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On Rhythmical Layering of Rocks Formed From Basaltic Magma[1]

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Formation of rhythmical layering in intrusive basic and ultrabasic rock bodies is explained in different ways, in particular, by the movement of microparticles (mineral clusters) of plagioclase and pyroxene within the basalt melt under the influence of thermal and gravitational forces. A model of cluster movement is proposed as a consequence of the forces appearing when ultrasonic elastic fluctuations pass through the melt. The model is based on fundamental dynamic equations. Depending on cluster density, wave parameters, and magmatic chamber size, in the melt there can form different combinations of rhythmically alternating layers of different composition.

KEY WORDS: melt, cluster, acoustic wave.

INTRODUCTION

The problem of layering in basic and ultrabasic rocks is being studied by many geologists. There are several hypotheses explaining this phenomenon including some based on the principles of physics. It is supposed, in particular, that layering could appear as a result of the influence of forces upon the magmatic melt, which cause the motion of separate particles (mineral clusters) to distances large enough to form visually identifiable monomineral layers.

In the case of a basaltic melt, the clusters are represented mainly by pyroxene and plagioclase. The leucocratic (plagioclase) layers are enriched in, or are entirely composed of, frame silicates, i.e. minerals especially inclined to polymerization. Melanocratic layers are composed of chain silicates, mainly pyroxene, sometimes with olivine. But, as it is known from observations, such differentiation produces no dunite or olivine layers. Along with the monomineral layers there are layers of mixed composition close to leucocratic or melanocratic gabbro.

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The observed thicknesses of the layers vary from millimeters to meters. The character of layering can be varied. So, a frequently occurring type of alternation with similar thicknesses of the leucocratic and melanocratic layers is observed in the Skaergaard sections (Irvin, 1992) and in the ophiolites described by Nicolas (1989). Another type of layering is alternation of a series of two melanocratic layers divided by a thin leucocratic layer and with thick leucocratic layers. Such layering is characteristic of the Stillwater massif, where it is observed for dozens of kilometers (Czamanske and Zientek, 1985). The borders between the layers are clear without any traces of interaction. Turbulent vortices inevitable with horizontal movement of molten material were not observed. Of physical forces which could cause differentiation of the substance within the melt, those which are relevant during crystallization include thermal and gravitational fields. Equations of heat conduction and directed crystallization are used in constructing corresponding models (Frenkel, 1995).

This paper presents an attempt to model the processes of layering of basaltic melt with at least two types of mineral clusters, based on the principles of continuum mechanics (Sedov, 1984). It is supposed that the fluctuating state of the medium (suspension consisting of melt and grain-clusters) was caused by the passage through this medium of elastic vibrations with acoustic (ultrasonic) frequencies and formation of standing waves as a result of reflection from the roof. Vibrations with such frequencies are constantly observed in nature, for example, in regions of volcanic activity. Appearance of standing waves in the melt volume leads to the formation within it of zones of compression and tension and movement of clusters under the influence of vibrational force into the zones corresponding to stationary states for certain densities. This is how the alternation of layers of minerals with different density appears; or, at least, offers a theoretical premise for this phenomenon.

It can be supposed that the source of vibrations was at a considerable depth underneath a magmatic chamber, longitudinal elastic waves moved through the melt along the normal to the surface and the set of frequencies depended upon the form and size of the chamber (such frequencies are resonant frequencies of the given volume and their presence requires investigation of equations of motion under corresponding boundary conditions).

We discuss the simplest case and study dispersal of melt of relatively low viscosity (basalt with, perhaps, volatile components) the liquid phase of which has a density ρ_0. The melt is in a chamber of height L with solid walls and rigid roof. ρ_i is the density of a solid phase of composition i, V is velocity, t is time, and ω is frequency. $\underline{L} = L\omega/c$ and $\overline{V_i} = V_i/c$ are dimensionless variables, $\overline{\rho} = \rho/\rho_0$ (ρ is the density of a mixture of the melt and solid phase), c is the velocity of sound in the carrying phase, and g is the gravitational constant. We assume that both volumetric and mass contents of the cluster phase are small. Let us discuss the averaged cluster movement in the carrying phase the movement of which is described by the following equation (nonlinear effects are taken into consideration) (Ganiev and Ukrainsky, 1975).

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \frac{\partial \bar{\rho}\bar{V}}{\partial \bar{x}} = 0, \quad \frac{\partial \bar{V}}{\partial \bar{t}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{x}} = -\frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial \bar{x}} + \bar{g}$$

$$\bar{p} = \bar{p}_0 + (\bar{\rho} - 1) + \frac{1}{2}(\gamma - 1)(\bar{\rho} - 1)^2 \tag{1}$$

where γ is an empirical factor of nonlinearity of the medium.

The equation of the cluster phase movement is

$$\frac{d_i V_i}{d\bar{t}} = \frac{3\bar{\rho}}{\bar{\rho} + 2\bar{\rho}_i}\frac{\partial \bar{V}}{\partial \bar{t}} + \frac{3\bar{\rho}}{\bar{\rho} + 2\bar{\rho}_i}\bar{V}\frac{\partial \bar{V}}{\partial \bar{x}} + \frac{9t^{(\omega)}}{t^{(\mu)}}\frac{\bar{V} - \bar{V}_i}{\bar{\rho} + 2\bar{\rho}_i} - 2\bar{g}\frac{\bar{\rho} - \bar{\rho}_i}{\bar{\rho} + 2\bar{\rho}_i} \tag{2}$$

$t^{(\mu)} = \rho_0 a^2/\mu_1$ is the time necessary for obtaining the Stokes conditions of the flow around particles, where a = particle radius, μ_1 = dynamic viscosity (pa/s),

$$\frac{d_i}{dt} = \frac{\partial}{\partial t} + v_i^k \frac{\partial}{\partial x_k}$$

is the substantive derivatives connected with the movement of the ith phase.

Suppose, at the chamber bottom ($\bar{x} = \bar{L}$) there are small simple harmonic oscillations of pressure p_0 with amplitude Δp_0 and frequency ω, and near the roofing ($\bar{x} = 0$) velocity of the carrying phase is 0, then

$$p(\bar{t}, \bar{L}) = \bar{p}_0 + \varepsilon \sum_n (a_n \sin(nt) + b_n \cos(nt)) = \bar{p}_0 + \varepsilon f(\bar{t})$$

where $f(\bar{t})$ is the arbitrary periodic function,

$$\bar{x} = \bar{L}, \quad p(\bar{t}, \bar{L}) = \bar{p}_0 + \varepsilon f(\bar{t}); \ \varepsilon = \frac{\Delta p_0}{\rho_0 c^2}; \ \varepsilon^2 \ll 1;$$

$$\bar{x} = 0, \quad \bar{V}(\bar{t}, 0) = 0.$$

Solution of these equations of the motion of the carrying phase at rather small amplitudes of perturbations ε can be submitted as expansion on a small parameter. Averaged motion is described by function $\xi(t)$. The equation for the parameters of the averaged motion in the third approximation is

$$\xi' + \mu\tau_0\xi'' = -\mu^4\frac{3}{2}\varepsilon^2\chi_0^2\left(2\bar{\rho}_i^2 - 1\right)\sum_{j=1}^{\infty} j\,\frac{a_j^2 + b_j^2}{\sin^2(Lj)}\sin(2j\xi) + \mu^4 2\varepsilon^2\chi_0(1-\bar{\rho}_i)\delta_0 \tag{3}$$

μ is a small parameter, $j = 1, 2, \ldots, \tau_0 = 9\chi_0 t^{(\omega)}/t^{(\mu)}$, $\chi_O = (1 + 2\rho_i/\rho_0)^{-1}$, $\delta_O = g/(c\omega\varepsilon^2)$.

A generalized vibrational force can be included in the equation of the averaged motion (3) together with viscous resistance and resultant force of weight and the Archimedes' force (the second member in the right-hand side expression);

$$F = \frac{3\varepsilon^2\chi_0^2(1-2\bar{\rho}_i)}{2}\sum_{j=1}^{\infty} j\,\frac{a_j^2 + b_j^2}{\sin(Lj)}\sin(2j\xi)$$

This vibrating force reflects the influence of the carrying phase oscillations upon the translational movement of the particles. The sign of the vibrating force depends upon the crystal density ρ_i. Stationary solutions of the Eq. (3) occur when

$$\xi' = \xi'' = 0 \Rightarrow \sum_{j=1}^{\infty} j\,\frac{a_j^2 + b_j^2}{\sin(Lj)}\sin(2j\xi) = \frac{4}{3}\frac{2\bar{\rho}_i + 1}{2\bar{\rho}_i - 1}(1-\bar{\rho}_i)\frac{g\rho_0^2 c^3}{\omega(\Delta p_0)^2}$$

The elementary example is $j = 1$ (Krasilstchikov and Krylov, 1984). The stationary solution can be written in the following form:

$$\sin(2\xi) = \frac{4}{3}\frac{2\bar{\rho}_i + 1}{2\bar{\rho}_i - 1}(1-\bar{\rho}_I)\frac{\sin^2 L}{\left(a_1^2 + b_1^2\right)(\Delta p_0)}\frac{g\rho_0^2 c^3}{\omega} = A$$

$$\xi = \xi^*(n), \quad \xi^*(n) = (-1)^n \arcsin(A)/2 - \pi n/2, \quad n = 1, 2 \ldots n_{\max}$$

$$\xi(n) = \frac{c}{\omega}\xi^*(n)$$

The condition for stability of the obtained stationary solutions is the following (Bogolyubov and Mitropolsky, 1963)

$$(2\bar{\rho}_i - 1)\cos(2\xi) > 0$$

A condition of applicability of the presented arguments is

$$\omega \gg \frac{\mu}{\rho_0 a^2}.$$

Now, we discuss cases of stationary solution derivation at various values of the density ρ_i.

Case 1

Values of the constants are chosen as the following:

$$L = 50\ m,\ g = 9.8\ m/s,\ c = 6000\ m/s,\ \Delta p_0 = 10^7\,pa,\ \omega = 52360.$$

$\rho_0 = 3000$ kg/m^3, $\rho_{pl} = 2700$ kg/m^3, ρ_{px} = 3400 kg/m^3 are densities of the melt, plagioclase, and pyroxene, respectively.

Stationary solutions for the averaged movement at such values are

$$\xi_{pl} = 0.04 + 0.3n,\ \xi_{px} = 0.26 + 0.3n \text{ (in meters)}.$$

So, if this case is realized, there will be only pyroxene and plagioclase layers in the section.

Case 2

Values of the constants are chosen as the following:

$$L = 50\ m,\ g = 9.8\ m/s,\ c = 6000\ m/s,\ \Delta p_0 = 10^7\,pa,\ \omega = 52360.$$

$\rho_0 = 3000$ kg/m^3, $\rho_{pl} = 2600$ kg/m^3, $\rho_{px} = 3400$ kg/m^3 are densities of the melt, plagioclase, and pyroxene, correspondingly.

Stationary solutions for the averaged movement at these values are as follows: for ξ_{pl} there are no stationary solutions, and $\xi_{px} = 0.26 + 0.3n$ (in meters)
So in this case there will be only layers of pyroxene separated by gabbro layers. Plagioclase layers will be absent.

The proposed method of formation of rhythmical layering in massifs of basic and ultrabasic rocks should be considered as a possibility for solution of the problem based on fundamental dynamic equations. The model allows reconstruction in general of any distribution of layers observed in a certain object.

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