again, solving eqs (1)–(4) for the circulation period and employing formulae (31) and (32) at the shut-in stage we obtain at the end of the next shut-in period $t_{s,2} = t_s(r, \tau_c + \tau_{s,2})$, etc. Further evaluations can be continued without any difficulty.


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## APPENDIX

The generalized integral-balance method is briefly illustrated below by means of application to a specific problem, say, problem (1)–(4). According to Volkov *et al.* (1988) the approximate solution of the problem (1)–(4) is sought in the form

$$T(r,\tau) = \begin{cases} T_{\mathrm{L}} + \sum_{k=1}^{n} a_k(\tau) f_k(r), & 1 \le r \le l(\tau), \\ 0, & l(\tau) < r < \infty, \end{cases} \tag{A1}$$

where functions $f_k$(for $k = 1, 2, 3, \ldots$) constitute a complete and linearly independent system for every finite interval $[0, l]$ and satisfy the following relationships:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_k}{\partial r}\right) = f_{k-1}, \quad f_0 = 0, \tag{A2}$$

$$r = 1, \quad \partial f_k/\partial r = B \cdot f_k. \tag{A3}$$

It is approximately assumed that function $T$ and its derivative $\partial T/\partial r$ for each specified moment of time differs from zero only for $r$ within the finite interval $(0, l)$ and at $r = l$ the following conditions for $r = l$ are valid:

$$T|_{r=l} = \partial T/\partial r|_{r=l} = 0. \tag{A4}$$

So in this sense the function $l(\tau)$ can be referred to as a radius of thermal influence.

Multiplying eq. (A1) by $r f_k(r)$ and integrating over the interval $[0, l]$ while accounting for boundary conditions (A2) and (A4), yields the recurrent system

$$\frac{d\nu_k}{d\tau} = \nu_{k-1} + T_{\mathrm{L}} B f_k(r)|_{r=1}, \quad (\nu_0 = 0) \tag{A5}$$

with initial conditions at $\tau = 0$, $\nu_k = 0$, where

$$\nu_k = \int_1^{l(\tau)} r f_k(r) T(r,\tau)\, dr. \tag{A6}$$

Hence, from eq. (A5) $\nu_k$ can be readily obtained,

$$\nu_k = B \sum_{j=1}^{k} f_j(1) \underbrace{\int_0^{\tau} \cdots \int_0^{\tau}}_{k-j+1} T_{\mathrm{L}}(\tau)\, d\tau. \tag{A7}$$

On the other hand, substituting eq. (A1) into eq. (A6), $\nu_k$ can be presented in the following form:

$$\nu_k = T_{\mathrm{L}} M_k + \sum_{j=1}^{n} a_j(\tau) M_{jk}, \tag{A8}$$

where

$$M_k(l) = \int_1^{l(\tau)} r f_k(r)\, dr, \quad M_{jk}(l) = \int_1^{l(\tau)} r f_j(r) f_k(r)\, dr, \quad (k, j = 1, 2, \ldots). \tag{A9}$$

Functions $f_k(r)$, which should be substituted into eq. (A9), satisfy the recurrent relationships (A2) and (A3). From the latter, for instance for $k = 1, 2$ and 3, the first three functions are

$$f_1(r) = \ln(r) + 1/B, \tag{A10}$$

$$f_2(r) = \frac{r^2}{4}\left[\ln(r) - 1 + \frac{1}{B}\right] + \frac{B^2 - 2B + 2}{4B^2}, \tag{A11}$$

$$f_3(r) = \frac{r^4}{64}\left(\ln r - \frac{3}{2} + \frac{1}{B}\right) + \frac{r^2}{4}\frac{B^2 - 2B + 2}{4B^2} - \frac{5B^3 - 20B^2 + 40B - 32}{128B^3}. \tag{A12}$$

Comparing formulae (A7) and (A8), yields

$$\sum_{j=1}^{n} a_j(\tau) M_{jk} = -T_{\mathrm{L}} M_k + B \sum_{j=1}^{k} f_j(1) \underbrace{\int_0^{\tau} \cdots \int_0^{\tau}}_{k-j+1} T_{\mathrm{L}}(\tau) d\tau, \quad (k = 1, \ldots, n-1) \tag{A13}$$

$$\sum_{k=1}^{n} a_k(\tau) f_k(l) = -T_\mathrm{L}, \quad \sum_{k=1}^{n} a_k(\tau) f_k'(l) = 0. \tag{A14}$$

In eq. (A14) and below the derivatives, $df_k/dr$, at $r = l$ are denoted as $f_k{}'(l)$ $(k = 1, \ldots, n)$.

The system of $(n + 1)$ equations (A13) and (A14) is used for the calculation of $n$ unknown coefficients $a_k(\tau)$ $(k = 1, 2, \ldots, n)$ and the function $l(\tau)$. To a first approximation $(n = 1)$

$$a_1 = -T_\mathrm{L}/f_1(l), \tag{A15}$$

$$M_1 - \frac{M_{11}(l)}{f_1(l)} = Bf_1(1)\frac{1}{T_\mathrm{L}}\int_0^\tau T_\mathrm{L}(p)\,dp. \tag{A16}$$

and, hence, eq. (A1) reduces to

$$T(r, \tau) = T_\mathrm{L}\frac{B\ln(l/r)}{1 + B\ln l,} \quad 1 \le r \le l(\tau), \tag{A17}$$

Since, as can be readily shown, $M_{11}(l) = l[f_1(l)f_2'(l) - f_1'(l)f_2(l)]$, and $M_1(l) = lf_2'(l) - Bf_2(1)$, eq. (A16), which defines function $l(\tau)$, can be converted to

$$\frac{f_2(l)}{f_1(l)} - Bf_2(1) = \frac{1}{T_\mathrm{L}}\int_0^\tau T_\mathrm{L}(p)\,dp. \tag{A18}$$

After a bit more tedious but straightforward manipulation, the second approximation $(n = 2)$ for the temperature distribution in the rocks, eq. (A1), can be presented by equation

$$\begin{aligned} T(r, \tau) = T_\mathrm{L}\Bigg\{1 - \Bigg[&\frac{r^2}{4}\left(\ln r + \frac{1}{B} - 1\right) + \frac{B^2 - 2B + 2}{4B^2} \\ &- \frac{l^2}{2}\left(\ln l + \frac{1}{B} - \frac{1}{2}\right)\left(\ln r + \frac{1}{B}\right)\Bigg] \\ &\Bigg/\Bigg[\frac{l^2}{4}\left(\ln l + \frac{1}{B} - 1\right) + \frac{B^2 - 2B + 2}{4B^2} \\ &- \frac{l^2}{2}\left(\ln l + \frac{1}{B} - \frac{1}{2}\right)\left(\ln l + \frac{1}{B}\right)\Bigg]^{-1}\Bigg\}, \end{aligned} \tag{A19}$$

which is valid for $1 \le r \le l(\tau)$.

In eq. (A19) the function $l(\tau)$ is defined by

$$\begin{aligned} &\frac{f_2(l)}{f_1(l)} - Bf_2(1) + \frac{lf_1(l)\,f_3'(l) - f_3(l)}{lf_1(l)\,f_2'(l) - f_2(l)} - \frac{f_2(l)}{f_1(l)} \\ &\quad = \frac{1}{T_\mathrm{L}}\int_0^\tau T_\mathrm{L}(p)\,dp. \end{aligned} \tag{A20}$$